Triplet pairing in nuclei

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The superfluid phases for p pairing in the presence of strong spin–orbit coupling are classified. It is found that the quasiparticle excitation spectrum in some phases is of a double-gap nature. New branches of the collective nuclear excitations are indicated. The asymptotic angular-momentum dependence of the moment of inertia of a nucleus in a high-spin rotational-band state is obtained. The results of the theory are compared with the experimental data.

INTRODUCTION

Triplet Cooper pairing (pair spin $S = 1$) in nuclei has thus far not been considered.[1)] At the same time we do not, on the basis of the properties of the interaction between nucleons, see any grounds for ruling out triplet pairing. It is of interest to identify the physical nuclear effects that could be produced by triplet pairing. Attracting particular attention are three groups of qualitative consequences of triplet pairing:

1) enrichment of the collective nuclear excitation spectrum as a result of the multiphase nature of a superfluid Fermi liquid with triplet pairing;

2) the anisotropy of the triplet nuclear superfluid liquid, as inferred from the presence of a physically preferred axis (let us note that the latter does not stem from any nonspherical mean field introduced without justification);

3) a two-gap structure of the quasiparticle excitation spectrum for a number of superfluid phases.

The theory of triplet pairing was developed specifically for superfluid ^{3}He (see Ref. 3). The nucleus, as a Fermi liquid, differs significantly from ^{3}He mainly because of the fact that triplet pairing in nuclei should be accompanied by strong spin–orbit coupling (in the case of ^{3}He it is extremely weak, so that it is neglected in the first approximation).[2)] As a consequence, the superfluid phases that are possible for ^{3}He are different from those for nuclear matter.

Notice that in nuclear physics we are interested not only in the ground states (corresponding to the absolute minimum of the free energy), but also in the excited (quasistationary) states. Among them could be the states pertaining to the various superfluid phases. Therefore, it is important in nuclear theory to know the entire set of possible superfluid phases.[3)] Accordingly, in Sec. 1 we carry out a complete phase analysis for triplet p pairing in the presence of strong spin–orbit coupling.

The nuclear states corresponding to the various superfluid phases should, in a sense, be "highly orthogonal" to each other (e.g., the radiative transitions between them should be inhibited). This should lead to the appearance of a band of nonintercombining levels. In order to determine the degree of inhibition of the interphase nuclear transitions, we must have the superfluid-state wave functions in the various phases. This problem is solved in Sec. 2 with the aid of the group formalism often called the "quasispin method."

In Sec. 3 we consider the quasiparticle-excitation spectrum. Here we indicate the phases that give a two-gap spectrum, elucidate the structure of the two-gap multiplet for even-A and odd-A nuclei, and determine the multipole orders of the electromagnetic transitions between the components of the multiplet.

Section 4 is devoted to the rotational spectra of nuclei in states with triplet superfluidity. Here we obtain asymptotic dependences of the moment of inertia on the angular momentum. The results are compared with the experimental data.

In Sec. 5 we discuss those boundary conditions for the order parameter which are specifically for the superfluid phases under consideration.

The final section contains a summary of the principal results and conclusions.

Let us emphasize that, in the present paper, we do not consider models of the interactions leading to triplet pairing. What we aim is to identify the observable consequences of the assumption that triplet pairing occurs in nuclei.

1. THE PHASE ANALYSIS

For the benefit of the reader, we shall briefly recall the basic points of the theory that are relevant to our subject, although they are expounded in the papers cited above.[2]

We define the wave function of a Cooper pair as the matrix element

$$\langle N|\hat{\Psi}_\alpha(\mathbf{r}_1)\hat{\Psi}_\beta(\mathbf{r}_2)|N+2\rangle = f_{\alpha\beta}(\boldsymbol{\rho}, \mathbf{r}). \qquad (1.1)$$

Here N is the number of identical fermions (neutrons or protons); the $\hat{\Psi}(\mathbf{r}_i)$ are the anticommuting annihilation operators for the fermions located at the points $\mathbf{r}_i$; the spinor indices α and β assume the values 1 and 2; and the variables $\boldsymbol{\rho}$ and $\mathbf{r}$ are the coordinates of the relative position and center of mass of the particles of the pair:

$$\boldsymbol{\rho} = \mathbf{r}_1 - \mathbf{r}_2, \quad \mathbf{r} = (\mathbf{r}_1 + \mathbf{r}_2)/2.$$

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To separate from the direct product of the two spinors the scalar (pair spin $S=0$) and the vector ($S=1$), let us expand the matrix $f=(f_{\alpha\beta})$ in terms of the basis matrices:

$$f=a(\rho,\mathbf{r})i\sigma_2+\mathbf{b}(\rho,\mathbf{r})i\sigma_2\boldsymbol{\sigma} \tag{1.2}$$

(the $\boldsymbol{\sigma}$ are the Pauli matrices). It follows from the equality (1.1) that

$$f_{\alpha\beta}(\rho,\mathbf{r})=-f_{\beta\alpha}(-\rho,\mathbf{r}) .$$

Hence (with allowance for the fact that σ_2 is antisymmetric and the matrices $\sigma_2\boldsymbol{\sigma}$ are symmetric) we obtain

$$a(-\rho,\mathbf{r})=a(\rho,\mathbf{r}),\quad \mathbf{b}(-\rho,\mathbf{r})=-\mathbf{b}(\rho,\mathbf{r}). \tag{1.3}$$

These formulas indicate that the singlet pair (the wave function is the scalar a) can be in states with only even orbital angular momenta L for the relative motion, while the triplet pair (the wave function is the vector $\mathbf{b}$) can be in states with only odd L. Below we consider only triplet p pairing ($L=1$).

In the case of strong spin–orbit coupling the orbital and spin angular momenta combine into a total pair angular momentum J that assumes the values 0, 1, and 2. The state of the Cooper pair is thus characterized by the angular momentum J and its component M along the axis of quantization. The superfluid state is a Bose condensate of Cooper pairs, i.e., an ensemble of identical (with one and the same J) pairs in one and the same state (the same M for all pairs with a given J).

It follows therefore that each phase is specified by giving the values of J and M. Here the phases differing only in the sign of M are equivalent (since $\mathbf{b}$ and its complex conjugate $\mathbf{b}^*$ are equivalent). On the whole we obtain six superfluid phases specified by the possible values of the pair of numbers J and $|M|$. This result,[5] which is based on qualitative physical arguments, should be supplemented by a formal derivation that allows the establishment of the explicit form of the wave functions $\mathbf{b}$ for each of the phases.

For a fixed orbital angular momentum for the pairing, it is expedient to separate out the angle variables $\hat{\boldsymbol{\rho}}=\boldsymbol{\rho}/\rho$. Discarding the unimportant—for the phenomenological analysis—function of the scalar ρ, we have

$$b_j(\rho,\mathbf{r})=\hat{\rho}_i B_{ij}(\mathbf{r});\quad i,j=1,2,3. \tag{1.4}$$

The complex tensor B is the order parameter (OP). The system's energy $\mathscr{E}$ can be written in the form of a functional of the OP:

$$\mathscr{E}=F[B].$$

The functional F contains real scalars formed from B_{ij} and B^*_{ij}. In the case of strong spin–orbit coupling the tensor indices of the OP are indistinguishable (contractions over the orbital and spin indices are possible). The scalars thus obtained are invariants of the SO(3) group.

Moreover, the real functional F is invariant under the transformation $B\rightarrow e^{i\alpha}B$ from the group U(1). Therefore, the full symmetry group for triplet pairing in the presence of strong spin–orbit coupling is the direct product $G=\mathrm{SO}(3)\otimes\mathrm{U}(1)$:

$$F[gB]=F[B],\quad g\in G$$

Two OP's B and B' that cannot be transformed into each other by a $G(B'\neq gB)$-group operation clearly describe different phases. In particular, this means that the irreducible second-rank tensors B^J_{ij} ($J=0,1,2$) correspond to different superfluid phases when the J values are different (the weight J of an irreducible representation of the SO(3) group is the angular momentum of the Cooper pair).

The degeneracy space $\{gB\}$ (g runs through the entire group G and B is a fixed tensor) i.e., all the OP's pertaining to the same phase coincide with the factor space G/H of the group G with respect to the maximal stationary subgroup H of the tensor B:

$$HB=B. \tag{1.5}$$

The subgroup H contains discrete and continuous elements. In the present case the continuous part of H depends on one parameter: the angle of rotation about an axis specified by unit vector $\hat{\mathbf{z}}$. Let us denote by h_z the generator of the subgroup H. Then instead of (1.5) we can write

$$h_zB=0. \tag{1.6}$$

The phase analysis amounts to the solution of Eq. (1.5) or (1.6) for all the subgroups H (notice that the discrete components of these subgroups can be determined if the explicit form of the B tensors satisfying Eq. (1.6) is known).

Let us find the generators h_z. The continuous subgroup of SO(3) is the group of rotations about the axis $\hat{\mathbf{z}}$. Let us denote that generator of the irreducible SO(3) representation of weight J which corresponds to these operations by I^J_z and its eigenfunctions by B^{JM}:

$$I_z^JB^{JM}=MB^{JM},\quad M=J,\ldots,-J. \tag{1.7}$$

It is clear that $h_z=I^J_z$ only when $M=0$. In order to satisfy Eq. (1.6) for $M\neq 0$, we must supplement a rotation about $\hat{\mathbf{z}}$ through an angle φ, i.e., the transformation

$$B^{JM}\rightarrow e^{iM\varphi}B^{JM} \tag{1.8}$$

by a transformation ($B^{JM}\rightarrow e^{-iM\varphi}B^{JM}$) from U(1), to which transformation corresponds the generator $iM\partial/\partial\Phi$ ($e^{i\Phi}$ is a phase factor that appears for all components of B^{JM}). Thus,

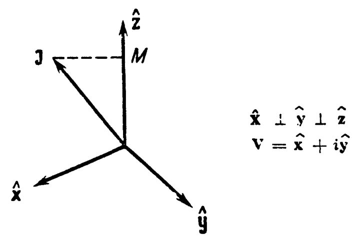
$$h_z=I_z^J+iM\partial/\partial\Phi. \tag{1.9}$$

Evidently, it is sufficient to consider the solutions to Eq. (1.6) with the generators (1.9) only for the M values of one sign.

The formal analysis carried out above leads to the same results as the qualitative arguments used earlier: the possible superfluid phases are specified by the pairs of numbers $J,|M|$. But besides that, we now have Eq. (1.7), the solution to which (with the generators (1.9)) allows us to determine the tensor structure of the OP for each phase, which in turn allows us to find the discrete elements H_D of the stationary

TABLE I. Structure of the order parameter in the case of triplet p pairing with strong spin-orbit coupling.

J,	$\lvert M\rvert$	B_{ij}	H_D	G/H	π_1	π_2
0,	0	$\delta_{ij}e^{i\Phi}$	–	S^1	Z	0
1,	0	$e_{ij\rho}\hat{z}_\rho e^{i\Phi}$	$\hat{\mathbf{z}}\to-\hat{\mathbf{z}}$ $\Phi\to\Phi+\pi$	$S'\otimes S^2/Z_2$	Z	Z
1,	1	$e_{ij\rho}V_\rho$	–	P^3	Z_2	0
2,	0	$(3\hat{z}_i\hat{z}_j-\delta_{ij})e^{i\Phi}$	$\hat{\mathbf{z}}\to-\hat{\mathbf{z}}$	$S^1\otimes P^2$	$Z+Z_2$	Z
2,	1	$\hat{z}_iV_j+\hat{z}_jV_i$	–	P^3	Z_2	0
2,	2	V_iV_j	–	P^3/Z_2	Z_4	0



subgroup H and, thus, completely determine the degeneracy space G/H.

The procedure for solving the equations (1.7) is a standard one, and does not require special explanations. The results are presented in Table I (the scalar factors in the expressions for the B tensors are omitted). Also given in the table are the first and second homotopy groups π_1 and π_2 for the degeneracy space. The nontriviality of these groups indicates the possibility of the appearance in nuclear matter of stable OP inhomogeneities determined by singular lines (π_1) and singular points (π_2).

The principal result of the present section consists in the fact that, in all the phases with $J\neq 0$, the superfluid nuclear liquid is anisotropic: the axis of quantization of the components of the angular momentum of the Cooper pairs is physically distinct. Thus, in nuclei with triplet pairing the spherical symmetry is spontaneously broken. The ($J,M=0$) phases retain their axial symmetry.

2. THE WAVE FUNCTIONS OF NUCLEI IN STATES WITH TRIPLET PAIRING

The nuclear wave functions $|JM\rangle$ corresponding to different superfluid phases (J,M) should, as has already been noted, be "strongly orthogonal" to each other, since the transition from one state into another requires the reconstruction of the entire condensate.

The strong orthogonality implies that the off-diagonal matrix elements

$$\langle J'M'|a_1{}^+\dots a_n{}^+a_{n+1}\dots a_{2n}|JM\rangle \tag{2.1}$$

of the product of any finite number of quasiparticle creation and annihilation operators go to zero exponentially in N as $N\to\infty$ (N is the number of identical fermions), as do the scalar products $\langle J'M'|JM\rangle$ themselves. In macroscopic condensed media N is very large, and it is clear that the quantities $\langle J'M'|JM\rangle$ are negligibly small. In nuclei, however, N does not, in order of magnitude, exceed 10^2. Therefore, the question of the degree of inhibition of the interphase transitions (e.g., the radiative transitions) needs to be investigated.

To do this, we shall use the quasispin formalism. In it the Hamiltonian $\hat{H}$ of the system is expressed in terms of the generators $\mathbf{I}=(I_1,\dots,I_l)$ of some Lie group:

$$\hat{H}=\sum_{\mathbf{k}}\alpha_{\mathbf{k}}\mathbf{I}_{\mathbf{k}}. \tag{2.2}$$

Here $\mathbf{k}$ is the quasiparticle momentum (as usual, we assume that fermions with opposite momenta pair up, and therefore the sum is a single one) and the $\alpha_{\mathbf{k}}$ are numerical functions of $\mathbf{k}$. Let us introduce the operators $D(\omega_{\mathbf{k}})$ that diagonalize the terms $\alpha_{\mathbf{k}}\cdot\mathbf{I}_{\mathbf{k}}$:

$$D(\omega_{\mathbf{k}})(\alpha_{\mathbf{k}}\mathbf{I}_{\mathbf{k}})D^+(\omega_{\mathbf{k}})=(\dots\lambda_n(\mathbf{k})\dots). \tag{2.3}$$

They are clearly the operators from the quasispin group that correspond to the finite (fairly large) values of the group parameters $\omega_{\mathbf{k}}$. The eigenfunctions $|JM\rangle$ of the Hamiltonian (2.2) are obtained from the particleless state $|0\rangle$ through the "rotation"

$$|JM\rangle=\prod_{\mathbf{k}}D(\omega_{\mathbf{k}})\,|0\rangle. \tag{2.4}$$

In the paper of Hasegawa *et al.*[6] on the theory of superfluidity of ^{3}He, the Lie algebra of the SO(5) group is used as the quasispin. It will be seen below that, in the general case of triplet pairing, we can manage with the six generators of the SO(3) $\otimes$ SO(3) group [instead of the ten generators of the SO(5) group].

The linear dimensions of nuclei are much greater than $1/k_f$ (k_f is the radius of the Fermi sphere). This allows us to consider the properties of nuclear matter, classifying the quasiparticle states according to the momenta. On the other hand, the radii of even the heavy nuclei are smaller than the correlation length. This means that we can be interested mainly in the homogeneous superfluid states (the order parameter B does not depend on $\mathbf{r}$). The Hamiltonian corresponding to such states with ^3P pairing has the form

$$\hat{H}=\hat{H}_0-G_{lml'm'}\times\sum_{\mathbf{k}\mathbf{k}'}\hat{k}_l(-i\sigma_2\sigma_m)_{\alpha\beta}a_{\mathbf{k}\alpha}{}^+a^+_{-\mathbf{k}\beta}\hat{k}_{l'}{}'(i\sigma_{m'}\sigma_2)_{\alpha'\beta'}a_{-\mathbf{k}'\alpha'}a_{\mathbf{k}'\beta'},\quad(2.5)$$

$$\hat{H}_0=\sum_{\mathbf{k}}\varepsilon_{\mathbf{k}}a_{\mathbf{k}\alpha}{}^+a_{\mathbf{k}\alpha}.\quad(2.6)$$

Here $\mathbf{k}$ is the quasiparticle momentum ($\hat{\mathbf{k}}=\mathbf{k}/k$), $\varepsilon_{\mathbf{k}}$ is the quasiparticle energy (measured from the Fermi energy ε_f), the Greek subscripts indicate the quasiparticle spin components, G is a tensor containing the effective constants g_J of the short-range pair forces in the pair states with different $J=0, 1, 2$:

$$G_{lml'm'}=g_0\delta_{lm}\delta_{l'm'}+g_1e_{lmn}e_{l'm'n}+g_2(\delta_{ll'}\delta_{mm'}+\delta_{lm'}\delta_{l'm}-{}^2/_3\delta_{lm}\delta_{l'm'}).\quad(2.7)$$

In the formula (2.5) the $\mathbf{k}$ and $\mathbf{k}'$ sum is over the half-space $k_3, k_3'>0$ (the normalization volume has been taken to be equal to unity, and summation over repeated tensor and spinor indices is implied). Next, let us go over to the approximate Hamiltonian $\hat{H}'$, the eigenvectors of which are coherent states $|f\rangle$:

$$\hat{\Psi}_\alpha(\mathbf{r}_1)\hat{\Psi}_\beta(\mathbf{r}_2)|f\rangle=f_{\alpha\beta}(\rho,\mathbf{r})|f\rangle,\quad(2.8)$$

$$\hat{H}'|f\rangle=E_f|f\rangle.\quad(2.9)$$

For a given value J of the pair angular momentum the Hamiltonian $\hat{H}'$ has the form

$$\hat{H}'=\hat{H}_0-g_J\sum_{\mathbf{k}}\{[-i\sigma_2\sigma\mathbf{b}(\mathbf{k})]_{\alpha\beta}a_{\mathbf{k}\alpha}{}^+a^+_{-\mathbf{k}\beta}+\mathrm{H.c.}\}+g_J\,\mathrm{Sp}\,B^+B.\quad(2.10)$$

Here the order parameter B_{ij} is normalized as follows:

$$B_{ij}=\frac{1}{2k_F}\frac{\partial\mathbf{b}_j(\rho)}{\partial\rho_i}\Big|_{\rho\to0}.\quad(2.11)$$

The vectors $\mathbf{b}(\mathbf{k})$ are connected with the B_{ij} by a relation similar to the equation (1.4):

$$b_j=\hat{k}_iB_{ij}.\quad(2.12)$$

Of importance for the determination of the appropriate quasispin group that diagonalizes the Hamiltonian (2.10) are the commutation relations between the terms in the Hamiltonian. Triplet pairing is characterized by the presence in the Hamiltonian of the spin matrices $\sigma\cdot\mathbf{b}$. Therefore, an important role is played by the commutator

$$[(\sigma\mathbf{b}),\ (\sigma\mathbf{b}^*)]=2i\sigma[\mathbf{b}\mathbf{b}^*].\quad(2.13)$$

Under rotations about the quantization axis $\hat{\mathbf{z}}$, the tensor B_{ij}^{JM} transforms according to the law (1.8). It follows from (1.8) and (2.12) that, when $M=0$, the vectors $\mathbf{b}$ and $\mathbf{b}^*$ are collinear, and $[\mathbf{b}\times\mathbf{b}^*]=0$. It is clear that the quasispin group should be the same as for singlet pairing (although the spin matrices are present, they do not affect the required commutation relations). According to (1.8) and (2.12), for $M\neq0$, $\mathbf{b}$ and $\mathbf{b}^*$ change under rotations about $\hat{\mathbf{z}}$, and besides they change differently. The latter fact indicates that $\mathbf{b}$ and $\mathbf{b}^*$ are not collinear. The vector $[\mathbf{b}\times\mathbf{b}^*]\neq0$, and the commutator (2.13) also does not vanish, and this can affect the Lie algebra of the quasispin group. It is therefore advisable to consider the indicated two cases ($M=0$ and $M\neq0$) separately.

Let us introduce the operators:

$$I_+(\mathbf{k})=\frac{1}{\sqrt{2}}a_{\mathbf{k}\alpha}{}^+a^+_{-\mathbf{k}\beta}(-i\sigma_2\sigma)_{\alpha\beta}(\mathbf{z}+i\hat{\mathbf{y}}),\quad I_-=I_+{}^+,\quad(2.14)$$

$$I_3(\mathbf{k})=\frac{1}{4}\sum_{\mathbf{q}=\pm\mathbf{k}}(a_{\mathbf{q}\alpha}{}^+a_{\mathbf{q}\alpha}+\sigma_{\alpha\beta}\hat{\mathbf{x}}a_{\mathbf{q}\alpha}{}^+a_{\mathbf{q}\beta})-\frac{1}{2}.\quad(2.15)$$

Here the set of three basis vectors $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ are defined as in Table I ($[\hat{\mathbf{x}}\times\hat{\mathbf{y}}]=\hat{\mathbf{z}}$).

The commutation relations between the operators (2.14), (2.15) correspond to the Lie algebra of the SO(3) group; moreover here $I_+^2=0$, so that the irreducible representations with weights 0 and 1/2 are realized. The Hamiltonian (2.10) can be written in the form of a linear combination of the generators of this representation:

$$\hat{H}'=2\sum_{\mathbf{k}}\varepsilon_{\mathbf{k}}+g_J\,\mathrm{Sp}\,B^+B+\hat{H}(I),\quad(2.16)$$

where

$$\hat{H}(I)=\sum_{\mathbf{k}}(2\varepsilon_{\mathbf{k}}I_3(\mathbf{k})-\sqrt{2}\,g_J|\mathbf{b}(\mathbf{k})|[I_+(\mathbf{k})+I_-(\mathbf{k})].$$

The finite-rotation operators $D(\omega_{\mathbf{k}})$ that diagonalize the terms of the Hamiltonian (2.16) have the form

$$D(\omega_{\mathbf{k}})=\prod_\alpha[\cos(\omega_{\mathbf{k}}/2)-a_{\mathbf{k}\alpha}{}^+a^+_{-\mathbf{k}-\alpha}\cdot2\alpha\sin(\omega_{\mathbf{k}}/2)],\quad(2.17)$$

where $\alpha=\pm1/2$ is the quasiparticle spin component along the axis $\mathbf{z}$ of symmetry of the condensate, and

$$\cos\omega_{\mathbf{k}}=\frac{\varepsilon_{\mathbf{k}}}{[\varepsilon_{\mathbf{k}}{}^2+g_J{}^2\mathbf{b}^*(\mathbf{k})\mathbf{b}(\mathbf{k})]^{1/2}}.\quad(2.18)$$

The particleless state $|0\rangle$ is the eigenvector of the generators $I_3(\mathbf{k})$ corresponding to the eigenvalue $-1/2$.

The ground-state wave function $|J0\rangle$ of the condensate in the phases with $M=0$ can be found from the formula (2.4) by letting the operators (2.17) act on the vector $|0\rangle$. The ground-state energy can be found through direct computation of the mean value of the Hamiltonian $\hat{H}'$:

$$E_{J0}=\langle J0|\hat{H}|J0\rangle=g_J\,\mathrm{Sp}\,B^+B+\sum_{\mathbf{k}}2\{\varepsilon_{\mathbf{k}}-[\varepsilon_{\mathbf{k}}{}^2+g_s{}^2\mathbf{b}(\mathbf{k})\mathbf{b}^*(\mathbf{k})]^{1/2}\}.\quad(2.19)$$

As was expected, for $M=0$, the results are similar to the formulas for singlet S pairing ($l=0$). In both cases the SO(3) group serves as the quasispin group. The $D(\omega_{\mathbf{k}})$ operators for singlet pairing are obtained from (2.17) through the replacement of 2α by 1, while the $\omega_{\mathbf{k}}$ parameters are obtained from the equality (2.18) through the replacement of the vector $\mathbf{b}$ by the corresponding scalar.

In the phases with $M \neq 0$ the vectors **b** and **b*** are not collinear, and $[\mathbf{b}\times\mathbf{b}^*]\neq 0$. Using Table I, we find, for example, for $J = M = 2$:

$$i[\mathbf{b}\mathbf{b}^*]=\hat{\mathbf{z}}|[\mathbf{b}\mathbf{b}^*]|.$$

Accordingly, the vector $\mathbf{b}\perp\hat{\mathbf{z}}$. The Hamiltonian (2.10) can, as before, be written in a form similar to (2.16), but another quasispin group must be chosen. Let us consider the operators

$$I_+(\mathbf{k},\mu)=\frac{1}{\sqrt{2}}a_{\mathbf{k}\alpha}{}^+a^+_{-\mathbf{k}\beta}(-i\sigma_2\boldsymbol{\sigma})_{\alpha\beta}\frac{\mathbf{b}(\mathbf{k})-2i\mu[\mathbf{b}\hat{\mathbf{z}}]}{[\mathbf{b}\mathbf{b}^*-2\mu|[\mathbf{b}\mathbf{b}^*]|]^{1/2}}, \quad (2.20)$$

$$I_3(\mathbf{k},\mu)=\frac{1}{4}\sum_{\mathbf{q}=\pm\mathbf{k}}(a_{\mathbf{q}\alpha}{}^+a_{\mathbf{q}\alpha}+2\mu a_{\mathbf{q}\alpha}{}^+a_{\mathbf{q}\beta}\boldsymbol{\sigma}_{\alpha\beta}\hat{\mathbf{z}})-\frac{1}{2}. \quad (2.21)$$

Here $\mu = \pm 1/2$ is the quasiparticle spin component along the $\hat{\mathbf{z}}$ axis. It is not difficult to verify that, for a fixed μ, the set of three operators I_+, $I_- = I_+^+$, and I_3 are the generators of the SO(3) group, while operators corresponding to different values of μ commute. Thus, we have the Lie algebra of the SO(3) ⊗ SO(3) group. As before, the irreducible representations with weights 0 and 1/2 are realized, the particleless state $|0\rangle$ being the eigenvector of the generators $I_3(\mathbf{k},\mu)$ belonging to the eigenvalue $-1/2$.

The Hamiltonian (2.10), expressed in terms of the generators (2.20) and (2.21), has the form (2.16) with the following operator $\hat{H}(I)$:

$$\hat{H}(I)=\sum_{\mathbf{k}\mu}\{2\varepsilon_{\mathbf{k}}I_3(\mathbf{k},\mu)$$
$$-\sqrt{2}g_J[\mathbf{b}^*(\mathbf{k})\mathbf{b}(\mathbf{k})-2\mu|[\mathbf{b}(\mathbf{k})\mathbf{b}^*(\mathbf{k})]|]^{1/2}$$
$$\times(I_+(\mathbf{k},\mu)+I_-(\mathbf{k},\mu))\}. \quad (2.22)$$

The finite-quasispin-rotation operators that diagonalize the terms of $\hat{H}(I)$ are given by the formula (2.17), with the only difference that the rotation parameters $\boldsymbol{\omega}$ now depend not only on the momentum **k**, but also on the spin variable $\boldsymbol{\alpha}$:

$$\cos\omega_{\mathbf{k}\alpha}=\varepsilon_{\mathbf{k}}/[\varepsilon_{\mathbf{k}}^2+g_J^2(\mathbf{b}(\mathbf{k})\mathbf{b}^*(\mathbf{k})-2\alpha|[\mathbf{b}(\mathbf{k})\mathbf{b}^*(\mathbf{k})]|)]^{1/2}. \quad (2.23)$$

The condensate wave function $|JM\rangle$ in the phases with $M\neq 0$ can, as before, be obtained from the formula (2.4) by letting the operators (2.17) with the parameters (2.23) act on the vector $|0\rangle$. Notice that the state $|JM\rangle$ is P-even, since the operators (2.17) acting on the vector $|0\rangle$ are invariant under space reflections.

The ground-state energy E_{JM} of the condensate with $M\neq 0$ is given by the expression

$$E_{JM}=g_J\,\mathrm{Sp}\,BB^+ +\sum_{\mathbf{k}\mu}\{\varepsilon_{\mathbf{k}}-[\varepsilon_{\mathbf{k}}^2+g_J^2(\mathbf{b}\mathbf{b}^*-2\mu|[\mathbf{b}\mathbf{b}^*]|)]^{1/2}\}. \quad (2.24)$$

It can be seen from the formula (2.24) that the energy of the quasiparticles in the phases with $M\neq 0$ depends on their spin component along the axis of symmetry of the condensate. This circumstance gives rise to a two-gap quasiparticle-excitation spectrum structure. This question is discussed in greater detail in Sec. 3.

Knowing the condensate wave functions in the various superfluid phases, we can compute the scalar products $\langle J'M'|JM\rangle$, and, as noted at the beginning of this section, thus determine the degree of inhibition of the interphase transitions (including the radiative transitions) in nuclei. To estimate the order of magnitude of the effect, it is sufficient to consider the scalar product $\langle {}^1S_0|{}^3P_0\rangle$ of the condensate wave function with singlet S pairing ($|{}^1S_0\rangle$) and the wave function with triplet p pairing in the $J = M = 0$ phase ($|{}^3P_0\rangle$). As noted above, the SO(3) group serves as the quasispin group in both cases. In order to obtain an estimate for the degree of orthogonality in the case when the characteristics of the two superfluid states are as close as possible, let us assume that the quasispin-rotation parameters $\boldsymbol{\omega}$ [formula (2.18)] for these states are identical. Using the preceding results for the wave functions, we obtain then:

$$\langle {}^1S_0|{}^3P_0\rangle=\prod_{\mathbf{k}}\cos\omega_{\mathbf{k}}=\exp\left\{\sum_{\mathbf{k}}\ln\frac{\varepsilon_{\mathbf{k}}}{(\varepsilon_{\mathbf{k}}^2+\Delta_{\mathbf{k}}^2)^{1/2}}\right\}. \quad (2.25)$$

Here we have set

$$\Delta_{\mathbf{k}}^2=g^2\mathbf{b}(\mathbf{k})\mathbf{b}^*(\mathbf{k}). \quad (2.26)$$

Performing the integration over **k** in (2.25) in the standard approximation [with upper cutoff of the integral at k_F and the replacement of $\Delta_{\mathbf{k}}$ by the constant $\Delta(k = k_F)$], we find

$$\langle {}^1S_0|{}^3P_0\rangle=\exp\left\{-\frac{3\pi}{8}\frac{\Delta}{\varepsilon_F}N\right\}. \quad (2.27)$$

(N is the number of nucleons of the same kind in the nucleus). In real nuclei $N\approx 10^2$, $\varepsilon_F\approx 40$ MeV, and the gap $\Delta\approx 1$ MeV. Substituting these numbers into (2.27), we obtain the estimate:

$$\langle {}^1S_0|{}^3P_0\rangle\approx 5\cdot 10^{-2}.$$

This means that the probability suppression factor for the interphase radiative transitions is, in order of magnitude, equal to 10^{-3}.

Thus, assuming the existence in nuclei of the various types of Cooper pairing, we arrive at the qualitatively new conclusion that there exist groups of levels corresponding to different superfluid phases, and practically not intercombining with each other.

3. THE QUASIPARTICLE EXCITATIONS

The wave functions of the excited n-quasiparticle states of the condensate are obtained through quasispin rotations of the vector $|0\rangle$:

$$\left(\prod_{i=1}^{n}a^+_{\mathbf{k}_i\alpha_i}\right)|0\rangle.$$

The diagonalization of the Hamiltonian (2.10) by the method expounded in Sec. 2 yields the quasiparticle energy spectrum

$$\varepsilon_{JM}(\mathbf{k},\mu)=\{\varepsilon_{\mathbf{k}}^2+g_J^2(\mathbf{b}\mathbf{b}^*-2\mu|[\mathbf{b}\mathbf{b}^*]|)\}^{1/2}. \quad (3.1)$$

Let us recall that $\mu = \pm 1/2$ is the quasiparticle spin component along the axis $i[\mathbf{b}\times\mathbf{b}^*]$. As follows from (3.1), the quasiparticle excitation spectrum is a two-gap spectrum if $[\mathbf{b}\times\mathbf{b}^*]\neq 0$. According to the foregoing, this is the case for the phases with $M\neq 0$. Notice that this spin splitting of the terms of the quasiparticle spectrum has nothing to do with the spin–orbit coupling in the pairing forces. It is not difficult to show that it occurs also in the so-called β and γ phases in the case of triplet pairing without spin–orbit coupling, which is characteristic of superfluid ^{3}He. This circumstance has not been discussed in detail in the literature probably because of the fact that other phases are realized in the homogeneous state of the superfluid state of ^{3}He.[4)] But recently Volovik and Salomaa[8] put forward the hypothesis that the β phase exists at the core of a ^{3}He–B vortex. Let us emphasize that the existence of double-gap phases is a qualitative characteristic of triplet pairing. In the case of singlet pairing with any admissible (i.e., even) orbital angular momentum the quasiparticle excitation spectrum will always be a single-gap spectrum. The outwardly similar ($L=2$, $M\neq 0$) singlet and ($J=2, M\neq 0$) triplet phases, for example, differ from each other by, in particular, the fact that the quasiparticle excitation spectrum is a single-gap spectrum in the first case and a double-gap one in the second. The equations for the gaps are also naturally different. Amundsen and Østgaard, in the cited paper on the superfluidity of neutron stars,[1] consider just the triplet pairing with $J=2$ and $M=0$, 2, using for the $M=2$ case the equations corresponding to the single-gap spectrum, which cannot be considered to be correct.

Let us now consider those quasiparticle excitations of an even-even nucleus in the triplet superfluid phase with a double-gap spectrum (i.e., with $M\neq 0$) which are closest to the ground state. Such excitations are obtained through the breakup of one Cooper pair, and are therefore two-quasiparticle excitations:

$$E_{JM}^*=\varepsilon_{JM}(\mathbf{k}_1,\mu_1)+\varepsilon_{JM}(\mathbf{k}_2,\mu_2). \qquad (3.2)$$

In the single-gap phases [$\varepsilon_{JM}(\mathbf{k},\mu)$ does not depend on the component of the quasiparticle spin μ], including the singlet ones, to this two-quasiparticle excitation corresponds one level. In the double-gap phases the picture will be different: for fixed $\mathbf{k}_1$ and $\mathbf{k}_2$ there will arise four nondegenerate levels corresponding, according to the formula (3.2), to the following possible sets of values of the number pair μ_1 and μ_2: $\mu_1=\mu_2=\pm 1/2$ and $\mu_1=-\mu_2=\pm 1/2$ (Fig. 1).[5)]

The radiative transitions between the ($\mu_1=\mu_2$) and ($\mu_1=-\mu_2$) levels, transitions which correspond to the flipping ($\Delta\mu=\pm 1$) of the spin of one quasiparticle, are magnetic dipole ($M1$) transitions. As to the transition between the two extreme (the top $\mu_1=\mu_2=-1/2$ and bottom $\mu_1=\mu_2=+1/2$) levels of the double-gap multiplet, it should be suppressed: it requires a two-unit change in the angular momentum component, and, consequently, cannot belong to the $M1$ class. According to the selection rules, such a transition can be an electric quadrupole ($E2$) transition. But an $E2$ transition without a change in the orbital angular momentum of the quasiparticle can occur only as a result of the presence of relativistic corrections (for nuclei the sup-

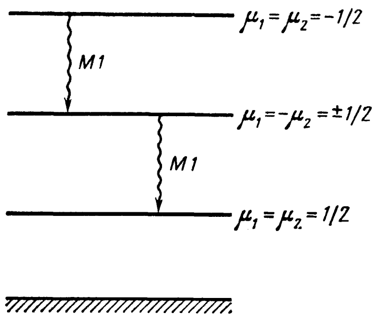


FIG. 1. The double-gap multiplet of the two-quasiparticle excitations in even-even nuclei and the radiative-transition scheme (μ_1 and μ_2 are the quasispin components along the axis of symmetry of the condensate).

pression factor due to this circumstance is, in order of magnitude, equal to 10^{-2}).

It follows from the foregoing that the triplet pairing in nuclei in the phases with $M\neq 0$ should, in particular, manifest itself in the occurrence of successive single-particle $M1$ transitions accompanied by the release of energies close to the magnitudes of the gaps given by the formula (3.1). The expected numerical values of the level separations in the double-gap multiplet lie in the range from 0.1 to 1 MeV.

4. THE ROTATIONAL SPECTRA

The combination of the anisotropy of the superfluid Fermi liquid in the phases with $J\neq 0$ and the smallness of the nuclear dimensions (compared to the correlation length) leads to the possibility of the appearance of rotational excitations corresponding to the rotation of the nucleus as a whole. The existence, as a result of the anisotropy, of distinct spatial directions allows us to indicate what precisely rotates. But the smallness of the drop dimensions (in the scale indicated above) prevents the appearance of rotation-related volume textures (of the quantized-vortex type).

Our aim in the present section is to determine the dependence of the moment of inertia of a nucleus on the rotational angular momentum. We shall also find out how the rotational angular momentum is oriented relative to the axis of quantization of the angular momentum of the Cooper pair in the triplet phases with different M values.

Traditionally, in nuclear physics the investigation of these problems is based on the introduction of a rotating nonspherical (but axially symmetric) mean field that ensures the quasiparticle energy quantization stemming from the finiteness of the nuclear size.

The basic object in the approach expounded below is the order parameter (OP). We use (without special justification in the particular problem under consideration) the Ginzburg-Landau approximation (the energy is assumed to be a polynomial function of the OP). We shall show that a physically transparent and satisfactory description of the relationships observed in rotational nuclear spectra can be achieved on this basis.

The energy $E_{JM}(\mathbf{I})$ of a rotating nucleus with a rotational angular momentum $\mathbf{I}$ can be written in the following form:

$$E_{JM}(\mathbf{I})=E_{JM}(\boldsymbol{\Omega})+\mathbf{I}\boldsymbol{\Omega}. \qquad (4.1)$$

Here $\boldsymbol{\Omega}$ is the angular velocity vector and $E_{JM}(\boldsymbol{\Omega})$ is the energy of the nucleus in the reference frame rotating together with the nucleus. This latter quantity can be expressed in terms of scalars constructed from the OP and the angular velocity vector.

For the $(J, M \neq 0)$ phases the expansion of $E_{JM}(\boldsymbol{\Omega})$ in powers of $\boldsymbol{\Omega}$ begins with a linear term. This is due to the fact that, as indicated above, the vector $e_{ijl}B_{jn}B^*_{ln}$, which is parallel to the quantization axis $\hat{\mathbf{z}}$, is not a null vector in the case when $M \neq 0$. Using the formulas given in Table I, we easily obtain

$$\Omega_i e_{ijl} B_{jn} B_{ln}^* = c\boldsymbol{\Omega}\hat{\mathbf{z}}\,\mathrm{Sp}\,BB^+, \quad c=\begin{cases}1 & J=M\\ {}^1/_2 & J=2,\ M=1\end{cases}.$$

Let us set

$$A^2=\mathrm{Sp}\,BB^+. \qquad (4.2)$$

Then, assuming $\boldsymbol{\Omega}$ is a small quantity, we can write

$$E_{JM}(\boldsymbol{\Omega})=(-\alpha+\beta\boldsymbol{\Omega}\hat{\mathbf{z}})A^2+\gamma A^4 \qquad (4.3)$$

[notice that all the invariants of fourth order in B can be expressed in terms of (4.2)]. In order for the energy to have a minimum at $A \neq 0$, $\boldsymbol{\Omega}\to 0$, the coefficient α should be positive. We can, without loss of generality, assume that $\beta > 0$. It can be seen from (4.3) that the energy has its minimum value at $\boldsymbol{\Omega}\cdot\hat{\mathbf{z}}/\Omega = -1$. This means that the steady-state rotation of the nucleus will occur about the internal quantization axis $\hat{\mathbf{z}}$. This unusual—for nuclear physics—orientation of the axis of rotation of a nucleus is due to the fact that the axial symmetry of the nuclear matter is broken in the case when $M \neq 0$: the order parameter B is not invariant under rotations about $\hat{\mathbf{z}}$ [see the formula (1.8)], and therefore the rotation about $\hat{\mathbf{z}}$ has a real physical meaning [the orientations of the nuclear-matter-anisotropy basis vectors $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ perpendicular to $\hat{\mathbf{z}}$ (see Table I) vary]. Thus, in the $M \neq 0$ case the approach under consideration differs greatly from the standard cranking model. The minimization of the energy (4.3) yields the equilibrium value of $A(\boldsymbol{\Omega})$:

$$A^2(\Omega)=(\alpha+\beta\Omega)/2\gamma, \quad \gamma>0. \qquad (4.4)$$

An important conclusion following from (4.4) consists in the fact that the superfluidity in the phases with $M \neq 0$ is not destroyed by the rotation (at least in first order in $\boldsymbol{\Omega}$), since there are no values of $\boldsymbol{\Omega}$ that make the OP minimizing (4.3) vanish.

Let us now compute $E_{JM}(I)$. By definition we have

$$\mathbf{I}=-\partial E_{JM}(\boldsymbol{\Omega})/\partial\boldsymbol{\Omega}. \qquad (4.5)$$

Combining the formulas (4.1)–(4.5), we find

$$E_{JM}(I)-E_{JM}(0)=\frac{1}{2\mathscr{I}_0}\left((\mathbf{I}^2)^{1/2}-I_0\right)^2. \qquad (4.6)$$

Here we have introduced the notation

$$\mathscr{I}_0=\beta^2/2\gamma, \quad I_0=\alpha\beta/2\gamma. \qquad (4.7)$$

Notice that I_0 can be obtained from (4.5) by substituting into it the equilibrium value $A^2(\boldsymbol{\Omega}=0)=\alpha/2\gamma$. To this value of A^2 corresponds the energy $E_{JM}(0)$ of the lowest rotational-band level (it is negative because it is measured from the energy of the state in which there is no condensate in the $(J, M \neq 0)$ phase: to such a reference point corresponds the absence in (4.3) of an A^2-independent term).

Differentiating the energy (4.6) with respect to $\mathbf{I}^2$, we find the dependence of the moment of inertial $\mathscr{I}(I)$ on $\mathbf{I}^2$ $(\mathbf{I}^2 \gg I_0^2)$:

$$\frac{1}{2\mathscr{I}(I)}=\frac{\partial E_{JM}(I)}{\partial(\mathbf{I}^2)}=\frac{1}{2\mathscr{I}_0}\left(1-\frac{I_0}{(\mathbf{I}^2)^{1/2}}\right). \qquad (4.8)$$

This expression differs qualitatively from the traditional formula (see, for example, Ref. 9) used in the theory of rotational nuclear spectra at sufficiently large (but not very large) values of $\mathbf{I}^2$:

$$1/2\mathscr{I}(I)=C+D\mathbf{I}^2 \qquad (4.9)$$

(C and D are positive constants). We show below that the formula (4.9) is valid for the $(J,M=0)$ phases. In this case the asymptotic behavior, as I increases, of the moment of inertia in the indicated phases turns out to be considerably different from the behavior prescribed by the formula (4.8), according to which in the $(J,M \neq 0)$ superfluid phases

$$\mathscr{I}(I\gg I_0)\approx\mathscr{I}_0=\mathrm{const.} \qquad (4.10)$$

The phases whose asymptotic behavior is described by (4.10) can be called rotationally stable phases (since in these phases the superfluidity is not destroyed by the rotation).

The assertion made above about the rotational spectra of the $(J,M \neq 0)$ triplet phases is valid also for the $(J=L=2,M \neq 0)$ singlet phases, since the OP in such phases is identical to the OP of the $(J,M \neq 0)$ triplet phases (let us also recall that the quasiparticle excitation spectrum in any singlet phase will be a single-gap spectrum, in contrast to that of an $M \neq 0$ triplet phase).

For axially symmetric $(J,M=0)$ phases $e_{ikl}B_{kn}B^*_{ln}$ is a null vector. Therefore, the formula expressing $E_{J0}(\boldsymbol{\Omega})$ in terms of the OP will not contain a term linear in $\boldsymbol{\Omega}$. The rotation of the homogeneous (textureless) condensate as a whole in such phases can occur only about a direction perpendicular to the symmetry axis $\hat{\mathbf{z}}$ ($\mathbf{I}\cdot\hat{\mathbf{z}}=0$; the mutual orientation of $\mathbf{I}$ and $\hat{\mathbf{z}}$ is the same as in the standard cranking model with an axially symmetric mean field).

In terms of the OP, the energy $E_{J0}(\boldsymbol{\Omega})$ defined by (4.1) has the form

$$E_{J0}(\Omega)=(-\alpha-\eta\Omega^2)A^2+\gamma A^4. \qquad (4.11)$$

The coefficients α, η, and γ are positive. The condition $\eta > 0$ corresponds to the requirement that the second-order perturbation-theory correction to the energy of the system be negative (let us recall that we are considering a sufficiently slow rotation: the rotational-level spacing is assumed to be much smaller than the quasiparticle-excitation energies). The significant difference between the expressions (4.3) and (4.11) is apparent here. In the first case the sign of the term of lowest order in the frequency is no way connected with perturbation theory; in the second, it is entirely due to the smallness of the angular velocity. Minimizing the energy (4.11), we obtain

$$A^2(\Omega)=(\alpha+\eta\Omega^2)/2\gamma. \qquad (4.12)$$

Using, as before, the formulas (4.1), (4.5), (4.11), and (4.12) [the two last formulas replacing (4.3) and (4.4)], we find the levels of the rotational band corresponding to the $(J,M=0)$ phase:

$$E_{J0}(I)-E_{J0}(0)$$
$$=\frac{E_{J0}(0)}{3}\left\{\left[\sum_{\lambda=\pm1}\left(\frac{\mathbf{I}^2}{I_c^2}+1\right.\right.\right.$$
$$\left.\left.\left.+\lambda\left[\left(\frac{\mathbf{I}^2}{I_c^2}+1\right)^2-1\right]^{1/2}\right)^{1/3}-1\right]^2-1\right\}. \qquad (4.13)$$

Here $E_{J0}(0)$ is given by the formula (4.7), and

$$I_c^2={}^2/_{27}\alpha^3\eta/\gamma^2. \qquad (4.14)$$

For $\mathbf{I}^2 \ll I_c^2$, the expansion in powers of $\mathbf{I}^2$ (right up to the terms of second order in $\mathbf{I}^2$ inclusively) leads to the formula (4.9), with the coefficients given by the expressions

$$C=\gamma/2\alpha\eta, \quad D={}^{19}/_4\gamma^3/\alpha^4\eta^2.$$

In the case of asymptotically large values of $\mathbf{I}^2 \gg I_c^2$, we obtain from (4.13) [with I_c^2 and E_{J0} respectively given by (4.14) and (4.7)]

$$E_{J0}(I)-E_{J0}(0)={}^3/_4(\gamma/\eta^2)^{1/3}(\mathbf{I}^2)^{2/3}. \qquad (4.15)$$

Let us note that an asymptotic formula similar to (4.15) is obtained in Ref. 10 on the basis of a completely different physical approach (a variant of the model of interacting bosons). In Ref. 11 a fit formula is proposed (without any physical justification) for the description of the rotational bands with a variable moment of inertia. This formula does not coincide analytically with (4.13), although it does give in certain cases nearly the same numerical values and the asymptotic form (4.15). Differentiating (4.15) with respect to $\mathbf{I}^2$, we find the asymptotic value of the moment of inertia $\mathscr{I}$:

$$\mathscr{I}(I)=(\eta^2/\gamma)^{1/3}\mathbf{I}^{2/3}. \qquad (4.16)$$

Notice that the formulas (4.11)–(4.16) are valid also for singlet pairing ($L=0,2$) in the phases with $M+0$.

It follows from all that has been said in the present section that multiphase superfluidity should give rise to rotational bands with a qualitatively different rotational-angular-momentum dependence of the moment of inertia. This in turn makes probable the intersection of the levels at some I values, which gives rise to the well-known feedback effect.

Let us now proceed to compare the formulas (4.6) and (4.13) with the experimental data. We carried out such a comparison for the rotational bands of the nuclei of 34 isotopes: ^{130}Xe, ^{128}Ba, Ce (128, 130), ^{156}Gd, Dy (156, 158), Er (158, 164, 166), Yb (164, 166, 174), Hf (166, 168, 170, 174), W (166, 168, 170, 172, 174, 176), Hg (184, 186), Th (222, 228, 232), U (232, 234, 236, 238), and Pu (242, 244). The quantities α, β, γ, and η served as the adjustable parameters that did not vary along the bands described by them. In all the cases the discrepancy between the theoretical and experimental data ranged from 1 to 5%; for the majority of the nuclei, from 1 to 2%.

In Table II we give a significant example of the ^{156}Dy spectrum (the experimental data were taken from Ref. 12) with two rotational bands, one of which (based on the ground state of the nucleus) is described by the formula (4.13), while the other is described by the formula (4.6) [with the constant asymptotic value of the moment of inertia (see the formulas (4.8) and (4.10)].

Figure 2 shows the behavior of the moment of inertia in the indicated rotational bands of the ^{164}Er and ^{168}Hf nuclei (the experimental data were taken from Ref. 13).

TABLE II. Rotational spectrum of the ^{156}Dy nucleus. The experimental data were taken from Ref. 12. The theoretical level-energy values obtained in the present paper are given in brackets. The band *a* is based on the ground state of the nuclei, and corresponds to the formula (4.13). The band *b* is rotationally stable [formula (4.6)].

I+	a	*E*, MeV	b	*E*, MeV
32	–	(9.619)	–	(9.742)
30	–	(8.768)	8.650	(8.707)
28	–	(7.936)	7.738	(7.746)
26	7.130	(7.129)	6.877	(6.857)
24	6.329	(6.347)	6.069	(6.061)
22	5.573	(5.590)	5.319	(5.300)
20	4.859	(4.861)	4.635	(4.630)
18	4.178	(4.164)	4.025	(4.034)
16	3.523	(3.497)	3.499	(3.511)
14	2.887	(2.867)	3.066	(3.061)
12	2.286	(2.275)	2.707	(2.684)
10	1.725	(1.727)	–	(2.380)
8	1.211	(1.228)	–	(2.150)
6	0.770	(0.787)	–	(1.992)
4	0.404	(0.417)	–	(1.907)
2	0.138	(0.141)		

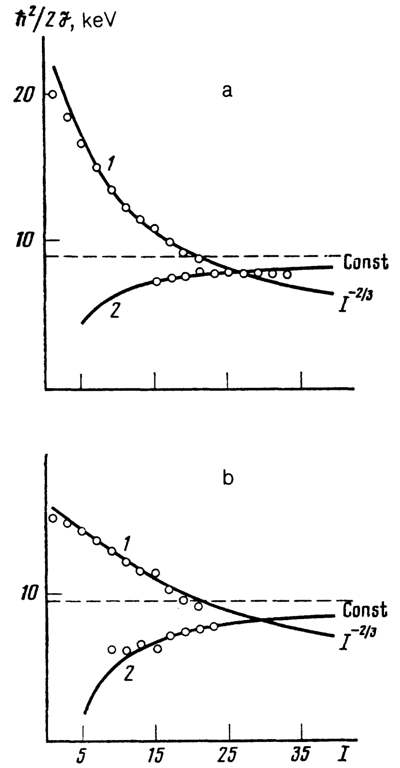


FIG. 2. Angular-momentum dependence of the moment of inertia $\mathscr{I}(I)$ for the rotational bands of the nuclei ^{168}Hf (a) and ^{164}Er (b). The points correspond to the experimental data (see Ref. 13) and the curves 1 and 2 are the results of computations with the formulas (4.13) and (4.6).

5. THE BOUNDARY CONDITIONS

In the vicinity of the nuclear boundary (on the scale of internucleon distances) the OP is inhomogeneous in the direction of the normal **n** to the surface because of the continuity of the normal OP component $n_i B_{ij}(\mathbf{r})$ and its first derivative along **n**. Since $B_{ij} = 0$ outside the nucleus, we should have at the boundary $\mathbf{r} = \mathbf{R}$.

$$n_i B_{ij}(\mathbf{R}) = 0. \tag{5.1}$$

It follows from (5.1) that, in the case of the axially symmetric ($J,M = 0$) phases, the OP itself vanishes at the nuclear boundary:

$$B_{ij}(\mathbf{R}) = 0 \tag{5.2}$$

as is the case for the OP of singlet s pairing. This circumstance is due to the fact that, in the vicinity of the surface, the spontaneous axis $\hat{\mathbf{z}}$ of quantization of the angular momentum of the Cooper pairs can be oriented only along **n**. On the other hand, as can be seen from Table I, the OP corresponding to the phases with $M = 0$ contains only the vector $\hat{\mathbf{z}}$. A different situation obtains for the ($J,M \neq 0$) phases. In this case the tensor B_{ij} contains the vectors perpendicular to $\hat{\mathbf{z}}$, and therefore, when $\hat{\mathbf{z}} \| \mathbf{n}$,

$$B_{ij}(\mathbf{R}) \neq 0, \quad M \neq 0. \tag{5.3}$$

In other words, the anisotropic phases survive at the surface. As noted in Ref. 14 for the analogous situation in superfluid ^{3}He, the realization and coexistence of different phases in the interior and at the surface are possible. The existence of a surface superfluid phase with $M \neq 0$ will change the value of the surface energy of the nucleus (as compared to the value obtained in the standard liquid-drop model), and this can, in particular, affect the equilibrium shape of the nucleus. Moreover, since in the case (5.3) a tangential vector field $V_i B_{ij}^{JM}$ (**V** is a vector introduced in Table I) occurs at the surface, there arises the question of the singularities of this field, singularities which, according to the well-known Poincaré theorem, must certainly exist. Either the theorem nullifies B_{ij}^{JM}, or the OP of the superfluid surface phase should be inhomogeneous. In the latter case the possible linear scales of such an inhomogeneity are not quite apparent, since the dimension of the nucleus is smaller than the expected correlation length determined by the pairing. If, however, we suppose that the indicated topological singularities exist, then their motion on the surface should lead to the appearance of a new branch of surface excitations.

CONCLUSION

On the basis of the foregoing, we arrive at the conclusion that the most spectacular observable consequences of the existence in nuclear matter of the proposed triplet superfluidity are the existence of nonintercombining collective-excitation branches and the existence of a double-gap quasiparticle-excitation spectrum with successive radiative $M1$ transitions between the levels. If the above-presented interpretation of the experimental data on the rotational spectra bears any relation to reality, then the double-gap quasiparticle-excitation multiplets should be sought in the region of energies at which the rotationally stable bands appear (see Sec. 4).

Let us note that quite recently Rekstad and his coworkers[15] found indications of the existence of quite widely separated groups of nonintercombining nuclear energy levels (the physical nature of which has as yet not been elucidated).

In the paper we have not considered the space quantization, stemming from the finite nuclear size, of the quasiparticle motion. The above-enumerated principal results do not depend on the specific primordial-level diagram that arises as a result of this quantization. At the same time, in our opinion, a microscopic analysis of the possibility of the occurrence of the various types of pairing in nuclei requires a self-consistent treatment of the effective mean field and the collective (in particular, the superfluid) modes (the fixed potential-well model as the basis for the quantization of the particle motion, with the subsequent solution of the problem of the type of pairing, seems quite unconvincing today).

The authors are grateful to G. E. Volovik for the useful information.

[1)]The possibility of triplet pairing in neutron stars has been investigated (see Ref. 1 and the references cited therein; let us note that the possibility of superfluidity in neutron star matter was first pointed out in the papers cited in Ref. 2). Apropos of these papers, see the remark made in Sec. 3 of the present paper.

[2)]Here and below we mean by spin–orbit coupling both the spin–orbit interaction proper and the tensor forces.

[3)]Natually, besides the triplet phases, we must also consider singlet pairing ($S = 0$), which entered into nuclear theory long ago.[4]

[4)]The possibility of ambiguity of the gap in the quasiparticle excitation spectrum of superfluid ^{3}He is noted in Ref. 7 (without specifying the phase and elucidating the nature of the term splitting). The formula obtained in this paper for the gap contains the matrix (1.2) in its general form (including singlet and triplet parts).

[5)]Notice that, for the lowest-energy excitations, $k_1 = k_2 = k_F$ and $\mathbf{k}_1 = -\mathbf{k}_2$. Since $\varepsilon_{JM}(\mathbf{k}, \mu) = \varepsilon_{JM}(-\mathbf{k}, \mu)$, the two levels with $\mu_1 = -\mu_2$ will be degenerate, and there should be three levels instead of four.

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