Tunnel-activated motion of a string across a potential barrier

B. I. Ivlev and V. I. Mel'nikov

L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR, Chernogolovka, Moscow Province

(Submitted 18 March 1986)
Zh. Eksp. Teor. Fiz. **91**, 1944–1954 (November 1986)

A study is made of the temperature dependence of the coefficient representing penetration of a weakly asymmetric semiclassical potential barrier by a string. At low temperatures the barrier is overcome by tunneling, at the temperature $T_0 \sim F$ this mechanism changes to tunnel-activated, and at $T > T_c \sim F^{1/2}$ the motion across the barrier results from activation processes (F is the asymmetry parameter of the barrier, which obeys $F \ll 1$). Determination of the optimal paths of a string in imaginary time is analogous to the problem of the shape of a critical nucleus in the theory of phase transitions. Equations are derived for the boundaries of a nucleus allowing for the interaction of boundaries with one another and expressions describing the change of regimes near T_0 and T_c are obtained. The influence of an hf field on the tunneling of a string at zero temperature is investigated.

1. INTRODUCTION

Calculation of the probabilities of tunnel-activated decay of metastable states has been the subject of many papers. In a fairly general form this problem was first investigated by Lifshitz and Kagan,[1] who extended the concept of quantum-mechanical tunneling to a system with macroscopic degrees of freedom. A similar problem was also solved by Iordanskiĭ and Finkel'shteĭn[2] for a more specific situation.

The present paper deals with decay of a metastable state of a string in a potential barrier. This model is of direct relevance to dislocations in a crystal, in which the velocity of tunnel-activated motion was investigated in detail by Petukhov and Pokrovskiĭ,[3] and to the motion of charged density waves in a Peierls insulator.[4] The tunneling of a string at zero temperature is analogous to the quantum formation of nuclei in phase transitions.[1,2,5] The same mechanism results in decay of metastable vacuum.[6] At finite temperatures the rate of decay increases because of thermal activation.

Our aim will be to determine in detail the temperature dependence of the coefficient representing the penetration of a string across a potential barrier. We shall assume the asymmetry of the potential to be weak, which makes it possible to solve the problem completely by employing the macroscopic approach in which a nucleus of a new phase with large dimensions is described only by the position of its boundary. We shall derive an equation for the position of the boundary of a nucleus allowing for the interaction with adjacent boundaries.

The penetration coefficient is known to be described with exponential accuracy by the following expression:

$$D = \exp\left[-\int_{-\frac{1}{2}T}^{\frac{1}{2}T} L\left\{\frac{\partial y}{\partial \tau}, y\right\} d\tau\right],$$

which corresponds to the maximum, as a function of the energy ε, of the product of the Gibbs factor $\exp(-\varepsilon/T)$ and of the tunnel factor $\exp[-S(\varepsilon)]$, where $S(\varepsilon)$ is the action for subbarrier motion. The energy ε is defined by the equation $\partial S/\partial\varepsilon = -1/T$ (Ref. 1).

The Lagrangian of a string is

$$L = \int_{-\infty}^{\infty} \left[\frac{\rho}{2}\left(\frac{\partial y}{\partial \tau}\right)^2 + \frac{\varkappa}{2}\left(\frac{\partial y}{\partial x}\right)^2 + U_0 U\left(\frac{y}{a}\right) - F U_0 \frac{y}{a} - U_0 E\right] dx.$$

Here, $U(z)$ is a symmetric function of the order of unity with minima at the points $z = \pm 1$ and shown by the dashed curve in Fig. 1. The continuous curve represents the potential $V(z) = U(z) - Fz - E$. The constant energy shift E is introduced in order to reduce to zero the potential at the bottom of the initial valley. Usually potentials of the form $U(z) = (1 - z^2)^2/2$ and $U(z) = \cos^2(\pi z/2)$ are used. The quantity U_0 sets the height of the potential barrier; since F is finite, the degeneracy of the energy minima is lifted. Here $y(x,\tau)$ should be the solution of the classical equation of motion for an imaginary time τ, governed by the condition that the moments in time $(1/2)T$ correspond to the turning points of the path. Retaining the notation x and τ, we shall measure these quantities in units of

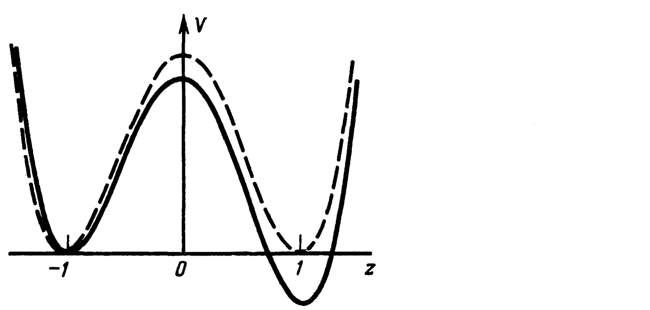

FIG. 1. Cross section of the potential relief $V(z)$. The dashed curve is the unperturbed potential $U(z)$.

 0038-5646/86/111153-06$04.00

$$x_0=a(\varkappa/U_0)^{1/2}, \quad \tau_0=a(\rho/U_0)^{1/2}, \tag{1}$$

and the temperature T in units of $1/\tau_0$. Then, the penetration coefficient becomes

$$D=\exp[-2a^2(\varkappa\rho)^{1/2}A],$$

where

$$A=\int_0^{1/2T}d\tau\int_{-\infty}^{\infty}dx\left[\frac{1}{2}\left(\frac{\partial z}{\partial\tau}\right)^2+\frac{1}{2}\left(\frac{\partial z}{\partial x}\right)^2+U(z)-Fz-E\right]. \tag{2}$$

Here, we have introduced $z=y/a$.

We can see that the semiclassical action is proportional to $V/\omega\gg 1$, where—as deduced from the relationships in Eq. (1)—we have $\omega\propto\tau_0^{-1}$ and V is proportional to the energy of a kink: $V\propto a(\varkappa U_0)^{1/2}$. The equation for the classical path $z(x,\tau)$ is obtained by variation of the action (2) and is of the form

$$\partial^2z/\partial\tau^2+\partial^2z/\partial x^2-U'(z)+F=0 \tag{3}$$

subject to the boundary condition

$$\partial z(x,\tau)/\partial\tau|_{\tau=0,\,1/2T}=0. \tag{4}$$

We are thus faced with a fairly complicated problem requiring the solution of a nonlinear equation in partial derivatives in a limited region. The present paper is concerned with the situation in which the neighboring minima have similar energies, which corresponds to the condition $F\ll 1$. Consistent use of a small parameter F makes it possible to calculate the temperature dependence of the action $A(T,F)$ throughout the temperature range investigated. It is remarkable that in this situation it is possible to identify some fine details of the behavior of this solution in the $x\tau$ plane.

The present paper is organized as follows. In Sec. 2 we shall consider the solution at zero temperature, characterized by a radial symmetry relative to the origin of the coordinates. In this case we have $z=1$ in a circle with large radius of the order to $1/F$, outside this circle we have $z=-1$, and the thickness of the transition layer between these two regions is of order unity. In Sec. 3 we shall discuss temperatures $T>T_0\sim F$, when the initial circle no longer fits a band of width $1/T$ in the $x\tau$ plane. In this case a nucleus is bounded by two arcs of the initial circle intersecting at points $\tau=\pm 1/2T$. The action A is now a universal function of the ratio T/F and the barrier is overcome by tunnel activation. The interaction of boundaries of a nucleus, resulting in changes of their curvature, is discussed in Sec. 4. An equation is derived for the shape of the boundary of a nucleus. The transition from the tunnel to the tunnel-activated regime near the temperature T_0 is discussed in detail in Sec. 5. Section 6 deals with the high-temperature limit $T_0\ll T\sim F^{1/2}$ and it is shown that at $T>T_c\sim F^{1/2}$ the barrier is overcome by purely activation process. The influence of an hf field on the tunneling of a string at zero temperature is considered in Sec. 7.

2. TUNNELING AT ZERO TEMPERATURE

At $T=0$ we have to seek the solution of Eq. (3) in the form of a centrally symmetric function which automatically satisfies the boundary condition (4). In polar coordinates, Eq. (3) becomes

$$\frac{d^2z}{dr^2}-U'(z)=-\frac{1}{r}\frac{dz}{dr}-F, \quad r^2=x^2+\tau^2. \tag{5}$$

The situation resembles the problem of finding a nucleus for first-order phase transition. The condition $F\ll 1$ corresponds to a small difference between the specific energies of two phases. Consequently, the function $z(r)$ should describe a thin-walled large-radius nucleus. The problem can therefore be tackled in two stages: finding of the structure of the boundary and determination of the radius of a nucleus R. Ignoring the right-hand side of Eq. (5), we obtain the equation

$$d^2z_0/dn^2-U'(z_0)=0, \quad n=r-R, \tag{6}$$

which describes the structure of the boundary. Allowance for the right-hand side of Eq. (5) then makes it possible to find the radius R of a nucleus. We shall initially consider the specific case when the line $z(x,\tau)=0$ can be regarded as the boundary of a nucleus. The structure of the boundary is given by the first integral of Eq. (6):

$$(dz_0/dn)^2=2U(z_0). \tag{7}$$

The radius of a nucleus can be found if we multiply Eq. (5) by dz_0/dr and integrate with respect to r from zero to infinity. Then, the integral of the left-hand side vanishes and on the right-hand side we can substitute $r=R$ and take it outside the integral. The result is then

$$R=\alpha/F, \tag{8}$$

where α is a number of order unity and is governed by the nature of potential:

$$\alpha=\int_0^1[2U(z)]^{1/2}dz$$

In the same approximation we find that the action of Eq. (2) is[6–9]

$$A=\pi\alpha^2/F. \tag{9}$$

The dependence of z_0 on r is shown schematically in Fig. 2.

In contrast to the tunneling of a particle, the penetration of a string across a barrier is entirely due to lifting of the degeneracy between the potential energy minima (Fig. 1) because F is finite. The point is this: the reduction in the energy of a string because of the formation of two kinks crossing a barrier of order unity should be compensated by the energy increase $RF\sim 1$ because part of a string of length $2R$ drops to the lower minimum.

3. TUNNEL-ACTIVATED REGIME

As shown above, in the limit of low temperatures a nucleus is a circle of radius α/F. The influence of the boundary

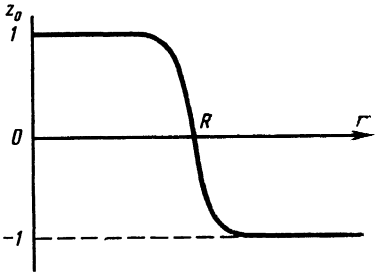

FIG. 2. Radial dependence of the function z_0 at zero temperature.

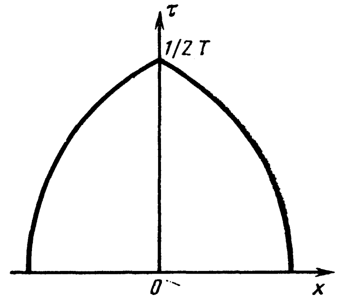

FIG. 3. Shape of the boundary of nucleus [curve $z(x,\tau) = 0$] in the tunnel-activated regime.

condition (4) on the shape of a nucleus can be ignored as long as the straight line $\tau = 1/2T$ lies sufficiently far from the circular boundary. At a temperature

$$T_0 = F/2\alpha \tag{10}$$

this straight line touches the boundary of the circle and at higher temperatures the circular solution becomes invalid. In this case the passage of a string across a barrier is determined by the simultaneous action of the tunneling and activation mechanisms. We shall calculate later the dependence $A(T,F)$ using the macroscopic approach based on the fact that the thickness of the boundary of a nucleus is considerably less than its size. Variation of z along the normal n to the boundary is described implicitly by an expression which follows from Eq. (7):

$$n = -\int_0^z [2U(y)]^{-1/2}\,dy.$$

Using this expression to integrate with respect to n in the equation for the action (2), we obtain a functional which contains only the shape of the boundary[8]:

$$A = 2(\alpha l - FS), \tag{11}$$

where l and S are the length of the boundary and the area of the nucleus (Fig. 3). If the boundary is described by the curve $x(\tau)$, then the action of Eq. (11) can be written in the form[6–9]

$$A = 4\int_0^{1/2T} \{\alpha[1 + (dx/d\tau)^2]^{1/2} - Fx\}\,d\tau. \tag{12}$$

The condition for an extremum of the action (12) shows that the boundary of a nucleus is still an arc of a circle of radius α/F. This arc approaches normally the axis x at $\tau = 0$ and the height of the arc is $1/2T$ (Fig. 3), which fixes the length of the arc l. An extremum of the action (11) corresponds to the minimum area of the nucleus when the two arcs join at $\tau = 1/2T$. This gives[10]

$$A(T,F) = \frac{2\alpha^2}{F}\left\{\arcsin\frac{F}{2\alpha T} + \frac{F}{2\alpha T}\left[1 - \left(\frac{F}{2\alpha T}\right)^2\right]^{1/2}\right\}. \tag{13}$$

In limiting cases we obtain

$$A(T,F) = \frac{\pi\alpha^2}{F}\left[1 - \frac{2^{7/2}}{3\pi}\left(1 - \frac{F}{2\alpha T}\right)^{3/2}\right], \quad 1 - \frac{F}{2\alpha T} \ll 1. \tag{14}$$

$$A(T,F) = \frac{2\alpha}{T} - \frac{F^2}{12\alpha T^3}, \quad T \gg F, \tag{15}$$

where the first formula gives the small tunnel-activation correction to the tunneling probability. On the other hand, the main term in the second formula corresponds to the activation mechanism and the correction is due to the tunneling.

The expressions (9) and (13) cover the full temperature interval and are matched at $T_0 = F/2\alpha$. The use of the macroscopic approximation postulates a small curvature of the boundary and is therefore known to be inapplicable at the point where the boundary is a sharp bend (Fig. 3). The vicinity of this bend makes a relatively small contribution to the action so that Eqs. (9) and (13) are valid far from the point $T = T_0$. Determination of the action in the vicinity of this point requires a more detailed study of the shape of the boundary of a nucleus. The result is a formula which reduces to Eqs. (9) and (14) as we move away from T_0.

An approach of the same type is suitable for considering the high-temperature regime when the boundaries of a nucleus approach so closely that the interaction between them becomes important and the tunnel correction to the activation law differs from that given by Eq. (15).

4. INTERACTION OF THE BOUNDARIES OF A NUCLEUS

In this section we treat in a consistent manner the problem of determination of the shape of a nucleus on the assumption that the asymmetry parameter is small, $F \ll 1$. In this situation we can use the macroscopic analysis when a nucleus is represented by the position of the boundary $x = x(\tau)$ determined by the condition $z(x,\tau) = 0$ and the boundary thickness is considerably less than typical dimensions of the nucleus. The problem can therefore be divided into two parts: determination of the structure of the boundary in a narrow nonlinear region and derivation of the equation for $x(t)$.

The boundary of a nucleus can be described by its radius of curvature $R(\varphi)$, where φ is the angle between the normal to the boundary and the τ axis. When the angle φ is specified, the position of a point on the boundary is known and we shall denote the shift along the normal to the boundary by n. In accordance with this definition, the family of curves $n = \text{const}$ is set of evolvents of the same curve. In terms of the coordinates n and φ, Eq. (3) becomes

$$\frac{\partial^2 z}{\partial n^2}+\frac{1}{R(\varphi)+n}\frac{\partial z}{\partial n}+\frac{1}{[R(\varphi)+n]^2}\frac{\partial^2 z}{\partial\varphi^2}$$
$$-\frac{1}{[R(\varphi)+n]^3}\frac{dR}{d\varphi}\frac{\partial z}{\partial\varphi}-U'(z)+F=0. \quad (16)$$

This equation contains small parameters F and $1/R$. In zeroth order with respect to these parameters the solution of Eq. (16) is determined by the quadrature of Eq. (7). In next order we have $z = z_0 + z_1$ and the small correction z_1, linear F and $1/R$, is found from the equation

$$\frac{\partial^2 z_1}{\partial n^2}-U''(z_0)z_1=-\frac{1}{R(\varphi)}\frac{dz_0}{dn}-F. \quad (17)$$

We shall need the solution of this equation which is finite inside the nucleus and grows exponentially in the external region. We shall assume that $z_1 = -1 + \psi$ and then the asymptotic behavior of ψ for $n \gg 1$ is obtained from Eq. (17):

$$\psi=\frac{1}{\beta\gamma^2}\left[\frac{\alpha}{R(\varphi)}-F\right]e^{\gamma n}+\beta e^{-\gamma n}, \quad (18)$$

where the second term originates from z_0. The number α is defined above, and the other quantities are given by

$$\beta=\exp\left\{\int_0^1\left[\frac{\gamma}{(2U(z))^{1/2}}-\frac{1}{1-z}\right]dz\right\}, \quad \gamma=(U''(1))^{1/2}.$$

At absolute zero ($T=0$) the boundary satisfies the condition (4) and shifts to infinity and the radius of curvature of the boundary of a nucleus is given by Eq. (8), which corresponds to the vanishing of the coefficient of the increasing exponential function. The function ψ obeys a linear equation

$$\Delta\psi-\gamma^2\psi=0,$$

the boundary conditions for which (ψ and $\delta\psi/\delta n$ at $n = 0$) can be seen, on the basis of Eq. (18), to be governed by R and F.

A nucleus can be regarded as symmetric relative to the $\tau = 1/2T$ line. The shape of a nucleus should then be selected so that the increasing and decreasing solutions determined near the boundary by Eq. (18) are matched to the corresponding asymptote at the next boundary. In the case of practical importance for us it is sufficient to consider parts of boundaries close to two parallel lines. In particular, at high temperatures $T \gg F$ the boundaries of a nucleus are almost parallel to the τ axis. Ignoring terms of the order of

$$x(dx/d\tau)^2 \ll 1 \quad (19)$$

we find that the distance along the normal from the boundary to the τ axis can be replaced by the projection of the normal along the x axis. Matching of the increasing and decreasing solutions of the type given by Eq. (18) then gives an equation for $x(\tau)$:

$$\frac{d^2x}{d\tau^2}+\frac{F}{\alpha}=\frac{\beta^2\gamma^2}{\alpha}e^{-2\gamma x}, \quad (20)$$

where an allowance is made for the fact that in the adopted approximation we have $1/R = -d^2x/d\tau^2$. The boundary condition for Eq. (20) is

$$dx/d\tau|_{\tau=0,\,1/2T}=0.$$

Equation (20) is valid throughout the full range of τ from zero to $1/2T$. In accordance with the condition (19), the range of validity of Eq. (20) is given by the inequality

$$T \gg F[\ln(1/F)]^{1/2}. \quad (21)$$

When the distance between the boundaries is large, Eq. (20) defines a line of constant curvature F/α. This equation also has a solution corresponding to parallel boundaries separated by a distance

$$x_\parallel=\frac{1}{2\gamma}\ln\frac{\beta^2\gamma^2}{F}.$$

When temperature is increased, $x(1/2T)$ increased and $x(0)$ decreases, tending to $x_\parallel$. If the temperature obeys $T \ll [F/\ln(1/F)]^{1/2}$, then if we ignore the right-hand side of Eq. (20) we obtain the following expression describing the interaction between the boundaries:

$$x=\frac{F}{2\alpha}\left(\frac{1}{4T^2}-\tau^2\right), \quad x \gg x_\parallel, \quad (22)$$

which represents the initial part of a circle. If the boundaries approach one another, we can ignore the term F/α in Eq. (20). This gives

$$x=\frac{1}{\gamma}\ln\left\{\frac{2T}{F}\beta(\alpha\gamma)^{1/2}\operatorname{ch}\left[\frac{\gamma}{2\alpha}\frac{F}{T}\left(\frac{1}{2T}-\tau\right)\right]\right\},$$
$$x \ll x(0). \quad (23)$$

Equations (22) and (23) are matched in the common range of validity, the existence of which follows from the inequality $x_\parallel \ll x(0)$. At $x(0)$ and $x(1/2T)$, we find that Eqs. (22) and (23) yield

$$x(0)=\frac{F}{8\alpha T^2}, \quad x\left(\frac{1}{2T}\right)=\frac{1}{\gamma}\ln\left[\frac{2T}{F}\beta(\alpha\gamma)^{1/2}\right].$$

It is clear from the above solution that the boundary has no sharp bends and approaches normally the straight lines $\tau = 0,\ 1/2T$. The boundary consists of two symmetrically located smooth curves (Fig. 4b) separated from one another by the distance $2x(1/2T)$, which subject to the selected restriction given by Eq. (21) is large compared with unity. Our

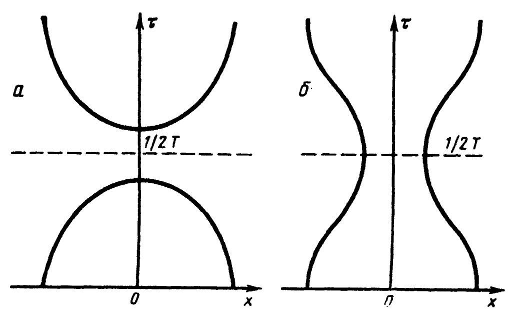

FIG. 4. Interacting boundaries of a nucleus: a) in the vicinity of T_0; b) in the vicinity of T_c.

results thus show that the macroscopic approach is applicable to determination of the shape of the whole nucleus when the boundaries intersect at a small angle if the interaction between them is ignored.

In the other limiting case of temperatures close to T_0 such noninteracting boundaries intersect at an angle close to π (Fig. 4a). In the vicinity of this point we can apply a similar analysis. An equation analogous to Eq. (20) is

$$\frac{d^2\tau}{dx^2}+\frac{F}{\alpha}=-\frac{\beta^2\gamma^2}{2}\exp\left[2\gamma\left(\tau-\frac{1}{2T}\right)\right]. \qquad (24)$$

It is valid near the vertex of a nucleus (Fig. 4a) and describes the transformation of its boundary into a circle as we move away from the point $\tau = 1/2T$. Equation (24) is valid from $T=0$ right up to temperatures defined by the inequality $0<T-T_0\ll T_0$.

At intermediate temperatures $T\gtrsim T_0$, when the solution exhibits bifurcation (transition from Fig. 4a to Fig. 4b), the boundaries of a nucleus at a point $\tau = 1/2T$ approach each other to a distance of order unity and the macroscopic approximation is invalid.

5. CHANGES OF REGIMES NEAR $T=T_0$

If we ignore the interaction between the boundaries, we find that the action $A(T,F)$ is described by different functions for $T\leqslant T_0$ [see Eq. (9)] and for $T\geqslant T_0$ [see Eq. (14)]. In fact, a singularity at the point $T = T_0$ disappears when we allow for the interaction between the boundaries of a nucleus.

The energy integral of Eq. (24) is

$$\frac{1}{2}\left(\frac{d\tau}{dx}\right)^2+\frac{F\tau}{\alpha}+\frac{\beta^2\gamma}{\alpha}\exp\left[2\gamma\left(\tau-\frac{1}{2T}\right)\right]=1, \qquad (25)$$

which away from $\tau = 1/2T$ gives the unperturbed shape of the boundary. The contribution to the action by a curved part of the boundary can easily be restored by writing the Lagrangian corresponding to the energy of Eq. (25) so that away from the point $\tau = 1/2T$ it reduces to the integrand in Eq. (12) when x is replaced with τ. For τ close to $1/2T$ the Lagrangian is

$$L=2\alpha\left[\frac{1}{2}\left(\frac{d\tau}{dx}\right)^2-\frac{F\tau}{\alpha}-\frac{\beta^2\gamma}{2\alpha}\exp\left[2\gamma\left(\tau-\frac{1}{2T}\right)\right]+1\right\}.$$

Calculating the action subject to Eq. (25), we obtain

$$A=\frac{\pi\alpha^2}{F}+\frac{2}{\gamma}\left(\frac{\alpha F}{\gamma}\right)^{1/2} f\left[\frac{4\alpha^2\gamma^2(T-T_0(F))}{F^2}\right], \qquad (26)$$

where the following corrected temperature is introduced [compare with Eq. (10)]:

$$T_0(F)=\frac{F}{2\alpha}-\frac{F^2}{4\alpha^2\gamma}\ln\frac{\beta^2\gamma^2}{F}.$$

The function f is defined as follows:

$$f(p)=\int_0^\infty [\theta(x-x_1)(x-e^{p-x})^{1/2}-x^{1/2}]\,dx, \qquad p\equiv x_1+\ln x_1,$$

and in the limiting cases this function becomes

$$f=-{}^1/_2\pi^{1/2}e^p, \quad p\to-\infty,$$

$$f=-{}^2/_3 p^{3/2}, \quad p\to\infty.$$

It therefore follows that Eq. (26) is matched to Eq. (14) above the temperature T_0 and it reduces to Eq. (9) at low temperatures. The width of the transition region is

$$|T-T_0(F)|\sim F^2.$$

6. TRANSITION TO THE ACTIVATION REGIME

We have studied above the transition from the purely tunnel crossing of a barrier to the tunnel-activated mechanism and we have obtained Eqs. (9), (13), and (26) for the action. At higher temperatures the size of a nucleus decreases. Exactly as in the vicinity of the temperature T_0, there is an increase in the relative contribution to the action associated with the interaction between the boundaries of a nucleus. The shape of the boundaries is then given by Eqs. (22) and (23).

Adopting, as before, the Lagrangian corresponding to the equation of motion (20), we find that the action is described by

$$A=4\alpha\int_0^{1/2T}\left[1+\frac{1}{2}\left(\frac{dx}{d\tau}\right)^2-\frac{Fx}{\alpha}-\frac{\beta^2\gamma}{2\alpha}e^{-2\gamma x}+\frac{F}{2\alpha\gamma}\ln\frac{e\beta^2\gamma^2}{F}\right]d\tau.$$

The constant term in the Lagrangian ensures that the minimum value of the potential energy vanishes. The final expression for the action is

$$A=\frac{2\alpha}{T}-\frac{2}{\gamma}\left(\frac{\alpha F}{\gamma}\right)^{1/2}\Phi\left(\frac{T}{T_c}\right), \qquad (27)$$

where the implicit function $\Phi(q)$ is given by the relationship

$$\Phi(q)=\frac{\pi Q}{2^{1/2}q}-\int_{x_1}^{x_2}(Q-x-e^{-x-1})^{1/2}\,dx,$$

$$\frac{2^{1/2}\pi}{q}=\int_{x_1}^{x_2}(Q-x-e^{-x-1})^{-1/2}\,dx.$$

Here, x_1 and x_2 are zeros of the radicand. The value of T_c is given by $T_c = \pi^{-1}(\gamma F/2\alpha)^{1/2}$. The asymptotic forms of the function Φ are

$$\Phi(q)=\frac{1}{3\cdot 2^{3/2}}\left(\frac{\pi}{q}\right)^3, \quad q\ll 1,$$

$$\Phi(q)=3\cdot 2^{1/2}\pi(1-q)^2, \quad 1-q\ll 1.$$

The first limiting case corresponds to matching with Eq. (15) in the region $T_c\ll T\ll T_c$. At temperatures close to T_c, when $T_c - T\ll T_c$, and also at all temperatures above T_c, we obtain

$$A=\frac{2\alpha}{T}-\frac{12\pi}{\gamma}\left(\frac{\alpha F}{2\gamma}\right)^{1/2}\left(1-\frac{T}{T_c}\right)^2\theta(T_c-T),$$

from which it follows that at temperatures $T>T_c$ the overcoming of a barrier is a pure activation process.

Therefore, Eqs. (13), (26), and (27) describe the action $A(T,F)$ at all temperatures.

7. TUNNELING UNDER THE INFLUENCE OF AN ALTERNATING FIELD

We shall assume that in addition to a static field F there is also a small alternating component $F_1 \cos \Omega t$ and we shall assume that the absolute temperature is zero. As before, we shall discuss the semiclassical situation when the penetration coefficient can be represented in the form

$$D=\exp(-A_0+A_1 \cos \Omega t),$$

where the correction to the action A should be small compared with A_0, but large compared with unity. We can find A_1 by a method developed in Refs. 11 and 12. Following these methods, we find that A_1 is given by

$$A_1=\frac{iF_1}{2}\int_C dt \int_{-\infty}^{\infty} dx z_0(x,t)\cos(\Omega t+\varphi). \tag{28}$$

Here, $z_0(x,t)$ is the solution of Eq. (10) where $n=(x^2-t^2)^{1/2}-\alpha/F$. The contour C at zero temperature follows the imaginary axis and is closed in a distant region of the left- and half-space. The integral over the whole contour is finite although the contribution by the imaginary axis diverges. The constant phase φ in Eq. (28) is selected from the condition that A_1 should be maximal.

We shall consider only the potential

$$U(z)=(1-z^2)^2/2,$$

where $z_0(x,t)=\tanh[(x^2-t^2)^{1/2}-2/3F]$. When considered as a function of t, this solution has poles at points t_k, which are described by the following expression for low values of x:

$$t_k=\pm\frac{2i}{3F}\left(1-\frac{9F^2x^2}{8}\right)-\frac{\pi}{2}(1+2k)\left(1+\frac{9}{8}F^2x^2\right),$$

where k is an integer. Integrating Eq. (28) with respect to time by the method of residues, we can see that in the limit $\Omega \gg F$ the values $x\sim(\Omega F)^{-1/2}$ are important and this justifies the expansion (29). Summing with respect to k from zero to infinity, we obtain the following expression for A_1:

$$A_1=\pi F_1\left(\frac{\pi}{3\Omega F}\right)^{1/2}\frac{\exp(2\Omega/3F)}{\sin(\pi\Omega/2)}.$$

Two features of this result should be noted. The field F_1 is exponentially enhanced because the tunnel motion time under the barrier is proportional to $1/F$ and it is long compared with the field period $1/\Omega$. The resonance denominator is related to the motion of a string in the classical region where in our units the frequency of small oscillations is $\omega=2$.

It is important to note that A_1 is calculated allowing implicitly for the structure of the boundary of a nucleus, since the integral (28) is governed by characteristic features of the function z_0 in the complex time plane. The use of the effective Lagrangian of the one-dimensional problem (12) when a nucleus is described only by the shape of its boundary $x(\tau)$, gives the same exponential dependence as the above equation but it cannot used to reproduce the resonance denominator of this equation. This macroscopic approach to the solution of the time-dependent problem was used in Ref. 7.

8. CONCLUSIONS

The problem of overcoming of a barrier by a string is much more difficult than the one-particle problem because an extremal path is described by a partial differential equation. A considerable simplification of the problem occurs in the case of a weakly asymmetric potential when a nucleus is large compared with the thickness of its walls. As shown above, the use of the fact that the asymmetry of the potential is weak ($F \ll 1$) allows us to calculate the penetration coefficient at all temperatures. When temperature is $T\sim F$, the tunneling chages tothe tunnel-activated mechanism and at $T>T_c\sim F^{1/2}$ the barrier is overcome by activation alone. The temperature dependences are described by universal functions of temperature, of the asymmetry parameter F, and of three parameters α, β, and γ governed by the shape of the unperturbed potential barrier. This answer is in a sense simpler than for the tunneling of a single particle, when the temperature dependence of the penetration coefficient is not universal but is governed by the actual shape of the barrier.

The authors are grateful to Yu. M. Kagan and V. L. Pokrovskiĭ for valuable discussions.

[1] I. M. Lifshitz and Yu. Kagan, Zh. Eksp. Teor. Fiz. **62**, 385 (1972) [Sov. Phys. JETP **35**, 206 (1972)].

[2] S. V. Iordanskiĭ and A. M. Finkel'shteĭn, Zh. Eksp. Teor. Fiz. **62**, 403 (1972) [Sov. Phys. JETP **35**, 215 (1972)].

[3] B. V. Petukhov and V. L. Pokrovskiĭ, Zh. Eksp. Teor. Fiz. **63**, 634 (1972) [Sov. Phys. JETP **36**, 336 (1973)].

[4] M. J. Rice, A. R. Bishop, J. A. Krumhansl, and S. E. Trullinger, Phys. Rev. Lett. **36**, 432 (1976).

[5] Yu. Kagan and L. A. Maksimov, Zh. Eksp. Teor. Fiz. **76**, 687 (1979) [Sov. Phys. JETP **49**, 343 (1979)].

[6] M. B. Voloshin, I. Yu. Kobzarev, and L. B. Okun', Yad. Fiz. **20**, 1229 (1974) [Sov. J. Nucl. Phys. **20**, 644 (1975)].

[7] H. J. Katz, Phys. Rev. D **17**, 1056 (1978).

[8] K. Maki, Phys. Rev. B **18**, 1641 (1978).

[9] K. Hida and U. Eckern, Phys. Rev. B **320**, 4096 (1984).

[10] I. V. Krive and A. S. Rozhavskiĭ, Fiz. Nizk. Temp. **6**, 1272 (1980) [Sov. J. Low Temp. Phys. **6**, 618 (1980)].

[11] B. I. Ivlev and V. I. Mel'nikov, Phys. Rev. Lett. **55**, 1614 (1985).

[12] B. I. Ivlev and V. I. Mel'nikov, Zh. Eksp. Teor. Fiz. **90**, 2208 (1986) [Sov. Phys. JETP **63**, 1295 (1986)].

Translated by A. Tybulewicz