Equivalence principle and zero-point field fluctuations

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Einstein's equivalence principle is considered in connection with processes that take place in accelerated detectors of elementary particles. It is shown that allowance for the vacuum ("zero-point") fluctuations of the fields does not give rise to violation of the equivalence principle, and from the behavior of the detector it is not possible to establish whether it is in a homogeneous gravitational field or is uniformly accelerated in Minkowski space. The temperature of a detector at rest on the surface of a static massive body is identically equal to zero (in disagreement with the opposite statement of Boyer[1]).

1. INTRODUCTION

It is well known (see, for example, Ref. 2) that a photon detector moving in Minkowski space with constant acceleration a (under the influence of an external force of nongravitational origin) emits photons and is also excited as if it were in a field of thermal photons with temperature $T_a = a/2\pi$ (we take $c = \hbar = k = 1$, where k is Boltzmann's constant). On the other hand, it follows from the equivalence principle that all physical phenomena take place in the same way in a uniformly accelerated elevator and in an elevator at rest in a homogeneous gravitational field (for example, one standing on the surface of the "almost flat" Earth). Naive application of the equivalence principle to the detection process led Boyer[1] to conclude that a detector on the surface of the Earth (even in the absence of the microwave background radiation) must also detect photons corresponding to the temperature $T_g = g/2\pi$, where g is the acceleration of free fall, $\mathbf{g} = -\mathbf{a}$ (see also Ref. 3). In other words, it is asserted in Ref. 1 that T_g is the minimal temperature to which bodies lying on the surface of the Earth can be cooled by radiation (in the absence of other sources of heat or cold). Thus, it would appear that not only black holes but all gravitating bodies possess a nonzero temperature. At the same time, direct quantization of fields in the Schwarzschild metric shows that thermal properties arise only in the case of black holes i.e., geometries with an event horizon (see, for example, the review of Ref. 4). On this basis, one might even conclude that in quantum field theory the equivalence principle is not satisfied and by means of a photon detector one could establish whether one is in an elevator in a homogeneous gravitational field or moves with uniform acceleration in Minkowski space.

The paradox is resolved by noting that the physical processes in different elevators will take place in exactly the same way only when the initial conditions specified for these processes are the same, as is assumed in the equivalence principle. In the case of quantum detectors, this assertion takes the following form: A detector accelerated in Minkowski space will behave in the same way as a detector at rest in a homogeneous gravitational field if the quantum state of the detected field in the Minkowski space corresponds to the quantum state of the detected field in the gravitational field. It is well known[5] that in Minkowski space two different vacuum states correspond to inertial and uniformly accelerated observers, namely, the Minkowski vacuum and the Rindler vacuum. We shall show that it is the Rindler vacuum in Minkowski space that must be associated locally with the vacuum in the gravitational field of a static body (not a black hole!) (see also Refs. 5 and 6). Then in complete agreement with the equivalence principle a uniformly accelerated detector in the Rindler vacuum and a detector at rest in the gravitational field of the static body will not be excited. We shall also show how the equivalence principle must be applied to physical processes taking place in the gravitational field of a black hole. A method capable of leading to experimental realizaton of the Rindler vacuum (cf. Ref. 7) is discussed at the end of the paper. Some technical details are put in the Appendix.

2. QUANTUM DETECTOR IN MINKOWSKI SPACE AND IN THE GRAVITATIONAL FIELD OF A STATIC BODY

Following Ref. 2, we shall, to simplify the expressions below, replace the electromagnetic field by a massless neutral scalar field φ. As detector of the field φ, we shall use a two-level point system whose interaction Hamiltonian with the field φ has the form $\hat{H}_{\text{int}} = \hat{\varphi}(x)\hat{V}$, where x is the coordinate of the detector and $\hat{V}$ is an Hermitian operator having transition matrix elements between the ground and excited levels of the detector.

As is shown in Ref. 4, the definition of the vacuum involves the choice of a locally timelike Killing vector field. In other words, it is necessary to choose coordinates $\tau,\boldsymbol{\rho}$ in which the metric tensor does not depend on the time coordinate τ. Then the operator $\hat{\varphi}(\tau,\boldsymbol{\rho})$ can be represented in the form of the sum

$$\hat{\varphi}(\tau,\boldsymbol{\rho}) = \sum_m \int_0^\infty d\omega\, e^{-i\omega\tau}\varphi_{\omega,m}(\boldsymbol{\rho})\hat{a}_{\omega,m} + \text{h.c.},$$
$$[\hat{a}_{\omega,m}, \hat{a}^+_{\omega',m'}] = \delta_{mm'}\delta(\omega-\omega'), \qquad (1)$$

where $\varphi_{\omega,m}(\boldsymbol{\rho})e^{-i\omega t}$ are the wave functions of the quanta of the field φ, satisfying the equation $\nabla^\mu\nabla_\mu(\varphi_{\omega,m}(\boldsymbol{\rho})e^{-i\omega\tau}) = 0$, $\hat{a}^+_{\omega,m}$ and $\hat{a}_{\omega,m}$ are the creation and annihilation operators, and m is the set of quantum numbers. The vacuum state is determined by the condition $\hat{a}_{\omega,m}|0\rangle = 0$. Note that in some frames of reference (for example, rotating frames) the lower limit of the integration over ω in Eq. (1) is nonzero.

In the Lorentz coordinates t, x, y, z the Minkowski space metric is obviously independent of the time, the functions $\varphi_{\omega,m}$ have the form $\varphi^{\text{M}}_{\omega,k}(\mathbf{r}) = e^{ikr}/2(\omega)^{1/2}$, and the corresponding vacuum is the Minkowski vacuum $|0_{\text{M}}\rangle$.

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To determine the Rindler vacuum, we introduce the Rindler coordinates η, ξ, y, z, which are defined in the region $x > |t|$ by

$$x = a^{-1} e^{a\xi} \cosh a\eta, \quad t = a^{-1} e^{a\xi} \sinh a\eta. \tag{2}$$

In these coordinates, the Minkowski space metric becomes

$$ds^2 = e^{2a\xi}(d\eta^2 - d\xi^2) - dy^2 - dz^2 \tag{3}$$

and does not depend on the time η. The wave functions $\varphi_{\omega,m}$ are determined in the region $x > |t|$ by the expression (see, for example, Ref. 8)

$$\varphi^R_{\Omega,\varkappa} = K_{i\Omega}(|\varkappa| e^{a\xi}) e^{i\varkappa \mathbf{w}} [\sinh(\pi\Omega)]^{1/2}/2\pi^2, \tag{4}$$

where $\varkappa = (\varkappa_y, \varkappa_z)$, $\mathbf{w} = (y,z)$, and $K_\nu(x)$ is a modified Bessel function of the second kind. Note that although we have defined the expressions (2) and (4) only for the region $x > |t|$ the corresponding expressions can also be defined in the remaining three quadrants of Minkowski space. Thus, the Rindler vacuum is defined globally. But if we restrict ourselves to considering physical processes taking place in the region $x > |t|$, then, as is shown in Ref. 2, the Minkowski vacuum represents a thermal bath excited above the Rindler vacuum. The local temperature of this bath depends on the spatial Rindler coordinates:

$$T(\xi) = T_{a(\xi)} = a(\xi)/2\pi = a e^{-a\xi}/2\pi, \tag{5}$$

where $a(\xi)$ is the acceleration to which nonmoving Rindler observers are subject, and it also depends on the coordinate ξ. The temperature $T(\xi)$, defined in accordance with (5), naturally satisfies the condition of thermodynamic equilibrium in the gravitational field: $T(\xi) g^{\frac{1}{2}}_{\eta\eta} = \text{const}$.

Thus, the definition of the vacuum involves the choice of a stationary coordinate system. In the case of a spherically symmetric gravitating body, Schwarzschild coordinates are, to within substitutions that do not affect the choice of the vacuum, such coordinates. In Refs. 8 and 9 the connection between these coordinates and Rindler coordinates in flat Minkowski space was already pointed out. A detector on the surface of the Earth has constant Schwarzschild spatial coordinates, while a uniformly accelerated detector in Minkowski space is at rest in the Rindler coordinates. In addition, the metric coefficient g_{00} near the surface of the Earth has in the approximation of a weak gravitational field the form

$$g_{00} = 1 + 2gh, \tag{6}$$

where h is the height above the surface of the Earth. This expression corresponds to expansion of the exponential in formula (3) up to the linear terms, $g_{\eta\eta} = e^{2a\xi} \approx 1 + 2a\xi$, whereas in Lorentz coordinates we should have $g_{00} \equiv 1$. Therefore, the Rindler coordinates correspond to Schwarzschild coordinates, and the Rindler vacuum in Minkowski space corresponds to the Schwarzschild vacuum near the Earth (the so-called Boulevard vacuum).

A body that is at rest in Rindler coordinates has nevertheless a nonvanishing absolute (invariant) acceleration $a(\xi)$. We show that, in complete agreement with the equivalence principle, a detector that is in the Rindler vacuum and at rest in the Rindler coordinates is not excited. This assertion is almost obvious. The probability amplitude for the process consisting of excitation of the detector and simultaneous emission of a quantum of the field φ is

$$A = \langle 1 | \hat{a}^+_{\Omega,\varkappa} | 0_R \rangle \langle 1 | \hat{V} | 0 \rangle \varphi^R_{\Omega,\varkappa}(\xi, \mathbf{w}) \times \int_{-\infty}^{+\infty} \exp(isE + is\Omega e^{-a\xi})\, ds \sim \delta(E + \Omega e^{-a\xi}), \tag{7}$$

where ξ and $\mathbf{w}$ are the constant spatial coordinates of the detector, E is its excitation energy, and the δ function expresses the energy conservation law in the Rindler coordinate system. Since $\Omega \geq 0$ and $E > 0$, the amplitude satisfies $A = 0$, and the detector is not excited.

Summarizing our calculations, we formulate assertions that can also be regarded as a consequence of the equivalence principle: If an observer is in an elevator and feels a force of gravity corresponding to the acceleration g and a photon detector he is holding does not detect any radiation, then either the elevator is on the surface of the (cold) Earth or it moves with constant acceleration g through Minkowski space in which the Rindler vacuum is realized. But if the detector detects photon radiation corresponding to the temperature $T_g = g/2\pi$, then either the elevator is on the surface of the Earth and is surrounded by thermal radiation of temperature T_g emitted by some external source, or it moves with acceleration g in Minkowski space in which the Minkowski vacuum is realized. It must be borne in mind that the proof of the equivalence principle has here, so to speak, a theoretical nature, i.e., we establish that this principle is indeed contained in the formalism of the theory. By itself, this neither replaces nor ensures experimental verification of the equivalence principle, a task that was in essence already begun by Galileo.

Thus, the quantum vacuum with which we are dealing in the gravitational field of the Earth is in fact the Rindler vacuum; more precisely, it is identical to the Rindler vacuum in a region of space whose dimensions are determined by the experimental apparatus and the acceleration $a(\xi)$ by the acceleration of free fall on the Earth. To obtain from this vacuum the true Minkowski vacuum it is necessary to "heat" the laboratory absolute vacuum to the temperature $T_{g_0} \approx 4 \cdot 10^{-20}$ °K. Since this temperature is extremely low, the difference between these vacuums in the region considered can be ignored from the practical (but not the fundamental) point of view. If our apparatus cannot distinguish the temperature T_{g_0} from the absolute zero, it can be assumed that on the Earth we are dealing with the Minkowski vacuum.

Can an experimentalist in the Minkowski vacuum create in a finite volume a state corresponding to the Rindler vacuum for some large given acceleration a? This is possible. It is necessary to make an elevator with mirror walls that perfectly reflect the quanta of the field (photons) and give it an increasing acceleration up to the acceleration a. As the acceleration is increased, the walls of the elevator will radiate photons, but when the acceleration of the elevator becomes constant, this radiation will cease. The photons radiated in the process of reaching the constant acceleration must be absorbed by some absorber previously placed in the elevator. As is noted in Ref. 7, if the elevator is brought to the acceleration a adiabatically (i.e., slowly), the reflecting walls will not radiate photons, and one will be able to dispense with the absorber. In any case, when no photons remain in the elevator moving with constant acceleration the

Rindler vacuum corresponding to this acceleration will be realized in it.

3. THE EQUIVALENCE PRINCIPLE AND NONUNIFORM GRAVITATIONAL FIELDS

Hitherto, we have assumed that the inhomogeneity of the spherically symmetric gravitational field does not prevent application of the equivalence principle to the behavior of the detector, since the size of the region of space in question can be taken sufficiently small. Indeed, the inhomogeneity scale of the gravitational field on the surface of a static body is usually determined by means of the curvature tensor $R_{\alpha\beta\gamma\delta}$ (or, more precisely, the value of its invariants) and is characterized by a radius of curvature $L_1 = R^{3/2}R_g^{-1/2}$, where R is the radius of the body (the radial Schwarzschild coordinate), and R_g is its gravitational radius. When we compare a thermal bath in a homogeneous gravitational field with the Minkowski vacuum, the important thing for us is the constancy in this field of $T_{\rm inv} = T_g g_{00}^{1/2}$, where g_{00} is the corresponding coefficient of the metric tensor. This leads to the appearance of one further quantity with the dimensions of length, L_2, which characterizes the gradient of the gravitational field, with $L_2 < L_1$. Let us find this quantity.

In an inhomogeneous spherically symmetric gravitational field, $T_{\rm inv}$ depends on the spatial coordinates. Therefore, if we excite above the surface of a gravitating body a thermal bath whose temperature on the surface of this body is $T_{g(R)}$, then at the characteristic distance $L_2 \approx R$ from the surface of the body there will be a discrepancy between the temperature $T(r)$ of this bath and $T_{g(r)}$. Since in the case we consider $R > R_g$, it follows that $L_2 < L_1$ and the influence of the gradient of the spherically symmetric gravitational field on the results of the measurements is determined by $\alpha = l/L_2$, where l is the characteristic scale of the measuring apparatus. In our case, $l = l(E,g)$, where E is the excitation energy of the detector. The excitation intensity of the detector, i.e., the magnitude of the effect to which we wish to apply the equivalence principle, is determined by the parameter $\beta = \exp(-2\pi E/g)$. For given g, we can make the radius R of the body arbitrarily large, since these quantities are related by

$$g(R)=\frac{R_g}{2R^2}\left(1-\frac{R_g}{R}\right)^{-1/2}. \tag{8}$$

Increasing R_g, for a given g we can increase R without limit, i.e., for unchanged value of the observed effect β we can make the gravitational field gradient tend to zero. Thus, there exist bodies so massive that the equivalence principle can be applied to the behavior of detectors on their surface. Therefore, the paradox described at the beginning of this note did indeed require resolution, since it could not be eliminated by referring to the inapplicability of the equivalence principle due to the inhomogeneity of the gravitational field.

4. QUANTUM DETECTOR IN THE GRAVITATIONAL FIELD OF A BLACK HOLE

We now consider the application of the equivalence principle to the behavior of a detector in the gravitational field of a static (eternal) black hole. In such a gravitational field, three different quantum states of the fields are called vacuum states (see, for example, Ref. 5). The first vacuum, called the Boulevard vacuum, is determined using the Killing vector field associated with shifts along the Schwarzschild time, and it therefore corresponds to the vacuum around a cold Earth. In other words, a detector in the Boulevard vacuum at a constant distance from the black hole does not detect photons. But a detector at rest in the Boulevard vacuum above which thermal radiation of temperature T_g is excited will behave like a uniformly accelerated detector in the Minkowski vacuum.

The two remaining vacuums (the Unruh and Hartle-Hawking vacuums) simulate the physical situation which arises when a black hole is formed as a result of a collapse taking place, respectively, in empty space or within a cavity with reflecting walls. These vacuums are not associated with any globally defined Killing vector field, and therefore they do not correspond to any vacuum of Minkowski space. The quantum field states known as the Unruh and Hartle-Hawking vacuums represent radiation of thermal nature (with Hawking temperature $T_H = 1/4\pi R_g(1 - R_g/R)^{1/2}$, where R is the radial Schwarzschild coordinate) excited above the Boulevard vacuum, just as the Minkowski vacuum is thermal radiation of temperature $T_a(\xi)$ excited above the Rindler vacuum. A detector at rest in the Unruh or Hartle-Hawking vacuum will be excited (by analogy with a detector at rest in the Rindler coordinates in the Minkowski vacuum), but the temperature $T_H(R)$ detected by such a detector differs from the temperature corresponding to the invariant acceleration of the detector. Specifically, the excitation temperature is $T_H(R) = T_g R^2/R_g^2$. Therefore, with the Unruh or Hartle-Hawking vacuums it is necessary to associate thermal radiation fields of temperature T_H excited above the Rindler vacuum, and it is only near the horizon of the black hole, where $T_H(R) \approx T_{g(R)}$, that they correspond to the Minkowski vacuum. In other words, a detector at rest near the horizon in the Unruh or Hartle-Hawking vacuum is excited in the same way as a detector in the Minkowski vacuum that has the same invariant acceleration. Thus, the equivalence principle also applies to physical phenomena taking place in the gravitational field of a black hole.

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APPENDIX: DEFINITION AND PROPERTIES OF THE RINDLER VACUUM

We consider in more detail the indeterminacy in the procedure of quantizing a scalar (or any other) field φ which relates to the possibility of speaking of the existence in Minkowski space of different vacuums. This procedure is based on postulating equal-time commutation relations between the field operators $\hat\varphi(\mathbf{r},t)$ and the momentum operators $\hat\pi(\mathbf{r},t) = \partial\hat\varphi(\mathbf{r},t)/\partial t$ corresponding to them:

$$[\hat\varphi(\mathbf{r},t),\ \hat\varphi(\mathbf{r}',t)]=0,\quad [\hat\pi(\mathbf{r},t),\ \hat\pi(\mathbf{r}',t)]=0,$$
$$[\hat\varphi(\mathbf{r},t),\ \hat\pi(\mathbf{r}',t)]=i\delta(\mathbf{r}-\mathbf{r}'), \tag{A.1}$$

where $\mathbf{r}$ and t are the Lorentz spatial and time coordinates. These commutation relations uniquely fix the operator algebra, but there is an indeterminacy in the choice of the representation of this algebra. To construct a definite representation, the procedure already described in our paper is used. First of all, one chooses a locally timelike Killing vector field or, which is the same thing, a system of coordinates ρ,τ in which the Minkowski metric does not depend on the time τ. Then the operator $\hat{\varphi}(\rho,\tau)$ is decomposed in accordance with formula (1) with respect to the basis of functions that is formed from the solutions of the equation of motion for the field φ (namely, $\Box\varphi = 0$) that depend harmonically on the time τ. The representation of the algebra of the operators $\hat{\varphi}(\rho,\tau)$ is then determined by the representation of the algebra of the creation and annihilation operators $\hat{a}^+_{\omega,m}, \hat{a}_{\omega,m}$, and this representation, in its turn, is constructed in the standard manner in the Fock space. The possibility of choosing a complete set of solutions of the equation for the field φ that depend harmonically on the time τ is related to the fact that the metric does not depend on this coordinate. The state of the Fock space annihilated by all the annihilation operators $\hat{a}_{\omega,m}$ is called the vacuum associated with the coordinates ρ,τ, since a detector at rest in these coordinates does not detect any particles in this state.

To construct the vacuum states in Minkowski space, we consider two locally timelike Killing fields. The first of them generates shifts along the Lorentz time, the second along timelike hyperbolas, i.e., it corresponds to Lorentz transformations (hyperbolic rotations in the x,t plane). To the first vector field there correspond the Lorentz coordinates t, x, y, z (i.e., in these coordinates the field has the components $\xi^i = (1,0,0,0)$ and the Minkowski vacuum $|0_M\rangle$. To the second vector field there correspond the Rindler coordinates η, ξ, y, z determined in accordance with formula (2). In the Rindler coordinate system the Lorentz transformations are equivalent to shifts along the Rindler time τ, and therefore the Minkowski metric, expressed in the Rindler coordinates (see formula (3)), does not depend on η. The Rindler frame of reference is a relativistic generalization of a uniformly accelerated reference frame. A body at rest in the Rindler coordinates is subject to an invariant acceleration $a(\xi) = (d^2x^i/ds^2)^2 = ae^{-a\xi}$ (cf. Eq. (5)), which does not depend on its proper time s. The dependence of the acceleration $a(\xi)$ on the spatial coordinate ξ is a consequence of the relativistic definition of rigidity of the coordinate system.

To construct the representation of the algebra of the operators $\hat{\varphi}(\mathbf{r},t)$ and the vacuum $|0_R\rangle$ corresponding to the Rindler coordinates, we expand the operator $\hat{\varphi}(\mathbf{r},t)$ with respect to the special set of functions ${}^R\varphi^R_{\Omega,\varkappa}(\mathbf{r},t)$ and ${}^L\varphi^R_{\Omega,\varkappa}(\mathbf{r},t)$. To fix uniquely any function $f(\mathbf{r},t)$ that solves the wave equation $\Box f(\mathbf{r},t) = 0$, is is sufficient to specify what it is equal to in the regions $x > |t|$ and $x < -|t|$. The functions ${}^R\varphi^R_{\Omega,\varkappa}$ are determined in the region $x > |t|$ by

$$ {}^R\varphi^R_{\Omega,\varkappa}(\eta,\xi,y,z) = e^{-i\Omega\eta}\varphi^R_{\Omega,\varkappa}(\xi,y,z), \tag{A.2} $$

where the function $\varphi^R_{\Omega,\varkappa}$ is given by the expression (4), while in the region $x < -|t|$ they are identically equal to zero. The functions ${}^L\varphi^R_{\Omega,\varkappa}$ are determined by the relation ${}^L\varphi^R_{\Omega,\varkappa}(t,x,y,z) = {}^R\varphi^R_{\Omega,\varkappa}(t,-x,y,z)$ and are identically equal to zero in the region $x > |t|$. The coefficients in the expansion of the operator $\hat{\varphi}(\mathbf{r},t)$ with respect to the functions ${}^L\varphi^R_{\Omega,\varkappa}$ and ${}^R\varphi^R_{\Omega,\varkappa}$ are, respectively, the operators ${}^L\hat{a}^R_{\Omega,\varkappa}$ and ${}^Ra^R_{\Omega,\varkappa}$.

Equating to each other the expansions of the operator $\hat{\varphi}(\mathbf{r},t)$ with respect to the wave functions corresponding to the Minkowski and Rindler coordinate systems, we obtain expressions for the operators of creation and annihilation of particles defined above the Minkowski vacuum (Minkowski particles) $\hat{a}^{M+}_{\omega,k}$ and $\hat{a}^M_{\omega,k}$, and in terms of the operators of creation and annihilation of the Rindler particles, ${}^{L,R}\hat{a}^{R+}_{\Omega,\varkappa}$ and ${}^{L,R}\hat{a}^R_{\Omega,\varkappa}$. Such expressions are called Bogolyubov transformations. We introduce the notation

$$ {}^{L,R}\hat{a}^R_{-\Omega,\varkappa} = {}^{L,R}\hat{a}^{R+}_{\Omega,\varkappa}, \tag{A.3} $$

$$ \hat{b}_{\Omega,\varkappa} = ({}^R\hat{a}^R_{\Omega,\varkappa}e^{\pi\Omega/2} - {}^L\hat{a}^R_{-\Omega,\varkappa}e^{-\pi\Omega/2})\,\varepsilon(\Omega)/(2\sinh\pi|\Omega|)^{1/2}, \tag{A.4} $$

where $\epsilon(\Omega) = 1$ for $\Omega > 0$ and $\epsilon(\Omega) = -1$ for $\Omega < 0$. It can be shown that the operators $\hat{b}_{\Omega,\varkappa}$ defined in this manner can be expanded only with respect to the annihilation operators of the Minkowski particles, and therefore

$$ \hat{b}_{\Omega,\varkappa}|0_M\rangle = 0. \tag{A.5} $$

In addition, the operators $\hat{b}_{\Omega,\varkappa}$ have the standard commutation relations

$$ [\hat{b}_{\Omega,\varkappa}, \hat{b}_{\Omega',\varkappa'}] = 0, \qquad [\hat{b}_{\Omega,\varkappa}, \hat{b}^+_{\Omega',\varkappa'}] = \delta(\Omega-\Omega')\delta(\varkappa-\varkappa'). \tag{A.6} $$

Using Eqs. (A.5) and (A.6), we can calculate the number of Rindler particles present in the Minkowski vacuum:

$$ {}^Ln_{\Omega,\varkappa} = {}^Rn_{\Omega,\varkappa} = \langle 0_M|{}^Ra^{R+}_{\Omega,\varkappa}\,{}^Ra^R_{\Omega,\varkappa}|0_M\rangle \sim 1/(e^{2\pi\Omega}-1). \tag{A.7} $$

It can be seen from this formula that the Rindler particles present in the Minkowski vacuum have a Planck spectrum. It is these particles that are detected by a detector moving with constant acceleration in the Minkowski vacuum.

Thus, from the point of view of Rindler quantization (and a uniformly accelerated detector), the Minkowski vacuum is a many-particle state. It can be shown that if this state is averaged between the states of the Rindler particles in the region $x < -|t|$ (or $x > |t|$) then the obtained density matrix is identical to the density matrix of a thermal bath in a constant gravitational field and having temperature $T(\xi)$ determined in accordance with Eq. (5). For a recent discussion of this question, see also Ref. 10.

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