Kinetic properties of two-dimensional ferromagnets

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The classical fluctuation dynamics of two-dimensional ferromagnets with easy-plane anisotropy is considered, and the form of the fluctuation corrections to the spin-wave spectrum is found. It is shown that allowance for fluctuation–dissipation corrections leads to a spectrum in the long-wavelength region that is substantially different from the result obtained using a phenomenological dissipative term of the Landau-Lifshitz form in the equation of motion for the magnetic-moment density. An effective equation of motion for vortex excitations is also presented.

I. INTRODUCTION

In this paper we study the kinetic properties of a two-dimensional Heisenberg ferromagnet with easy-plane anisotropy. The ratio of the anisotropy constant λ to the exchange integral J is assumed small, in accordance with the usual experimental situation.[1] As was originally established by Kosterlitz and Thouless,[2] this system has a low-temperature phase transition at $T_c \sim T/\ln(J/\lambda)$ at which the vortex pairs dissociate, with the result that free vortex excitations are present at $T > T_c$.

In Refs. 3–5, the equations of motion of the vortices were obtained and the parametric resonance with the vortex excitations in an rf field was studied. The initial equation was the Landau-Lifshitz equation with the phenomenological dissipative term.

$$\frac{\partial}{\partial t}\mathbf{n} = -\left[\mathbf{n}, \frac{\delta H}{\delta \mathbf{n}}\right] + \frac{B}{A}\left[\mathbf{n}\left[\mathbf{n}, \frac{\delta H}{\delta \mathbf{n}}\right]\right] \qquad (1)$$

[the square brackets denote the vector (cross) product], where the Hamiltonian H is of the form

$$H = {}^1/_2 \int d^2r \{A(\nabla \mathbf{n})^2 + Am^2(\mathbf{nv})^2\}. \qquad (2)$$

Here $\mathbf{n}$ is a unit vector corresponding to the normalized magnetization, which is treated as a classical vector of constant length, and $\mathbf{v}$ is a unit vector in the direction of the anisotropy axis. We use a system of units in which $J = 1$ and $\lambda = m^2$.

Linearizing Eq. (1), we get the following excitation spectrum:

$$\omega(k) = A[k^2(k^2+m^2) - B^2m^4/4A^2]^{1/2} - iB(k^2+m^2/2). \qquad (3)$$

We see from (3) that for $k \gg m$ the spin-wave spectrum is the same as in an isotropic magnet, $\omega = \pm Ak^2 - iBk^2$, while in the intermediate region $mB/A \ll k \ll m$ the spin-wave dispersion relation is linear, $\omega = \pm Amk - iBm^2/2$, and for $k \ll Bm/A$ the propagation of the spin mode becomes diffusive, $\omega = -iA^2k^2/B$. The spin diffusion at small k has not been observed experimentally.

Let us consider the question of whether phenomenological equation (1) is confirmed by a microscopic treatment.

As is shown below, the onset of spin diffusion can be due solely to "bare" dissipative terms in the macroscopic dynamical equation. Since our goal in the following is to study the dissipative terms of fluctuational origin, we shall not consider the region of smallest k, where spin diffusion can occur and, hence, fluctuation corrections are unimportant.

Another property of Eq. (1) is the logarithmic divergence of the integral of the dissipative term taken over the area occupied by the vortex excitations. For a system of vortices this gives rise to a coefficient $\ln R$ in the equations of motion, where R is the average distance between vortices,[3] and in particular this makes it impossible for an isolated vortex to move (at $T > T_c$). These properties of Eq. (1) follow from the fact that the phenomenological dissipative term becomes the leading term in the long-wavelength limit.

Below we shall obtain the form of the dissipative term in the limit $k \ll m$, assuming that the dimensionless charge $g = T/J$ is small. The magnetic-moment density $\mathbf{n}$ will be treated as a classical vector, since the governing contribution is from large regions of linear dimension $R \sim 1/m$. In addition, we assume that the occupation numbers f_k are large for $k \sim m$, as follows from the condition $\lambda \ll T \ll J$, which is possible for weak anisotropy. In this case we are dealing with classical thermal fluctuations of a chiral field $\mathbf{n}$, and the hydrodynamic description applies over a wide range of wave vectors. We restrict discussion to excitations of the spin-wave type, assuming the contribution of vortex pairs to be small, and we consider damping that is due entirely to the exchange interaction.

For a hydrodynamic description of the fluctuation dynamics we use the formalism developed in Ref. 6. This method has been applied previously[7] to analysis of the properties of isotropic two-dimensional ferromagnets, and the renormalization-group equations have been obtained for the charges g, A, and B of a renormalizable dynamic Lagrangian with a phenomenological dissipative term corresponding to Eq. (1). Here the bare value of the charge B was assumed to stem from the contribution of the nonhydrodynamic region and was determined outside the framework of the given method.

At the same time, in the case of a small bare B the leading contribution to the spin-wave damping arises in the hydrodynamic region, and the dissipation is governed by the fluctuation terms rather than the kinetic terms in the equation of motion. The corresponding value of the kinetic coeffi-

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cient is determined solely by the parameters appearing in Hamiltonian (2) and does not depend on the bare dissipative terms, to the extent that they are small. Fluctuational spin-wave damping arises in second order in $g \ll 1$ (Ref. 8).

More precisely, we require that the expansion parameter

$$g_R = g\left[1 - \frac{1}{2\pi} g \ln \frac{\Lambda}{m}\right]^{-1},$$

and not only g, be small compared to 1 (here Λ is the upper boundary of the hydrodynamic region). We shall not consider the neighborhood of the pole of the renormalized (in the one-loop approximation) charge g_R.

For the bare value of the coefficient B, which originates from the nonhydrodynamic region, we can take the following estimate. The introduction of anisotropy into the macroscopic Hamiltonian (2) is equivalent to allowing for the interaction of the spin system with degrees of freedom which are not considered in the purely exchange approximation. Even if we assume that the dissipative terms arising in the macroscopic equation of motion upon allowance for this interaction are of the same size at wave vectors $k \sim \Lambda$ as the nondissipative anisotropic term, we obtain for the bare coefficient B_0 the maximum value $B_0 \sim A(m/\Lambda)^2$. By virtue of the logarithmic character of the renormalization of the coefficient B in the presence a phenomenological dissipative term in an isotropic ferromagnet,[7] the corrections to this coefficient will not substantially alter this estimate. We shall assume below that there is a rather wide region in which the fluctuation terms are the leading terms, as is possible for weak anisotropy.

In the isotropic situation an analogous approach was taken in Ref. 9, but the numerical value of B obtained in that study is incorrect. In Sec. 4 we give the result of a calculation in the isotropic limit.

II. DYNAMIC LAGRANGIAN AND THE FLUCTUATION-DISSIPATION THEOREM

Following the method elaborated in Ref. 6, we write the boson part of the dynamic Lagrangian, using the field $\mathbf{n}$ and the auxiliary vector field $\mathbf{p}$, which satisfies the condition $\mathbf{p}\cdot\mathbf{n} = 0$:

$$\mathscr{L} = \mathscr{L}_r + \mathscr{L}_d, \tag{4}$$

$$\mathscr{L}_r = \mathbf{p}\left(\frac{\partial}{\partial t}\mathbf{n} + \left[\mathbf{n}, \frac{\delta H}{\delta \mathbf{n}}\right]\right). \tag{5}$$

Here $\mathscr{L}_r$ is the nondissipative part of the Lagrangian, and $\mathscr{L}_d$ is the contribution of the dissipative terms, whose form we must determine. We note that by virtue of the analytic properties of the fermion propagators,[7] the fermion vertices do not contribute to the purely bosonic counterterms of $\mathscr{L}$.

For the isotropic case Lebedev[7] used for $\mathscr{L}_d$ the local operator

$$\mathscr{L}_d = \frac{B}{A}\left(\mathbf{p}\frac{\delta H}{\delta \mathbf{n}}\right) + igB\mathbf{p}^2 \tag{6}$$

and demonstrated the renormalizability of Lagrangian (4) with $\mathscr{L}_d$ from (6). In the limit $B \to 0$ the local counterterms which renormalize $\mathscr{L}_d$ vanish. In this case the corrections to the Lagrangian are due to power-law diagrams and are not local. Of course, this statement also applies to the anisotropic case at momenta $k \gg m$. Nevertheless, in the long-wavelength limit $k \ll m$ the dissipative terms become local, so that the modified equation of motion for the field $\mathbf{n}$ assumes a local form, though different from (1).

The real part of the spectrum is renormalized in the limit $B \to 0$ independently of the imaginary part, and the equation obtained in Ref. 7 for the coefficient A in this case becomes

$$\frac{d \ln A}{d \ln \Lambda} = -\frac{g^2}{4\pi^2}. \tag{7}$$

Since the fields $\mathbf{n}$ and $\mathbf{p}$ are subject to constraints, it will be necessary to use a definite parametrization of these fields, introducing two independent components for each of them. We choose the coordinate system such that the spatially uniform ground state $\mathbf{n}_0$ about which the small fluctuations are considered is directed along the x axis, while the vector $\boldsymbol{\nu}$ giving the direction of the anisotropy axis is along z. As independent variables we take φ and $\psi = \cos\theta$, where θ and φ are the angles of the vector $\mathbf{n}$ in a spherical coordinate system with polar axis z, and

$$\begin{aligned} p_\varphi &= (1-\psi^2)^{1/2}(p_2 \cos\varphi - p_1 \sin\varphi), \\ p_\psi &= p_3(1-\psi^2)^{-1}, \end{aligned} \tag{8}$$

where p_μ are the component of the vector $\mathbf{p}$ in this same system. In the equilibrium state at $T = 0$ we have $\varphi = \psi = 0$. At small T the restrictions on the range of variation φ and ψ are unimportant, and these variables, like p_φ and p_ψ, can be treated as unrestricted variables.

In the chosen parametrization expression (2) for H becomes

$$H = A/2 \int d^2r \{(\nabla\psi)^2/(1-\psi^2) + (\nabla\varphi)^2(1-\psi^2) + m^2\psi^2\}. \tag{9}$$

The reactive part of the dynamic Lagrangian (5) is written in the form

$$\mathscr{L}_r = p_\varphi\left(\frac{\partial\varphi}{\partial t} - \frac{\delta H}{\delta\psi}\right) + p_\psi\left(\frac{\partial\psi}{\partial t} + \frac{\delta H}{\delta\varphi}\right). \tag{10}$$

Let us expand this expression in a power series in the fields φ and ψ to fourth order:

$$\mathscr{L}_r = \mathscr{L}_r^{(2)} + \mathscr{L}_r^{(4)}, \tag{11}$$

$$\mathscr{L}_r^{(2)} = p_\varphi \partial\varphi/\partial t + p_\psi \partial\psi/\partial t + A(p_\varphi\nabla^2\psi - p_\psi\nabla^2\varphi - m^2 p_\varphi\psi), \tag{12}$$

$$\mathscr{L}_r^{(4)} = A\{p_\varphi[\nabla^2(\psi^3)/3 - \psi(\nabla\psi)^2] + p_\varphi\psi(\nabla\varphi)^2 - \nabla p_\varphi \nabla\varphi\psi^2\}. \tag{13}$$

The dissipative terms are written to lowest order in the fields as

$$\begin{aligned} &\mathscr{L}_d = \mathscr{L}_{d1} + \mathscr{L}_{d2}, \\ &\mathscr{L}_{d1} = ip_a\Sigma_{ab}\varphi_b, \quad \mathscr{L}_{d2} = ip_a\Pi_{ab}p_b/2, \end{aligned} \tag{14}$$

where we have introduced the notation $\varphi_1 = \varphi$, $\varphi_2 = \psi$, $p_1 = p_\varphi$, $p_2 = p_\psi$. The additional vertices deriving from dissipative terms are unimportant for our purposes.

The quadratic part of the Lagrangian $\mathscr{L}_r^{(2)} + \mathscr{L}_d$ governs pairings of the type $\langle \varphi_a p_b \rangle$ and $\langle \varphi_a \varphi_b \rangle$. The averages

$\langle \varphi_a p_b \rangle$ form a matrix of linear response functions G_{ab} (Ref. 6), the poles of which determine the excitation spectrum:

$$\langle \varphi_a p_b \rangle = iG_{ab}(\omega, \mathbf{k}). \tag{15}$$

Similarly, for the binary averages of the order parameter we obtain

$$\langle \varphi_a \varphi_b \rangle = -D_{ab}(\omega, \mathbf{k}) = G_{ac}(\omega, \mathbf{k}) \Pi_{cd} G_{bd}(-\omega, \mathbf{k}). \tag{16}$$

The corrections to the spectrum are determined by the single-particle irreducible self-energy part Σ_{ab} according to

$$\det G^{-1} = \det(G_0^{-1} + i\Sigma) = 0. \tag{17}$$

The self-energy function Σ_{ab} and the polarization operator Π_{ab} for configurations close to equilibrium are connected by a relation that follows from the fluctuation-dissipation theorem. To formulate this condition we note that the variables φ and ψ are Hamiltonian conjugates of each other.[4] In arbitrary coordinates Φ_a parametrizing the space of order-parameter values (in this case it is the sphere S^2) and the function F_a in a nondissipative equation of the form

$$\frac{\partial}{\partial t} \Phi_a + F_a = 0$$

is generally not a total derivative. In addition to the derivative of the Hamiltonian with respect to the corresponding variable, F_a contains the metric of the space in which the order parameter varies in the given parametrization. Because of this it is difficult to obtain a concrete form for the fluctuation-dissipation theorem.

If, however, the parametrization is such that the nondissipative equations of motion can be written in the form

$$\frac{\partial}{\partial t} \Phi_a = W_{ab} \frac{\delta H}{\delta \Phi_b} = W_{ab} H_{,b} \tag{18}$$

with a matrix W_{ab} that does not depend on Φ_a, then the fluctuation-dissipation theorem assumes the form

$$\Pi_{ab}(\omega) = -(g/\omega)(\Sigma_{ac}(\omega) W_{cb} - \Sigma_{bc}(-\omega) W_{ca}), \tag{19}$$

which is equivalent to a relation between G_{ab} and D_{ab} of the form

$$D_{ab}(\omega) = (g/i\omega)(G_{ac}(\omega) W_{cb} - G_{bc}(-\omega) W_{ca}). \tag{20}$$

These relations are valid for a system of type (17) with an arbitrary N-component order parameter Φ_a under the condition that the matrix W_{ab} is antisymmetric and specifies a "Hamiltonian structure" on the space of variables Φ_a.

For the case when equations (18) are Hamilton's equations and describe a conservative system in a state of thermal equilibrium, the fluctuation-dissipation theorem in form (19) has been proved in general.[9] Here it is more convenient to use the time representation, in which the fluctuation-dissipation theorem (19) is written in the form

$$\theta(t) \frac{\partial}{\partial t} \Pi_{ab}(t) = ig \Sigma_{ac}(t) W_{cb}. \tag{21}$$

The series for $\Sigma_{ab}(t)$ and $\Pi_{ab}(t)$, which are formed by skeleton diagrams containing the complete G and D functions, represent an expansion in powers of the renormalized charge g_R, which is also present in expressions (19)–(21). The contributions which arise include the state renormalization of the parameters of the Hamiltonian in addition to the charge g. We will be convinced of this from the example of an easy-plane ferromagnet.

To the given order in g_R, the differentiation with respect to t of an arbitrary diagram contributing to $\Pi_{ab}(t)$, leads to a $\Sigma_{ab}(t)$ in accordance with (21). Conversely, every contribution to $\Sigma_{ab}(t)$ in the given order can be obtained in this way. Without giving the whole proof here, let us demonstrate, for example, how Eq. (21) is satisfied for the lowest contributions to $\Sigma_{ab}(t)$ and $\Pi_{ab}(t)$. Suppose the Hamiltonian H can be expanded in a series in even powers of Φ_a (in equilibrium $\Phi_a = 0$). Then, keeping the fourth-order terms in the dynamic Lagrangian for Eq. (17), we obtain

$$\mathscr{L}_r = \pi_a \left(\frac{\partial}{\partial t} \Phi_a + F_{a,b} \Phi_b + \frac{1}{6} F_{a,bcd} \Phi_b \Phi_c \Phi_d + \ldots \right). \tag{22}$$

Here π_a are auxiliary boson fields, $F_{a,b} = H_{,bc} W_{ca}$, $F_{a,bcd} = H_{,ebcd} W_{ea}$, and the vertex matrix is symmetrized with respect to the last three indices.

In the time representation, Eq. (20) becomes

$$G_{ab}(t) = -\frac{1}{g} \frac{\partial}{\partial t} D_{ac}(t) W_{cb}^{-1} \theta(t). \tag{23}$$

The first fluctuation corrections to $\mathscr{L}$ comes from the diagrams of Fig. 1 for the contributions to Σ_{ab} and Π_{ab}, respectively. Here the solid line corresponds to the field φ_a and the dashed line to the field p_a. The arrangement of the momenta $\mathbf{q}_1$, $\mathbf{q}_2$, and $\mathbf{q}_3 = \mathbf{q}_1 + \mathbf{q}_2 - \mathbf{k}$ is indicated in Fig. 1.

The corresponding analytical expressions are of the form

$$\delta\Sigma_{aq}(t) = \frac{i}{2} \int F_{a,bcd} F_{m,npq} D_{bn}(1) D_{cp}(2) G_{dm}(-3) \frac{d^3q_1\, d^3q_2}{(2\pi)^4}, \tag{24}$$

$$\delta\Pi_{am}(t) = -\frac{1}{6} \int F_{a,bcd} F_{m,npq} D_{bn}(1) D_{cp}(2) D_{dq}(-3) \frac{d^3q_1\, d^3q_2}{(2\pi)^4}. \tag{25}$$

Using (23) and symmetrizing by relabelling the momenta, we transform (24) into

$$\begin{aligned} \delta\Sigma_{aq}(t) &= -\frac{i}{2g} \int F_{a,bcd} F_{m,npq} D_{bn}(1) D_{cp}(2) \\ &\quad \times \frac{\partial}{\partial t} D_{dh}(-3) W_{hm}^{-1} \theta(t) \\ &= \frac{i}{6g} \theta(t) \int F_{a,bcd} H_{,mnpq} \frac{\partial}{\partial t} [D_{bn}(1) D_{cp}(2) D_{dm}(-3)]. \end{aligned} \tag{26}$$

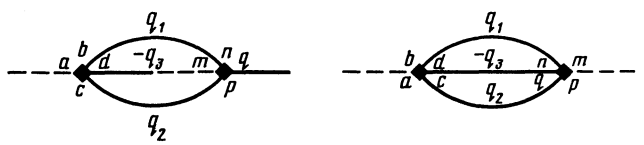

FIG. 1.

Comparison of this expression with (25) with allowance for the relation between $F_{m,npq}$ and $H_{,nmpq}$ demonstrates that Eq. (21) holds for the corrections in question.

We have said that if the charge g is among the charges which are renormalizable in statics, expression (21) contains its renormalized value g_R due to higher corrections. Inasmuch as the nature of the static renormalizations depends on the specific system, in verifying this fact below we shall consider only problems of ferromagnetics.

III. DIAGRAM TECHNIQUE

In our case W_{ab} is simply the two-component antisymmetric tensor ε_{ab}. The matrix of G functions is written

$$G_{ab}=\frac{i}{2}(\omega-\varepsilon-\Sigma_+)^{-1}\begin{pmatrix}1 & ip^2/\varepsilon\\ -ik^2/\varepsilon & 1\end{pmatrix}+\frac{i}{2}(\omega+\varepsilon-\Sigma_-)^{-1}\begin{pmatrix}1 & -ip^2/\varepsilon\\ ik^2/\varepsilon & 1\end{pmatrix}, \tag{27}$$

where $\varepsilon=Ak(k^2+m^2)^{1/2}$ is the unperturbed spectrum, $p^2=k^2+m^2$, and $\Sigma_\pm$ is the imaginary part of the spectrum, which we assume to be negligible. The functions D_{ab} are constructed from G_{ab} according to (19). Neglecting the bare $\Sigma_\pm$, we obtain

$$\begin{aligned}D_{\varphi\varphi}&=-(\pi g/k^2)[\delta(\omega-\varepsilon)+\delta(\omega+\varepsilon)],\\ D_{\psi\psi}&=-(\pi g/p^2)[\delta(\omega-\varepsilon)+\delta(\omega+\varepsilon)],\\ D_{\varphi\psi}&=-D_{\psi\varphi}=-(ig\pi/\varepsilon)[\delta(\omega-\varepsilon)-\delta(\omega+\varepsilon)].\end{aligned} \tag{28}$$

Naturally, we can neglect $\Sigma_\pm$ only if the functions $\Sigma_\pm(k)$ fall off at small k no slower than the real part of the spectrum $\varepsilon(k)$. As we shall see, the fluctuation contributions $\Sigma_\pm$ retain a k^2 dependence even in the long-wavelength limit and can therefore be neglected.

The integral of D_{ab} over frequency reproduces the static correlators:

$$\int D_{ab}(\omega,\mathbf{k})\frac{d\omega}{2\pi}=-g\begin{pmatrix}k^{-2} & 0\\ 0 & p^{-2}\end{pmatrix}. \tag{29}$$

The nonzero components of the vertex function are

$$\begin{aligned}&F_{\varphi,\varphi\varphi\psi}(4;1,2,3)=-2(\mathbf{q}_1\mathbf{q}_2),\quad F_{\varphi,\psi\psi\psi}(4;1,2,3)=2\sum_{i<j}^{4}(\mathbf{q}_i\mathbf{q}_j),\\ &F_{\psi,\psi\psi\varphi}(4;1,2,3)=2(\mathbf{q}_3\mathbf{q}_4).\end{aligned} \tag{30}$$

The choice of variables φ and ψ to parametrize $\mathbf{n}$ permits exact allowance of the anisotropy.

Calculation of the logarithmic corrections to $\mathscr{L}$ leads to a renormalization of the coefficients of the various operator structures contained in $\mathscr{L}$. To obtain a relation between these coefficients that is analogous to the bare relation under the condition that the factor multiplying the combination $p_\varphi\partial\varphi/\partial t+p_\psi\partial\psi/\partial t$, is normalized to 1, we must assign multiplicative Z factors to the fields φ_a and p_a and to the parameters m and g. For example, in the one-loop approximation we have

$$Z_\varphi=Z_{p_\varphi}^{-1}=1,\quad Z_\psi=Z_{p_\psi}^{-1}=1+\frac{g}{2\pi}\ln\frac{\Lambda}{m},$$

$$\begin{aligned}Z_m&=1-\frac{g}{2\pi}\ln\frac{\Lambda}{m},\\ Z_g&=1+\frac{g}{2\pi}\ln\frac{\Lambda}{m}.\end{aligned} \tag{31}$$

All the Z factors in (31) are defined as ratios of the renormalized quantities to the bare quantities. We show below that in this case formulas (24) and (25) give expressions for $\operatorname{Im}\Sigma_{ab}$ and Π_{ab} that do not contain $\ln(\Lambda/m)$. However, the fluctuation-dissipation theorem should apply to the operators Σ and Π with allowance for the Z factors of the "brackets" in which they are taken. Therefore, in the presence of Z factors multiplying the fields φ_a and p_a, relation (19) applied to the lowest order contributions (24) and (25) becomes

$$\begin{aligned}&Z_{p_a}^{-1}\Pi_{ab}Z_{p_b}^{-1}=-(g/\omega)Z_{\varphi_c}^{-1}\\ &\times[Z_{p_a}^{-1}\Sigma_{ac}(\omega)W_{cb}-Z_{p_b}^{-1}W_{ca}\Sigma_{bc}(-\omega)].\end{aligned} \tag{32}$$

In our case, the explicit expression (31) imply the possibility of collecting the Z factors of the fields into the renormalization of g. This action is equivalent to allowing for the logarithmic corrections to expressions (24) and (25); these corrections are of the next higher order in g and come from more-complicated diagrams. The appearance of a renormalized value g_R on account of the presence of nontrivial Z factors is not accidental but is due to the geometric nature of the model.

According to (25), the contribution to Π_{ab} of second order in g is, after integration over frequencies,

$$\Pi_{ab}=\frac{\pi g^3}{2}\int\frac{d^2q_1\,d^2q_2}{(2\pi)^4}M_{ab}[\delta(\omega+\xi)+(-1)^{a+b}\delta(\omega-\xi)], \tag{33}$$

where $\xi=\varepsilon_3-\varepsilon_1-\varepsilon_2$, $\varepsilon_i=\varepsilon(q_i)$, and the functions M_{ab} are of the form

$$\begin{aligned}M_{\varphi\varphi}=&\frac{(\mathbf{q}_1\mathbf{q}_2)^2}{q_1^2q_2^2p_3^2}+\frac{(\mathbf{q}_1\mathbf{q}_3)^2}{q_1^2q_3^2p_2^2}+\frac{(\mathbf{q}_2\mathbf{q}_3)^2}{q_2^2q_3^2p_1^2}+\frac{S^2}{p_1^2p_2^2p_3^2}\\ &+2\frac{(\mathbf{q}_1\mathbf{q}_3)(\mathbf{q}_2\mathbf{q}_3)}{\varepsilon_1\varepsilon_2q_3^2}+2\frac{(\mathbf{q}_2\mathbf{q}_3)(\mathbf{q}_1\mathbf{q}_2)}{\varepsilon_1\varepsilon_3q_2^2}+2\frac{(\mathbf{q}_1\mathbf{q}_3)(\mathbf{q}_1\mathbf{q}_2)}{\varepsilon_2\varepsilon_3q_1^2}\\ &+2S\left[\frac{(\mathbf{q}_1\mathbf{q}_3)}{p_2^2\varepsilon_1\varepsilon_3}+\frac{(\mathbf{q}_2\mathbf{q}_3)}{p_1^2\varepsilon_2\varepsilon_3}+\frac{(\mathbf{q}_1\mathbf{q}_2)}{p_3^2\varepsilon_1\varepsilon_2}\right],\end{aligned} \tag{34}$$

$$\begin{aligned}M_{\psi\psi}=&\frac{(\mathbf{q}_1\mathbf{k})^2}{p_3^2p_2^2q_1^2}+\frac{(\mathbf{q}_2\mathbf{k})^2}{p_1^2p_3^2q_2^2}+\frac{(\mathbf{q}_3\mathbf{k})^2}{p_1^2p_2^2q_3^2}\\ &+2\frac{(\mathbf{q}_2\mathbf{k})(\mathbf{q}_3\mathbf{k})}{\varepsilon_2\varepsilon_3p_1^2}+2\frac{(\mathbf{q}_1\mathbf{k})(\mathbf{q}_2\mathbf{k})}{\varepsilon_1\varepsilon_2p_3^2}+2\frac{(\mathbf{q}_1\mathbf{k})(\mathbf{q}_3\mathbf{k})}{p_2^2\varepsilon_1\varepsilon_3},\end{aligned} \tag{35}$$

$$\begin{aligned}M_{\varphi\psi}=-M_{\psi\varphi}=i\Bigg\{&\frac{(\mathbf{q}_1\mathbf{k})(\mathbf{q}_1\mathbf{q}_3)}{q_1^2p_2^2\varepsilon_3}+\frac{(\mathbf{q}_2\mathbf{k})(\mathbf{q}_1\mathbf{q}_3)}{\varepsilon_1\varepsilon_2\varepsilon_3}\\ &+\frac{(\mathbf{q}_3\mathbf{k})(\mathbf{q}_1\mathbf{q}_3)}{q_3^2p_2^2\varepsilon_1}\\ &+\frac{(\mathbf{q}_1\mathbf{q}_2)(\mathbf{q}_1\mathbf{k})}{q_1^2p_3^2\varepsilon_2}+\frac{(\mathbf{q}_1\mathbf{q}_2)(\mathbf{q}_3\mathbf{k})}{\varepsilon_1\varepsilon_2\varepsilon_3}+\frac{(\mathbf{q}_1\mathbf{q}_2)(\mathbf{q}_2\mathbf{k})}{q_2^2p_3^2\varepsilon_1}\\ &+\frac{(\mathbf{q}_2\mathbf{q}_3)(\mathbf{q}_1\mathbf{k})}{\varepsilon_1\varepsilon_2\varepsilon_3}\end{aligned}$$

$$+\frac{(\mathbf{q}_2\mathbf{q}_3)(\mathbf{q}_3\mathbf{k})}{\varepsilon_2 q_3^2 p_1^2}+\frac{(\mathbf{q}_2\mathbf{q}_3)(\mathbf{q}_2\mathbf{k})}{\varepsilon_3 q_2^2 p_1^2}$$

$$+S\left[\frac{(\mathbf{q}_3\mathbf{k})}{p_1^2 p_2^2 \varepsilon_3}+\frac{(\mathbf{q}_1\mathbf{k})}{\varepsilon_1 p_2^2 p_3^2}+\frac{(\mathbf{q}_2\mathbf{k})}{\varepsilon_2 p_1^2 p_3^2}\right]\Bigg\}. \tag{36}$$

Here $S=\mathbf{q}_1\mathbf{q}_2-\mathbf{q}_1\mathbf{q}_3-\mathbf{q}_2\mathbf{q}_3-\mathbf{k}^2$ and $p_i^2=q_i^2+m^2$. We see from (31)–(34) that the differential cross sections for magnon scattering processes fall off rapidly with increasing momenta for $q_i>m$. It is for this reason that the classical treatment is self-consistent.

We note that Eqs. (34)–(36) can also be obtained by the procedure of Halperin andHohenberg[10] in analogy with the calculation done in Ref. 8 or by direct analytical continuation for the temperature technique in a theory with a Lagrangian

$$L=\psi\partial\varphi/\partial t-H.$$

Here we shall use the dynamic Lagrangian method to establish unambiguously the relation between the G and D functions—a nontrivial problem for an arbitrary parametrization of $\mathbf{n}$.

In expressions (34)–(36) we have kept only the terms corresponding to the scattering of a magnon of momentum k by a thermal magnon and have neglected the contribution of the decay of one magnon into three magnons in view of the small statistical weight of such processes. The reason for this is that the governing contribution in integrals (33) is from the region with momenta of integration $q_i\sim m$. In decay processes the energies of the outgoing magnons do not exceed the energy $\varepsilon(k)$ of the decaying magnon, so that these contributions are smaller by $\sim(k/m)^2$ than the scattering contribution. For this reason we can neglect ω in the arguments of the δ functions for $k\ll m$ (from now on we are assuming that ω and k are not too far from the unperturbed mass shell, so that $[\omega-\varepsilon(k)]/\varepsilon(k)\sim 1$). Therefore, we shall not take into account the contribution $\Pi_{\varphi\psi}=-\Pi_{\psi\varphi}$, which contains an extra small factor $\sim\omega/Am^2\sim k/m$ as compared with $\Pi_{\varphi\varphi}$ and $\Pi_{\psi\psi}$. One can see that allowance for $\Pi_{\varphi\psi}$ will lead to corrections of the form k^4 to the imaginary part of the spectrum for $k\ll m$. In addition, the bare spectrum is sufficiently nonlinear that we can neglect k together with ω in the resonance condition $\omega\pm\xi=0$. This is most easily seen by establishing the finiteness of the contribution of small momenta to the expression

$$\int d^2q_1\, d^2q_2\, \delta(\omega\pm\xi),$$

in which one can pass to the limit $\omega,k\to 0$ without having an IR divergence (this expression is understood to be cut off at the upper limit, e.g., at $q_i\sim m$). For $q_i>m$ integral (31) falls off as the denominator $1/p_i^2$ comes into play. The integral for $\Pi_{\varphi\varphi}$ has an ultraviolet divergence of degree zero. We shall show below that this integral is finite.

The reader is reminded once again that all these remarks refer to the limit in which the bare B goes to zero. For finite $B\ll A$ the analogous integrals, as was shown in Ref. 7, give a logarithmic contribution

$$\delta B\sim g^2 B\ln(A/B)\ln(\Lambda/m),$$

which comes from the region $q_i\gg m$.

In addition to the simplifications mentioned, in our case we can directly average the integrand in $\Pi_{\psi\psi}$ over the directions of q_1 and q_2, and to leading order we can neglect the k dependence in $\Pi_{\varphi\varphi}$. As a result we obtain

$$\Pi_{\varphi\varphi}=2gB_\varphi,\qquad \Pi_{\psi\psi}=2g\frac{k^2}{m^2}B_\psi, \tag{37}$$

where the coefficients B_φ and B_ψ are given by the integrals

$$B_a=\frac{g^2}{2}\frac{A}{(2\pi)^2}\int\frac{dq_1\,dq_2\,\varepsilon_3}{(4\varepsilon_3^2+m^4)^{1/2}}\frac{M_{aa}(\theta_{12}^0)}{|\sin\theta_{12}^0|}, \tag{38}$$

in which the value of the angle θ_{12}^0, determined from the resonance condition $\xi=0$, is

$$\theta_{12}^0=\arccos\{(2q_1q_2)^{-1}(-q_1^2-q_2^2-m^2/2+(4\varepsilon_3^2+m^4)^{1/2})\}. \tag{39}$$

The expression for $M_{\varphi\varphi}$ is given by (34), while $M_{\psi\psi}$ is obtained by averaging (35) over the angle between k and q_1+q_2:

$$M_{\psi\psi}=\frac{m^2}{2}\Bigg\{\frac{1}{p_1^2p_2^2}+\frac{1}{p_1^2p_3^2}+\frac{1}{p_2^2p_3^2}+2\frac{(\mathbf{q}_2\mathbf{q}_3)}{\varepsilon_2\varepsilon_3p_1^2}$$
$$+2\frac{(\mathbf{q}_1\mathbf{q}_3)}{\varepsilon_1\varepsilon_3p_2^2}+2\frac{(\mathbf{q}_1\mathbf{q}_2)}{\varepsilon_1\varepsilon_2p_3^2}\Bigg\}. \tag{40}$$

An approximate evaluation of integrals (38) gives

$$B_\varphi=0.04g^2A,\qquad B_\psi=0.05g^2A. \tag{41}$$

According to what was said above, the leading contributions arising in higher orders can be taken into account by replacing g in (37) and (41) by g_R and m by m_R and using the solution of Eq. (7) for A. This remark also pertains to the damping in the isotropic limit, as presented in Sec. 4 [see Eq. (56)]. In the formulas which follow below, the parameters g, m, and A are understood to mean the renormalized values.

Using relation (19) between Σ_{ab} and Π_{ab}, we obtain the contributions to the components Σ_{ab} that give rise to the imaginary part of the spectrum (these are Re $\Sigma_{\varphi\psi}$, Re $\Sigma_{\psi\varphi}$, and also Im $\Sigma_{\varphi\varphi}=$ Im $\Sigma_{\psi\psi}$, which contain an extra small factor $\sim k/m$). To leading order we have

$$\Sigma_{ab}(\omega,\mathbf{k})=\begin{pmatrix}0 & B_\varphi\\ -k^2B_\psi/m^2 & 0\end{pmatrix}\omega. \tag{42}$$

The contribution to the imaginary part of the spectrum from Re $\Sigma_{\varphi\psi}$ and Re $\Sigma_{\psi\varphi}$ are of the same order. To see this, let us solve equation (17), neglecting the terms quadratic in $\Sigma_{ab}\sim g^2$:

$$\omega(k)\approx\pm\varepsilon(k)-ik^2(B_\varphi+B_\psi)/2. \tag{43}$$

Recall that spectrum (43) obtains under the condition $k\ll m$. The dissipative term (14) with Σ_{ab} from (42) can also be represented in a different form obtained from the equation of motion with dissipative corrections after it is solved for $\frac{\partial}{\partial t}\Phi_a$. For the general system (17) the quadratic form in the dynamic Lagrangian with allowance for (21) becomes

$$\mathscr{L}^{(2)}=\pi_a\Big[\frac{\partial}{\partial t}\Phi_a(t)+\frac{1}{g}\int dt'\frac{\partial}{\partial t}\Pi_{ac}(t-t')W_{cb}^{-1}\theta(t-t')$$
$$\times\Phi_b(t')-W_{ac}H_{,cb}\Phi_b(t)\Big]+\frac{i}{2}\pi_a\Pi_{ab}\pi_b. \quad (44)$$

In the case of a local (in the time arguments) operator Π_{ab} transformation of the variables π_{ab} according to

$$\pi_a=\tilde{\pi}_bE_{ba},$$
$$E_{ab}^{-1}=\delta_{ab}+(1/2g)\Pi_{ac}W_{cb}^{-1}, \quad (45)$$

will reduce quadratic form (44) to

$$\mathscr{L}^{(2)}=\tilde{\pi}_a(\partial\Phi_a/\partial t-E_{ac}W_{cd}H_{,db}\Phi_b)$$
$$+^1/_2i\tilde{\pi}_aE_{ac}\Pi_{cd}E_{bd}\tilde{\pi}_b. \quad (46)$$

Let us assume that the product $(\Pi W^{-1})^2$ is a multiple of the unit matrix. Then Eq. (46) can be written

$$\mathscr{L}^{(2)}=\tilde{\pi}_a\Big(\frac{\partial}{\partial t}\Phi_a-CW_{ak}H_{,kb}\Phi_b+\frac{C}{2g}\Pi_{ak}H_{,kb}\Phi_b\Big)+\frac{C}{2}\tilde{\pi}_a\Pi_{ab}\tilde{\pi}_b, \quad (47)$$

where C is the coefficient in the relation

$$\hat{1}=C\Big[\hat{1}-\frac{1}{4g^2}(\Pi W^{-1})^2\Big]. \quad (48)$$

Matrix equation (48) is satisfied, for example, for an arbitrary Hamiltonian system in the case when the operator Π_{ab} is isotropic ($\Pi_{ab}=\Pi\delta_{ab}$). Quadratic form (47) corresponds to the dissipative corrections arising in a system described by the equation[6]

$$\partial\Phi_a/\partial t+F_a-\gamma_{ab}S_{,b}+f_a=0, \quad (49)$$

in which

$$F_a=-CW_{ab}H_{,b},\quad S_{,a}=-CH_{,a}/g,$$
$$\gamma_{ab}=C\Pi_{ab}/2 \quad (50)$$

and the random forces f_a are correlated as $\langle f_af_b\rangle=2\gamma_{ab}$.

For system (49) a fluctuation-dissipation theorem of the form[6,9,11]

$$2\gamma_{ab}(t)S_{,bc}\theta(t)=-i\Sigma_{ac}(t) \quad (51)$$

is satisfied for the dissipative terms in (47) with allowance for (50). Conservative system (18) with the first fluctuation corrections in the case in which (48) holds is thus renormalized subsequently like a system with random forces that is described by Eq. (49) in which the form of the dissipative terms is "induced" by the fluctuation contribution.

It has been shown for system (18), as was done in Ref. 6 for Eq. (49) and fluctuation-dissipation theorem (51), that the fluctuation-dissipation theorem in form (19) leads to the correct static limit for the distribution function, $P_{st}\sim\exp(-H/g)$.

In the case when sufficient condition (48) holds, this essentially follows from the aforementioned equivalence to system (49). A more detailed comparison of the various forms of the fluctuation-dissipation theorem is planned for a separate paper.

In the present case condition (48) holds, and for the operator E_{ab} we have

$$E_{ab}=C\begin{pmatrix}1 & \Pi_{\varphi\psi}/2Ag\\ -\Pi_{\psi\psi}/2Ag & 1\end{pmatrix}$$
$$C=(1+\Pi_{\varphi\varphi}\Pi_{\psi\psi}/4g^2A^2)^{-1},$$

and the local operators $\Pi_{\varphi\varphi}$ and $\Pi_{\psi\psi}$ in the region of interest $k\ll m$ are given in (37). Transformation (45) leads to

$$\Sigma_{ab}=-ik^2\begin{pmatrix}B_\varphi & 0\\ 0 & B_\psi\end{pmatrix}.$$

This expression holds to the same accuracy as in (42).

IV. ISOTROPIC LIMIT

For $k\gg m$ all the channels in (33), including the decay channel, and all the components Π_{ab} are of the same order of magnitude. The reason is that in this case the governing region is for values $q_i\sim k$. In this limit Π_{ab} is expressed as

$$\Pi_{ab}=\frac{\pi}{2}g^3\int\frac{d^2q_1\,d^2q_2}{(2\pi)^4\varepsilon_1\varepsilon_2\varepsilon_3}\Big\{[\delta(\omega+\xi)+(-1)^{a+b}\delta(\omega-\xi)]M_{ab}$$
$$+\frac{1}{3}\Big[\delta\Big(\omega+\sum_{i=1}^{3}\varepsilon_i\Big)+(-1)^{a+b}\delta\Big(\omega-\sum_{i=1}^{3}\varepsilon_i\Big)\Big]N_{ab}\Big\}, \quad (52)$$

where

$$M_{\varphi\varphi}=(2\mathbf{q}_1\mathbf{q}_2-\mathbf{k}^2)^2,\quad M_{\psi\psi}=(2\mathbf{q}_3\mathbf{k}+\mathbf{k}^2)^2,$$
$$M_{\varphi\psi}=-M_{\psi\varphi}=i(2\mathbf{q}_3\mathbf{k}+\mathbf{k}^2)(2\mathbf{q}_1\mathbf{q}_2-\mathbf{k}^2),$$
$$N_{\varphi\varphi}=\Big(\sum_{i=1}^{3}\varepsilon_i^2\Big)^2,\quad N_{\psi\psi}=k^4, \quad (53)$$
$$N_{\varphi\psi}=-N_{\psi\varphi}=ik^2\Big(\sum_{i=1}^{3}\varepsilon_i\Big).$$

In calculating the corrections to the spectrum in order g^2, the function $\Sigma_{ab}(\omega,\mathbf{k})$ should be taken on the corresponding part $\omega=\pm Ak^2$ of the unpertubed mass shell. We note that this statement remains valid when the three-loop contributions to Σ_{ab}, which are of order g^3, are taken into account.

Neglecting the terms quadratic in Σ_{ab}, we obtain the contribution to the imaginary part of the spectrum:

$$\delta\omega(k)=-(ik^2/4g)(\Pi_{\varphi\varphi}+\Pi_{\psi\psi}-2i\Pi_{\varphi\psi}\operatorname{sign}\omega). \quad (54)$$

On the mass shell Eq. (54) gives a correction $\delta\omega(k)=-iBk^2$ to the spectrum, with

$$B=\frac{\pi}{2}g^2A\int\frac{d^2q_1\,d^2q_2}{(2\pi)^4\varepsilon_1\varepsilon_2\varepsilon_3}(\mathbf{q}_1\mathbf{q}_2+\mathbf{q}_3\mathbf{k})^2\delta(Ak^2+\xi). \quad (55)$$

It should be noted that the contribution of the decay channels vanishes on the mass shell. This is a consequence of conservation of the total spin in an isotropic magnet.

The integral in (55) can be calculated explicitly; it gives

$$B=(3/16\pi)g^2A. \quad (56)$$

Let us make an additional remark here concerning the use of other parametrizations. The invariant result, independent of the choice of coordinates on S^2, is the excitation spectrum; this means that, to leading order in g is agreement among the

values of the functions $\Sigma_{\pm}(\omega,\mathbf{k})$ calculated using different parametrizations on the unperturbed mass shell $\omega = \pm Ak^2$, while the functions $\Sigma_{\pm}(\omega,\mathbf{k})$ themselves can be different in different representations. Let us consider, for example, the parametrization in terms of the transverse components n_2 and n_3 of the vector $\mathbf{n}$ and the components p_2 and p_3 of the auxiliary vector field $\mathbf{p}$. In the linear approximation the fields φ and ψ coincide with n_2 and n_3, and p_φ and p_ψ with p_2 and p_3. In an isotropic model there is explicit symmetry between n_2 and n_3, so that we have exactly

$$\Sigma_{p_2n_2}=-\Sigma_{p_3n_3}.$$

At the same time, however, Eqs. (52) and (53) imply that $\Sigma_{\varphi\varphi}$ is not the same as $-\Sigma_{\psi\varphi}$, on account of the contribution of the next higher terms of the expansion of n_2 and n_3 in powers of φ and ψ. In terms of the elementary fields φ and ψ, the fields n_2 and n_3 are compound operators, so that in calculating $\langle n_a p_b \rangle$ to order g^2 one must take into account the contributions illustrated in Fig. 2b in addition to those in Fig. 2a.

Despite the lack of symmetry between the variables φ and ψ, our choice is the most convenient in the anisotropic case, since diagram technique (27)–(30) does not involve vertices associated with the anisotropy or formal divergences of the individual diagrams at small momenta. On the other hand, when using complex coordinates,[9] which are more convenient in the isotropic situation, one must keep careful track of the cancellations of the factors of the type m^2/q_i^2 which arise in the integrands, being mindful of all the anisotropic vertices and anomalous pairings.

V. EQUATION OF MOTION OF VORTEX EXCITATIONS

Let us now consider an effective equation of motion with allowance for dissipative term (13) with Σ_{ab} from (42):

$$\mathcal{L}_{d1}=ip_\psi\Sigma_{\psi\varphi}\varphi+ip_\varphi\Sigma_{\varphi\psi}\psi=-B_\varphi p_\varphi\partial\psi/\partial t-m^{-2}B_\psi p_\psi\nabla^2\partial\varphi/\partial t. \quad (57)$$

To terms linear in the deviation of $\mathbf{n}$ from equilibrium, this expression can be rewritten in a vector representation as

$$\mathcal{L}_{d1}=B_\varphi\mathbf{p}\mathcal{P}[\mathbf{n},\partial\mathbf{n}/\partial t]-B_\psi m^{-2}(\mathbf{pv})(\mathbf{v}[\mathbf{n},\nabla^2\partial\mathbf{n}/\partial t]), \quad (58)$$

where $\mathcal{P}_{ab}=\delta_{ab}-\nu_a\nu_b$ is the operator for projection onto an orthogonal plane. The equation of motion for $\mathbf{n}$ at large scales $R\gg 1/m$ becomes

$$\partial\mathbf{n}/\partial t=A[\mathbf{n},\nabla^2\mathbf{n}]-Am^2[\mathbf{n},\mathbf{v}](\mathbf{nv})-B_\varphi\mathcal{P}[\mathbf{n},\partial\mathbf{n}/\partial t]+B_\psi m^{-2}\mathbf{v}(\mathbf{v}[\mathbf{n},\nabla^2\partial\mathbf{n}/\partial t]). \quad (59)$$

In solving this equation for B_φ and B_ψ by pertubation theory, one should take into account in the dissipative terms only the part that is linear in the deviations of $\mathbf{n}$ from equilibrium.

In accordance with the discussion of Sec. 3, Eq. (59) can be written in the equivalent form

$$\partial\mathbf{n}/\partial t=A[\mathbf{n},\nabla^2\mathbf{n}]-Am^2[\mathbf{n},\mathbf{v}](\mathbf{nv})+B_\varphi\mathcal{P}\nabla^2\mathbf{n}+B_\psi\mathbf{v}(\mathbf{v}\nabla^2\mathbf{n}). \quad (60)$$

We stress that we are considering scales $R\gg 1/m$. At scales $R\ll 1/m$, as follows from the results of Sec. 4, the dissipative term is the same as the phenomenological Landau-Lifshitz term, the coefficient of which is given by (56).

Vector-multiplying Eq. (59) by $\mathbf{n}$ and then scalar-multiplying by $\nabla_\mu\mathbf{n}$ and afterward integrating over the area occupied by vortex excitations, we obtain equations in the collective degrees of freedom, which are the coordinates $\mathbf{R}_i$ of the vortices (the subscript i numbers individual vortices):

$$\frac{\partial}{\partial t}R_i^\lambda\varepsilon_{\mu\lambda}q_i\sigma_i+\sum_{k\neq i}q_iq_k\frac{R_{ik}^\mu}{R_{ik}^2}+B_\varphi\frac{\partial R_i^\mu}{\partial t}I_1-B_\psi\frac{\partial R_i^\mu}{\partial t}I_2=0, \quad (61)$$

where the coefficients $I_{1,2}$ are given by the integrals

$$I_1=(1/4\pi)\int d^2r\{(\nabla\mathbf{n})^2-(\mathbf{v}[\mathbf{n},\nabla\mathbf{n}])^2\}, \quad (62)$$

$$I_2=(1/4\pi m^2)\int d^2r(\mathbf{v}[\mathbf{n},\nabla_\mu\mathbf{n}])(\mathbf{v}[\mathbf{n},\nabla^2\nabla_\mu\mathbf{n}]). \quad (63)$$

Here q and σ are the topological charges of the vortices, which can be classified relative to $\pi_2(S^2,S^1)$ (Ref. 4). The second term in (61) is the force acting on the ith vortex on the part of all the others. In contrast to the phenomenological dissipative term in form (1), which can also be written $AB[\mathbf{n},\partial\mathbf{n}/\partial t]/(A^2+B^2)$, the dissipative terms in (59) do not lead to logarithmic divergences of the integrals $I_{1,2}$. Let us consider, for example, the potential of a single vortex, which can be written[4]

$$\varphi=q\chi,\qquad \psi=\begin{cases}c_1(mr)^{-1/2}e^{-mr}, & mr\gg 1,\\ 1-c_2(mr)^{2|q|}, & mr\ll 1,\end{cases} \quad (64)$$

where the coefficients $c_{1,2}$ are of the order of 1. Here r is the radius in the two-dimensional plane, and χ is the aximuthal angle. The long-range Coulomb force due to the field φ cancels out in the integral I_1 and is absent from I_2 by virtue of the equation $\nabla^2\varphi=0$ in a vortex-free space. Since expressions (62) and (63) incorporate the contribution to the dissipative terms only from scales large compared to $1/m$, they cannot be used for exact determination of the dissipative coefficients in (61). By virtue of the cancellations mentioned above, the contributions of scales $R<1/m$ and $R>1/m$ are quantities of the same order. For an estimate of the last term in (61) we use the approximate one-vortex solution (64) for $R<1/m$ and the value of B from (56), assuming roughly that the dynamics at distances smaller than the size of the vortex is isotropic. As a result we find that the coefficient of $\partial\mathbf{R}_i/\partial t$ in the third term in (61) is of the order of

$$\frac{B}{A}c_2^2|q_i|\approx\frac{3g^2}{16\pi}|q_i|c_2^2.$$

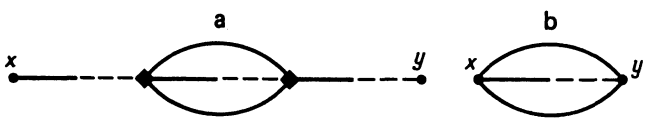

FIG. 2.

If follows from (41) that the use of an isotropic dissipative coefficient $B \approx 0.06 g^2 A$ can only give an overestimate. The coefficient c_2 in asymptotic expression (64) is also close to unity, so that even near the transition temperature kT, where $g \sim 1/\ln(\Lambda/m) \sim \frac{1}{5}$ (Ref. 2), the above quantity is of order 10^{-3} for $|q_i| \sim 1$ (the formation of vortices having a large circulation is suppressed by the Boltzmann factor).

In conclusion we note that the result $\Pi_{\psi\psi} \sim k^2/m^2$ and the consequent absence of a diffusion pole in the spectrum and the cancellation of the divergence ln R in the equation of motion of the vortices agree with the usual view[10] that the kinetic coefficient for a conserved quantity vanishes for spatially uniform distributions. In an anisotropic magnet the component of the moment along the anisotropy axis ψ is a conserved quantity, and the corresponding kinetic coefficient is $\Pi_{\psi\psi}$.

For Hamiltonian (2) we can see this directly, since the field φ appears in H only through gradients, giving rise to an external momentum k at each vertex having an external line p_ψ. An important element here is the lack of IR divergence in the present diagram technique.

The conclusions reached could be affected by the presence of various neglected factors, such as in-the-plane anisotropy leading to corrections to H of the form cos $(l\varphi)$, the presence of a dipole interaction, and scattering of magnons by vortices. In this case our treatment is valid only for $k_0 \ll k$, where k_0 specifies the momentum boundary beyond which the neglected terms in H become the leading terms.

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