Interpretation of the wave function for a strongly bound ($p_0 < 0$) state

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(Submitted 6 May 1986)
Zh. Eksp. Teor. Fiz. **91**, 1565–1572 (November 1986)

The idea that the solution of the so-called one-particle wave equation actually describes not a single-particle system but a many-particle system with given charge equal to zero or one is extended to bound states. It is suggested that, crudely speaking, the electron can be found only in the region V_+ in which its kinetic energy satisfies $\pi_0(x) = p_0 - eA_0(x) > 0$, and the positron in the region V_- in which $\pi_0(x) < 0$. In the case of the strong field of a deep potential well and finite motion of the particle (for example, in a narrow well), the integral $c_2^2 = \int dV |\psi|^2$ over V_- may be comparable with the integral $c_1^2 = \int dV |\psi|^2$ over V_+ ($c_1^2 + c_2^2 = 1$). The meaning of $|\psi|^2$ must then be reinterpreted. In particular, c_2^2 must be looked upon as the probability of finding the pair in an uncharged (in the usual language, unoccupied) level. Pauli's principle plays an essential part in this interpretation. For bosons, Pauli's principle does not apply, and the interpretation must be accordingly modified.

1. INTRODUCTION

The quantum electrodynamics of phenomena occurring in strong fields is undergoing rapid development and pair-creating fields are attracting particular attention (see, for example, the reviews given in Refs. 1–4). When the pair-creating field is turned off both in the past and future, i.e., for $t \to \pm \infty$, there can no longer be any doubt about the ability of the theory to describe all the phenomena. This cannot be said about fields that are not turned off for $t \to \pm \infty$. It is quite possible, however, that, in strong time-independent fields localized in a finite region of space, outside which all the particle and antiparticle states are well defined, the theory is, in principle, capable of answering all questions raised by the idealized experiment in which instruments record particles in a large region outside the strong-field region. On the other hand, when we consider measurements exploring the behavior of particles in the strong-field region, it is not clear how we could distinguish between a particle and an antiparticle. In particular, there is a difficulty with specifying the state of a vacuum electron in a level immersed in the lower continuum of the supercritical atom.[4]

In this paper, we shall discuss the interpretation of a strongly-bound state in a deep and narrow well with steep walls, in which virtual pair production by the field has a significant influence on the wave function $\psi(x)$. This question is closely related to the above problem. In this case, for a spin 1/2 particle, the integral $c_2^2 = \int dV |\psi|^2$ is not negligible when evaluated over a region V_- in which the kinetic energy of the particle is $\pi_0(x) = p_0 - eA_0(x) < 0$. We shall consider that, in this region, the wave function describes the positron that appears as a result of the virtual pair production by the field. The electron, on the other hand, can be found only in the region V_+ in which $\pi_0(x) > 0$. For a normalized function, the two integrals satisfy the condition $c_1^2 + c_2^2 = 1$. If the state is uncharged i.e., has zero charge, c_2^2 gives the probability of finding the pair in the particular state. For a charged state, i.e., a state with charge e, we have to suppose that the electron has unit probability of being in the region in which $\pi_0(x) > 0$. Virtual pair production by the field in the well is then prevented by Pauli's principle. The region in which $\pi_0(x) < 0$ remains unoccupied.

This does not, however, mean that an external photon of suitable energy will not be able to create a pair in which the electron enters some other state and the positron enters the same level as the bound electron. The system occupying this state will have zero charge, i.e., the state will be uncharged. In the more familiar language, it is said that virtual production of the pair by the photon contributes to the process of excitation of the electron.

For a spin 0 particle, this interpretation must be modified because the Pauli principle does not then apply.

The interpretation makes it clear that the same potential well can bind both the particle and antiparticle in the boson case, but it can bind only the particle in the fermion case.[5–8] In the latter case, the positively charged state (the charge of the system is then equal to 1) signifies the presence of the positron outside the well, and the absence of the electron from the well. Pauli's principle then ensures that the electron inside the well cannot appear because there is no place outside the well for the additional positron that compensates the charge of the electron. The well does not, therefore, reduce the energy of the system of charge 1. On the other hand, the energy of a boson system of charge 1 can be reduced by virtue of the presence at the bottom of the well of particles of charge -1 (Ref. 7).

2. SPIN 1/2 PARTICLE

We first note that the Feynman propagator, constructed from the so-called single-particle solutions, takes into account pairs in the intermediate state in the relativistic case. We also note that the theory of pair production by an external field that is turned off for $t \to \pm \infty$ (Refs. 1–3), the explanation of the Klein paradox,[1,9] and all the experience gained by working with strong fields (see, especially, Section 3 of the first paper by Ritus[3]) teach us that the so-called single-particle solutions of wave equations describe not so

 0038-5646/86/110922-05$04.00

much particle states as processes (in a sense, mutliparticle processes) occurring in states with charge 0 or $\pm e$. Let us illustrate this by some examples.

Consider the complete orthonormal sets of *in*-solutions $_{\pm}\psi_n$ and out-solutions $^{\pm}\psi_n(x)$ of the Dirac equation in the case of fields that vanish for $t \to \pm\infty$. The symbols $\pm$ indicate the sign of the frequency of the solution and n represents the set of quantum numbers of a state. For conserved quantum numbers, we have a simple relation between the *in*- and *out*-solutions:[1-3,10]

$$_{+}\psi_n(x) = c_{1n}\,{}^{+}\psi_n(x) + c_{2n}\,{}^{-}\psi_n(x), \tag{1}$$

$$_{-}\psi_n(x) = -c_{2n}^{*}\,{}^{+}\psi_n(x) + c_{1n}^{*}\,{}^{-}\psi_n(x), \qquad |c_{1n}|^2 + |c_{2n}|^2 = 1. \tag{2}$$

The last of these describes the conservation of the norm of the wave function (or charge).

For pair-producing fields, $|c_{2n}|^2 \neq 0$. This quantity can be interpreted as the absolute probability that a pair will be produced in a state n if the initial state was free. The quantity $|c_{1n}|^2$ is then the probability that the pair will not be produced, i.e., the probability that the vacuum will remain in the n-th state. According to Feynman's theory,[11,1] the same solution $_{+}\psi_n(x)$ for a charged initial state gives, with probability 1, the final state $^{+}\psi_n$ (and not $c_{1n}\,{}^{+}\psi_n$) if we take into account the difference between the relative and absolute amplitudes. Pair-production of any kind does not occur in this case, in accordance with the Pauli principle.

Thus, if we deal only with the solutions given by (1), we can use simple rules to extract from them information about processes occurring in the particular state. The consistency of these rules with the second quantization theory is discussed in Refs. 1–3.

Bearing in mind the extrapolation of these rules to the case of virtual pair-production by the strong field in the well, let us consider a closely-related example, namely, that of the creation of a pair by a time-independent potential wall (the Klein paradox[1,9]). For a step of the form

$$eA_0 = ea_0[1 + \tanh k_3x_3],\ a_0,\ k_3 > 0 \tag{3}$$

we can readily find the complete set of solutions of the Dirac equation (see Ref. 1). The electric field corresponding to the potential $A_0(x_3)$ is

$$\mathscr{E}(x) = -\partial A_0/\partial x_3 = -a_0k_3/\cosh^2 k_3x_3,$$

which will retard an electron ($e < 0$) moving from right to left. In reality, the specific shape of the step is unimportant. The only significant quantity is its height $2|e|a_0$, which we shall assume is greater than $2m$, and the effective electric field a_0k_3, which we assume to be strong ($a_0k_3 \gtrsim m^2/|e|$) in order to ensure that pair-production effects are considerable.

The solutions are labeled by quantum numbers $n = (p_1, p_2, p_0, \mu)$, where μ is the spin component. For simplicity, we set $p_1 = p_2 = 0$, so that the kinetic momentum and kinetic energy are given by

$$\pi_3(x_3) = \pm[\pi_0^2(x_3) - m^2]^{1/2}, \quad \pi_0(x) = p_0 - eA_0(x_3). \tag{4}$$

We note that the quantum number p_0 can be interpreted as the conserved total, i.e., kinetic plus potential, energy.

The electric field vanishes for $k_3|x_3| \gg 1$. In these regions, we have plane waves with momentum $\pi_3(x_3) \to \pi_3(\pm)$ as $x_3 \to \pm\infty$. Since $A_0(x_3) \to 0$ for $x_3 \to -\infty$, we have

$$\pi_3(-\infty) \equiv \pi_3(-) = (p_0^2 - m^2)^{1/2} = i(m^2 - p_0^2)^{1/2}. \tag{5}$$

When $|p_0| < m$, the electron wave incident on the step from the right is completely reflected. Let us examine barrier penetration by the wave from right to left, assuming that the height of the step is greater than $2m$. A step of this kind can create real pairs but, for the moment, we shall confine our attention to states with p_0 such that real pair-production is not possible in these states ($|p_0| < m$). For sufficiently large negative x_3, the x_3 dependence of the wave function is $\exp[-|x_3|(m^2 - p_0^2)^{1/2}]$. As p_0 falls from m to 0, there is less penetration by the electron to the left of the step. We note that, on the right of the step, the electron has positive kinetic energy $\pi_0(+) = p_0 + 2|e|a_0$. A reduction in this energy reduces penetration, as in the case of a nonrelativistic particle. Actually, if we write $p_0 = \pi_0(+) - U = m + E - U$, $U = 2|e|a_0$ and suppose that E, $U \ll m$, we obtain the same result as in the nonrelativistic theory, i.e., $m^2 - p_0^2 \simeq 2m(U - E)$, which may be compared with equation (25.1) in Ref. 12. Hence, it is clear that a reduction in E gives rise to an increase in the attenuation by the barrier $\exp[-|x_3|(m^2 - p_0^2)^{1/2}]$.

Further reduction in p_0 from 0 to $-m$ is accompanied by an increase in the penetration of the step. This property of the relativistic solution is a manifestation of virtual pair-production by the strong field. (For a weak field, $a_0k_3 \ll m^2/|e|$, the coefficient of the tunneling part of the solution becomes exponentially small.)

A strong field will not only give rise to efficient pair-production, but will also reduce the size of the region in which particles and antiparticles are not well separated, i.e., the region in which $|\pi_0(x)| \ll m$. To the left of the step, $\pi_0(-) = p_0$. Negative p_0 signifies negative $\pi_0(-)$. Such values of $\pi_0(-)$ are closer to the lower continuum of states than to the upper continuum. In this region, the wave function describes the virtual positron of the pair of the level is uncharged. To the right, we can then only have an electron—the partner in the pair. However, if the state is charged, the wave function to the right of the step describes the state of the electron occupying the level. Pair-production does not then occur, and the electron cannot penetrate the left region because of the effect of the field associated with the wall.

It is natural to ask why, with the chosen step for which the potential vanishes to the left of the step, virtual pair-production by the strong field occurs only for $p_0 < 0$ but not for $p_0 > 0$. The answer to this question is that, in the latter case, the field does not have the energy even to produce the below the barrier pair. The work that must be done by the field is equal to the height of the step, i.e., $2|e|a_0$, and the kinetic energy of the pair must, at any rate, be greater than the kinetic energy of the electron to the right of the step, i.e., greater than $\pi_0(+) = p_0 + 2|e|a_0$. Hence, it is clear that it is only for $p_0<0$ that the field can communicate positive energy to the virtual positron. We recall that, in the region in which $\pi_0(x)<0$, the quantity $-\pi_0(x)$ is the energy of the positron inside the barrier; the positron lies above the barrier if $-\pi_0(x)\geqslant m$. Since $\pi_0(-)=p_0$ to the left of the step, we can look upon $\pi_0(+)=p_0+2|e|a_0$ as the law of conservation of energy during the formation of the pair below the barrier if we rewrite it in the form $\pi_0(+)+[-\pi_0(-)]=2|e|a_0$.

Thus, virtual pair-production effect occurs in the solution even when we consider the solution of the Dirac equation for a step. It does not appear in the idealized analysis in which the motion of the electron from the right is looked upon as infinite. In principle, the effect is observable. Thus, the pair can be detected by checking near the wall whether the level is charged. This does not contradict the usual representation: a relatively small perturbation of the system in which virtual pair-production plays a significant part may convert the virtual pair into a real pair. If we reduce p_0 still further, we enter the Klein region in which $\pi_0(-)\equiv p_0<-m$, $\pi_0(+)=p_0+2|e|a_0 m$. (When p_1, $p_2\neq 0$, the quantity m must be replaced with $m_\perp=[m^2p_1^2+p_2^2]^{1/2}$.) The field then creates real pairs, and the wave previously attenuated in the left part becomes converted into a propagating positron wave.

It can be shown that the evaluation of the matrix elements for pair production and scattering on the potential step will converge efficiently to the well-known case of a field that vanishes for $t\to\pm\infty$ (cf., Ref. 1 and the boson case, below).

We are now very close to the extrapolation of the above interpretation of barrier scattering to the less pure case of the wave function of a strongly-bound state. We consider that the well field confines the electron to region V_+ in which $\pi_0(x)>0$, and the positron to region V_- in which $\pi_0(x)<0$. Roughly speaking, we may consider that the boundary between V_+ and V_- is given by the condition $\pi_0(x)=0$. In a strong field, the region in which $|\pi_0(x)|\ll m$ and particles and antiparticles are poorly distinguishable is relatively small. The integrals of $|\psi|^2$ over V_+ and V_- will be represented by c_1^2 and c_2^2, respectively, and will be normalized by the condition $c_1^2+c_2^2=1$. When the level is uncharged, we consider that it is occupied by a pair with probability c_2^2 and unoccupied with probability c_1^2. We are referring here to a bound virtual pair. Migdal has discussed,[5] from another point of view, bound pairs in a supercritical atom, taking into account the interaction between the electron and the positron.

When the level is charged, the electron is found in the region V_+ with probability 1. This means that it is then described by the previous wave function $\psi(x)$, which is now multiplied by c_1^{-1}. The region V_- remains free because the well field cannot create a pair in this state. The basic idea of a partially occupied level is used even in the quasiclassical treatment of pair production by a constant electric field (see problem 2 in Section 129 of Ref. 13).

In the supercritical atom, the levels in the upper part of the lower continuum contain an electron in the region V_+. The fact that the electron in a state immersed in the lower continuum is dissolved among the levels of this continuum is demonstrated in Ref. 6 (see also Ref. 2 and 4). The state of this vacuum electron has not as yet been adequately investigated. It is basically represented by solutions with p_0 lying in the neighborhood of the real part of the energy of the quasi-discrete level.[6] Positrons undergo resonance scattering by the field of the atom for such values of p_0 (Ref. 6). To a lesser extent, the vacuum electron is represented by solutions with other p_0 in the upper part of the lower continuum, namely, solutions for which there exists a region with $\pi_0(x)>0$.

The question is: can the state of the vacuum electron be described by a wave packet with p_0 from the admissible interval? Evidently, the answer is no because it is difficult to imagine how this packet could be prevented from leaking into the region V_-. Its norm would not then be conserved in V_+. Moreover, according to the usual ideas, the packet would describe in V_+ a type of motion that would be difficult to interpret because we are considering the lowest state of the system, i.e., the so-called charged vacuum. This may be used as a basis for concluding that the phases of the functions with different p_0 in the packet are random and, in effect, do not interfere. It is possible that the state of the vacuum electron is described by the density matrix. This question requires further investigation.

However, let us return to our problem. A time-independent state with $c_2^2>c_1^2$ is possible in a narrow well with steep walls. This is readily understood because a strong field (steep walls) will keep the electron inside the well, while the positron part of the wave function will slowly fall outside the well if p_0 is close enough to $-m$. It is precisely in this well, that both particles and antiparticles can have bound states in the boson case.

To summarize the results of this section, we emphasize once again that the solution of the so-called single-particle Dirac equation describes processes occurring in a system of charge 0 or $+e$ rather than the state of a particle. In the matrix element of a particle process, the wave function describes possible paths for the process. For example, if the level is uncharged, it is empty with probability c_1^2, and the incident electron will occupy it with probability α after emitting a photon. The probability of this path for the process is therefore proportional to αc_1^2. On the other hand, the well field can create a pair with probability c_2^2. The electron in the pair occupies the level, and the positron, having emitted a photon, annihilates the arriving electron with probability α. The probability of this path is proportional to αc_2^2. When the second path provides an appreciable contribution, it seems to us that the interpretation of the solution must incorporate the elements considered above.

3. SPIN 0 PARTICLE

Let us now consider the differences that are specific for a spin 0 particle.[1,2,9,10] Returning to fields that vanish for $t\to\pm\infty$, we recall that the *in*- and *out*-solutions are related by

$$
\begin{aligned}
{}_+\psi_n(x)&=c_{1n}\,{}^+\psi_n(x)+c_{2n}\,{}^-\psi_n(x),\\
{}_-\psi_n(x)&=c_{2n}^{*}\,{}^+\psi_n(x)+c_{1n}^{*}\,{}^-\psi_n(x).
\end{aligned}
\tag{6}
$$

Conservation of norm (charge) now yields

$$|c_{1n}|^2-|c_{2n}|^2=1, \tag{7}$$

i.e., $|c_{1n}|^2$ is always greater by one than $|c_{2n}|^2$.

It is readily shown that $|c_{2n}|^2$ can be interpreted as the mean number of pairs produced by the field in state n when the initial state (prior to the introduction of the field) was free. The probability that the pair will not be produced in this state is $c_{vn}=|c_{1n}|^{-2}$. The total probability of all the events in this state is equal to unity:

$$c_{vn}[1+w_n+w_n^2+\cdots]=c_{vn}(1-w_n)^{-1}=1, \tag{8}$$

$$w_n=|c_{2n}/c_{1n}|^2, \quad c_{vn}=|c_{1n}|^{-2}. \tag{9}$$

The second relatin in (8) is simply another form of (7). The mean number of pairs created in the nth initially unoccupied state is

$$\bar{n}=c_{vn}[w_n+2w_n^2+3w_n^3+\ldots]$$

$$=c_{vn}w_n\frac{d}{dw_n}(1-w_n)^{-1}=c_{vn}w_n(1-w_n)^{-2}=|c_{2n}|^2. \tag{10}$$

The presence of a particle (in general, any number of particles) in the initial state does not prevent the creation of pairs in this state. The absolute probability of the scattering of a particle, subject to the condition that m pairs are created in the same state, is equal to the product of the probability that m pairs will be produced [$|c_{1n}|^{-2}w^m$, according to (8)] and the probability of scattering in the presence of m pairs, which is equal to $|c_{1n}|^{-2}(m+1)$. The total probability of scattering accompanied by the appearance of any number of pairs is equal to unity:[10]

$$|c_{1n}|^{-4}\sum_{m=0}^{\infty}(m+1)w_n^m=|c_{1n}|^{-4}(1-w_n)^{-2}=1. \tag{11}$$

The mean number of pairs in this state is

$$\bar{n}=|c_{1n}|^{-4}\sum_{m=1}^{\infty}m(m+1)w_n^m=2|c_{2n}|^2. \tag{12}$$

We note that, in the theory of second quantization, the expectation values are found in the Heisenberg picture without averaging over the number of pairs. Thus, for (12), we have (see Refs. 1 and 2)

$$\bar{n}=\langle 0_{in}|a_{n\,in}\overset{+}{b}_{n\,out}b_{n\,out}\overset{+}{a}_{n\,in}|0_{in}\rangle=2|c_{2n}|^2, \tag{13}$$

where a_n (b_n) is the particle (antiparticle) annihilation operator, and so on. To obtain the right-hand side, we can use the relation between the *in*- and *out*-operators:

$$\begin{aligned} b_{n\,out}&=c_{2n}^*\overset{+}{a}_{n\,in}+c_{1n}b_{n\,in},\\ \overset{+}{b}_{n\,out}&=c_{2n}a_{n\,in}+c_{1n}^*\overset{+}{b}_{n\,in}. \end{aligned} \tag{14}$$

The many-particle interpretation of the coefficients c_{1n}, c_{2n} of the so-called single-particle solution of the Klein-Gordon equation remains valid for the time-independent solutions in the case of the pair-creating potential step. We shall show how c_{1n}, c_{2n} can be determined in this case. We are interested in solutions with p_0 lying in the Klein region: $\pi_0(-)<-m_\perp$, $\pi_0(x)>m$. We shall classify the wave functions in accordance with the sign of the momentum $\pi_3(x_3)$ at the infinity at which there is only one wave (that has penetrated the barrier). Thus, $_+\psi_n(x)$ means that, as $x_3\to-\infty$ (lower position of the symbol $+$), the wave is characterized by positive momentum $\pi_3(-\infty)$; $^-\psi_n(x)$ means that, as $x_3\to+\infty$, we have negative momentum $\pi_3(+\infty)$ and so on. We shall normalize the solution to unit flux through the plane xy throughout the time T of observation:[1]

$$\int {}^{\varepsilon'}\psi_{n'}^*(x)\left(-i\frac{\overleftrightarrow{\partial}}{\partial x_3}\right){}^{\varepsilon}\psi_n(x)\,d\tau=\pm\delta_{nn'}\delta_{\varepsilon\varepsilon'}, \tag{15}$$

$$d\tau=dx\,dy\,dt, \quad \varepsilon,\ \varepsilon'=\pm,$$

$$\delta_{nn'}=\delta_{p_1p_1'}\delta_{p_2p_2'}\delta_{p_0p_0'}, \quad -T/2\leqslant t\leqslant T/2.$$

Functions with the symbols $\pm$ in the inferior position will be normalized in the same way.

It is readily verified that

$$\begin{aligned} {}_+\psi_n(x)&=b_{1n}{}^+\psi_n(x)+b_{2n}{}^-\psi_n(x),\\ {}_-\psi_n(x)&=b_{2n}^*{}^+\psi_n(x)+b_{1n}^*{}^-\psi_n(x). \end{aligned} \tag{16}$$

Conservation of current along the x_3 axis yields

$$|b_{1n}|^2-|b_{2n}|^2=1. \tag{17}$$

The first relation in (16) will now be written in the form

$$^-\psi_n=c_{1n}{}^+\psi_n+c_{2n\,+}\psi_n, \tag{18}$$

$$c_{1n}=-b_{1n}/b_{2n}, \quad c_{2n}=b_{2n}^{-1}. \tag{19}$$

In the Klein region that we are considering, relation (18) can be interpreted in the single-particle sense, as follows. A particle in the state $^-\psi_n$ is incident on the barrier from the right. It is reflected by the step in the state $^+\psi_n$ and penetrates the barrier in the state $_+\psi_n$. Since only the antiparticle can penetrate the barrier [this is also clear from the sign of $\pi_0(-\infty)$], the positive value of $\pi_3(-)$ can be interpreted as the motion of the antiparticle away from the barrier with momentum $-\pi_3(-)$. Consequently, (18) describes the same situation as the analogous relation for fields that vanish for $t\to\pm\infty$. (A finite wave packet associated with the incident particle will not interact with the step field for $t\to\pm\infty$.) Accordingly, (7) follows from (19) and (17). According to (10), the quantity $|c_{2n}|^2$ is the mean number of pairs in the particular state throughout the time T of observation, provided the charge of the system is zero, i.e., the level is uncharged. All the possible processes in the state n, whatever its charge,[1] can be determined from the given c_{1n}, c_{2n}.

Finally, the many-particle interpretation of the wave function can be extrapolated to a bound state and virtual pair-production. It is readily seen that, in this case,

$$c_1^2=\max\left\{\int_{V_+}dV\,j_0,\ -\int_{V_-}dV\,j_0\right\}, \tag{20}$$

$$c_1^2-c_2^2=1.$$

The quantity j_0 is the fourth component of the current:

$$j_0=\psi_n^*(x)2\pi_0(x)\psi_n(x). \qquad (21)$$

The possibility that $c_1^2 = -\int dV j_0$ (integration over V_-) corresponds to the bound antiparticle state, whose norm is -1:

$$-c_1^2+c_2^2=\int_{V_-} dV j_0+\int_{V_+} dV j_0=-1. \qquad (22)$$

Suppose that this state is charged, i.e., its charge is equal to that of the antiparticle. In contrast to the fermion case, the system now contains particles at the bottom of the well. Because of this, the system can be bound. This state is unusual in that the antiparticle part of the wave function, which determines the sign of the norm in (22), does not have a classically accessible region of motion anywhere.

Finally, let us consier the case where the gap between the particle and antiparticle levels is small. We then have $c_1 \simeq c_2$ (see Refs. 7 and 8 and the text below). By virtue of the charge conservation condition (7), this means that the mean number of pairs in this state must increase:

$$2c_2 \approx c_1+c_2=1/(c_1-c_2) \gg 1.$$

This increase continues without limit as the level approaches the lower continuum (in a wide and/or shallow well, there are no bound antiparticle states), or, as the bound state of the particle approaches the bound state of the antiparticle. This is not surprising because the situation can no longer be regarded as time-independent at the time of merging of the levels. Actually, according to the solution of the Klein-Gordon equation, a laser-type exponential increase in the wave function and in the number of pairs with time begins as the well depth is increased still further. This type of increase, and even its initial stage, are inconsistent with the law of conservation of charge when the charge of the system can be different from zero or infinitly. It may be shown that the wave function of a quasistationary level is normalized for any fixed instant of time. Consequently, the only way of avoiding the contradiction with conservation of charge is $c_1 = c_2$. This is, in fact, what happens. This exact equation is not obeyed in the spinor case. Consequently, the norm of the quasistationary state is infinite.

In the boson case, it can be proved that the norm of the solution will vanish at the time the bound particle and antiparticle levels merge [see Eqs. (5) and (10) in Ref. 6 and the detailed discussion in Refs. 7 and 8].

Thus, even as we approach the point of merging of the particle and antiparticle levels, the state is essentially a many-particle state. A large number of particles in a finite part of the well signifies a high density of particles, which means that the interaction between them must be taken into account when the level energy is determined. At this point, the theory must be developed further by a procedure similar to that adopted by Migdal[5] for the evolution of a condensate in a supercritical well.

The author is indebted to V. I. Ritus for valuable discussions and advice and to E. L. Feinberg and M. S. Marinov for useful suggestions.

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Translated by S. Chomet