Incommensurate phases with point defects: the phason gap under strong-pinning conditions

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We investigate three-dimensional degenerate systems (incommensurate phases, crystals with charge density waves, two-component Heisenberg magnets) with frozen-in point defects which interact with the phase of the order parameter. A generalized susceptibility is determined for such a system relative to a homogeneous change in the phase of the order parameter—the "phason gap"—in the case where the value of the phase at the location of the defect is determined primarily by the defect field (the "strong-pinning" case). Using an effective Hamiltonian involving only the phase change, we show that the system in question possesses a large number of metastable states which are essentially collective in character with respect to the defects. We obtain conditions for strong pinning in a three-dimensional system, and determine the phason gap in the limit where transitions between metastable states are neglected. We discuss the conditions under which these transitions can be neglected. In order to find the temperature dependence and to estimate numerically the coefficients of the effective Hamiltonian, we make use of a continuum model for the incommensurate phase. The model we have chosen is the sort most commonly encountered in dielectric crytals, namely, that of potassium selenate. We show that defects like these can suppress scattering of light by phasons even at low concentrations, making the observation of this scattering extremely problematical in real crystals.

It is well-known that a degenerate system (here we are concerned with incommensurate phases, charge density waves and the x-y model) offers no resistance to a homogeneous change in the phase of its order parameter (i.e., a shift in the position of a displacement or charge density "wave," or a rotation of the magnetization vector). In other words, gapless modes of oscillation (phasons or spin waves) are present in the spectra of such systems. It is natural to expect (and many authors have noted) that a gap will appear in the spectrum of such oscillations in the presence of defects. The presence of such a gap should be detectable in a number of experimentally observable phenomena: neutron and light scattering, local magnetic relaxation, etc. At the same time, there are serious difficulties connected with theoretical calculations of this gap. The theory of one-dimensional systems has been the most successful,[1–3] while for three-dimensional systems the situation is much more ill-defined. Different expressions for the gap have been derived by various authors; the conditions under which these expressions are applicable remain unclear.

Let us explain the latter statement in more detail. We distinguish two regimes in which defects interact with the phase φ of the order parameter—the strong- and the weak-pinning regimes. In the first case, the value of φ at the location of a point defect is determined only by the characteristics of this defect. In the second case, the value of φ is determined not so much by a given defect as by all of the defects collectively. However, there is confusion even in the formulation of conditions for strong and weak pinning in a three-dimensional system. The appropriate conditions for a three-dimensional system were first presented (without proof) in the frequently cited Ref. 4. In contrast to analogous conditions for a one-dimensional system,[2,3] these conditions do not involve the defect concentration N. At the same time, the formulae (actually estimates) for the threshold fields presented in Ref. 4 for these two regimes contain different powers of N. For some reason, this contradiction has not been noted in subsequent papers: thus, for example, in the recent Ref. 5 and the review Ref. 6, these formulae are cited for the threshold field, and it is tacitly assumed that the strong-pinning conditions are the same in one and in three dimensions. In a recent paper[7] an attempt was made to obtain a strong-pinning condition in the same form as in Ref. 4; however, the author's argument is not compelling (see Section 1). The expression for the gap given in Ref. 7 for the case of strong pinning differs in its dependence on N from that presented in review Ref. 6 (for the same case). In the well-known Ref. 8 an expression is obtained for the gap as the limit of the frequency-dependent part of the inverse susceptibility when the frequency approaches zero. Although this procedure is clear, the authors do not point out that in fact the results pertain to the case of weak pinning. We note that in this case also the corresponding expression in the review Ref. 6 differs substantially from the results of Ref. 8.

In the present work, we present calculations from first principles of the magnitude of the phason gap in the strong-pinning regime. The paper is organized as follows: in the first section we use an effective Hamiltonian which is suitable to describe (in the long-wavelength approximation) both sinusoidal incommensurate phases (including charge density waves) and the x-y model in the presence of defects. We demonstrate explicitly that such systems possess a large number of metastable states which are essentially collective in character with respect to the defects. For the low-energy states we calculate the magnitude of the phason gap. We prove the correctness of the strong-pinning condition given

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in Ref. 4. In the second section, the parameters of the long-wavelength Hamiltonian are estimated for a specific kind of incommensurate phase which is often encountered in dielectric crystals. We elucidate the question of how these defects can lead to strong pinning. In the third section, we make estimates of the intensity of light scattering by phasons. It turns out that even for small impurity concentrations ($N = 10^{15}$–10^{16} cm^{-3}) this scattering can turn out to be unobservable.

1. LONG-WAVELENGTH APPROXIMATION

Since a degenerate system presents no resistance to a homogeneous change in the order parameter phase φ, the influence of defects can lead only to changes in φ in the long-wavelength approximation. The corresponding effective Hamiltonian is well-known[1–4,8]:

$$H = \int d\mathbf{r} \left\{ \frac{C}{2} (\nabla\varphi)^2 - \sum_i V_i \delta(\mathbf{r}-\mathbf{r}_i) \cos(\varphi_i - \psi_i) - h_{ext}\varphi \right\}. \tag{1}$$

Here, C is a constant of the ideal crystal, ψ_1 is the phase of the "field" of the ith defect, while V_1 is its amplitude; h_{ext} is a generalized external field coupled to the phase. The Hamiltonian (1) is suitable for describing sinusoidal incommensurate phases, charge density waves, and the x-y model. For charge density waves, h_{ext} has the sense of an electric field; in this case, it is necessary to add to (1) a long-range Coulomb term,[8] the expression for which is not needed by us here. The form of the term corresponding to defects is most easily clarified by the example of the x-y model with a randomly oriented local magnetic field $\mathbf{h}_i$. Each term in the sum (1) is then simply $-\mathbf{h}_i\mathbf{M}(\mathbf{r}_i)$, where the magnitude of the vector $\mathbf{M}$ is assumed to be the same everywhere in the crystal.

For point defects of the "random anisotropy" type (for this terminology see, e.g., Ref. 9), $\cos(\varphi_i - \psi_i)$ in formula (1) should be replaced by $\cos[m(\varphi_i - \psi_i)]$, where m is the degree of anisotropy; the investigation then proceeds in an analogous fashion.

It is necessary to note at once that questions regarding the actual parameter values of the Hamiltonian (1), their dependences on temperature, etc., must be answered separately for each specific system.[1] In the following sections we will study these questions for a specific type of incommensurate phase.

Let us turn to the Hamiltonian (1) and begin our analysis with the limiting case $V_1 \to \infty$. Then the value of φ at the defects coincides with the phases of the random field

$$\varphi_i = \psi_i + 2\pi n_i. \tag{2}$$

Refs. 2–4 also refer to this case as strong pinning.

Let us assume that topological defects can be neglected, and consider the function φ to be uniquely defined over the whole crystal. In our case, the only possible topological defects are dislocation lines; a circuit around one of these lines leads to an "advance" of the phase by 2π—see, e.g., Ref. 10. The usual dislocations in the theory of elasticity lead to destruction of long-range order in the atomic positions, a situation which apparently occurs in spin glasses. It is believed that in general the states of a Heisenberg spin glass contain many dislocations,[11] so that there is no magnetic long-range order in these glasses. As for incommensurate phases near structural phase transitions, any washing out of the reflex of the incommensurate structure in such systems is practically unnoticeable.[12,13] This can be considered as indirect evidence of the fact that, if there are dislocations in these structures their number is not large.

In order to find the function φ we must solve the Laplace equation

$$\Delta\varphi = 0 \tag{3}$$

with the boundary conditions (2) at each of the defects. (For charge density waves, we find a more complicated equation due to the Coulomb term[8]).

The various choices of the n_i correspond exactly to the various possible states. These states are separated by barriers; in fact, we will investigate changes in phase along a line connecting two defects. In the language of the x-y model, the change in n_i at one of the defects corresponds to a change in the number of "windings" of the magnetization vector around the dislocation line. In order to accomplish this, it is necessary to "detach" the vector $\mathbf{M}$ from the direction of the local field at one of the defects, which implies a transition through a barrier. Thus, the states differ in the number of windings of the vector $\mathbf{M}$ between any pair of defects, i.e., they have a collective character. Each of them correspond to a minimum energy, which is as yet undetermined.

Let us now turn to the case of large but finite V_1. At the same time, we will also obtain an expression for the gap. The variational equation corresponding to the Hamiltonian (1) has the form

$$C\Delta\varphi = \sum_i V_i \delta(\mathbf{r}-\mathbf{r}_i) \sin(\varphi_i - \psi_i) - h_{ext}. \tag{4}$$

For large values of V_1 it is natural to assume that the solution to Equation (4) is small relative to the solution to Equation (3) with boundary condition (2). We remark that for any particular choice of n_i, the problem (2), (3) is simply a problem of electrostatics for the "potential" $\varphi(\mathbf{r})$. As is well-known, any problem with given potentials at the defects (2) can be cast as a problem with given charges q_i at each defect:

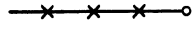
$$\Delta\varphi = -4\pi q(\mathbf{r}), \tag{5}$$

$$q(\mathbf{r}) = \sum_i q_i \delta(\mathbf{r}-\mathbf{r}_i). \tag{6}$$

Let φ_1 be the solution to Equation (5), while φ is the solution to Equation (4). Then for $\varphi_2 = \varphi - \varphi_1$, taking into account (2) we obtain

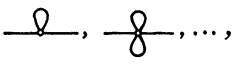
$$C\Delta\varphi_2 = \sum_i V_i \delta(\mathbf{r}-\mathbf{r}_i) \sin\varphi_2 + 4\pi C q(\mathbf{r}) - h_{ext}. \tag{7}$$

Since we are seeking φ_2 as a small correction to φ_1, we replace $\sin\varphi_2$ by φ_2, and then do perturbation theory with respect to V_1. In the usual "cross" technique[14] the terms in the perturbation series can be cast in diagrammatic form

 (8)

The lines correspond to $G_0(\mathbf{k}) = (Ck^2)^{-1}$. The term of order n contains n crosses, each of which is associated with $V(\mathbf{r}) = \sum_i V_i \delta(\mathbf{r} - \mathbf{r}_i)$. The small circles are associated with $q(\mathbf{r})$, given by (6) (we assume that $h_{ext} = 0$ for the time being).

The principle contribution to the mean square of the local fluctuation φ_2 comes from small wave vectors $\mathbf{k}$. Having calculated the correlation functions for long-wavelength fluctuations, we must exclude the most strongly divergent graphs for $\mathbf{k}\to 0$. In the three-dimensional case these are graphs which in general do not contain intermediate integrations over $\mathbf{k}$, in which we can simply replace $V(\mathbf{r})$ by $\overline{V} = NV_1$ (here and below, the bar denotes averaging over all defect configurations). These we will re-sum; it is necessary, however, to take into account two inessential complications connected with the fact that the quantity V_1 is not small in our case. First of all, there is a renormalization of $\overline{V}$ in the self-energy part of the Green's function by graphs of the form



(9)

obtained by joining together two crosses, three crosses, etc., in (8). In (9), we include the fact that the correlation functions for $V(\mathbf{r})$ are δ-functions. The open circles in (9) correspond to the factor $NV_1^{n/2}$, where n is the number of lines which converge on a given open circle. The graphs (9) are usually associated with the process of multiple-scattering by one and the same center. In order to evaluate these graphs, we proceed to introduce a cutoff wave vector $k_m = \pi/2r_0$. The meaning of the cutoff parameter r_0 will be clarified in the next section when we investigate the case of small spacings. The result of summing these graphs is to replace the original Green's function G_0 by the following:

$$G_0(\mathbf{k}) = [NV_1/(1+\xi) + Ck^2]^{-1}, \tag{10}$$

$$\xi = V_1/4\pi Cr_0. \tag{11}$$

Another complication connected with the large value of V_1 consists in the fact that in joining crosses in (8) we can also link with a small circle, i.e., all refer to one and the same defect. This will no longer give the renormalized Green's function, but will renormalize the charge q_i. It is not hard to calculate this:

$$q_i \to q_i/(1+\xi). \tag{12}$$

As a result, we find for the mean square fluctuation

$$\overline{\varphi_2^2} = \sum_{\mathbf{k}} \frac{(4\pi C)^2 \overline{q_{\mathbf{k}} q_{-\mathbf{k}}}}{(1+\xi)^2 [NV_1(1+\xi)^{-1} + Ck^2]^2}. \tag{13}$$

Let us clarify the meaning of the above calculation using different language: if it is meaningful to add to the right side of Equation (7) $A\varphi_2 + B\varphi_2^3$ (and, as previously, to set $\sin\varphi_2 \approx \varphi_2$), this equation coincides with the equation for the distribution of a single-component order parameter $\varphi_2(\mathbf{r})$ near a phase transition point in a crystal with defects. These defects have a component of the "random phase transition temperature" type with a "strong" V_1, which therefore leads in particular to a shift in the phase transition point. In other words, the singularity in the susceptibility is "shifted" from the point $A = 0$ to a new transition point $A \neq 0$. As for the susceptibility at the point $A = 0$, which is the only point of interest here, it is given by a simple expression from Landau's theory, taking into account only the renormalized temperature of the phase transition.[15,16] That is, this circumstance allows us to limit ourselves only to the simplest graphs. Expression (10) for the inverse susceptibility (at $k = 0$) corresponds exactly to the magnitude of the shift in the phase transition obtained in Refs. 15,16 (a detailed derivation of this expression was not presented there, so we will give one here). We note that for a charge density wave the original Green's function has an additional angular dependence in $\mathbf{k}$-space due to the Coulomb interaction.[8] However, for these simple graphs, which we have re-summed, this is not important.

Let us now estimate the magnitude of the fluctuations (13). The following inequality holds:

$$\overline{\varphi_2^2} \leqslant \frac{32\pi^2}{NV_1(1+\xi)} \left\{ \frac{C}{2} \sum_{\mathbf{k}} \frac{\overline{q_{\mathbf{k}} q_{-\mathbf{k}}}}{k^2} \right\} = \frac{32\pi^2}{NV_1(1+\xi)} E_1. \tag{14}$$

Here, we denote by E_1 the "electrostatic" energy of the field φ_1. Inequality (14) is obtained by discarding in one of the brackets in the denominator (13) the term $NV_1(1+\xi)^{-1}$, and in the other bracket the term Ck^2. There is at least one state for which E_1 can be calculated. This is the state in which all the n_i in formula (2) are zero. The calculation of E_1 for this case was in fact carried out in Ref. 15, and we make use of the result derived there:

$$E_1 \approx 2\pi Cr_0 N. \tag{15}$$

Comparing (14) and (15), we can convince ourselves that

$$\overline{\varphi_2^2} \ll (2\pi)^2, \tag{16}$$

if

$$\xi \gg 1. \tag{17}$$

We do not know what state corresponds to a minimum in the electrostatic energy; however, for this state E_1 can only be smaller than the value (15), and the inequality (16) under condition (17) is all the more correct. Inequality (16) justifies using the approximation $\sin\varphi_2 \approx \varphi_2$. Thus, the susceptibility (7) of the system with respect to h_{ext} does not depend on q_i and is defined by formula (10).

Thus, when condition (17) holds the correction φ_2 is small, at least for the low-lying states of the system. This implies that in place of the value at the position of the defect the phase of the order parameter coincides with the phase of the "random field" of a given defect (2). Hence, condition (17) is the condition for strong pinning as it is usually understood.[2-4]

It was pointed out to us in private discussions that the strong-pinning condition (17) was obtained by A. I. Larkin and L. P. Gor'kov. Condition (17) and an expression for the gap in the form (10) were also obtained in a recent paper.[7] However, in this paper an unjustified assumption was introduced into the problem, reducing it at once to the single-particle case. As our analysis shows, in fact, the states of the system have a collective character. Nevertheless, the similarity between the two formulae warrants attention, although the cutoff parameter introduced in (10) is certainly not the lattice parameter, as the authors of Ref. 7 assumed. The determination of this parameter requires a first-principle analysis in the regime of small defect spacing. For our case, it is now clear (see next section) that r_0 is determined by the magnitude of the force at the defect itself. There is also no way to reconcile these results with the frequency dependence of the susceptibility given in Ref. 7.

2. SMALL SPACINGS

As we have already mentioned, any determination of realistic values of the parameters C, V_1 and r_0, as well as their temperature dependences, must originate with an analysis of the small-spacing regime. We now turn to this problem. Let us note that a detailed analysis of vortex lattices in superconductors[17,18] leads to a number of nontrivial consequences. Thus, for example, the constant which is analogous to our V_1 can in this case have an oscillatory dependence of magnetic field.[18]

We will investigate this question within the framework of the basic continuum model for the incommensurate phase (see, e.g., Refs. 19,20). We will regard the incommensurate phase as a distinct region of some commensurate phase. The effective Hamiltonian as a function of the two-component order parameter ($\eta_1 = \rho\cos\tilde{\varphi}, \eta_2 = \rho\sin\tilde{\varphi}$) for the phase transition from a high-symmetry phase to the above-mentioned commensurate phase has in the simplest case the form[19]

$$H = \int d\mathbf{r}\left[\frac{\tilde{A}}{2}\rho^2 + \frac{B}{4}\rho^4 + B'\rho^n\cos n\tilde{\varphi} + r\rho^2\frac{\partial\tilde{\varphi}}{\partial x} + \frac{D}{2}(\nabla\rho)^2 + \frac{D}{2}\rho^2(\nabla\tilde{\varphi})^2\right]. \quad (18)$$

Here the term with coefficient B' is "responsible" for the subsequent transition from incommensurate to commensurate phase (for the basic family of incommensurate dielectrics, i.e., crystals of the potassium selenate type, $n = 6$); the term with coefficient r is a Lifshitz invariant. The presence of the Lifshitz invariant shows that a second-order phase transition from high-symmetry to commensurate phase is impossible, and that for small values of $\tilde{A}$ the inverse phase transition occurs. It is significant that in this case it is not necessary to include terms with higher derivatives up to values of $\mathbf{k}$ on the order of an inverse lattice vector.

Assume that defects give rise to the appearance of a local random field relative to one of the order parameters, say η_1. Then

$$\Delta H = -\sum_i h_1\eta_1(\mathbf{r}_i) = -\int d\mathbf{r}\sum_i h_1\delta(\mathbf{r}-\mathbf{r}_i)\rho\cos\tilde{\varphi}. \quad (19)$$

Let us neglect terms proportional to B' in (18). For the sinusoidal regime, which is realized near the symmetric-incommensurate phase transition, its role is insignificant due to the smallness of ρ. We will study just this regime. Then the equations which correspond to a minimum in the energy (18),(19) take the form

$$D\nabla(\rho^2\nabla\varphi) = \sum_i h_1\delta(\mathbf{r}-\mathbf{r}_i)\rho\sin(\varphi_i-\psi_i), \quad (20)$$

$$A\rho + B\rho^3 - D\Delta\rho + D\rho(\nabla\varphi)^2 = \sum_i h_1\delta(\mathbf{r}-\mathbf{r}_i)\cos(\varphi_i-\psi_i).$$

Here we have gone to the variables $\varphi = \tilde{\varphi} + k_0x, k_0 = r/D$; $A = \tilde{A} - r^2/D, \psi_i = k_0x_i$. As usual, we let $A = A_0(T - T_i)$, where T_i is the transition temperature from the incommensurate to the high-symmetry phase for a defectless crystal.

Due to the presence of a finite "stiffness" A, the perturbation of the amplitude ρ near each defect decays over a distance $r_A = [D/(-2A)]^{1/2}$. Therefore, for moderate defect concentrations and for temperatures not too close to the phase transition point, when

$$Nr_A^3 < 1, \quad (22)$$

it is sufficient to investigate the problem of a single defect to determine the parameters C, V_1 and r_0. Since we are interested in the case of strong pinning, i.e., when $\varphi_i \approx \psi_i + 2\pi n_i$, we set $\cos(\varphi_i - \psi_i) = 1$ in Eq. (21). We also discard the term with $\nabla\varphi$ in (21) (we will justify this approximation below). Then the solution to Eq. (21) (including only one defect at the coordinate origin) takes the simple form

$$\rho(\mathbf{r}) = \rho_\infty + \frac{h_1}{4\pi D}\frac{1}{r}\exp\left(-\frac{r}{r_A}\right), \quad \rho_\infty^2 = -\frac{A}{B}, \quad (23)$$

the condition that these expressions be applicable is the inequality[15]

$$\frac{h_1}{h_{1aT}}\ln\left(\frac{r_A}{d}\right) \ll 1. \quad (24)$$

Here $h_{1aT} = 4\pi D^{3/2}B^{-1/2}$ is the atomic, i.e., the maximum possible value of h_1, while d is the size of the "nucleus" of the defect—a quantity which usually appears in the continuum theories of defects in crystals (see, e.g., Refs. 4,15). The size d (on the order of a lattice constant) serves as a cutoff parameter for the distribution $\rho(r)$ for small spacings, i.e.,

$$\rho(0) - \rho_\infty = \sum_{0<k<\pi/2d}\rho_k = h_1/4\pi Dd. \quad (24a)$$

If along with (24) the condition which is the inverse of

$$(h_1/h_{1aT})(\rho_{aT}/\rho_\infty) \gg 1, \quad \rho_{aT}^2 = D/Bd^2, \quad (25)$$

is also fulfilled, then the perturbations ρ and φ at each defect are in general independent.[4] This case of extremely small fields h_1 necessarily corresponds to weak pinning and is of no interest to us. Therefore we will assume that although the field is small compared to its atomic value (24), condition (25) holds. We now substitute $\rho(\mathbf{r})$ given by (23) into Eq. (20); substitution of $\rho(\mathbf{r})$ into the right-hand part of Eq. (20) gives the value of the parameter V_1 from formula (1). Taking into account (24a) and (25), we have

$$V_1 = h_1^2/4\pi Dd. \quad (26)$$

Substituting $\rho(\mathbf{r})$ into the left-hand side of (20) determines the "bare" Green's function

$$D\nabla\left\{\left[\rho_\infty + \frac{h_1}{4\pi D}\frac{1}{r}\exp\left(-\frac{r}{r_A}\right)\right]^2\nabla G_0\right\} = 4\pi\delta(\mathbf{r}). \quad (27)$$

An exact solution to (27) cannot be found; however, this is not required to estimate the values of the parameters C, V_1 and r_0. It is not hard to convince oneself that terms with h_1 in general have no bearing on the behavior of G_0 for large spacings; therefore they are important only in determining the cutoff parameter r_0, while $C = D\rho_\infty^2$. Turning to the determination of the quantity r_0, we remark that a naturally important distance contained in $\rho(\mathbf{r})$ is the spacing a for which the two terms in Eq. (23) are comparable in magnitude:

$$a = h_1/4\pi D\rho_\infty. \quad (28)$$

For conditions (24) and (25), $d \ll a \ll r_A$. The spacing a also serves as the cutoff parameter r_0. Actually, in determining the asymptotic behavior of G_0 for $r \ll a$ we find

$$G_0(\mathbf{r}) = c_1 - c_2 r \tag{29}$$

and thus verify that it ensures convergence of the integrals at short distances.

The correction $\rho'(\mathbf{r})$ to solution (23), connected with the discarded terms with $\nabla\varphi$ in (21), takes the form

$$\rho'(\mathbf{r}) = \int d\mathbf{r}' \frac{1}{4\pi|\mathbf{r}-\mathbf{r}'|} \rho(\mathbf{r}') [\nabla\varphi(\mathbf{r}')]^2. \tag{30}$$

Since at a distance a the phase φ cannot change by more than 2π, we estimate $\nabla\varphi$ to be $2\pi/a$ for $r < a$ and $2\pi a/r^2$ for $r > a$. Estimating the integral (30) using $\rho(\mathbf{r})$ in the form (23) we confirm the fact that $\rho' \ll \rho$ both for $r \ll a$ and $r \gg a$ when condition (25) holds. For $r \sim a$ it turns out that $\rho' \sim \rho$; however, this situation can change only the numerical coefficients in estimates of C, V_1 and r_0, which these estimates cannot claim to predict.

Thus, we have the following estimates for the parameters introduced in the previous section:

$$C = D\rho_\infty^2, \tag{31}$$

$$r_0 = h_1/4\pi D\rho_\infty, \tag{32}$$

$$V_1 = h_1^2/4\pi Dd. \tag{33}$$

Rewriting the strong-pinning condition in the new notation, we verify that it coincides with condition (25). Incidentally, the strong-pinning condition was also presented in just this form in Ref. 4. We also present the complete formula for the susceptibility relative to h_{ext} (4), i.e., the Green's function. Using formula (10) and estimates (31)–(33) including condition (25), we have

$$G(\mathbf{k}) = (Nh_1\rho_\infty + D\rho_\infty^2 k^2)^{-1}. \tag{34}$$

Let us discuss (34). It is noteworthy that the stiffness of the system $G^{-1}(k=0)$ turns out to be additive in the defects, although the field $\varphi(\mathbf{r})$ over large length scales satisfies the Laplace equation and the states carry an essentially collective character. Let us try to explain this result qualitatively. It is natural to suppose that the stiffness of the system is determined by contributions from those regions in which the phase φ is most weakly varying under the action of the field. In the case of strong pinning, this region is the immediate neighborhood of each defect. Consequently, the stiffness of the system is proportional to the defect concentration N, the size r_0 (32) of a region of fixed phase at one defect, and, naturally, the crystal constant C (31). We note that in formula (34) the defect parameter d does not enter in; however, it figures into the conditions of applicability of this formula.

The estimates presented here allow us, in particular, to clarify under what conditions strong pinning can take place. The quantity ρ_∞ appearing in (25) reduces to zero at the incommensurate-symmetric phase transition point (at $T = T_i$). Therefore, near T_i and in the absence of defects which have a component of the random-field type (19), the strong-pinning regime is always realized. The width of this region is especially large for systems of displacive type, in which the parameter ρ_{at}/ρ_∞ is large even far from the transition. For defects which do not have the random-field type of components because of their high symmetry, the situation is entirely different. Thus, if at a defect there is only a component of the random-anisotropic type, in place of (19) we need to write

$$\Delta H = \sum A_1 \eta_1^2(\mathbf{r}_i) = \frac{1}{2}\int d\mathbf{r} \sum_i A_1 \rho^2 \delta(\mathbf{r}-\mathbf{r}_i)(1+\cos 2\tilde{\varphi}). \tag{35}$$

All the analysis is carried out in full analogy with the case investigated earlier. As a result, in place of condition (25) we obtain

$$A_1/4\pi Dd \gg 1, \tag{36}$$

which in fact implies that the quantity A_1 must significantly exceed its atomic value. This is improbable, i.e., for random anisotropy we are dealing with weak pinning.

Let us emphasize that we have not included the possibility of thermal "hopping" between the various states (2), i.e., we have in fact investigated the case $T = 0$. For $T \neq 0$ the gap, strictly speaking, is absent and the results for the susceptibility (34) are correct only for sufficiently high frequencies, when jumps between states do not occur. Clarifying the character of the dispersion of the susceptibility due to jumps is a very difficult task, and in this article we will not consider it. It is possible only to confirm that for $T = T_i$ the barriers between states disappear, whereas sufficiently far from T_i the relaxation time must be extremely long. We note that in experiments on incommensurate phases one often observes slowing-down of the relaxation (in the course of many hours). It is entirely possible that this is precisely a consequence of hopping.

We also draw attention to the fact that the susceptibility (34) is related only to the low-lying states. One may expect that for slow cooling from the symmetric phase the system necessarily finds itself in one of these low-lying states. It is reasonable to suppose, however, that the temperature and concentration dependences of the high-frequency susceptibility will be qualitatively the same for all states.

3. SCATTERING OF LIGHT BY PHASONS

As we noted in the Introduction, the presence of a gap must make itself felt in many observable quantities. This must lead to especially strong effects in scattering of light by phasons. The scattering of light by phasons is difficult to observe even in an ideal crystal, since it must lead to a very narrow and weak-intensity central peak in the scattering spectrum.[21] Experimental errors (misprint?) of such a peak have not yet been crowned with success.[22] We will show that under conditions in which the "gap relaxation time" is sufficiently long, defects of the type we have discussed here, even at low concentrations, can make light scattering by phasons practically unobservable.

In the simplest case the coupling of the dielectric permittivity in the region of optical frequencies with the phase fluctuations has the form[21]

$$\varepsilon = \varepsilon_0 + a\rho^2 \partial\varphi/\partial x, \tag{37}$$

from which, taking (34) into account, it follows that

$$\langle|\varepsilon(\mathbf{k})|^2\rangle = a^2\rho_\infty^4 k^2 T/(Nh_1\rho_\infty + D\rho_\infty^2 k^2). \tag{38}$$

This quantity determines the integrated intensity of light scattered by phasons. Equation (38) differs from the analogous expression for scattering in a perfect crystal by the presence of the first term in the denominator. As a result, the scattering intensity of light off of phasons turns out to be δ times smaller than in an ideal crystal:

$$\delta \approx \frac{Nh_1}{D\rho_\infty k^2} = \frac{4\pi Nd}{k^2}\left(\frac{\rho_{aT}}{\rho_\infty}\right)\left(\frac{h_1}{h_{1aT}}\right). \quad (39)$$

For a concentration $N = 10^{18}\ \text{cm}^{-3}$ characteristic of nominally pure samples and for $h_1/h_{1aT} = 0.1$, $\rho_\infty/\rho_{aT} = 10^{-2}$, $d = 10^{-7} - 10^{-8}$ cm, $k = 10^{-5}\ \text{cm}^{-1}$ we obtain $\delta \approx 10^2$–10^3 and correspondingly $\delta \approx 1$ for $N = 10^{15}$–$10^{16}\ \text{cm}^{-3}$. As far as we know, crystals of such perfection are practically unobtainable at the present time. Let us also note that for concentrations of 10^{15}–$10^{16}\ \text{cm}^{-3}$, in a nondegenerate system defects play no role either in generating anomalies of thermodynamic quantities or in elastic light scattering.[15,16] Naturally, we cannot claim this is true for all defects: it applies only to those which have a random-field component like (19). We emphasize that defects need not suppress the scattering of neutrons by phasons so strongly, since in this case the characteristic value of **k** is considerably larger. In this case the phason can be observed as a phonon mode with a gap (34).

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