Electron spin relaxation at composition irregularities in semiconducting compounds

F. T. Vas'ko and Yu. N. Soldatenko

Institute of Semiconductors, Academy of Sciences of the Ukrainian SSR
(Submitted 16 May 1986)
Zh. Eksp. Teor. Fiz. **92**, 199–207 (January 1987)

A theory is derived for the spin relaxation of conduction electrons at variations in the composition in semiconducting compounds. A mechanism for spin relaxation in a magnetic field due to fluctuations of the g-factor is also studied. A balance equation is derived for the spin density vector $\mathbf{S}_{\mathbf{r}t}$ for various relations among the length scale of the composition fluctuations, l_c, the de Broglie wavelength λ_e, and the mean free path l_m. Distinctive features of this relaxation mechanism are its tensor nature (for the spatially nonuniform case or in a magnetic field), the importance of drift, and the renormalization of the spin precession frequency because fluctuations of the bottom of the band and of the g-factor become entangled. For $\lambda_e < l_c < l_m$, the spin relaxation is determined by the precession of the fluctuating part of the distribution function in the quasielectric field and by damping (analogous to Landau damping) which does not depend on the specific scattering mechanism. A comparison is made with experimental data for $Al_x Ga_{1-x} As$.

1. INTRODUCTION

The spin relaxation of a conduction electron is being studied in connection with experiments on ESR,[1] on the use of circularly polarized light to orient spins,[2] and on distinguishing the contribution of quantum corrections to the kinetic coefficients of disordered semiconductors.[3] The mechanism for the flipping of the electron spin due to an effective spin-orbit interaction is determined by the length scale of the potential, which varies smooth over distances on the order of the lattice constant a. If this scale is shorter than the de Broglie wavelength λ_e, then the spin flipping will occur in individual scattering events (the Elliot-Yafet mechanism). This situation has been studied in detail.[4] If, on the other hand, the potential varies over distances greater than the momentum relaxation length l_m, then the D'yakonov-Perel' mechanism[5] will operate. This mechanism results from a simultaneous precession of the spin due to a spin-orbit interaction with the electric field and a scattering of momentum (see Ref. 6 for some calculations for this case). A distinctive feature of this mechanism in an external electric field (in contrast with the precession which results from the lifting of the spin degeneracy of the conduction band of semiconductors which lack a center of inversion[5]) is that the field creates a preferred direction. As a result, a tensor relaxation time arises in the balance equation for the spin density, and the spin flux is determined by drift in the electric field as well as by diffusion. Another possible case is that in which the length scale l_c of the potential fluctuations has an intermediate value: $\lambda_e < l_c < l_m$. Under these inequalities, the flipping of the spin results from its precession in the electric field and from a damping (analogous to Landau damping) which does not depend on the particular scattering mechanism.

In this paper we analyze these aspects of the spin relaxation for the case in which an electron is scattered by composition irregularities in as substitutional semiconducting compound. Such irregularities give rise to fluctuations of the bottom of the conduction band. The quasielectric fields which arise are strong, and the spin relaxation is quite effective. This model is more general than spin relaxation due to a spin-orbit interaction with the electric field of the irregularities, since in this case it is necessary to consider the change in the effective mass and the g-factor as functions of the composition in addition to the fluctuating quasielectric field. This gives rise to an additional mechanism for spin relaxation in a strong magnetic field.

The dynamics of a conduction electron in an $A_x \bar{A}_{1-x} B$ compound (III-V semiconducting compounds are also being studied widely[7]) is determined by the Hamiltonian of the effective-mass approximation[1)]:

$$\mathscr{H}={}^1/_4(\pi^2 m_\mathbf{r}^{-1}+m_\mathbf{r}^{-1}\pi^2)+\varepsilon_\mathbf{r}+(\hbar/4m_s)\boldsymbol{\sigma}[\nabla x_\mathbf{r},\boldsymbol{\pi}]+{}^1/_2 g_\mathbf{r}\mu_B\boldsymbol{\sigma}\mathbf{H}, \qquad (1)$$

where $m_\mathbf{r}$, $\varepsilon_\mathbf{r}$, and $g_\mathbf{r}$ are the effective mass, the energy of the bottom of the conduction band, and the g-factor at the point $\mathbf{r}$, all of which vary linearly with the composition; the operator $\boldsymbol{\pi}$ represents the kinematic momentum; μ_B is the Bohr magneton; σ is the Pauli matrix; and the parameter m_s determines the intensity of the spin-orbit interaction with the composition gradient in the alloy. The composition fluctuations $\delta_\mathbf{r} = x_\mathbf{r} - \bar{x}$ in a homogeneous and isotropic compound are determined by the correlation function $W(|\mathbf{r}-\mathbf{r}'|) = \langle \delta_\mathbf{r}\delta_{\mathbf{r}'}\rangle$, which we will assume to be Gaussian in our calculations (with a correlation length l_c and a mean square composition fluctuation σ^2). We will also use the linear relations $\varepsilon_\mathbf{r} = \bar{\varepsilon} + W_0\delta_\mathbf{r}$ and $g_\mathbf{r} = \bar{g} + \lambda\delta_\mathbf{r}$, for which parameters are given for several materials in Refs. 7 and 10. The mass m_s which determines the efficiency of the spin flipping, can be estimated in order of magnitude from the results of the model calculation of Ref. 8. For $Al_x Ga_{1-x} As$, these estimates agree with experimental results on the spin splitting of the spectrum of asymmetric heterostructures (see the discussion in Ref. 11).

In Section 2 below we write kinetic equations for the scattering of a spin by composition fluctuations or the precession of a spin in the case of a smooth variation of the composition. We derive (in the approximation of a weak spin-orbit interaction) balance equations for the spin density $\mathbf{S}_{\mathbf{r}t}$ We then take an average of $\mathbf{S}_{\mathbf{r}t}$ over fluctuations of intermediate scale (Section 3) and write a hydrodynamic-

 0038-5646/87/010112-05$04.00 © 1987 American Institute of Physics

approximation equation for $\mathbf{S}_{rt}$ in the case of smooth fluctuations (Section 4). This analysis is carried out for nondegenerate electrons (because of the Pauli principle, the average energy of the degenerate electrons also undergoes a relaxation over long spin times[12]). The hydrodynamic fluctuations are also assumed to be small when the averaging is carried out in Section 4 (this is the effective-medium approximation[13]). In the Conclusion we discuss the experimental results of Refs. 14 and 15 and the possibilities of a more detailed study of these relaxation mechanisms.

2. KINETIC EQUATIONS AND SPIN DENSITY BALANCE

For small, short-wave ($a < l_c < \lambda_e$) fluctuations of the composition it is convenient to single out in Hamiltonian (1) the increment proportional to δ_r, i.e., w_r, and to write a quantum kinetic equation for the statistical operator averaged over the fluctuations, $\hat{\rho}_t$:

$$\frac{\partial \hat{\rho}_t}{\partial t} + \frac{i}{\hbar}[h, \hat{\rho}_t] = \frac{1}{\hbar^2}\int_{-\infty}^{0} d\tau \langle [e^{ih\tau/\hbar}[w_r, \hat{\rho}_t]e^{-ih\tau/\hbar}, w_r]\rangle, \quad (2)$$

$$h = \pi^2/2m + {}^1/_2\hbar\sigma\Omega_B, \quad \Omega_B = g\mu_B\mathbf{H}/\hbar, \quad \Delta\Omega_B = \lambda\mu_B\mathbf{H}/\hbar,$$
$$w_r = W_0\delta_r + (\hbar/4m_s)\sigma[\nabla\delta_r, \pi] + (\sigma, \Delta\Omega_B)\delta_r,$$

The contribution from fluctuations of the kinetic energy due to the dependence of the effective mass on δ_r has been omitted since it is small in comparison with $W_0\delta_r$; here $\langle ... \rangle$ means an average over fluctuations. For long-wave fluctuations ($\lambda_e < l_c$) we need to consider a Wigner distribution function (which will be a 2×2 matrix), which satisfies the kinetic equation

$$\left\{\frac{\partial}{\partial t} + \mathbf{v}_{pr}\nabla + \mathbf{F}_{pr}\frac{\partial}{\partial \mathbf{p}} - \nabla(\varepsilon_{pr} + W_0\delta_r)\frac{\partial}{\partial \mathbf{p}}\right\}\hat{f}_{prt}$$
$$+ \frac{i}{2}[\sigma\Omega_{pr}, \hat{f}_{prt}] = I_c(\hat{f}\,|\,\mathbf{p}\mathbf{r}t). \quad (3)$$

In addition to the usual terms, $\mathbf{F}_{pr}$ is the Lorentz force, I_c is a collision integral which ignores spin flip, $\mathbf{V}_{pr} = \partial\varepsilon_{pr}/\partial\mathbf{p}$, and $\varepsilon_{pr} = p^2/2m_r$. An addition force arises here,[16] from the interaction with the composition gradient, $-\nabla(\varepsilon_{pr} + W_0\delta_r)$. The commutator on the left side of (3) (c.f. Ref. 17, for example) determines spin precession with a frequency

$$\Omega_{pr} = [\nabla\delta_r, \mathbf{p}]/2m_s + g_r\mu_B\mathbf{H}/\hbar. \quad (4)$$

Equations (2) and (3) rapidly impose an equilibrium distribution in the kinetic degrees of freedom, so that the relaxation which results from spin flip can be described by examining the spin density vector

$$\mathbf{S}_{rt} = \frac{1}{V}\sum_{\mathbf{p}} \mathrm{tr}\, \sigma\hat{f}_{prt} \quad (5)$$

(tr is a trace over the spin variable, and V is the normalization volume). For the limiting case of short-wave fluctuations described by (2), a spatially uniform distribution $\hat{f}_{pt} = (\mathbf{p}|\hat{\rho}t|\mathbf{p})$ appears in (5). We find a balance equation for the spin density by convolving (2) and (3) in accordance with definition (5). Calculations are carried out below for the case of weak magnetic fields, $\omega_c\bar{\tau} < 1$ (ω_c is the cyclotron frequency, and $\bar{\tau}$ is the momentum relaxation time) and $\exp(-\hbar\Omega_B/T) \simeq 1$ (in this case the energy relaxation is inconsequential, and in equilibrium we have $\mathbf{S}_{t\to\infty} = 0$).

From (2) we find an equation for $\mathbf{S}_t$:

$$\frac{\partial \mathbf{S}_t}{\partial t} - [\Omega_B\mathbf{S}_t] = \frac{1}{\hbar^2}\int_{-\infty}^{0} d\tau \frac{1}{V}\sum_{\mathbf{p}} \mathrm{tr}\, f_{pt}$$
$$\times\langle(\mathbf{p}\,|\,[e^{ih\tau/\hbar}[w_r, \sigma]e^{-ih\tau/\hbar}, w_r]\,|\,\mathbf{p})\rangle. \quad (6)$$

The right side of this equation is evaluated by substituting a distribution which is equilibrium in terms of momentum:

$$f_{pt} = \frac{n + \mathbf{S}_t\sigma}{n}e^{-\varepsilon_p/T}, \quad n_0 = \frac{1}{V}\sum_{\mathbf{p}} e^{-\varepsilon_p/T}, \quad (7)$$

where T is the lattice temperature, and n the electron density. Carrying out the summation over the spin variable on the right side of (6), we find

$$\frac{\partial \mathbf{S}_t}{\partial t} - \alpha[\Omega_B\mathbf{S}_t] + \nu_\parallel\mathbf{S}_t - \beta[\Delta\Omega_B[\Delta\Omega_B\mathbf{S}_t]] = 0,$$
$$\alpha = 1 - \frac{\lambda}{\bar{g}}\frac{8}{V^2}\sum_{\mathbf{pp}_1} W\left(\frac{|\mathbf{p}-\mathbf{p}_1|}{\hbar}\right)\frac{e^{-\varepsilon_p/T}}{n_0}\frac{W_0}{\varepsilon_p - \varepsilon_{p_1}}$$
$$\beta = \frac{8\pi\hbar}{V^2}\sum_{\mathbf{pp}_1} W\left(\frac{|\mathbf{p}-\mathbf{p}_1|}{\hbar}\right)\frac{e^{-\varepsilon_p/T}}{n}\delta(\varepsilon_p - \varepsilon_{p_1}), \quad (8)$$
$$\nu_\parallel = \frac{\pi}{3\hbar V^2}\sum_{\mathbf{pp}_1} W\left(\frac{|\mathbf{p}-\mathbf{p}_1|}{\hbar}\right)\frac{e^{-\varepsilon_p/T}}{n_0}\frac{|[\mathbf{pp}_1]|^2}{m_s^2}\delta(\varepsilon_{p_1} - \varepsilon_p),$$

where α determines the renormalization of the spin precession frequency which results from the interference of fluctuations of the band bottom, $W_0\delta_r$, and of the g-factor; the frequency $\nu_\parallel$ describes the spin flip due to the spin-orbit interaction; and β determines the anisotropic relaxation due to fluctuations of the g-factor. Using the inequality $\lambda_e \sim \hbar/(2mT)^{1/2} > l_c$ for the longitudinal component of the spin vector the component parallel to Ω_B, $S_t^\parallel$, and for the transverse component, $\mathbf{S}_t^\perp$, to calculate the coefficients in (8), we find

$$\frac{\partial S_t^\parallel}{\partial t} + \nu_\parallel S_t^\parallel = 0, \quad \frac{\partial \mathbf{S}^\perp}{\partial t} + \nu_\perp\mathbf{S}_t^\perp - \alpha[\Omega_B\mathbf{S}_t^\perp] = 0,$$
$$\alpha = 1 - 4\sigma^2\frac{W_0}{\varepsilon_c}\frac{\lambda}{\bar{g}}, \quad \nu_\parallel = \frac{8}{3}\left(\sigma\frac{m}{m_s}\right)^2\frac{T}{\hbar}\left(\frac{T}{\varepsilon_c}\right)^{1/2},$$
$$\varepsilon_c = \frac{(\hbar/l_c)^2}{2m}, \quad (9)$$
$$\nu_\perp = \nu_\parallel + \beta(\Delta\Omega_B)^2, \quad \beta = 4\sigma^2(\hbar/\varepsilon_c)(T/\varepsilon_c)^{1/2}.$$

The fluctuations of the g-factor may contribute significantly to the relaxation of $\mathbf{S}_t^\perp$ (despite the condition $\hbar\Omega_B/T \ll 1$) if $m/m_s < 1$.

From the kinetic equation (3) we find a balance equation for the spin density:

$$\frac{\partial S_{rt}^i}{\partial t} + \sum_j \frac{\partial Q_{rt}^{ij}}{\partial r_j} + \left(\frac{\partial S_{rt}^i}{\partial t}\right)_{so} - [\Omega_{B,r}\mathbf{S}_{rt}]_i = 0. \quad (10)$$

This equation has contributions from the divergence of the spin flux tensor Q_{rt}^{ij} and from the spin precession, which are given by the relations ($\xi_r = \nabla\delta_r$)

$$Q_{rt}^{ij} = \frac{1}{V}\sum_{\mathbf{p}} \mathrm{tr}\, \sigma_i v_{pr}^j \hat{f}_{prt}, \quad (11)$$

$$\left(\frac{\partial \mathbf{S}_{rt}}{\partial t}\right)_{so} = \frac{i}{4m_sV}\sum_{\mathbf{p}} \mathrm{tr}\,\sigma[\sigma[\xi_r\mathbf{p}], \hat{f}_{\mathbf{p}rt}]$$

$$= \frac{m_r}{2m_sV}\sum_{\mathbf{p}} \mathrm{tr}\,\hat{f}_{\mathbf{p}rt}\{\xi_r(\sigma\mathbf{v}_{\mathbf{p}r}) - \mathbf{v}_{\mathbf{p}r}(\sigma\xi_r)\}. \qquad (12)$$

The relationship between these expressions and $\mathbf{S}_{rt}$, i.e., a closed balance equation for the spin density, is found below with the help of approximate solutions of Eq. (3) for the cases of fast and slow (on the scale 1_m) changes in x_r.

3. AVERAGE OVER FLUCTUATIONS OF INTERMEDIATE SCALE

In the case $\lambda_e < l_c < l_m$ a solution of Eq. (3) is given by the distribution (7) plus a small fluctuating increment, whose spatial Fourier transform $\delta\hat{f}_{\mathbf{p}\mathbf{k}t}$ is determined by the following equation [$\delta\mathbf{\Omega}_{oj} = \mathbf{\Omega}_{\mathbf{p}\mathbf{k}} - \mathbf{\Omega}_{\mathbf{B}}$ is found from (4)]:

$$\left\{\frac{\partial}{\partial t} + i\mathbf{k}\mathbf{v} + \frac{e}{c}[\mathbf{v}\mathbf{H}]\frac{\partial}{\partial \mathbf{p}}\right\}\delta\hat{f}_{\mathbf{p}\mathbf{k}t} + \frac{W_0}{T}(\xi_r\mathbf{v})e^{-\varepsilon_p/T}\frac{\sigma\mathbf{S}_t}{n_0}$$

$$+\frac{i}{2}\left[\sigma\delta\mathbf{\Omega}_{\mathbf{p}\mathbf{k}}, \frac{\sigma\mathbf{S}_t}{n_0}\right]e^{-\varepsilon_p/T} = I_c(\delta f|\mathbf{p}\mathbf{k}t). \qquad (13)$$

We are interested in a solution of this equation for $k \sim l_c^{-1}$, so that the time dependence is the same for $\delta\hat{f}_{\mathbf{p}\mathbf{k}t}$ and $\mathbf{S}_t$, and we can ignore gyration in the magnetic field (the condition $\omega_c < k\bar{v}$ reduces to $\omega_c\tau < l_m/l_c$, and the magnetic field can be strong here). The same estimate allows us to replace the relaxation contribution from I_c by an infinitely small increment $i\delta$, which determines the rule for circumventing the pole in the denominator of the solution:

$$\delta\hat{f}_{\mathbf{p}\mathbf{k}t} = \frac{ie^{-\varepsilon_p/T}}{n_0(\mathbf{k}\mathbf{v}-i\delta)}\left\{\frac{W_0}{T}(\xi_k\mathbf{v})(\sigma\mathbf{S}_t) + \frac{i}{2}[\sigma\delta\mathbf{\Omega}_{\mathbf{p}\mathbf{k}}, \sigma\mathbf{S}_t]\right\}. \qquad (14)$$

Substituting this expression into (11) and (12), we can write Eq. (8) for the spatially uniform spin density $\mathbf{S}_t$; the coefficients in this equation, α_i, β_i, and ν_i, are now given by

$$\alpha_i = 1 - \frac{\lambda}{\bar{g}}\frac{2}{V^2}\sum_{\mathbf{k}\mathbf{p}} W(k)\frac{e^{-\varepsilon_p/T}}{n_0}\frac{W_0}{T},$$

$$\beta_i = \frac{2\pi}{V^2}\sum_{\mathbf{k}\mathbf{p}} W(k)\frac{e^{-\varepsilon_p/T}}{n_0}\delta(\mathbf{k}\mathbf{v}), \qquad (15)$$

$$\nu_{\parallel,i} = \left(\frac{m}{m_s}\right)^2\frac{2\pi}{3V^2}\sum_{\mathbf{k}\mathbf{p}} W(k)\frac{e^{-\varepsilon_p/T}}{n_0}k^2v^2\delta(\mathbf{k}\mathbf{v}).$$

As in the short-wave limit, the renormalization of the spin precession frequency results from interference of fluctuations of the bottom of the band and the g-factor. The parameters $\nu_{\parallel,i}$ and β_i, which determine the relaxation of $\mathbf{S}_t$, now arise from damping—which is independent of the particular scattering mechanim—of the fluctuations of the electron distribution, $\delta\hat{f}_{\mathbf{p}\mathbf{k}t}$. The explicit expressions for these coefficients,

$$\alpha_i = 1 - 2\sigma^2\frac{W_0}{T}\frac{\lambda}{\bar{g}}, \qquad \nu_{\parallel,i} = \frac{16}{3}\left(\frac{m}{m_s}\right)^2\sigma^2\frac{(\varepsilon_cT)^{1/2}}{\hbar},$$

$$\beta_i = \frac{\sigma^2\hbar}{(\varepsilon_cT)^{1/2}}, \qquad (16)$$

differ from those in (9), so that the scale of the composition fluctuations can be estimated from the temperature dependence (at $\lambda_e \sim l_c$ there is a transition between the limiting cases we have considered here).

4. HYDRODYNAMIC EQUATIONS FOR THE SPIN DENSITY

To calculate (11) and (12) in the long-wave limit,[2)] we replace the sum of expressions (7) and (14) by an equilibrium barometric distribution (n_{rt} is the electron density) and a small fluctuating increment $\Delta\hat{f}_{\mathbf{p}\mathbf{k}}$:

$$\hat{f}_{\mathbf{p}rt} = \frac{n_{rt}+\sigma\mathbf{S}_{rt}}{\bar{n}_r}e^{-\varepsilon_p/T} + \Delta\hat{f}_{\mathbf{p}rt}, \qquad \bar{n}_r = \frac{1}{V}\sum_{\mathbf{p}} e^{-\varepsilon_p/T}. \qquad (17)$$

The relaxation of the fluctuations is now controlled by the collision integral I_c in (3) (we assume that the external fields are weak), and for $\Delta\hat{f}_{\mathbf{p}rt}$ we find the equation

$$\{{}^1/_2i[\sigma\mathbf{\Omega}_{\mathbf{p}r}, \hat{N}_{rt}] + \mathbf{v}_{\mathbf{p}r}\nabla\hat{N}_{rt}\}e^{-\varepsilon_{pr}/T}$$

$$= I_c(\Delta f|\mathbf{p}rt), \qquad \hat{N}_{rt} = (n_{rt} + \mathbf{S}_{rt}\sigma)/\bar{n}_r. \qquad (18)$$

Following this equation, and evaluating (11) and (12), we find a closed balance equation for the spin density. The part of Δf which is asymmetric in terms of the momentum contributes to (11) and (12), so that a momentum relaxation time $\tau_{\mathbf{p}r}$ appears in the expressions for $Q_{rt}{}^{ij}$ and $(\partial \mathbf{S}_{rt}/\partial t)_{so}$, and the spin precession frequency in (4) drops out of (11) and (12). After summing over the spin variable, we find

$$Q_{rt}{}^{ij} = \frac{1}{V}\sum_{\mathbf{p}} v_{\mathbf{p}r}{}^j\tau_{\mathbf{p}r}e^{-\varepsilon_p/T}\left\{\frac{m_r}{2m_s\bar{n}_r}[[\xi_r\mathbf{v}_{\mathbf{p}r}]\mathbf{S}_{rt}]_i - (\mathbf{v}_{\mathbf{p}r}\nabla)\frac{S_{rt}{}^i}{\bar{n}_r}\right\} \qquad (19)$$

$$\left(\frac{\partial \mathbf{S}_{rt}}{\partial t}\right)_{so} = \frac{m_r}{2m_sV}\sum_{\mathbf{p}}\tau_{\mathbf{p}r}e^{-\varepsilon_p/T}\left\{\frac{m_r}{2m_s\bar{n}_r}[\xi_r([[\xi_r\mathbf{v}_{\mathbf{p}r}]\mathbf{S}_{rt}]\mathbf{v}_{\mathbf{p}r}) - \mathbf{v}_{\mathbf{p}r}([[\xi_r\mathbf{v}_{\mathbf{p}r}]\mathbf{S}_{rt}]\xi_r)] + [\mathbf{v}_{\mathbf{p}r}\xi_r{}^j(\mathbf{v}_{\mathbf{p}r}\nabla) - \xi_r v_{\mathbf{p}r}{}^j(\mathbf{v}_{\mathbf{p}r}\nabla)]\frac{S_{rt}{}^j}{\bar{n}_r}\right\}.$$

It remains to take an average over the momenta. A diffusion coefficient D_r and a drift velocity $\mathbf{V}_r$, defined by

$$D_r = \frac{1}{V}\sum_{\mathbf{p}}\frac{v_{\mathbf{p}r}{}^2\tau_{\mathbf{p}r}}{3\bar{n}_r}e^{-\varepsilon_{pr}/T}, \qquad \mathbf{V}_r = -D_r\bar{n}_r\nabla(\bar{n}_r{}^{-1}), \qquad (20)$$

appear in (19). We can relate D_r, V_r, and the quasielectric field by means of Einstein's equation (we assume that the spatial variation of m_r^{-1} is smooth). The contributions to (19) which ae proportional to m_r/m_s are conveniently collected in the spin flux density tensor $\tilde{\mathbf{Q}}_{rt}{}^{ij}$, so that (10) is replaced by the equation

$$\frac{\partial S_{rt}{}^i}{\partial t} + \sum_j\left(\frac{\partial \tilde{Q}_{rt}{}^{ij}}{\partial r_j} + \nu_r{}^{ij}S_{rt}{}^j\right) - [\mathbf{\Omega}_{B,r}\mathbf{S}_{rt}]_i = 0,$$

$$\tilde{Q}_{rt}{}^{ij} = -(D_r\nabla_j - V_r{}^j)S_{rt}{}^i + D_r\frac{m_r}{m_s}[\delta_{ij}(\xi_r\mathbf{S}_{rt}) - \xi_r{}^iS_{rt}{}^j], \qquad (21)$$

$$\nu_r{}^{ij} = (m_r/2m_s)^2D_r[\delta_{ij}\xi_r{}^2 + \xi_r{}^i\xi_r{}^j].$$

In this equation, the spin flux density tensor $\tilde{\mathbf{Q}}_{rt}{}^{ij}$, is determined by diffusion and drift and also by an increment from the spin-orbit interaction. Since ξ_r establishes a preferred direction, the spin relaxation frequency in (21) is the tensor ν_r^{ij} (an analogous situation arises in the case of two-dimensional electrons,[18] in which case the preferred direction is normal to the layer). The smooth coordinate dependence has been taken into account exactly, and Eq. (21) describes the dynamics of the spins in a varigap semiconductor (in interpreting an experiment on spin transport in a varigap semiconductor, Volkov *et al.* examined the simplified case of a constant diffusion coefficient).

Taking an average of (21) over small fluctuations δ_r again gives us Eqs. (9) for the spin density components $S_t^{\parallel}$ and $S_t^{\perp}$; the coefficients in these equations are now given by

$$\tilde{\alpha}=1-\frac{1}{V}\sum_{\mathbf{k}} W(k)\left[(\Delta\Omega_B)^2+\frac{\lambda}{\bar{g}}\frac{W_0}{T}Dk^2(Dk^2+\tilde{\nu}_{\parallel})\right]$$

$$\times[(Dk^2+\tilde{\nu}_{\parallel})^2+\Omega_B^2]^{-1},$$

$$\tilde{\nu}_{\perp}=\tilde{\nu}_{\parallel}+\frac{1}{V}\sum_{\mathbf{k}} W(k)\left[(\Delta\Omega_B)^2(Dk^2+\tilde{\nu}_{\parallel})\right.$$

$$\left.-\frac{\lambda}{\bar{g}}\frac{W_0}{T}\Omega_B^2Dk^2\right][(Dk^2+\tilde{\nu}_{\parallel})^2+\Omega_B^2]^{-1}, \qquad (22)$$

$$D\equiv\langle D_r\rangle, \quad \langle \nu_r^{ij}\rangle=\delta_{ij}\tilde{\nu}_{\parallel}.$$

In a weak field ($\Omega_B < Dl_c^{-2}$) we have

$$\tilde{\alpha}\approx 1-\sigma^2\frac{W_0}{T}\frac{\lambda}{\bar{g}}, \quad \tilde{\nu}_{\parallel}=2\left(\sigma\frac{m}{m_s}\right)^2\frac{D}{l_c^2}, \qquad (23)$$

$$\tilde{\nu}_{\perp}=\tilde{\nu}_{\parallel}+O\left[\left(\frac{\Omega_B}{\tilde{\nu}_{\parallel}}\right)^2\right],$$

and in a strong field we find from (22)

$$\tilde{\alpha}\approx 1-\left(\sigma\frac{\lambda}{\bar{g}}\right)^2, \quad \tilde{\nu}_{\perp}=\tilde{\nu}_{\parallel}\left[1+(\lambda\sigma)^2-6\lambda\sigma^2\frac{W_0}{T}\frac{Dl_c^{-2}}{\tilde{\nu}_{\parallel}}\right]. \qquad (24)$$

In the case of a spatially nonuniform distribution $\langle \mathbf{S}_{rt}\rangle$, which arises if (for example) there is a nonuniform initial spin distribution, we find an equation with a nonlocal relaxation contribution, so that we can write the following equation for the Fourier transform of the spin density $\mathbf{S}_{\mathbf{k}t}$ (we are again assuming small fluctuations, and we are restricting the analysis to the case $\Omega_B = 0$):

$$(\partial/\partial t+\Gamma_{\mathbf{k}})\mathbf{S}_{\mathbf{k}t}-d_{\mathbf{k}}\mathbf{k}(\mathbf{k}\mathbf{S}_{\mathbf{k}t})=0,$$

$$\Gamma_{\mathbf{k}}=\nu_{\parallel}+\frac{D}{V}\sum_{\mathbf{q}}\frac{W(q)}{(\mathbf{k}+\mathbf{q})^2}[(\mathbf{qk})^2+q^2(\mathbf{kq})]+(D-d_{\mathbf{k}})k^2, \qquad (25)$$

$$d_{\mathbf{k}}=\frac{D}{2V}\left(\frac{m}{m_s}\right)^2\sum_{\mathbf{q}}\frac{W(q)}{(\mathbf{k}+\mathbf{q})^2}\left[q^2-\frac{(\mathbf{kq})^2}{k^2}\right].$$

Here $\Gamma_{\mathbf{K}}$ describes the damping and diffusion of spins without a change in their direction, while $d_{\mathbf{K}}$ determines the depolarization of the spatially nonuniform distribution of the spin density. In the long-wave limit, $kl_c < 1$, we have

$$\Gamma_{\mathbf{k}}\approx\tilde{\nu}_{\parallel}+\tilde{D}k^2, \quad \tilde{D}=D\left\{1-\frac{\sigma^2}{3}\left[\left(\frac{W_0}{T}\right)^2+\left(\frac{m}{m_s}\right)^2\right]\right\},$$

$$d_{\mathbf{k}}\approx D\frac{\sigma^2}{3}\left(\frac{m}{m_s}\right)^2, \qquad (26)$$

so that the change in the spin orientation stems from the anisotropic nature of the diffusion.

5. CONCLUSION

Equations (9), (21), and (25) have been derived for the spin density vector of conduction electrons in a semiconducting compound which are described by Hamiltonian (1) [if m_r and g_r are constant, Eq. (1) corresponds to spin precession in an electric field[6]]. We have considered various scales of the composition variation. Distinctive features of the coefficients which appear in Eqs. (8), (21), and (25) stem from the entanglement of various fluctuating quantities and from the collisionless spin relaxation mechanism for fluctuations of intermediate scale. Analogous features arise in several other problems in which an average is taken over several parameters which depend on the composition (e.g., in a study of the effect of a random drift on diffusion[20]). When a magnetic field is imposed, an additional relaxation mechanism comes into play for the perpendicular component of the spin density vector. This new mechanism results from fluctuations of the g-factor. In addition, there is a renormalization of its precession frequency [see (9), (23), and (24)].

No detailed experimental studies have been made of spin relaxation or other kinetic phenomena in which variations in composition were monitored simultaneously. The contribution of this relaxation mechanism can be distinguished only by making a comparison with other mechanisms (e.g., on the basis of differences in a temperature dependence). Let us examine in more detail the experimental results of Ref. 14, where the spin relaxation time was measured for several $Al_xGa_{1-x}As$ samples ($0.1<x<0.3$) at $T\sim 77$ K. This time was found to be $(0.5–1.7)\cdot 10^{-9}$ s, and it was found to fall off with the temperature as $T^{-(2-3)}$. This temperature dependence led Clark *et al.*[14] to suggest that the relaxation occurs by a D'yakonov-Perel' mechanism. However, recent measurements of the constant α, which determines the magnitude of the spin splitting of the conduction band of a semiconductor lacking a center of inversion,[3] yield values an order of magnitude smaller than those which would be required to explain the experimental results of Ref. 14 (the relaxation time is two orders of magnitude longer). The relaxation mechanisms which we have discussed here provide the appropriate temperature dependence $\tilde{\nu}_{\parallel}\propto T^{5/2}$ only in the case of long-wave fluctuations [Eq. (23) in the case of momentum relaxation at a charged impurity, which was dominant under the conditions of Ref. 14]. The value of $\tilde{\nu}_{\parallel}$ agrees with the experimental results if we assume $\sigma/l_c\sim 2\cdot 10^3$ cm^{-1}; this value would correspond, at an amplitude of the composition fluctuations on the order of 1%, to a length $l_c\sim 5\cdot 10^{-6}$ cm (and the condition $l_c>l_m$ is already satisfied). Judging from these estimates and from the scatter in the experimental values for the different samples, we might assume that relaxation involving long-wave composition fluctuations was observed in Ref. 14. A corresponding analysis and an order-of-magnitude comparison with the experimental results of Ref. 15 can be carried out by writing equations like (9) and (21) for degenerate electrons (here we are assuming that the quantum corrections to the conductivity are determined by the same time as the relaxation of the average spin).

A separate determination of σ and l_c would be possible in experiments with a magnetic field, from the renormalization of the precession frequency and the relaxation time $\nu_{\parallel,\perp}^{-1}$. Another qualitative distinctive feature is that according to Eq. (25) not only relaxation but also rotation of the spin density vector occurs in the spatially nonuniform case, because of the anisotropic nature of the diffusion. This situation can be observed by studying the damping of the spin in a plate with a given spin density, making an angle of φ_0 with the normal, on one of the surfaces (this spin density would be set by an interband excitation of electrons by circularly polarized light). Solving Eqs. (25) and (26), we find the following profile of the increment in φ_0 along the plate thickness z:

$$\Delta\varphi \approx \frac{z}{l_\parallel}\frac{d}{4\bar{D}}\sin 2\varphi_0, \qquad l_\parallel=(v_\parallel \bar{D})^{1/2}. \tag{27}$$

The spin rotates toward the plane of the plate. An estimate for $Al_{0,3}Ga_{0,7}As$ with $z \approx l_\parallel$ yields $\Delta\varphi \sim 10^{-2}$ at the maximum composition fluctuations which are possible for this analysis $\sigma \sim 0.1$. A corresponding rotation of the spin density, due in this case to an anisotropy of the relaxation in (21), would be possible in a varigap layer. Estimates for such layers on the basis of $Al_xGa_{1-x}As$, however (see Ref. 19 for the parameter values), yield $\Delta\varphi \sim 10^{-4}$. The effect increases significantly and can apparently be observed in varigap structures of narrow-gap materials (ternary compounds of In, etc.), in which the spin-orbit interaction is more effective. The spin relaxation by composition fluctuations would also be more effective in such compounds.

[1)] A derivation of this Hamiltonian for the model of a virtual crystal is given in Ref. 8. In writing (1) we are assuming that the extremum does not shift away from the center of the Brillouin zone as the composition of the compound changes. Other models have also been studied (in connection with the description of varigap semiconductors and heterojunctions[9]). In several papers, the first terms in (1) have been replaced by the contribution $(\pi m_r^{-1}\pi)/2 + \tilde{\varepsilon}_r$, with the result that a change in the relationship between $\tilde{\varepsilon}_r$ and x_r is introduced phenomenologically. These distinctions are inconsequential for the analysis below.

[2)] If $\hbar\Omega_B > T$, or if the external fields are strong, it is necessary to consider the contribution to Δf which is symmetric in momentum. In the case of quasielastic energy relaxation, a separate study is also required of the case in which the scale of the fluctuations is limited by the energy and momentum lengths.

[3)] The experiments of Ref. 21 were carried out for GaAs, but it may be assumed that the parameter α, like the other band parameters, varies slowly with the composition in $Al_xGa_{1-x}As$.

[1]R. M. White, *Quantum Theory of Magnetism*, McGraw-Hill, New York, 1970 [Russ. transl. Mir, Moscow, 1985].

[2]B. P. Zakharchenya, D. N. Mirlin, V. I. Perel', and I. I. Reshina, Usp. Fiz. Nauk **136**, 459 (1982) [Sov. Phys. Usp. **25**, 143 (1982)].

[3]B. L. Altshuler, A. G. Aronov, and D. E. Khmel'nitzkii, in: Quantum Theory of Solids (ed. I. M. Lifshitz), 1982, p. 130; G. Bergmann, Phys. Rep. **107**, 1 (1984).

[4]V. F. Gantmakher and I. B. Levinson, Rasseyanie nositeleĭ toka v metallakh i poluprovodnikakh (Current-Carrier Scattering in Metals and Semiconductors), Nauka, Moscow, 1984.

[5]M. I. D'yakonov and V. I. Perl', Zh. Eksp. Teor. Fiz. **60**, 1954 (1971) [Sov. Phys. JETP **33**, 1053 (1971)]; Fiz. Tverd. Tela (Leningrad) **13**, 3581 (1971) [Sov. Phys. Solid State].

[6]E. N. Gr"ncharova and V. I. Perel', Fiz. Tekh. Poluprovodn. **10**, 2272 (1976) [Sov. Phys. Semicond. **10**, 1348 (1976)].

[7]M. Jaros, Rep. Prog. Phys. **48**, 1091 (1985); N. C. Casey and M. B. Panise, *Heterostucture Lasers*, Academic, New York, 1978 [Russ. transl. Mir, Moscow, 1981].

[8]L. Leibler, Phys. Rev. **B16**, 863 (1977).

[9]O. Roos, Phys. Rev. **B27**, 7547 (1983); P. J. Price and F. Stern, Surf. Sci. **132**, 577 (1983).

[10]C. Hermann and C. Weisbuch, Phys. Rev. **B15**, 823 (1977).

[11]F. T. Vas'ko, Fiz. Tekh. Poluprovodn. **19**, 1958 (1985) [Sov. Phys. Semicond. **19**, 1207 (1985)].

[12]F. T. Vas'ko and N. A. Prima, Fiz. Tekh. Poluprovodn. **13**, 521 (1979) [Sov. Phys. Semicond. **13**, 308 (1979)].

[13]L. D. Landau and E. M. Lifshitz, Ēlektrodinamika sploshnykh sred, Nauka, Moscow, 1982 (Electrodynamics of Continuous Media, Pergamon, New York).

[14]A. H. Clark, R. D. Burnham, D. J. Chadi, and R. M. White, Phys. Rev. **B12**, 5758 (1975).

[15]D. J. Newson, M. Pepper, H. J. Hall and J. H. Marsh, J. Phys. **C18**, L1041 (1985).

[16]A. H. Marshak and C. M. van Vliet, Proc. IEEE **72**, 5 (1984).

[17]M. I. D'yakonov and A. V. Khaetskiĭ, Zh. Eksp. Teor. Fiz. **86** 1843 (1984) [Sov. Phys. JETP **59**, 1072 (1984)].

[18]M. I. D'yakonov and V. Yu. Kachorovskiĭ, Fiz. Tekh. Poluprovodn. **20**, 178 (1986) [Sov. Phys. Semicond. **20**, 178 (1986)].

[19]A. S. Volkov, A. L. Lipko, Sh. M. Meretliev, and B. V. Tsarenkov, Pis'ma Zh. Eksp. Teor. Fiz. **41**, 458 (1985) [JETP Lett. **41**, 557 (1985)]; A. S. Volkov, A. L. Lipko, S. E. Minakov, and B. V. Tsarenkov, Fiz. Tekh. Poluprovodn. **19**, 1277 (1985) [Sov. Phys. Semicond. **19**, 780 (1985)].

[20]S. G. Petrosyan and V. V. Chaldyshev, Fiz. Tekh. Poluprovodn. **18**, 1565 (1984) [Sov. Phys. Semicond. **18**, 980 (1984)].

[21]A. G. Aronov, G. E. Pikus, and A. N. Titkov, Zh. Eksp. Teor. Fiz. **84**, 1170 (1983) [Sov. Phys. JETP **57**, 680 (1983)].

Translated by Dave Parsons