Filamentation of the Hall current in a two-dimensional nonlinear electron system in a quantizing magnetic field

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An experimental investigation was made of the nonlinear characteristics of silicon [Si(100)] MIS structures subjected to a quantizing magnetic field in the Corbino geometry. The observed nonlinearities were attributed to the appearance of a Hall current filament. A new method for the determination of the density of states half-way between two Landau levels was developed.

Nonlinear phenomena in two-dimensional electron systems subjected to a quantizing magnetic field have been investigated experimentally on a number of occasions.[1–4] The main attention has been concentrated on the breakdown of the quantum Hall effect, i.e., the strong (by 4–5 orders of magnitude) increase in the values of σ_{xx} ρ_{xx} when the voltage or current in a two-dimensional layer reach a critical value. There is as yet no agreement on the breakdown mechanisms. It has been suggested that the breakdown is due to transitions between electron levels accompanied by phonon emission,[5–7] injection of hot electrons from contact regions,[8] thermal instability,[1,3,9] an increase in the number of extended states on increase in the electric field intensity,[10] and the Zener effect.[2] However, in the first experimental investigation[1] it was found that nonlinear effects are observed before the critical fields and currents are reached. We shall report the results of an investigation of nonlinear characteristics of MIS transistors under magnetic quantization conditions in the prebreakdown region. We shall show that the nonlinearity observed in this region cannot be explained by any one of the above mechanisms, but is due to the formation of a narrow Hall current filament.

The possibility of the appearance of a current filament had been considered earlier[11] on the basis of a numerical model calculation. It should be pointed out particularly that an inhomogeneous distribution of the current in MIS structures discussed in Ref. 11 is an essentially nonlinear effect (a current filament appears only if ρ_{xx} depends on the potential difference), which is conceptually similar to the nonlinearities observed and interpreted in the present study.

The primary cause of these nonlinear effects is a strong dependence of σ_{xx} and ρ_{xx} on the electron density N_s exhibiting deep minima, characteristic of two-dimensional electron systems subjected to quantizing magnetic fields. These minima appear at electron densities corresponding to integer values of the occupancy factor $n = N_s hc/eH$. Small changes in the occupancy factor can alter σ_{xx} and ρ_{xx} by orders of magnitude, whereas σ_{xy} and ρ_{xy} remain practically unchanged. On the other hand, an MIS structure is a parallel-plate capacitor and the electron density is governed by the potential difference between its metal gate and a layer of two-dimensional electrons. Variation of the potential along the two-dimensional layer, which occurs during the flow of the current, gives rise to a dependence of the electron density on the coordinates so that the gate remains an equipotential surface. We are then faced with a typical nonlinear problem of the distribution of the potential in a current-carrying layer the conductivity of which depends on the potential.

It is not difficult to predict the qualitative features of the solution of this problem. By way of example, we shall consider a layer of two-dimensional electrons representing an infinite ribbon elongated along the y axis (Fig. 1a). We shall assume that an external source drives a current of constant density j_x. A magnetic field is applied along the z axis; it is selected in such a way that under linear conditions at low current densities j_x an integral number of the Landau levels is always filled with electrons within the two-dimensional layer. We shall assume specifically that the potential of the points at which $x = 0$ is known and it corresponds to an occupancy factor which is an integer, but is independent of the current density j_x. An increase in the current density j_x alters the potential of the layer in the region $x > 0$ and, consequently, increases σ_{xx}. The conductivity σ_{xx} remains minimal in the vicinity of the points where $x = 0$ and the electric field near these points is a maximum. Under nonlinear conditions the electric field and the Hall current density j_y vary as shown in Fig. 1b. The gradient of the Hall current increases as the dependence of σ_{xx} on the potential difference increases. Therefore, as temperature is lowered, an increase in the magnetic field or the current should reduce the characteristic width of the filament.

EXPERIMENTS

Our measurements were carried out on two Si(100) samples with a ring-shaped gate electrode (Corbino geome-

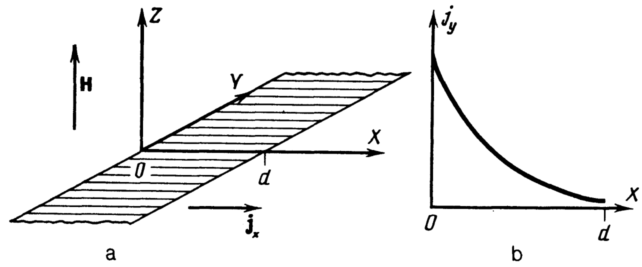

FIG. 1.

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try) with an internal diameter $2r_1 = 225\,\mu$m and an external diameter $2r_2 = 675\,\mu$m. The electron mobility at the maximum was 2×10^4 cm^2 V^{-1} sec^{-1}. The oxide thickness was 1400 Å. The results obtained for different samples were qualitatively similar. The nonlinear effects reported in the present paper were observed in the temperature interval from 4.2 to 0.3 K. The results reported below were obtained at 1.55 K.

In the Corbino geometry measurements it is usual to set the voltage across a sample and to measure the current proportional to σ_{xx}. In the case of nonlinear effects it is convenient to set a constant current through the sample. With this in mind we included not only a voltage source but also a resistance $R = 2.2\times 10^{11}\ \Omega$ in the drain-source circuit. We measured the voltage drop ΔU between the drain and source as a function of the voltage on the gate V_g (Fig. 2). The input resistance of the voltmeter used in these measurements was $10^{14}\ \Omega$. The range of the gate voltages V_g was selected so that the occupancy factor n varied in the region of $n = 4$.

Examples of experimental records are shown in Figs. 2 and 3. In the linear case and for an ideal system such measurements give the quantity

$$\Delta U = (2\pi)^{-1} I \sigma_{xx}^{-1}(V_g) \ln(r_2/r_1) \tag{1}$$

which can be used to find the dependence $\sigma_{xx}^{-1}(V_g)$. Therefore, the $\Delta U(V_g)$ curve should have a symmetric extremum at a voltage corresponding to an integer value of the occupancy factor. We can see from the figures that for the currents used in the present study the $\Delta U(V_g)$ curves were not symmetric and a change in the direction of the current not only altered the sign of the ΔU but also changed the whole curve, which underwent reflection relative to the point V_g^0 corresponding to $n = 4$ (Fig. 2). The measurements on the same sample could be carried out by setting a voltage relative to the inner (as shown in Fig. 2) or outer electrodes. The experimental curves were then identical apart from reflection relative to the point V_g^0 (Fig. 3).

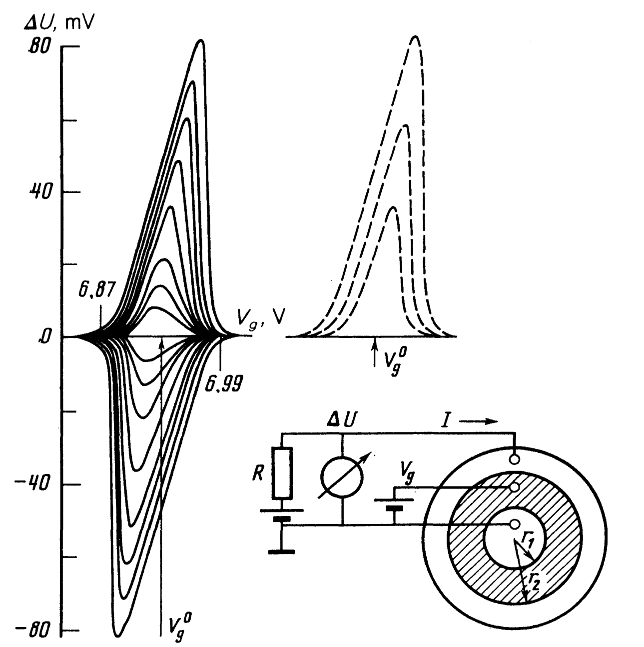

FIG. 2. Continuous curves represent the experimental records of $\Delta U(V_g)$ in a magnetic field $H = 10.6$ T obtained using the following currents I(A): 2.1×10^{-11}, 1.0×10^{-11}, 5.5×10^{-12}, 3.1×10^{-12}, 1.6×10^{-12}, 8.2×10^{-13}, 4.2×10^{-13}, 2.1×10^{-13}. The dashed curves represent calculations carried out using Eq. (5) and assuming the following currents I(A): 2.1×10^{-11}, 5.5×10^{-12}, 1.6×10^{-12} A. The experimental setup used in the measurements is also shown.

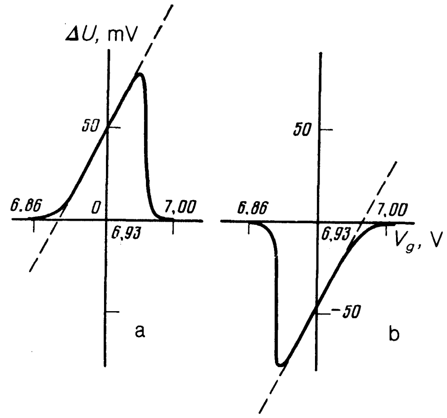

FIG. 3. Experimental dependences $\Delta U(V_g)$ obtained for $I = 2.1\times 10^{-11}$ A and $H = 10.6$ T. The gate voltage was set relative to the inner (a) and outer (b) electrodes. The dashed line corresponds to $\delta\Delta U/\delta V_g = 1$.

The shape of the $\Delta U(V_g)$ curves obtained at high values of the current I was quite specific: in a wide range of the gate voltages V_g it was found that $\partial\Delta U/\partial V_g \approx 1$ (see Fig. 3) and the separation between the extrema $V_g(\Delta U_{\max}, I) - V_g(\Delta U_{\min}, -I)$ coincided with the amplitude of the extremum $\Delta U_{\max}$. As shown in Fig. 4, the dependence $\Delta U(I)$ was nonlinear for a fixed gate voltage. This behavior was not affected by the rate of variation of the gate voltage or by the direction of the magnetic field.

CALCULATIONS

The distributions of the fields and currents could easily be calculated on the assumption that the Hall current was of the surface rather than edge type. A detailed discussion of

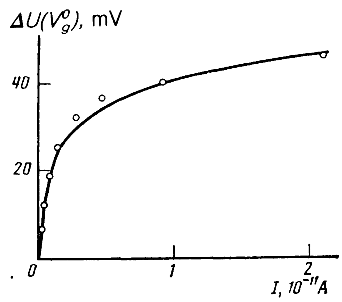

FIG. 4. Dependence of $\Delta U(V_g^0)$ on the current. The continuous curve is plotted in accordance with Eq. (5).

this problem was made in the review of Rashba and Timofeev.[12] Here we shall point out that this assumption has already been used above in a qualitative discussion, when we introduced the conductivities σ_{xx} and σ_{xy} in the quantum Hall effect regime. Another assumption which will be used later is the independence of σ_{xx} of the coordinates under linear conditions, i.e., it will be assumed that the two-dimensional electron system is ideal. In the case of our samples we observed a strong coordinate dependence of σ_{xx} under linear conditions,[13] so that the last assumption was invalid. A comparison of the calculated and experimental curves made below will show that the variation of σ_{xx} with position in the linear regime is of little importance under nonlinear conditions. We must stress once again that we are ignoring all other nonlinear effects apart from the dependence of σ_{xx} on the potential of a point in a two-dimensional electron layer. The calculations will differ somewhat for the cases of the Corbino geometry and for a Hall transistor with a long gate, which is why we shall consider these two cases separately.

1. MIS structure with the Corbino geometry

In view of the cylindrical symmetry the density of the current flowing along the radius in a two-dimensional electron layer is

$$I/2\pi r=-\sigma_{xx}(U)\,dU/dr, \tag{2}$$

where U is the potential difference between the gate and the layer at a point r. If the Fermi level ε_F is between Landau levels, the conductivity is activated[14]:

$$\sigma_{xx}(\varepsilon_F)=\sigma_0 e^{-\Delta/T}\operatorname{ch}(\varepsilon_F/T), \tag{3}$$

where the Fermi energy is measured from the midpoint between the Landau levels and Δ is the activation ennergy corresponding to $\varepsilon_F=0$. We find the relationship between U and ε_F using the experimental observation[14,15] that the density of states $D(\varepsilon_F)=D$ is independent of ε_F when $\varepsilon_F \ll \Delta$. (The last statement is clearly true in a narrow range of $\varepsilon_F \ll \Delta$, since a minimum of the density of states corresponds to $\varepsilon_F=0$.) Then

$$U-V_g^0=\nu\varepsilon_F,\quad \nu=eD/C_0. \tag{4}$$

Here, V_g^0 is the gate voltage corresponding to $\varepsilon_F=0$ and C_0 is the capacitance of the MIS structure normalized to a unit area. We shall consider the specific case when the voltage drop between the innner contact and the gate is given and the current flows from the outer to the inner contact. Then $U(r_1)=V_g$. Substitution of Eqs. (3) and (4) into Eq. (2) and integration gives

$$U(r)=V_g^0+\nu T\operatorname{Arsh}\left[\operatorname{sh}\frac{V_g-V_g^0}{\nu T}-\delta^{-1}\ln\frac{r}{r_1}\right],$$
$$\delta=2\pi\sigma_0 e^{-\Delta/T}\,\nu T I^{-1}. \tag{5}$$

Hence we can find the experimentally determined drain-source voltage

$$\Delta U=U(r_1)-U(r_2)$$
$$=V_g-V_g^0-\nu T\operatorname{Arsh}\left[\operatorname{sh}\frac{V_g-V_g^0}{\nu T}-\delta^{-1}\ln\frac{r_2}{r_1}\right]. \tag{6}$$

The maximum voltage between the drain and source for a given current I is

$$\Delta U_{max}=2\nu T\operatorname{Arsh}\left(\frac{\delta^{-1}}{2}\ln\frac{r_2}{r_1}\right). \tag{7}$$

An extremum corresponds to $V_g=V_g^0+\Delta U_{\max}/2$.

The transition to the linear case occurs for $\delta \gg 1$:

$$\Delta U\approx\nu T\delta^{-1}\ln(r_2/r_1)\propto I. \tag{8}$$

Consequently, in the linear case we have $\Delta U \ll \nu T$. Under nonlinear conditions we find that $\Delta U \gtrsim \nu T$ and the conductivity depends strongly on position:

$$\sigma(r)=\sigma_0 e^{-\Delta/T}\left[1+\left(\operatorname{sh}\frac{V_g-V_g^0}{\nu T}-\delta^{-1}\ln\frac{r_2}{r_1}\right)^2\right]^{1/2}. \tag{9}$$

A minimum of $\sigma_{xx}(r)$ occurs at $r=r_1$ if $V_g<V_g^0$ and at

$$r_{min}=r_1\exp\{\delta\operatorname{sh}[(V_g-V_g^0)/\nu T]\},$$

if $V_g>V_g^0$. A filament of the Hall current flowing along a circle is located near the point $r_{\min}$ and δ describes the relative width of the filament. The current density in the filament is less than

$$j_\varphi=(I/2\pi r_{min})\sigma_{xy}/\sigma_0 e^{-\Delta/T}. \tag{10}$$

2. Hall transistor with a long gate

In this case a two-dimensional electron current is analogous to that shown in Fig. 1a, but the current flows along the y axis and the external source sets the total current I. We shall assume that the transistor size in the y direction is considerably greater than the channel width d. We then have

$$\partial E_x/\partial y=\partial E_y/\partial x\approx 0,\quad E_y=E=\text{const}. \tag{11}$$

The experimentally determined quantity is the electric field E_y, which depends on I and V_g. The electric field is related to the current density j_y:

$$E_x=\rho_{xy}j_y,\quad \rho_{yy}=\rho_0 e^{-\Delta/T}\operatorname{ch}(\varepsilon_F/T),$$
$$E_y=\rho_{yy}j_y,\quad U-V_g^0=\nu\varepsilon_F. \tag{12}$$

At low values of ε_F the quantity ρ_{xy} is independent of ε_F and, consequently, it is independent of the potential difference between the gate and the electron layer.

Expressing E_x in Eq. (12) in terms of E_y and integrating the resulting relationship, we find the dependence of the potential U on the coordinate x (at $x=0$, we have $U=V_g$):

$$U=\nu T\operatorname{Arsh}\left[\operatorname{sh}\frac{V_g-V_g^0}{\nu T}+\frac{\rho_{xy}xEe^{\Delta/T}}{\nu T\rho_0}\right]+V_g^0. \tag{13}$$

Since the total current is given in the direction y, we have

$$U(d)-U(0)=\rho_{xy}I. \tag{14}$$

Therefore, the experimentally determined quantity is described by

$$E=\frac{\nu T\rho_0 e^{-\Delta/T}}{\rho_{xy}d}\left\{\mathrm{sh}\left[\frac{V_g-V_g^0}{\nu T}+\rho_{xy}\frac{I}{\nu T}\right]-\mathrm{sh}\frac{V_g-V_g^0}{\nu T}\right\}. \tag{15}$$

The current density j_y, representing essentially the Hall current ($E_y \ll E_x$), varies greatly as a function of the coordinate x. The above calculations ignore the dependence of the potential U on the coordinate y. This is permissible for the middle part of the sample when $E_y \ll E_x$.

DISCUSSION

Since our experimental results apply only to samples in the Corbino geometry, we shall first consider this case. We can easily see that Eq. (6) provides a qualitative description of all the characteristic features of the experimental dependences $\Delta U(V_g)$. In fact, in the range of gate voltages such that

$$\delta^{-1}\ln(r_2/r_1)\gg \mathrm{sh}[(V_g-V_g^0)/\nu T], \tag{16}$$

we find that the derivative is $\partial\Delta U/\partial V_g=1$. It follows from the calculations that the separation between the extrema observed for different polarities of the current I is $\Delta U_{\max}$, which is again in full agreement with the experimental results. Finally, calculations describe the change in the symmetry of the experimental curve due to a change in the direction of the current I.

The dependence $\Delta U_{\max}(I)$ can be used to find the density of states half-way between two Landau levels. It follows from Eq. (7) that for the correct value of ν the dependence of $\sinh(\Delta U_{\max}/2\nu T)$ on the current should be linear. The experimental points fit best (with the smallest relative scatter) a straight line corresponding to $D=1.05\times10^{13}$ cm$^{-2}\cdot$eV^{-1} (see Fig. 5). We then have the ratio $D/D_0=6.6\times10^{-2}$, where D_0 is the density of states in the absence of a magnetic field: $D_0=2m/\pi\hbar^2$.

Measurements of $\Delta U_{\max}$ in various magnetic fields for a fixed current I can be used to find the activation energy Δ.

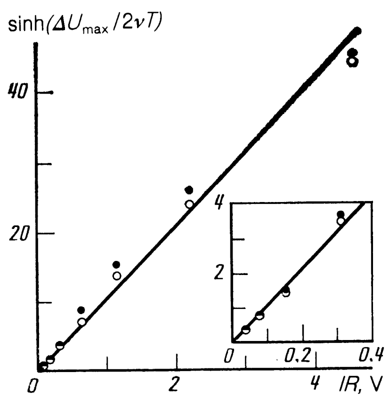

FIG. 5. Dependence of $\sinh(\Delta U_{\max}/2\nu T)$ on the current I. The positions of the experimental points are given for $D=6.6\times10^{-2}D_0$ (where D_0 is the density of states in the absence of a magnetic field). The open and black symbols corresponds to different directions of the current through the sample.

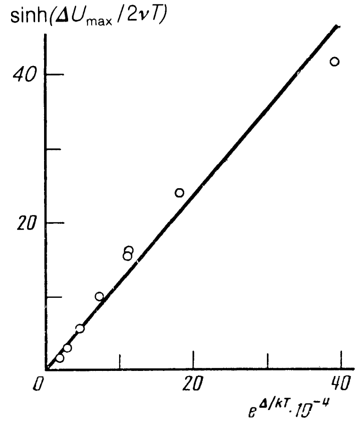

FIG. 6. Procedure used in the determination of Δ. The best positions (characterized by the smallest scatter) relative to a straight line are shown.

We applied this procedure subject to the simplest assumption that the density of states in the range of magnetic fields investigated depends weakly on H and that $\Delta\propto\hbar\omega_c$. The experimental points representing the dependence of $\sinh(\Delta U_{\max}/2\nu T)$ on $e^{\Delta/T}$ (Fig. 6) fit best a straight line if $\Delta=20$ K corresponding to $H=10$ T, in good agreement with the results of other investigations.[14,16] The slope of the straight line in Fig. 6 gives $\sigma_0=1.7\times10^{-6}\ \Omega^{-1}$.

Figure 2 shows the curves plotted using Eq. (6) for three values of the current I using the parameters given above. A comparison of the experimental and calculated curves shows that the experimental dependences have a smaller drop $\Delta U(V_g)$ than the calculated results. It is possible that this is a consequence of the dependence $\sigma_{xx}(r)$ under linear conditions.

Figure 7 shows the distribution of the Hall current

$$j_\varphi(r)=I\sigma_{xy}/2\pi r\sigma_{xx}(r) \tag{17}$$

in the radial direction found using Eq. (8) employing the parameters σ_0, Δ, and D obtained above from the experimental data. It is clear from this figure that a Hall current filament is exceptionally narrow under experimental conditions. Depending on V_g, it may be located at one of the edges of a sample or at its center. The widest, as a function of V_g, is the region when the Hall current filament is located at that edge of the sample relative to which the gate voltage is set.[1)]

The nonlinear characteristics in the prebreakdown region have been determined so far for the Hall transistor geometry using GaAs–$Al_xGa_{1-x}As$ heterostructures. It is not possible to compare the experimental data directly with Eq. (15), because in the case of heterostructures the relationship between U and ε_F is different than that given by Eq. (4). However, the experimentally observed exponential rise of the electric field with increasing current I or decreasing temperature is described qualitatively by Eq. (15) with $V_g=V_g^0$.

We shall conclude with the following comment. The relationship of σ_{xx} or ρ_{xx} to the voltage drop U was derived

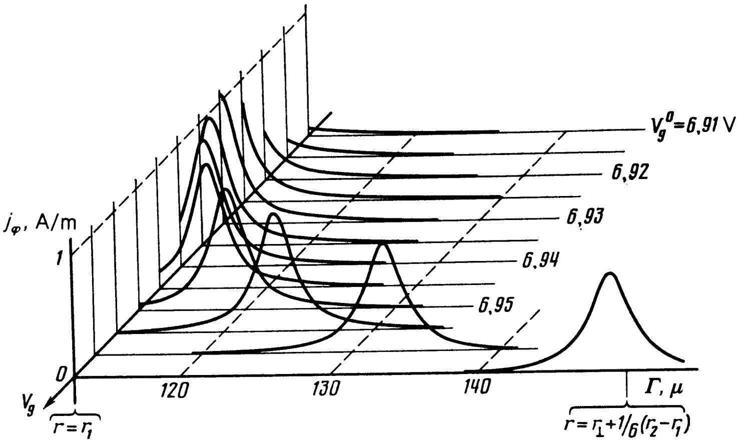

FIG. 7. Distribution of the Hall current along the ordinate r obtained for different gate voltages. The parameters σ_0, Δ, and D were determined from the experimental results (see Figs. 5 and 6); $I=2.1\times10^{-11}$ A.

using the formula for the parallel-plate capacitor [Eq. (4)]. This is justified as long as the characteristic scale (distance) in which an electric field peak is observed and, consequently, the width of the current filament are large compared with the thickness of the insulator in the MIS structure. It follows from Eqs. (5)–(9) or (13) that the filament can be made as narrow as we please by increasing the current. However, it follows from the above discussion that even in the absence of another physical mechanism limiting such reduction in the current filament width, we can expect Eqs. (5)–(9) and (13) to become invalid when the filament width becomes comparable with the thickness of the insulator in the MIS structure.

The authors are deeply grateful to V. F. Gantmakher for valuable discussions and advice.

[1)] It should be pointed out that filamentation of the current does not depend on the specific form of the dependence $\sigma_{xx}(V_g)$ [Eqs. (3) and (4)], but is the result of a deep minimum of this dependence. This choice also does not affect greatly the existence of a region in which $\Delta U(V_g)$, has a derivative equal to unity.

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