Vortical combination resonances

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Transfer of energy of vortical motion in multidimensional systems between low-frequency and resonantly excited high-frequency oscillations becomes possible, and under certain conditions predominant, in multidimensional systems. Energy exchange via the vortical mechanism has surprising features associated with breaking of the usual symmetry, inherent in systems with few dimensions, of the energy transfer over the Raman spectrum.

INTRODUCTION

The interaction between fast and slow forms of system motion is usually greatly enhanced if the fast subsystem contains high-Q oscillation modes excited by high-frequency fields near resonances. Raman resonances can produce in this case intense energy transfer from the fast to the slow subsystem or in the reverse direction. Phenomena of this type have long been under investigation and have found many applications. These include the operation of parametric amplifiers and oscillators, and also phenomena such as stimulated Raman scattering of light, or gas cooling by resonant optical fields. The many phenomena of this type include a special class that should display a rather unusual character of energy conversion up and down the Raman spectrum. This class is the subject of the present paper.

Waves of frequency ω in a resonant medium, with resonance frequencies $\{\omega_k\}$, are usually scattered, on account of the Raman resonances

$$\omega=\omega_k+\Omega, \tag{1}$$

$$\omega=\omega_k-\Omega \tag{2}$$

in such a way that, when ω exceeds the frequencies ω_k, the oscillations at the difference frequencies $\Omega=|\omega-\omega_k|$ are enhanced, and phenomena such as stimulated Raman scattering become possible; deexcitation and suppression of the oscillations of frequency Ω take place when ω is lower than ω_k.

This dependence of the energy exchange on the detunings $\omega-\omega_k$ takes place in simple systems (when the fast and slow subsystems are one-dimensional), such as the Mandel'shtam-Papaleksi parametric motor,[1] and in many complicated multidimensional and distributed systems (see Luisell[2] and Bloembergen[3]). For systems with reactive nonlinearities, in which only lower-order nonlinear processes are effective and level-population changes are inessential, this directivity of energy exchange is so prevalent and so widely invoked in the study of oscillation and wave phenomena, that it is regarded in fact as an indisputable rule.

This rule, however, does not hold for the discussed class of Raman resonance phenomena. It is possible, for example, to amplify oscillations of frequency Ω at all values of ω relative to the position of the $\{\omega_k\}$ spectrum, including $\omega<\min\{\omega_k\}$. What are meant here the same conditions as implied above, in which the nonlinearities are reactive, only the Raman resonances (1) and (2) are set in action, and system oscillations other than at the frequencies ω, $\omega\pm\Omega$, and Ω can be neglected.

I was probably the first to point out the possibility of such phenomena.[4] They are due to the special role played in the excitation of combined oscillations of a low-frequency system by the vortical reaction forces exerted by the field of the high-frequency resonance oscillations. For vortical reaction forces to set in, both the fast and the slow subsystem must be multidimensional (have not less than two dimensions). A similar symmetry-breaking mechanism in excitation and deexcitation of combined oscillations can be expected also for Raman resonances of higher order.

Since questions involving the symmetry of energy conversions up and down the Raman spectrum are connected with fundamental physical principles, it is expedient to discuss the general aspect of the problem in greater depth than in Ref. 4. The present article deals, from this viewpoint, with the character of energy exchange induced by resonances (1) and (2) in multidimensional systems.

The prevailing opinion is that the characteristic mechanisms of energy transfer via the resonances (1) and (2) are accounted for by the model of three nonlinearly coupled waves (three oscillators), which has been investigated in sufficient detail.[3] This model leads in natural fashion to the notion that energy conversions over the Raman spectrum are reactions in which photons of different frequencies decay and coalesce. The decays $\hbar\omega\rightarrow\hbar\omega_k+\hbar\Omega$ are related to energy transfer from a wave of frequency ω to waves of frequency ω_k and Ω, while the coalescences $\hbar\omega+\hbar\Omega\rightarrow\hbar\omega_k$ are related to energy extraction from waves of frequency Ω. Obviously that $\omega>\omega_k$ in the former and $\omega<\omega_k$ in the latter processes. The aforementioned rule follows therefore directly from quantum principles. Note that quantum interpretations of Raman processes are in essence classically corroborated in the Manley-Rowe theorem,[5] since this theorem is equivalent (see Ref. 6) to conservation laws for the energies of different frequencies transformed by a reactive nonlinearity, if they are regarded as quantized.

This raises the question: are not these fundamental premises negated by the considered vortical phenomena?

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Note that the proof of the Manley-Rowe theorem and the treatment of quantum decay and coalescence processes involve energy conversion by one reactive two-terminal element. Transfer of vortical-motion energy, however, occurs under conditions corresponding in fact to a branched network of such nonlinear elements loaded by finite-Q frequency filters. Such a situation is in general more representative of the physics of real systems. It will be clear from the analysis, however, that the point is not so much the ensuing deviations form the Manley-Rowe relations as the fact that the theorem and the directivity rule for the energy exchange are not uniquely related.

It turns out that from the standpoint of symmetry with respect to the detunings $\omega - \omega_k$ the energy exchange in Raman resonances can be divided into two parts. One is vortical and the other is not. There is no vortical part in the three-wave model. From this standpoint, "fundamental" for the mechanisms of energy transfer for resonances (1) and (2) in many-dimensional systems should be the four-wave rather than the three-wave model.

Energy exchange by the vortical mechanism can play a significant role when the fast motions have close frequencies ω_k and the frequencies of the slow motions are also close to one another. As applied to oscillations and waves in distributed system, this is the usual situation, and its inverse is more readily an exception.

WHAT IS MEANT BY ENERGY EXCHANGE OF VORTICAL TYPE

Consider energy exchange between nonlinearly interacting high- and low-frequency motions, produced by resonant excitation of high-frequency oscillations of frequency ω. We denote by $x = (x_1, x_2, \dots)$ the generalized coordinates of the slow forms of motion (of the frequencies $\Omega \ll \omega$), and by $c = (c_1, c_2, \dots)$ the variables of the high-frequency modes. The energy exchange during one cycle of the periodic variation of x amounts to

$$\oint F\,dx, \qquad (3)$$

where $F = (F_1, F_2, \dots)$ are the generalized forces corresponding to the coordinates and brought about by interactions with the c oscillations. Being interested in the energy exchange averaged over the time $2\pi/\omega$ for the slow motions x, we must replace the values of F in (3) by their averages. Assuming this done, we retain the earlier symbol F for the average forces.

A contribution to (3) is made, obviously, only by the x-dependent part[1)] of F. Since the reaction of the force of the c oscillations lags the x motions, F is a retarded functional of x. In the case of small harmonic changes δx the oscillations δF are harmonic in the linear approximation and are shifted in phase relative to δx. Separating the in-phase (proportional to δx) part of δF from that shifted by $\pi/2$ (and proportional to $\delta \dot{x}$), we represent them in the form

$$\delta F = (\delta F)_0 + (\delta F)_v + (\delta F)_d, \qquad (4)$$

where $(\delta F)_0$ contains the entire reversible part of δF, which does not contribute to (3). The irreversible terms are of the form

$$(\delta F)_v = -K\delta x, \quad (\delta F)_d = -\Gamma \delta \dot{x}, \qquad (5)$$

where K and Γ are certain real matrices that are even functions of Ω. From the irreversibility condition it follows that the matrix K should be antisymmetric and Γ symmetric. Physically, $(\delta F)_d$ are dissipative forces characterized by a friction-coefficient matrix Γ, and $(\delta F)_v$ are vortical forces characterized by a vorticity-coefficient matrix K.

In the general case of anharmonic perturbations

$$\delta x = \sum_\Omega b(\Omega) e^{-i\Omega t}$$

we arrive at the representation (45) with vortical and dissipative forces

$$(\delta F)_v = -\sum_\Omega K(\Omega) b(\Omega) e^{-i\Omega t},$$

$$(\delta F)_d = i \sum_\Omega \Omega \Gamma(\Omega) b(\Omega) e^{-i\Omega t}.$$

The parameters K and Γ are functions of the degree of tuning of the combination resonances. If the energy exchange between x and c is unilateral at certain tunings, so that only amplification or only damping of small x oscillations with frequency Ω takes place, regardless of their phase shift), the quadratic form $\langle \dot{x}\Gamma\dot{x}\rangle$ (where the angle brackets $\langle\ \rangle$ denote averaging over the time $2\pi/\Omega$) should be of definite sign, positive or negative. Moreover, we must also have

$$|\langle \dot{x}\Gamma\dot{x}\rangle| \geqslant |\langle \dot{x}Kx\rangle|, \qquad (6)$$

since the power $\langle \dot{x}Kx\rangle$ reverses sign, depending on the phase differences of the components $\{x_\alpha\}$.

Probably no attention was paid in the earlier physical investigations to the energy exchange generated by vortical forces in combined resonances. The condition (6) is generally not satisified, so that the energy fluxes can have arbitrary directions, and in particular opposite to those expected for combined resonances. We proceed to a detailed treatment based on the analysis of Ref. 4.

MODEL OF INTERACTIONS

Consider a class of systems for which the energy of the high-frequency oscillation modes c, including their coupling to the motions x and the external field of frequency ω, has the structure

$$H = \sum_{k,n} [\omega_{kn}(x) c_k^* c_n + e_k(t) c_k^* + e_k^*(t) c_k]. \qquad (7)$$

The connection between c and x is contained in the relations $\omega_{kn}(x) = \omega_{nk}^*(x)$, and $e_k(t) \sim \exp(-i\omega t)$. It is assumed (without loss of generality) that at fixed x, about which the considered oscillations δx take place, the matrix ω_{kn} is diagonal and the diagonal elements $\{\omega_k\}$ are the eigenfrequencies of the corresponding c modes.

Interaction models having this structure are widely used. They are employed to approximate conditions when the excitation of the high-frequency system is resonant and the excitation level is low, so that the anharmonicity of the c subsystem (at $x = \text{const}$) is still negligible. The case heretofore investigated in greatest detail is three-wave interaction, which corresponds in (7) to two-dimensional oscillations of c and one-dimensional of x, representing, as does also each of the c_k oscillations, an oscillator. Note that in the three-wave model, and for a one-dimensional x in general, the irreversible part of (4) can contain only the term $(\delta F)_d$, and there is no vortical term.

The power flux from c to x is equal to

$$P=-\frac{\partial H}{\partial x}\dot{x}=-\sum_{k,n,\alpha}\omega_{kn,\alpha}c_k^{*}c_n\dot{x}_\alpha, \tag{8}$$

where

$$\omega_{kn,\alpha}=\partial\omega_{kn}/\partial x_\alpha .$$

The coefficients of $\dot{x}$ in (8) are obviously the generalized forces F. The irreversible terms in F are due to the response of the c oscillations to the changes of x, since it can be seen from (8) that the forces F are potential if this response is neglected.

We are interested in the general structure of forms quadratic in δx and defined by matrices K and Γ representing the vortical and dissipative energy exchanges. There is no need to specify the dynamics of the x subsystem in this case.

We choose the variables x to be real. Note that the analysis pertains not only to interactions for which x in (7) are generalized coordinates. We can take x to mean a set of arbitrary variables of the slow system, including arbitrary combinations of coordinates and momenta. In this general case the energy exchange is given as before by Eq. (8). The quantities F, however, which are equal to coefficients of x in (8), will no longer be strictly speaking generalized forces. The energy flux from the fast to the slow system, however, is still determined by (3) in which, as previously, we can separate the vortical and dissipative parts in accordance with (4).

Considering resonant regimes, we assume the Q of the c modes to be finite, and use the standard substitution $\omega_k \to \omega_k - i\gamma_k$, where γ_k are the damping frequencies of the corresponding c modes.

Thus, when estimating the parameters K and Γ and the energy exchange at small δx, we start from Eq. (8) where the dynamics of c is defined by the set of equations

$$i\left(\frac{d}{dt}+\gamma_k\right)c_k=\omega_k c_k+e_k(t)+\sum_{n,\alpha}\omega_{kn,\alpha}c_n\delta x_\alpha. \tag{9}$$

ANALYSIS

The calculation yields

$$K=\sum_k \varkappa^{(k)}\,\mathrm{Re}\left\{\frac{1}{D_+^{(k)}}+\frac{1}{D_-^{(k)}}\right\},$$
$$\Gamma=\frac{1}{\Omega}\sum_k g^{(k)}\,\mathrm{Re}\left\{\frac{1}{D_+^{(k)}}-\frac{1}{D_-^{(k)}}\right\}, \tag{10}$$

where $g^{(k)}$ and $\varkappa^{(k)}$ are symmetric and antisymmetric matrices with elements

$$g_{\alpha\beta}^{(k)}=\mathrm{Re}\left\{\sum_{n,l}\omega_{nk,\alpha}\omega_{kl,\beta}a_n^{*}a_l\right\},$$

$$\varkappa_{\alpha\beta}^{(k)}=\mathrm{Im}\left\{\sum_{n,l}\omega_{nk,\alpha}\omega_{kl,\beta}a_n^{*}a_l\right\}.$$

Here $a=(a_1, a_2, \ldots)$ are the amplitudes of the c oscillations of frequency ω and are stationary at $\delta x = 0$. The parameters $D_+^{(k)}$ and $D_-^{(k)}$ are equal to

$$D_\pm^{(k)}=i[\omega_k-(\omega\pm\Omega)]+\gamma_k.$$

Expression (10) are exact (see Ref. 4).

For the power transferred from c to x at small oscillations

$$\delta x(t)=be^{-i\Omega t}+b^{*}e^{i\Omega t}, \tag{11}$$

we obtain, averaged over the period $2\pi/\Omega$,

$$P(\Omega)=\langle F\delta\dot{x}\rangle=\sum_k bg^{(k)}b^{*}R_-^{(k)}-i\sum_k b\varkappa^{(k)}b^{*}R_+^{(k)}, \tag{12}$$

where $R_\pm^{(k)}$ are nondimensional functions of the detuning $\omega-\omega_k$ and of the frequency Ω, and are given by

$$R_\pm^{(k)}=2\Omega\,\mathrm{Re}\{1/D_-^{(k)}\pm 1/D_+^{(k)}\}.$$

The behavior of their ratio is shown in Fig. 1.

The first and second sums in (12) are the energy exchanges due to the action of the dissipative and vortical forces, respectively. Note that $ib\eta^{(k)}b^{*}$ is a real quantity, as is also $bg^{(k)}b^{*}$. The two expression are of the same order of magnitude. The quadratic form $bg^{(k)}b^{*}$, however, is always non-negative, while $ib\varkappa^{(k)}b^{*}$ can be of either sign, depending on the relations between the $\{b_\alpha\}$ and $\{a_n\}$ phases, but always

$$bg^{(k)}b^{*}\geqslant|b\varkappa^{(k)}b^{*}|. \tag{13}$$

This result follows, for example, from the fact that the relations

$$P_\pm^{(k)}=2\gamma_k\omega_k|c_k^{\pm}|^2$$

are non-negative, and we obtain for them from (9)

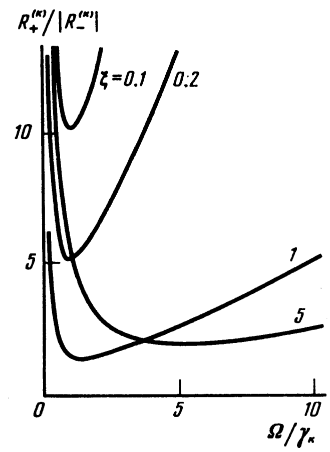

FIG. 1. Ratio of the factors $R_+^{(k)}$ and $R_-^{(k)}$ as a function of Ω/γ_k and of $\xi=(\omega-\omega_k)/\gamma_k$.

$$P_+^{(k)} = 2(bg^{(k)}b^* + ib\varkappa^{(k)}b^*)\omega_k \operatorname{Re}(1/D_+^{(k)}), \quad (14)$$

$$P_-^{(k)} = 2(bg^{(k)}b^* - ib\varkappa^{(k)}b^*)\omega_k \operatorname{Re}(1/D_-^{(k)}), \quad (15)$$

$P_\pm^{(k)}$ stands for the average power dissipated by the c oscillations of the k mode at the frequencies $\omega \pm \Omega$ in the oscillation process (11). We denote by c^+ and c^- the corresponding harmonics of the c oscillations.

We consider now the dissipative part of the energy exchange in (12). It is proportional to the factors $R_-^{(k)}$, which reverse sign simultaneously with the detunings $\omega - \omega_k$. From this and from the fact that $bg^{(k)}b^*$ are non-negative it follows that the dissipative energy exchange is determined by the signs of the detunings: the resonances with frequencies $\omega_k < \omega$ produce a positive energy flux from c to x, and the resonances with $\omega_k > \omega$ cause energy to be drawn from the x subsystem. This agrees fully with the usual viewpoints concerning the direction of the energy flow in the combined resonances (1) and (2).

A different character of energy exchange is brought about by vortical forces: the resonance denominators $D_+^{(k)}$ and $D_-^{(k)}$ are the same, but the vortical contribution to the energy exchange (12) is determined by factors $R_+^{(k)}$ that do not depend on the sign of the detunings $\omega - \omega_k$. Recognizing furthermore that the quantities $ib\varkappa^{(k)}b^*$ reverse sign, depending on the $\{b_\alpha\}$ phases, it becomes clear that for the vortical part it is immaterial whether the energy of the oscillations of frequency ω is transformed into the Stokes or anti-Stokes region of the spectrum. At $\omega = \omega_k$ the contribution to (12) from the kth terms comes only from the vortical forces.

We call attention to the fact that $R_+^{(k)} > |R_-^{(k)}|$ always, and that equalities are possible in (13) at all k simultaneously. For example, for a non-gyrotropic system with two-dimensional motions $c = (c_1, c_2)$ and $x = (x_1, x_2)$ the equalities take place, for example ast $a_1 = ia_2$, $b_1 = ib_2$, and

$$\omega_{11,1} = \omega_{22,1} = \omega_{12,2}, \quad \omega_{11,2} = \omega_{22,2} = -\omega_{12,1}. \quad (16)$$

The possibility of (13) turning into an equality at all k means that vortical energy exchange can be predominant at all values of ω and Ω. The power P can then be of either sign regardless of the location of ω relative to the $\{\omega_k\}$ spectrum, including, in particular, $P > 0$ at $\omega < \min\{\omega_k\}$. An example of a system with predominant energy exchange in the case of three-dimensional c oscillations and two-dimensional x is the vortical parametric motor whose idea is described in Ref. 4.

Let us compare $P(\Omega)$ with the powers P_+ and P_- dissipated by the system at the combined frequencies $\omega + \Omega$ and $\omega - \Omega$. We have

$$P_+ = \sum_k P_+^{(k)}, \quad P_- = \sum_k P_-^{(k)}. \quad (17)$$

It follows from (12), (14), and (15) that

$$P(\Omega) = \sum_k \frac{\Omega}{\omega_k} (P_-^{(k)} - P_+^{(k)}). \quad (18)$$

The presence of vortical terms in $P_\pm^{(k)}$ causes partial or total (if (13) represents equalities) suppression of the Raman oscillations c at the Stokes or anti-Stokes frequencies, regardless of the location of ω relative to $\{\omega_k\}$. This is indeed the cause of the breaking of the usual symmetry of $P(\Omega)$ relative to the detunings $\omega - \omega_k$, and is not due at all to violation of the Manley-Rowe theorem. In fact, Eq. (18) differs from the relation that follows from the theorem

$$P(\Omega) = \Omega P_-/(\omega - \Omega) - \Omega P_+/(\omega + \Omega), \quad (19)$$

in that the denominators ω_k of $P_-^{(k)}$ and $P_+^{(k)}$ in (18) are replaced by $\omega - \Omega$ and $\omega + \Omega$, respectively. For high-Q resonances ($\tau_k \ll \omega_k$) and at $\Omega \ll \omega$ the difference between (18) and (19) is immaterial.

BROADBAND ACTIONS

The difference between the dependences of the vortical and dissipative parts of the energy exchange on the detunings $\omega - \omega_k$ points to interesting and important differences between their behaviors under conditions of broadband actions $e(t) = \{e_k(t)\}$. Heretofore the $e(t)$ were assumed harmonic. Let the $e(t)$ contain high frequencies in a ceratin band $\Delta\omega$:

$$e(t) = \sum_\omega e_\omega e^{-i\omega t}. \quad (20)$$

If this sum has components that differ in frequencies by Ω, a nonzero energy exchange between c and x can occur also without allowance for the influence of the motions of x in the c-oscillation regime. Neglecting the reaction force, we have $F = F^0(t)$, where

$$F_\alpha^0(t) = -\sum_{k,n,\omega',\omega''} \omega_{kn,\alpha} \dot{a}_{k\omega'} a_{n\omega''} \exp\{i(\omega' - \omega'')t\}; \quad (21)$$

here $a_{k\omega} = (\omega - \omega_k + i\gamma_k)^{-1} e_{k\omega}$.

Oscillations of x stimulated by the forces $F^0(t)$ causes, besides the discussed effects of the reaction forces, a redistribution of the energy fluxes at the combined frequencies. In particular, they cause high-frequency oscillations at the anti-Stokes frequencies $\omega_a = 2\omega' - \omega''$ (here $\omega' > \omega''$). These effects can be related to phenomena called in nonlinear optics coherent anti-Stokes Raman scattering (CARS). To differentiate from them, we consider conditions when the sum (20) either contains no components that differ by Ω, or the number of such components is large and their phases are sufficiently uniformly distributed in the interval $(0, 2\pi)$. It suffices in this respect to have the sum (21) for F^0 vanish at $|\omega' - \omega''| = \Omega$. We shall assume that this is the case also at $|\omega' - \omega''| = 2\Omega$.

The results of the calculation of the parameters K and Γ and of the powers P and $P_\pm^{(k)}$ reduce then simply to the fact that the summation over k in their expression is supplemented by summation over ω. For example, for the power $P(\Omega)$ we now obtain

$$P(\Omega) = \sum_{k,\omega} bg^{(k,\omega)} b^* R_-^{(k,\omega)} - i \sum_{k,\omega} b\varkappa^{(k,\omega)} b^* R_+^{(k,\omega)}, \quad (22)$$

where $g^{(k,\omega)}$ and $\varkappa^{(k,\omega)}$ differ from $g^{(k)}$ and $\varkappa^{(k)}$ in that $a_n^* a_l$ are replaced by $a_{n\omega}^* a_{l\omega}$. The terms proportional to the factors $a_{n\omega'}^* a_{l\omega''}$ with $\omega' \neq \omega''$ vanish as a result of the time averaging. The same results are obtained if the pump (20) is regarded as a random stationary field and statistically averaged characteristics are considered.

Let the intensity of the $e(t)$ spectrum be relatively uniform in a wide band $\Delta\omega$ that overlaps the spectral band $\{\omega_k \pm \Omega\}$ for the c modes that participate in the interactions with x. The value of Γ and the energy-exchange part corresponding to it then become negligible. On the other hand, the vortical energy exchange may in this case not only decrease in intensity, but also increase. It can be controlled by varying the phases of the components $\{e_{k\omega}\}$ with different k.

Thus, energy exchange via the vortical mechanism should, generally speaking, become predominant in the case of broadband pumping of many-dimensional systems. Of course, for vortical Raman effects to set in, the slow forms of motion must have close eigenfrequencies. If there are no special resonant modes in the low-frequency system, vortical irregular motions can apparently be excited.

The analysis above was carried out for the case of harmonic perturbations δx. For perturbations δx containing a spectrum of frequencies Ω it is necessary to regard b and b^* in the expressions given above for the energy fluxes as functions of Ω and carry out additional summation over Ω. Terms containing factors $b_\alpha(\Omega) b^*_\beta(\Omega')$ with $\Omega' \neq \Omega$ will vanish as a result of the averaging. Thus, a generalization to the case of anharmonic δx encounters no difficulties and does not alter the statements above.

CONCLUSION

Thus, for resonances (1) and (2) we must distinguish, from the standpoint of the symmetry of energy conversions over the Raman spectrum, between two energy-exchange mechanisms, vortical and dissipative.

Only one dissipative mechanism is realized if the motions are one-dimensional in the fast or slow subsystem (or in both). Actually, only the dissipative mechanism served as the basis of the present views concerning the character of energy conversion over the Raman spectrum. The existence of the vortical mechanism upsets these views and makes possible excitation or suppression of low-frequency oscillations irrespective of the position of the frequency ω relative to the $\{\omega_k\}$ spectrum.

In the general case, energy exchange via the vortical mechanism is no less significant than via the dissipative one. Both are determined in fact by one and the same system of interaction matrix elements $\{\omega_{kn,\alpha}\}$ and the corresponding selection rules. Both are quadratic in these elements.

It is understandable that in many-dimensional systems close to degeneracy in the high and low frequencies, with more than one component of the pump $e(t)$, the excitation of combined oscillations should be manifest primarily in excitation of vortical motion. This is clear because the instability growth rates are extremal for those forms of low-frequency motion for which both forms of energy fluxes are significant. Vortical energy fluxes are quite important also from the standpoint of realization of inverse phenomena in such systems, i.e., deexcitation and suppression of slow motions by high-frequency fields. For suppression to take place it is necessary that the detunings $\omega - \omega_k$ be negative. Even in this case, however, the presence of a vortical energy-exchange channel can lead to warming-up of definite forms of motion, reducing the summary effect to zero.

A particularly important role can be played by energy exchange via the vortical mechanism if the low-frequency subsystem has enough inertia (inasmuch as the ratios $R^{(k)}_+ / R^{(k)}_- \to 0$ as $\Omega \to 0$), and also when the high-frequency pump has a large bandwidth.

The author thanks S. G. Rautian for helpful questions concerning the relation between CARS and the phenomena considered here.

[1)] An exception is the case when x are non-single-valued variables or have a certain period. For example, changing an angle coordinate x by 2π does not change the state. The constant part of F (the constant torque F_α) makes then a contribution to (3) when x_α goes through one complete revolution 2π. No contribution is made to (3) by constant forces if the motions of x take place in a region smaller than the size of the ambiguity, and it is this which we have in mind.

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