A model for relaxation dynamics in spin glass

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A model for relaxation dynamics in spin glasses is investigated, in which transitions between metastable states are treated as thermally-activated "hops." The space of these states is assumed to be ultrametric, and is described within mean-field theory. Possible types of temporal and temperature behavior are analyzed for the time-dependent correlation functions.

1. INTRODUCTION

A large number of experimental papers have shown that at low temperatures spin glasses are characterized by a very wide spectrum of relaxation times. The existence of this wide spectrum of relaxation times can be related to the appearance of a large number of metastable states at low temperatures. However, several questions remain about the characteristics of these metastable states and the system's relaxation dynamics when they are present. The construction of a dynamic theory for spin glasses is complicated by the circumstance that for low temperatures a phase transition is possible. Unfortunately, the problem of phase transitions in spin glasses remains unsolved: recently papers have appeared, both experimental and theoretical, which argue for the possibility of finite-temperature phase transitions; on the other hand there are experimental data and theoretical papers suggesting that phase transitions are possible only at zero temperature.

The most highly-developed method in the theory of spin glasses is the mean-field approximation, which for a model of the spin glass with an infinite interaction radius can be solved exactly[1] (the Sherrington-Kirkpatrick model[2]). Although the Sherrington-Kirkpatrick model is not realistic, there are nevertheless grounds for assuming that some important features of the low-temperature behavior of this model are also correct for more realistic models of spin glasses. There are already experimental data confirming the results of mean-field theory, e.g., the temperature independence of the static susceptibility in the low-temperature phase, and the dependence of the critical temperature on the magnitude of the magnetic field.[3,4] The conclusions derived from mean-field theory about the structure of the low-temperature phase are very important and interesting. Below the transition temperature, the state of a spin glass within the Ising-like Sherrington-Kirkpatrick model is strongly degenerate. There is a large number of equilibrium thermodynamic states,[5,6] each of which is characterized by a set of local magnetic moments $\{m_i\}$ defined by the Thouless-Anderson-Palmer equations, with a free energy F_α[7]; the subscript $i = 1,2,\ldots,N$ labels the spins while the subscript α labels the states. These states are separated from one another by high-energy barriers (infinite in the thermodynamic limit $N \to \infty$). To describe the low-temperature phase of a spin glass it is not sufficient to use one Edwards-Anderson order parameter

$$q_{EA} = \frac{1}{N}\sum_i (m_i^\alpha)^2, \tag{1}$$

it is also necessary to introduce other important characteristics,[8] such as the overlap between states

$$q_{\alpha\beta} = \frac{1}{N}\sum_i m_i^\alpha m_i^\beta \tag{2}$$

and the probability $\mathscr{P}(q)$ of finding two states with an overlap q:

$$\mathscr{P}(q) = \sum_{\alpha,\beta} P_\alpha P_\beta \delta(q - q_{\alpha\beta}), \tag{3}$$

where

$$P_\alpha = \frac{1}{Z} e^{-\beta F_\alpha}, \quad Z = \sum_\alpha e^{-\beta F_\alpha}. \tag{4}$$

The overlaps $q_{\alpha\beta}$ vary continuously within the interval $[q_0, q_{EA}]$ where q_0 is determined by the magnetic field. As was shown in Ref. 9, the space of overlaps possesses an ultrametric topology; a convenient representation of this space is a hierarchical tree.[10]

As a starting point for constructing a dynamic model of spin glasses which describes the slow relaxation processes, we can assume that for low temperatures the space of metastable states of a spin glass possesses the same property (ultrametricity) as the state space in the Sherrington-Kirkpatrick model. A transition from one of these metastable states to another can be viewed as thermally-activated hopping over a finite energy barrier. A similar approach to the relaxation dynamics of a spin glass was used not long ago in Refs. 11–13. In Ref. 11 a model of relaxation dynamics was investigated in a space of independent states; the free energies of these states were assumed to be independent random variables with exponential probability distributions. In Refs. 12, 13 a model of relaxation dynamics was investigated on a hierarchical tree with a fixed number of branches at each level; the energies of all the states were assumed to be identical. In this case, nontrivial relaxation functions were a consequence of ultrametricity, and not of the randomness of the energy of the metastable states.

In the present work we will investigate a model for relaxation dynamics in a state space described by the Sherrington-Kirkpatrick model. An important feature of this state space is the fact that it can be represented by a hierarchical

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tree with a random number of branchings at each level and for each branch of the tree. In addition, the energies of these states are randomly correlated variables.

The plan of the paper is as follows: in Section 2, a model for the relaxation dynamics of the spin glass is introduced and solved exactly; in Section 3, the character of the relaxation for long times is analyzed; in Section 4 we summarize and discuss the fundamental assumptions on which the model is based.

2. THE MODEL

We assume that a transition between metastable states can be treated as a thermally-activated "hop." Then the equation for the function $P_\alpha(t)$, which gives the probability for observing the system in the state α at a time t, takes the form

$$\frac{\partial}{\partial t}P_\alpha(t)=\sum_\beta w_{\alpha\beta}P_\beta(t)-\sum_\beta w_{\beta\alpha}P_\alpha(t). \tag{5}$$

Taking into account the detailed-balance condition, we can choose the quantity $w_{\alpha\beta}$ in the form

$$w_{\alpha\beta}=\nu_0 f_{\alpha\beta}\exp[-\beta(F_\alpha-F_0)], \tag{6}$$

where $f_{\alpha\beta}=f_{\beta\alpha}$; ν_0 is some characteristic microscopic frequency. Let us measure the energy of a state α from the system free energy, i.e., $F_0=-T\ln Z$. The symmetric matrix $f_{\alpha\beta}$ is not determined within the framework of the present model. We will assume that $f_{\alpha\beta}$ is a function only of the overlap $q_{\alpha\beta}$ between the states α and β, i.e.,

$$f_{\alpha\beta}=f(q_{\alpha\beta}). \tag{7}$$

The assumption (7) implies that for any pair of states α and β whose overlap is q, the quantity $f_{\alpha\beta}$ which determines the transition frequency between these states always takes the same value, i.e., $f_{\alpha\beta}=f(q)$. The assumption (7) allows us to include in the dynamics the extent to which two metastable states differ, which is determined by the magnitude of the overlap $q_{\alpha\beta}$, in a very simple way. If $f(q)$ is a monotonically increasing function, then this implies that the smaller the overlap $q_{\alpha\beta}$ between states α and β, i.e., the more they differ from one another, the smaller the transition frequency between them.

Before we turn to the solution of Eq. 5, let us briefly recall the basic properties of the state space in the Sherrington-Kirkpatrick model which are necessary to solve this equation.[9] The ultrametricity of this space implies the following: for any three states α,β, and γ, the overlaps between the $q_{\alpha\beta}$, $q_{\alpha\gamma}$, and $q_{\gamma\beta}$ are either equal to one another, i.e., $q_{\alpha\beta}=q_{\alpha\gamma}=q_{\gamma\beta}$, or only two of them are equal, e.g., $q_{\alpha\beta}=q_{\alpha\gamma}$, while the third overlap $q_{\gamma\beta}$ is larger, i.e., $q_{\gamma\beta}>q_{\alpha\beta}=q_{\alpha\gamma}$. Let us define the cluster $I_{q,\alpha}$ as the set of all states β for which $q_{\alpha\beta}\geqslant q$. Then under the conditions described above, either the clusters $I_{q,\alpha}$ and $I_{q,\beta}$ coincide fully or they are disjoint, i.e., have no state in common. It is obvious that as q increases the number of states in the clusters I_q increases (from here on, the subscript α for the cluster $I_{q,\alpha}$ will be omitted). For any $q'>q$, the cluster I_q can be represented as a set of nonintersecting clusters $I_{q'}$ of smaller size. Let us introduce the statistical weight of the cluster I_q:

$$W(I_q)=\sum_{\alpha\in I_q}P_\alpha, \tag{8}$$

where the statistical weight P_α of the state α is determined from Eq. (4). We will solve Eq. (5) for the probability $P(I_q;t)$ of observing the system in any state $\alpha\in I_q$ at a given time t:

$$P(I_q;t)\equiv\sum_{\alpha\in I_q}P_\alpha(t). \tag{9}$$

It is convenient to go from the continuous variable $q\in[q_0,q_{EA}]$ to a discrete variable q_j; $q_0<q_1<q_2<\cdots<q_R=q_{EA}$. In the end we will set $R\to\infty$. Equation (5) for $P(I_i;t)$, where $I_i\equiv I_{qi}$, can be cast in the form

$$\nu_0^{-1}\frac{\partial}{\partial t}P(I_i;t)=W(I_i)\sum_{j=0}^{i-1}[P(I_j;t)-P(I_{j+1};t)]f_j-P(I_i;t)\times\sum_{j=0}^{i-1}[W(I_j)-W(I_{j+1})]f_j, \tag{10}$$

where $f_j\equiv f(q_j)$ and $I_i\subset I_{i-1}\subset\cdots\subset I_1\subset I_0$. Since q_0 is the minimum overlap, it follows that I_0 contains all states in itself; hence,

$$W(I_0)=\sum_\alpha P_\alpha=1,\quad P(I_0;t)=\sum_\alpha P_\alpha(t)=1. \tag{11}$$

This latter equation is the normalization condition. Equation (10) can be solved exactly, and its solution has the form

$$P(I_i;t)=W(I_i)+W(I_i)\sum_{j=0}^{i-1}[P(I_{j+1};0)/W(I_{j+1})-P(I_j;0)/W(I_j)]\exp(-t/\tau_j), \tag{12}$$

$$(\tau_j\nu_0)^{-1}=\sum_{l=0}^{j-1}(W(I_l)-W(I_{l+1}))f_l+W(I_j)f_j, \tag{13}$$

where $I_i\subset I_{i-1}\subset\cdots\subset I_1\subset I_0$. Let the system be in state α at the initial time $t=0$, i.e.,

$$P_{\tilde\alpha}(0)=1;\ P(I_i;0)=1,\ \tilde\alpha\in I_i;\quad P(I_i;0)=0,\ \tilde\alpha\notin I_i.$$

Then the probability of finding the system at a time t in the same state α equals

$$P_{\tilde\alpha}(t)=P_{\tilde\alpha}+P_{\tilde\alpha}\sum_{j=0}^{R-1}[W^{-1}(I_{j+1})-W^{-1}(I_j)]\exp(-t/\tau_j). \tag{14}$$

Let us average (14) over all initial states α, taking into account their statistical weights P_α, and introduce the autocorrelation function

$$A(t)\equiv\sum_{\tilde\alpha}P_{\tilde\alpha}[P_{\tilde\alpha}(t)-P_{\tilde\alpha}]=\sum_{\tilde\alpha}P_{\tilde\alpha}^2\sum_{j=0}^{R-1}[W^{-1}(I_{j+1})-W^{-1}(I_j)]\exp(-t/\tau_j). \tag{15}$$

We define the function

$$F(W_1, W_2, \dots W_R)$$

$$\equiv \sum_{I_1 \subset I_0} \delta(W_1 - W(I_1)) \sum_{I_2 \subset I_1} \delta(W_2 - W(I_2)) \dots \sum_{\alpha \in I_{R-1}} \delta(W_R - P_\alpha). \tag{16}$$

Since $\alpha \in I_{R-1} \subset I_{R-2} \subset \dots \subset I_1 \subset I_0$, obviously the inequality

$$0 < W_R < W_{R-1} < \dots < W_1 < W_0 = 1$$

holds. Using the function $F(W_1, \dots, W_R)$, Equation (15) can be written in the form

$$A(t) = \sum_{j=0}^{R-1} \int \left(\prod_{l=1}^{R} dW_l \right) F(W_1 \dots W_R) W_R^2 (W_{j+1}^{-1} - W_j^{-1}) \times \exp(-t/\tau_j). \tag{17}$$

Up until now, all calculations have been carried out for frozen-in configurations of the random exchange integrals J_{ij}. The weights of the clusters $W(I_q)$ are randomly-correlated variables. The function $F(W_1, \dots, W_R)$ determines the joint distributions of the weights for clusters which are sequentially subsets of one another. Let us investigate the temporal behavior of the autocorrelation function $A(t)$ averaged over all configurations J_{ij}. This problem reduces to calculating the average value of the function $F(W_1, \dots, W_R)$. Using the averaging method discussed in Ref. 9, which consists of calculating moments of $\bar{F}(W_1, \dots, W_R)$

$$M(k_1, k_2, \dots k_R) = \int_0^1 \left(\prod_{i=1}^{R} dW_i W_i^{k_i} \right) \bar{F}(W_1 \dots W_R),$$

and then reconstructing the function $\bar{F}(W_1, \dots, W_R)$ from these moments, we obtain the following result:

$$\bar{F}(W_1 \dots W_R) = \frac{1}{\Gamma(x_1)\Gamma(1-x_R)} \frac{(W_0 - W_1)^{x_1 - 1}}{W_R^{1+x_R}} \times \prod_{i=1}^{R-1} \frac{\theta(W_i - W_{i+1})(W_i - W_{i+1})^{x_{i+1} - x_i - 1}}{\Gamma(x_{i+1} - x_i)}, \tag{18}$$

where $W_0 = 1$, $\theta(W)$ is a step function and x_i is the value of the Parisi function $x(q)$ at the point q_i, i.e., $x_i = x(q_i)$, $0 \leqslant x_0 < x_1 < x_2 \cdots < x_R = x_m < 1$. The points x_0 and x_m are the points which define the edges of the plateau in the function $q(x)$.[1] Substituting (18) into (17) and integrating over the variables $W_R, W_{R-1}, \dots W_{j+1}$, ($\tau_j^{-1}$ does not depend on these variables; see (13)), we obtain

$$\bar{A}(t) = (1 - x_R) \sum_{j=0}^{R-1} \frac{x_{j+1} - x_j}{(1-x_j)(1-x_{j+1})} \times \int_0^1 \left(\prod_{l=1}^{j} dW_l \right) \bar{F}(W_1 \dots W_j) W_j \exp\left(-\frac{t}{\tau_j} \right), \tag{19}$$

where the function

$$\bar{F}(W_1 \dots W_j) \equiv \int \left(\prod_{l=j+1}^{R} dW_l \right) \bar{F}(W_1 \dots W_R) \tag{20}$$

is determined by Eq. (18) with the subscript j replacing R. So as to calculate the integral over W_1, we perform a Laplace transform on the function

$$T_j(p, t) = \int_0^p dW_1 \int_0^{W_1} dW_2 \dots \int_0^{W_{j-1}} dW_j \bar{F}(W_0 = p,\ W_1 \dots W_j) \times W_j \exp(-t/\tau_j) \tag{21}$$

for the variable p. We obtain

$$T_j(s, t) = (\tilde{t} f_0 + s)^{-x_1} (\tilde{t} f_1 + s)^{-1+x_j} \prod_{l=1}^{j-1} (\tilde{t} f_l + s)^{x_l - x_{l+1}}, \tag{22}$$

where $\tilde{t} = \nu_0 t$ is a dimensionless time. When we perform the inverse Laplace transform and deform the contour of integration around the negative real half-axis, we are led to the result

$$T_j(t) \equiv T_j(p=1, t) = \frac{1}{\pi} \int_{f_0}^{f_j} ds\, e^{-s\tilde{t}} \varphi_j(s) \sin(\pi x(s)), \tag{23}$$

$$\varphi_j(s) = |f_0 - s|^{-x_1} |f_j - s|^{x_j - 1} \prod_{l=1}^{j-1} |f_l - s|^{x_l - x_{l+1}}. \tag{24}$$

The function $x(s)$ is defined by the equation

$$f(q(x)) = s. \tag{25}$$

Let us now go to the continuum limit $R \to \infty$. It is convenient to consider all functions as functions of the variable x:

$$x_j \to x, \quad x_1 \to x_0, \quad q_j \to q(x),$$
$$f_j = f(q_j) \to f(q(x)) = f(x), \quad T_j(t) \to T_x(t).$$

Using Eq. (19), we can rewrite the expression for the autocorrelation function $\bar{A}(t)$ in the form

$$\bar{A}(t) = T_{x_m}(t) - (1 - x_m) \int_{x_0}^{x_m} \frac{dx}{(1-x)} \frac{\partial}{\partial x} T_x(t) - (1 - x_m) \times \exp(-\tilde{t} f_0). \tag{26}$$

Let us now investigate the time variation of the average overlap between an initial state $\tilde{\alpha}$ at time $t = 0$ and states β in which the system finds itself at the instant t:

$$q_{\tilde{\alpha}}(t) \equiv \sum_{\beta} q_{\tilde{\alpha}\beta} P_\beta(t).$$

Averaging over all initial states $\tilde{\alpha}$, we are led to the function

$$q(t) \equiv \sum_{\tilde{\alpha}} q_{\tilde{\alpha}}(t) P_\alpha = q_{EA} \sum_{\tilde{\alpha}} P_{\tilde{\alpha}} P_{\tilde{\alpha}}(t) + \sum_{\tilde{\alpha}} P_{\tilde{\alpha}} \sum_{j=0}^{R-1} q_j (P(I_j; t) - P(I_{j+1}; t)), \tag{27}$$

where

$$\tilde{\alpha} \in I_{R-1} \subset I_{R-2} \subset \dots I_2 \subset I_1, \quad P(I_R; t) \equiv P_{\tilde{\alpha}}(t).$$

From the definition of $q(t)$ it is obvious that

$$q(t=0) = q_{EA}, \quad \bar{q}(t \to \infty) = \int_0^1 dx\, q(x).$$

The calculation of the function $\bar{q}(t)$ is fully analogous to the calculation of $\bar{A}(t)$. Therefore we present only the final result

$$\bar{q}(t)=\int_0^1 dx\, q(x)+\int_{q_0}^{q_{EA}} dq\,(1-x(q))\int_{x_0}^{x(q)}\frac{dx}{(1-x)^2}T_x(t)$$
$$+\frac{x_0}{(1-x_0)}\left(\int_0^1 dx\, q(x)-q_0\right)\exp(-\tilde{t}f_0). \qquad (28)$$

Thus, the time dependence of the functions $\bar{A}(t)$ and $\bar{q}(t)$ is determined by the time dependence of the function $T_x(t)$ (Eq. (23)).

3. ANALYSIS OF THE NATURE OF THE RELAXATION FOR LONG TIMES

In the continuum limit $R\to\infty$, Eq. (23) for the function $T_x(t)$ has the form

$$T_x(t)=\frac{1}{\pi}\int_{f_0}^{f(x)} ds\, e^{-s\tilde{t}}\varphi_x(t)\sin(\pi x(s)), \qquad (29)$$
$$\varphi_x(s)=\exp\left[-\ln|f(x)-s|-\int_{f_0}^{f(x)} ds'\frac{x(s')}{s'-s}\right],$$

where the integral over s' is understood in the sense of a principal value. We will assume that for long times $\tilde{t}=tv_0\gg 1$, the basic contribution to the integral comes from a saddle point $s^*(t)$, at which $x(s^*)\ll 1$; naturally $x(s^*)<x$. If this condition is fulfilled, then $T_x(t)$ depends very weakly on x for $x>x^*$. In this case, Equation (26) can be cast in the form

$$\bar{A}(t)\approx x_m\int_{f_0}^{f(x_m)} ds\, e^{-s\tilde{t}}\varphi_{x_m}(s)x(s). \qquad (30)$$

For simplicity, we will consider the case of zero magnetic field, when $x_0=0$ and the minimum overlap is $q_0=0$. In Section 2 we already discussed the fact that the function $f(q)$ can plausibly be assumed to be a monotonically increasing function; since $q(x)$ is also a monotonically increasing function of x on the interval $[x_0,x_m]$, the function $f(x)\equiv f[q(x)]$ is a monotonically increasing function of x. Let us consider the case $f(0)=f_0\ll(tv_0)^{-1}\ll 1$. In the integral (30), the main contribution for large times $t\gg 1$ comes from the region $s\ll 1$. This means we can confine ourselves to investigating the behavior of the function $x(s)$ for small s. Let us analyze two cases:

$$x(s)\sim as^\gamma,\quad \gamma>0, \qquad (31)$$
$$x(s)\sim as^p\exp(-bs^{-\gamma}),\quad \gamma>0. \qquad (32)$$

We will calculate the integral (30) by the saddle-point method. To leading order in $\tilde{t}^{-1}\ll 1$, the equation for the saddle point has the form

$$\tilde{t}=d\ln x(s)/ds, \qquad (33)$$

since the contribution from the function $\varphi_x(s)$ can be neglected. For the asymptotic forms (31) and (32), Eq. (33) gives a value $s^*\approx\tilde{t}^{-1}$ for the saddle point. The power-law asymptotic form (31) leads to a power-law decrease

$$\bar{A}(t)\sim A_0(\gamma/v_0t)^{\gamma+1}. \qquad (34)$$

The asymptotic behavior (32) leads to a "stretched" exponential law:

$$\bar{A}(t)\sim A_0'\exp[-(t/\tau)^{1-n}], \qquad (35)$$
$$\tau^{-1}=v_0\exp\{-(1/\gamma)\ln[b^{-1}(1+\gamma)^{-1}(1+\gamma^{-1})^{-\gamma}]\}, \qquad (36)$$
$$n=1/(1+\gamma),$$

where $0<n<1$ since $\gamma>0$. In the exponent of (35) we have retained only the leading term, discarding terms of order $\ln t$.

The long-time behavior of the function $\bar{q}(t)$ defined by Equation (28) can be analyzed in an analogous manner. It turns out that the asymptotic behavior of $\bar{q}(t)$ is the same as that of $\bar{A}(t)$, i.e., it is described by the functions (34) and (35).

The "stretched" exponential relaxation function (35) was recently observed for the remanent magnetization of a spin glass.[14,15] This relaxation law is observed over a broad temperature interval $T<T_g$. The temperature behavior of the exponent n is interesting[15]: in the temperature range $0.5\leqslant T/T_g\leqslant 0.8$, n is almost constant, while outside this interval it begins to increase, eventually approaching unity. This behavior of the exponent n can be explained in terms of the dynamic model described above, if it is assumed that the degree of the exponent γ depends on temperature. From Eqs. (25) and (32), it follows that the function $f(x)$ for small x takes the form

$$f(x)\sim\exp[-U(x)/\gamma], \qquad (37)$$
$$U(x)=\ln[b^{-1}\ln(a/x)].$$

If we assume that for low temperatures (far from T_c) contributions to the function $f(x)$ are related to jumps over the barrier $U(x)$, then we must set

$$\gamma\approx\gamma_0T/T_c. \qquad (38)$$

In this case, Eq. (36) takes the form

$$\tau^{-1}=v_0\exp\left\{-\frac{T_c}{T}\frac{\ln[b^{-1}(1+\gamma)^{-1}(1+\gamma^{-1})^{-\gamma}]}{\gamma_0}\right\},$$
$$n\approx\frac{1}{1+\gamma_0T/T_c}, \qquad (39)$$

i.e., $n\to 1$ as $T\to 0$. For T close to T_c we set

$$\gamma\approx\gamma_0'(T_c-T)/T_c. \qquad (40)$$

This dependence of γ on temperature corresponds to an assumption about the critical decrease of the transition frequencies between metastable states for T close to T_c. In this case

$$n\approx 1-\gamma_0'(T_c-T)/T_c,$$

i.e., $n\to 1$ as $T\to T_c$.

In Ref. 16, the character of the remanent magnetic moment relaxation was studied as a function of waiting time in the spin glass CuMn. It was observed that the law (35) describes the relaxation only within the limited time interval $5\leqslant t\leqslant 10^3$ sec. For longer times the relaxation law has an-

other form. At the same time, for the spin glass $CsNiFeF_6$ a similar investigation[17] showed that relaxation of the remanent magnetization for a broad range of waiting times is given by the function

$$\sigma=\sigma_0\left(\frac{t}{\tau_p}\right)^{-\alpha}\exp\left[-\left(\frac{t}{\tau_p}\right)^{1-n}\right] \tag{41}$$

for all times in the range $10^{-1}\leqslant t\leqslant 10^{-5}$ sec. The function (41) can be obtained in terms of the present model by setting the exponent $p=\alpha-1$ in (32).

The results of Ref. 16 show that the relaxation function (35) (or (41)) may not be universal for spin glasses, and that deviations from it are possible.

4. CONCLUSION

In this paper we have considered relaxation in spin glasses as diffusion in an ultrametric space of metastable states with hierarchical energy barriers. The features of this type of diffusion are presently being widely discussed as possible descriptions of slow relaxation phenomena not only in spin glasses but also in other systems.[13,18] Below, we will discuss the assumptions which underly our use of Eqs. (5)–(7) to describe relaxation in spin glasses.

The presence of a degenerate ground state in the various models of a spin glass with short-range interactions between spins is repeatedly confirmed by numerical methods. Investigations of the properties of the space of equilibrium configurations have only begun. In Ref. 19 a complete-frustration model was investigated with Heisenberg spins on a simple cubic lattice. This model had a strongly degenerate ground state, and in numerical calculations a phase transition was observed to the spin-glass state. The analysis in Ref. 19 showed that equilibrium spin configurations possessed ultrametric properties. Further investigations are required to determine whether or not ultrametricity is a general property of other spin-glass models.

We have considered the transitions between metastable states as thermally-activated "hops" over finite energy barriers. Each metastable state was characterized by a set of average local magnetic moments m_i. This approach is valid when the time τ_1 for establishing a quasi-equilibrium state is much smaller than the time τ_m for a transition through the minimum energy barrier separating two states. The time τ_1 is usually microscopic, whereas τ_m can be macroscopic since it can be related to the flipping of a large number of spins. In this case, the quantities m_i must be understood as values of the local magnetic moments averaged over a time interval $\tau_1\ll\Delta t\ll\tau_m$. If $\tau_1\ll\tau_m$ and we are interested in the time scale $t>\tau_m$, then the relaxation dynamics in the spin glass can be described by Eq. (5), i.e., as diffusion in the space of metastable states.

Let us consider a possible basis for Eq. (7). Usually the characteristic energy E which determines the relaxation time $\tau\sim\tau_0\exp(E/kT)$ of a cluster of $\mathcal{N}$ spins is proportional to $\mathcal{N}^{\nu}$, where the exponent ν is related to the dimensionality of the cluster. In our case the number of spins which must be flipped in order to go from state α to state β is proportional to the quantity

$$\mathcal{N}_{\alpha\beta}\sim\sum_i (m_i^{\alpha}-m_i^{\beta})^2/q_{EA}=2N(q_{EA}-q_{\alpha\beta})/q_{EA},$$

where q_{EA} and $q_{\alpha\beta}$ are defined by Eqs. (1) and (2). Consequently, the quantity $\mathcal{N}_{\alpha\beta}$, which we assume is macroscopic but finite, is a function of the state overlap $q_{\alpha\beta}$. It is natural to assume also that the energy barrier $E_{\alpha\beta}$ which determines the transition frequency between the states is a function of $\mathcal{N}_{\alpha\beta}$, i.e., a function of $q_{\alpha\beta}$; then we are led to Eq. (7). It is clear that $f(q_{\alpha\beta})$ must decrease as $q_{\alpha\beta}$ decreases, since this corresponds to growth of $\mathcal{N}_{\alpha\beta}$, which implies an increase in $E_{\alpha\beta}$.

We have described the energy distribution of metastable states within the mean-field approximation, using exact results obtained from the Sherrington-Kirkpatrick model, which, as we already noted in the Introduction, gives results that describe certain experimental data rather well. Of course, in realistic models of spin glasses, the character of the energy distribution of metastable states can differ from the mean-field distribution; this is related to spatial fluctuations which are not included in the mean-field approximation. However, at the present time it is not possible to go outside the framework of this approximation for the distribution function (16).

One feature of diffusion in an ultrametric space should be pointed out; it is necessary to admit the possibility of "hops" over any "distance," defined as $d_{\alpha\beta}=q_{EA}-q_{\alpha\beta}$.[9] If "hops" were possible only for distances d smaller than $d'=q_{EA}-q'$, then the system would not be able to grow beyond the limit of some cluster $I_{q'}$ of metastable states. This result is a consequence of ultrametricity, since the whole space can be decomposed into nonintersecting clusters of states of size q'.

As we showed in Section 3, to determine the long-time behavior of the autocorrelation function, it is sufficient to know the asymptotic behavior of the function $f(x)=f[q(x)]$ for smaller x. Let us recall that physical meaning attaches not to the function $q(x)$ but to its inverse function $x(q)$.[8,9] That is, it is $x(q)$ which represents the probability of finding two metastable states with overlap $q'\leqslant q$. This definition of the function $x(q)$ allows it to be used also in describing the structure of the space of metastable states of a realistic spin-glass model.

The assumptions about the asymptotic behavior (32) and the temperature dependence of the function $f(x)$ (Eqs. (38) and (40)) lead to results in agreement with certain experimental data.[14,15,17] In principle, the different temporal behavior of the relaxation in various time intervals, as was observed in Ref. 16, can also be explained in terms of this model, if we assume that there are two ranges for the variable x over which the function $f(x)$ behaves differently. Unfortunately, in order to define the function $f(x)$ for a realistic spin-glass model, it is necessary to go beyond the framework of this model of relaxation dynamics.

[1]G. Parisi, Phys. Rev. Lett. **43**, 1754 (1979); J. Phys. **A13**, L117, 1101, 1887 (1980).
[2]D. Sherrington and S. Kirkpatrick, Phys. Rev. Lett. **32**, 1792 (1975).

[3]R. V. Chamberlin, M. Hardiman, L. A. Turkevich, and R. Orbach, Phys. Rev. **B25**, 6720 (1982).
[4]I. A. Campbell, N. de Courtenay, and A. Fert, J. de Phys. Lett. **45**, L565 (1984).
[5]C. de Dominicis, M. Gabay, T. Garel, and H. Orland, J. de Phys. **41**, 923 (1980).
[6]A. J. Bray and M. A. Moore, J. Phys. **C13**, 419 (1980).
[7]D. J. Thouless, P. W. Anderson, and R. Palmer, Phil. Mag. **35**, 593 (1977).
[8]G. Parisi, Phys. Rev. Lett. **50**, 1946 (1983).
[9]M. Mezard, G. Parisi, N. Sourlas, G. Toulouse, and M. Virasoro, J. de Phys. **45**, 843 (1984).
[10]M. Mezard and M. Virasoro, J. de Phys. **46**, 1293 (1985).
[11]C. de Dominicis, H. Orland, and F. Lainee, J. de Phys. Lett. **46**, L469 (1985).
[12]A. T. Ogielski and D. L. Stein, Phys. Rev. Lett. **55**, 1634 (1985); G. Paladin, M. Mezard, and C. de Dominicis, J. de Phys. Lett. **46**, L985 (1985).
[13]S. Grossmann, F. Wegner, and K. H. Hoffman, J. de Phys. Lett. **46**, L575 (1985); B. A. Huberman and M. Kerszberg, J. Phys. **A18**, L331 (1985).
[14]R. V. Chamberlin, G. Mozurkewich, and R. Orbach, Phys. Rev. Lett. **52**, 867 (1984).
[15]R. Hoogerbects, Wei-Li Luo, and R. Orbach, Phys. Rev. Lett. **55**, 111 (1985).
[16]P. Nordbald, P. Svendlindh, P. Lundgren, and L. Sandlund, Phys. Rev. **B33**, 645 (1986).
[17]M. Ocio, M. Alba, and J. Hammann, J. de Phys. Lett. **46**, L1101 (1985).
[18]S. Teitel and E. Domany, Phys. Rev. Lett. **55**, 2176 (1985).
[19]H. T. Diep, A. Ghazali, and P. Lallemand, J. Phys. **C18**, 5881 (1985).

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