

# Tunnel transitions in the valence band of germanium and inversion of the populations of light-hole Landau levels

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The quasiclassical approximation is used to find the probability of a tunnel conversion of a light hole into a heavy one in crossed electric and magnetic fields. This process is considered as a possible reason for a population inversion of light-hole Landau levels in the passive region.

## 1. INTRODUCTION

Ivanov and Vasil'ev<sup>1</sup> observed stimulated emission from germanium as a result of transitions between light-hole Landau levels in crossed electric and magnetic fields. The population inversion mechanism responsible for such stimulated emission is not yet clear. Kozlev *et al.*<sup>2</sup> suggested a mechanism based on the energy dependence of the time for the conversion of light holes into heavy ones as a result of scattering by ionized impurities. This time increases with the energy and, therefore, the lower light-hole Landau levels are emptied faster than the higher ones.

The light-hole levels are populated in the passive region (at energies less than the optical phonon energy) by transitions from the band of heavy holes when they emit optical phonons. For sufficiently strong heating of holes in crossed fields,<sup>3,4</sup> the passive region is populated practically uniformly. Therefore, the scattering of impurities may result in a population inversion.

However, under the conditions of the experiments reported in Ref. 1 the impurity concentration was low ( $\sim 10^{13}$  cm<sup>-3</sup>) and the mechanism proposed in Ref. 2 was clearly ineffective, because the competing process of emptying of the levels of light holes—scattering by acoustic phonons—has an energy dependence which is the opposite of that in the case of impurity scattering.

We shall consider a different possible population inversion mechanism which is not associated with the scattering and is due to tunnel conversion of light holes into heavy ones. Such tunnel transitions were considered in Refs. 5 and 6 and it was shown that the tunneling probability increases the closer are the hole paths in the velocity space to the origin i.e., to the band degeneracy point). If the velocity of drift in crossed fields is less than the velocity of cyclotron motion of a light hole at the first Landau level, the tunneling probability should decrease as the level number increases. Consequently, the lower levels will be emptied faster than the higher ones.

The tunneling probability was calculated numerically in Ref. 6 for the case when the paths of light and heavy holes between which a transition takes place almost touch, which does not correspond to the experimental conditions in Ref. 1. We shall obtain an analytic expression for the argument of the exponential function governing the probability of tunneling in general.

If the shortest distance between the paths of light and heavy holes with the same energy is small compared with the path dimensions, we can obtain also the preexponential fac-

tor. In this case the problem reduces to calculation of the tunneling probability in an electric field (when the magnetic field is absent).

We shall use the expressions for the tunneling probability derived below to consider the conditions for a population inversion.

## 2. TUNNELING IN AN ELECTRIC FIELD

We shall consider tunnel transitions of holes in an electric field employing the semiclassical approximation. Such transitions were studied in Ref. 7. Asymptotic formulas were obtained there for the transition probability. The result for the semiclassical limit given in Ref. 7 is not quite correct. However, as shown below, the correct response differs from the result obtained in Ref. 7 only by a numerical factor close to unity.

We shall select a coordinate system with the  $x$  axis directed along an electric field  $E$  and the  $y$  axis along the component of the momentum which is perpendicular to the field. Then, in the representation in which the matrix  $\hat{J}_z$  is diagonal ( $\hat{J}$  are matrices of the momentum  $3/2$ ) a system of four Luttinger equations splits into two independent subsystems. One of them relates the states with  $J_z = 3/2$  and  $J_z = -1/2$ , and is of the form

$$[\mathcal{H}(\mathbf{p}) - eEx] \psi = 0, \quad (1)$$

where  $\mathbf{p}$  is the momentum operator;  $\psi$  is a two-component column with the components  $\Phi_{3/2}$  and  $\Phi_{-1/2}$ ; and the matrix  $\mathcal{H}$  is given by

$$\hat{\mathcal{H}} = \frac{1}{2m_0} \begin{pmatrix} (\gamma_1 + \gamma) p^2 & -3^{1/2} \gamma (p_x - ip_y)^2 \\ -3^{1/2} \gamma (p_x + ip_y)^2 & (\gamma_1 - \gamma) p^2 \end{pmatrix}. \quad (2)$$

Here,  $m_0$  is the mass of a free electron;  $\gamma_1$  and  $\gamma$  are the Luttinger constants related to the masses of heavy and light holes by  $\gamma_1 - 2\gamma = m_0/m_h$  and  $\gamma_1 + 2\gamma = m_0/m_l$ . The second system of equations for the functions  $\Phi_{-3/2}$  and  $\Phi_{1/2}$  is obtained from Eqs. (1) and (2) by the substitution  $p_y \rightarrow -p_y$ .

The quantity  $p_y$  is a quantum number which is conserved. In the semiclassical approximation we can separate the motion of light and heavy holes if we ignore tunnel transitions. Then, the relationship between  $\mathbf{p}$  and  $x$  is given by

$$\frac{p_x^2 + p_y^2}{2m_l} = eEx, \quad \frac{p_x^2 + p_y^2}{2m_h} = eEx. \quad (3)$$

Figure 1a shows the dependences of  $p_x^2$  on  $x$  for light and

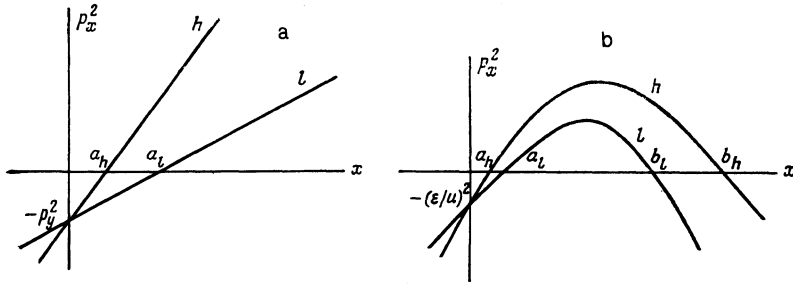


FIG. 1. Dependences of  $p_x^2$  on  $x$  for light ( $l$ ) and heavy ( $h$ ) holes: a) in an electric field; b) in crossed electric and magnetic fields.

heavy holes. The classically allowed regions ( $p_x^2 > 0$ ) are bounded by the turning points  $a_l = p_y^2/2m_l eE$  and  $a_h = p_y^2/2m_h eE$ . In the classically inaccessible region ( $p_x^2 < 0$ ) there is a singularity at  $x = 0$ , where the value of  $p_x^2$  is the same for light and heavy holes. Therefore, in the classically inaccessible region there is a path on which a light hole is transformed into a heavy one. It is known<sup>8</sup> that the action  $S$  calculated along this path determines the exponential dependence of the probability of such a conversion:  $W \propto \exp(-2S/\hbar)$ , where

$$S = \int_0^{a_l} |p_{x_l}| dx - \int_0^{a_h} |p_{x_h}| dx = \frac{|p_y|^3}{3eE} \left( \frac{1}{m_l} - \frac{1}{m_h} \right). \quad (4)$$

Our aim will be to calculate the preexponential factor. A special feature of the case under consideration is that the transition point  $x = 0$  is not a branching point of the function  $p(x)$ , as is usually the case, but a point at which two branches of the same function intersect (Fig. 1a).

We shall now adopt the momentum representation, i.e., we shall replace  $x$  in Eq. (1) with the operator  $i\hbar\partial/\partial p_x$ . The semiclassical solution of Eq. (1) corresponding to a light hole is<sup>9</sup>

$$\psi_l = \frac{b_l}{F^{1/2}} \chi_l \exp\left(-\frac{i}{\hbar} \int_0^{p_x} x_l dp_x\right), \quad (5)$$

where the dependence  $x_l(p_x)$  is governed by the first equation in the system (3) and  $\chi_l$  is the eigenfunction of the matrix  $\mathcal{H}$  satisfying the equation

$$\mathcal{H}\chi_l = \epsilon_l \chi_l, \quad \epsilon_l = p^2/2m_l.$$

Equation (5) includes  $F = dp_x/dt = eE$  instead of the usual  $v = dx/dt$  in the coordinate representation. The coefficient  $b_l$  satisfies the equation<sup>9</sup>

$$i\hbar\partial b_l/\partial t = -D_l E b_l. \quad (6)$$

Here,  $D_l = i\hbar e(\chi_l, \partial\chi_l/\partial p_x)$  is a matrix element of the dipole moment. In the momentum representation, we have  $db_l/dt = eE\partial b_l/\partial p_x$ . The semiclassical solution corresponding to a heavy hole is analogous to Eq. (5).

The columns  $\chi_l$  and  $\chi_h$  are given by

$$\chi_l = \frac{1}{2} \begin{pmatrix} -3^{1/4} \\ p_x + ip_y \\ p_x - ip_y \end{pmatrix}, \quad \chi_h = \frac{1}{2} \begin{pmatrix} 1 \\ 3^{1/4} p_x + ip_y \\ p_x - ip_y \end{pmatrix}. \quad (7)$$

Using these functions, we find  $D_h = 3D_l = e\hbar p_y/2p^2$ . Finally, we obtain the following expressions for the semiclassical wave functions of light and heavy holes:

$$\begin{aligned} \psi_l &= \chi_l \left( \frac{k-i}{k+i} \right)^{1/4} \exp\left(-iR_l \left( k + \frac{k^3}{3} \right)\right), \\ \psi_h &= \chi_h \left( \frac{k-i}{k+i} \right)^{1/4} \exp\left(-iR_h \left( k + \frac{k^3}{3} \right)\right), \end{aligned} \quad (8)$$

where a dimensionless variable  $k = p_x p_y$  is introduced and the notation  $R_{l,h} = p_y^3/(2eE\hbar m_{l,h})$  is employed.

We can find the probability of a tunnel transition converting a light hole into a heavy one by writing down the solution of Eq. (1) in the form  $\psi = A_l \psi_l + A_h \psi_h$ . Then, the coefficients  $A_l$  and  $A_h$  are described by the following system of equations<sup>7</sup>:

$$\begin{aligned} i \frac{dA_l}{dk} &= \frac{3^{1/2}}{2} \frac{k-i}{(k^2+1)^{1/2}} \exp\left[iR \left( k + \frac{k^3}{3} \right)\right] A_h, \\ i \frac{dA_h}{dk} &= \frac{3^{1/2}}{2} \frac{k+i}{(k^2+1)^{1/2}} \exp\left[-iR \left( k + \frac{k^3}{3} \right)\right] A_l, \end{aligned} \quad (9)$$

where  $R = R_l - R_h$  is a dimensionless parameter; in the quasiclassical approximation the condition  $|R| \gg 1$  is obeyed.

We shall first assume that  $p_y > 0$ . Then, we have  $R > 0$  and the system of equations (9) should be solved subject to the boundary conditions  $A_l(-\infty) = 1$ ,  $A_h(-\infty) = 0$ . The transition probability is  $W = |A_h(\infty)|^2$ .

The singularity at  $x = 0$  corresponds to the points  $k = \pm i$  in the momentum representation. We shall integrate the system (9) along a contour passing through the point  $k = -i$  and parallel to the real axis near this point. In the vicinity of this point we find that the assumption  $k = -i + sR^{-1/2}$  allows us to obtain the following results from the system (9):

$$\frac{dA_l}{ds} = \frac{3^{1/2}}{4} \frac{\exp(s^2)}{s^{1/2}} B, \quad \frac{dB}{ds} = \frac{3^{1/2}}{4} \frac{\exp(-s^2)}{s^{1/2}} A_l, \quad (10)$$

where instead of  $A_h$  we now have a function

$$B = 2^{1/2} R^{1/2} \exp(2R/3 - i\pi/4) A_h.$$

The system (10) reduces to a quadratic equation for  $B$  and the general solution of this equation can be expressed in terms of confluent hypergeometric functions

$$B = C_1 s^{-1/2} F(-1/8, 1/2, -s^2) + C_2 s^{3/2} F(3/8, 3/2, -s^2). \quad (11)$$

The phase  $s$  in the above expression should be selected in the same way as the phases of  $k+i$  in the system (8). We shall consider the specific case when the phase varies from 0 to  $2\pi$ .

Using the asymptotic formula for the functions  $F$  and the boundary conditions in the limit  $s \rightarrow -\infty$ , we obtain

$$B(\infty) = \frac{3^{1/2}}{4} \frac{\Gamma(-1/8)}{\Gamma(5/8)} \sin \frac{\pi}{8} \exp\left(\frac{3i\pi}{4}\right). \quad (12)$$

The function  $B(s)$  decreases exponentially in the direction of negative values of  $s$  and reaches a constant value described by Eq. (12) when  $s \gg 1$ . Therefore, the use of Eqs. (10), which is valid when  $s \ll |R|^{1/6}$  is justified in the semiclassical limit  $|R| \gg 1$ .

Therefore, if  $p_y > 0$ , the transition probability is

$$W_+ = \frac{3\pi}{[\Gamma(1/4)]^2} \frac{\exp(-4R/3)}{(2R)^{1/2}} = 0,51R^{-1/2} \exp\left(-\frac{4}{3}R\right). \quad (13)$$

We can calculate  $W$  similarly in the case when  $p_y < 0$  (in this case the contour should pass through the point  $k = 1$ ):

$$W_- = \frac{2}{3\pi} [\Gamma(1/4)]^2 (2|R|)^{1/2} \exp\left(-\frac{4}{3}|R|\right) = 3,95|R|^{1/2} \times \exp\left(-\frac{4}{3}|R|\right). \quad (14)$$

For the second pair of states with  $J_z = -3/2$  and  $J_z = 1/2$  the probabilities  $W_+$  and  $W_-$  are interchanged. Equations (13) and (14) differ from the results of Ref. 7 deduced from perturbation theory [it was assumed in Ref. 7 that  $A_l = 1$  in the second of the equations in the system (9)] simply as a result of numerical factors. This difference, however, is slight: in Ref. 7 these factors are 0.62 and 4.5, respectively.

### 3. TUNNELING IN CROSSED ELECTRIC AND MAGNETIC FIELDS

Let us assume that a magnetic field is directed along the  $z$  axis and an electric field along the  $x$  axis. We shall use the Landau gauge  $A = (0, Hx, 0)$ . In the semiclassical approximation when  $p_z = 0$  we still have the system (3) where the kinematic momenta  $p_x$  and  $p_y$  are related to the canonical momenta  $P_x$  and  $P_y$  by the expressions  $p_x = P_x$  and  $p_y = P_y - eHx/c = (eH/c)(x_0 - x)$ , where  $x_0 = cP_y/eH$  is the position of the center of a Larmor circle, which is the same for light and heavy holes. The integral of motion is now not  $p_y$  but  $P_y$  (i.e.,  $x_0$ ). Instead of the system (3), we now have

$$\frac{p_x^2}{2m_l} + \frac{m_l \Omega_l^2}{2} (x - x_0)^2 = eEx, \quad (15)$$

$$\frac{p_x^2}{2m_h} + \frac{m_h \Omega_h^2}{2} (x - x_0)^2 = eEx,$$

where  $\Omega_{l,h} = eH/m_{l,h}c$  are the cyclotron frequencies of light and heavy holes.

We shall introduce an energy  $\varepsilon = eEx_0$  which should be conserved in the course of conversion of a light hole into a heavy one. Using the system (15), we can represent this quantity in the form

$$\varepsilon = \frac{m_l}{2} (v_l^2 - u^2) = \frac{m_h}{2} (v_h^2 - u^2), \quad (16)$$

where  $\mathbf{v}_{l,h} = \mathbf{p}/m_{l,h} - \mathbf{u}$  are the velocities of light and heavy holes in a system of coordinates moving at the drift velocity  $\mathbf{u} = (0, -u, 0)$ ; here,  $u = cE/H$ . It should be noted that the quantities  $m_l v_l^2/2$  and  $m_h v_h^2/2$  are the energies of cyclotron motion of light and heavy holes in a coordinate system moving at a velocity  $\mathbf{u}$  where there is no electric field. These are the energies which in the semiclassical approximation

(when light and heavy holes are independent) assume values corresponding to the Landau levels. Equation (6) describes the change in the energy of cyclotron motion as a result of conversion of a light hole into a heavy one:  $m_h v_h^2/2 - m_l v_l^2/2 = (m_h - m_l)u^2/2$ .

Figure 1b shows the dependences of  $p_x^2$  on  $x$  for light and heavy holes which follow from the system (15). The classically allowed regions for light and heavy holes are found between the turning points  $a_l, b_l$  and  $a_h, b_h$ , respectively, where

$$eEa_l = \varepsilon + m_l u^2 - [(\varepsilon + m_l u^2)^2 - \varepsilon^2]^{1/2},$$

$$eEb_l = \varepsilon + m_l u^2 + [(\varepsilon + m_l u^2)^2 - \varepsilon^2]^{1/2}. \quad (17)$$

The expressions for  $a_h$  and  $b_h$  are obtained from the above formulas by replacing  $m_l$  with  $m_h$ .

As in Fig. 1a, in the classically inaccessible region there is a path passing through the point  $x = 0$  at which a light hole is converted into a heavy one. Calculating, by analogy with Eq. (4), the action  $S$  along this path and using the formulas in Eq. (15), we obtain the argument of the exponential function which governs the tunneling probability:

$$\frac{2S}{\hbar} = \left| N_h \left( \ln \frac{N_h}{\nu_h} - 1 \right) - N_l \left( \ln \frac{N_l}{\nu_l} - 1 \right) + \nu_h - \nu_l \right|. \quad (18)$$

We have introduced here the notation

$$N_l = m_l \nu_l^2 / 2\hbar \Omega_l, \quad \nu_l = m_l u^2 / 2\hbar \Omega_l, \quad (19)$$

where the quantities  $N_h$  and  $\nu_h$  are defined similarly, and are related to  $N$  and  $\nu$  by

$$N_h = \frac{m_h}{m_l} N_l + \frac{m_h}{m_l} \left( \frac{m_h}{m_l} - 1 \right) \nu_l, \quad \nu_h = \left( \frac{m_h}{m_l} \right)^2 \nu_l. \quad (20)$$

The numbers  $N_l$  and  $N_h$  (when large) are the numbers of the Landau levels. In the course of tunneling a light hole at a level  $N_l$  is converted into a heavy one at a level  $N_h$ . In the limit  $m_l/m_h \rightarrow 0$  Eq. (18) becomes

$$2S/\hbar = \nu_l | (1+x) \ln(1+x) - x(1+x/2) |,$$

$$x = (N_l/\nu_l) - 1 = 2\varepsilon/m_l u^2.$$

We shall now consider the case when paths in the momentum space of light and heavy holes are close to one another and have the same value of  $\varepsilon$ . This is true when  $|\varepsilon| \ll m_l u^2$  (Ref. 5). Then, Eq. (18) yields

$$\frac{2S}{\hbar} = \frac{4}{3} |\tilde{R}|, \quad \tilde{R} = \frac{(\varepsilon/u)^3}{2eE\hbar} \left( \frac{1}{m_l} - \frac{1}{m_h} \right), \quad (21)$$

which is in agreement with the argument of the exponential function that determines the tunneling probability in an electric field (see preceding section) if the expression for  $R$  is modified by replacing  $p_y$  with  $\varepsilon/u$ . Such a replacement is natural because in the region of the transition we have  $x \ll x_0$  and the kinematic momentum is  $p_y = (eH/c)(x_0 - x) \approx eHx_0/c = \varepsilon/u$ . This circumstance is obvious also from a comparison of Figs. 1a and 1b.

It is moreover clear that if  $|\varepsilon| \ll m_l u^2$  then the probabilities of tunnel conversion of a light hole into a heavy one within one cyclotron period in crossed fields (together with the preexponential factors) are governed by the formulas

(13) and (14) where  $R$  is replaced with  $\tilde{R}$ . These results are valid if

$$|N_l - \nu_l|^3 \gg \nu_l^2, \quad |N_l - \nu_l| \ll \nu_l. \quad (22)$$

Here the first inequality is the condition for the validity of the semiclassical approximation ( $|\tilde{R}| \gg 1$ ), whereas the second inequality is equivalent to the requirement that the orbits come closer in the momentum space:  $|\varepsilon| \ll m_l u^2$ .

We have assumed so far that the projection of the momentum  $p_z$  along the magnetic field direction vanishes. We shall now consider the dependence of the tunneling probability on  $p_z$ . We shall do this by allowing for the energy of longitudinal motion  $p_z^2/2m$  in the system (15). At low values of  $p_z$  this gives rise to an additional term on the right-hand side of Eq. (18):

$$2 \frac{\Delta S}{\hbar} = \left( \frac{p_z}{u} \right)^2 \left( \frac{N_l - \nu_l}{m_l^2} - \frac{N_h - \nu_h}{m_h^2} \right). \quad (23)$$

Therefore, an increase in  $p_z$  reduces the tunneling probability over a characteristic distance  $p_z \propto m_l u (N_l - \nu_l)^{-1/2}$ . In the case discussed above we have  $|N_l - \nu_l| \ll \nu_l$  and an allowance for  $p_z$  reduces to the replacement, in Eq. (21) for  $\tilde{R}$ , of the quantity  $\varepsilon/u$  with  $[(\varepsilon/u)^2 + p_z^2]^{1/2}$ .

#### 4. DISCUSSION OF RESULTS

A typical lifetime of a light hole in the process of conversion into a heavy hole can be written in the form

$$\tau = \frac{2\pi}{\Omega_l} \eta^{-1} \exp\left( \frac{2S}{\hbar} \right), \quad (24)$$

where  $S$  is describe (for  $p_z = 0$ ) by Eq. (18) and  $\eta$  is the preexponential factor in the probability of a transition per one period. This factor is calculated in the preceding section only for the special case when  $|\varepsilon| \ll m_l u^2$ . In the subsequent discussion we shall assume that  $\eta = 1$ . This can only overestimate the lifetime  $\tau$  [see Eq. (14)].

Figure 2 shows how  $\tau$  depends on the Landau level number for different values of the parameter  $\nu_l$  when  $m_l/m_h = 0.13$  (representing the case of germanium). If  $N_l = \nu_l$ , the orbits of light and heavy holes in the velocity space pass through the origin ( $\varepsilon = 0$ ). Near this point, when  $2S/\hbar \lesssim 1$ , the probability of a transition per cyclotron orbit is maximal and is also of the order of unity. In this region the

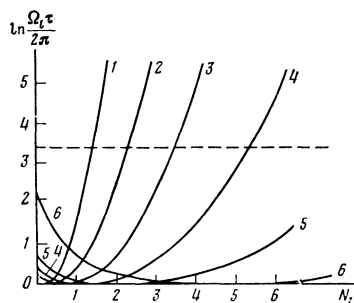


FIG. 2. Dependences of the time  $\tau$  for tunnel conversion of a light hole into a heavy one on the light-hole Landau level number for  $p_z = 0$ . Value of  $\nu_l$ : 1) 0.1; 2) 0.5; 3) 0.5; 4) 1.5; 5) 2; 6) 5.

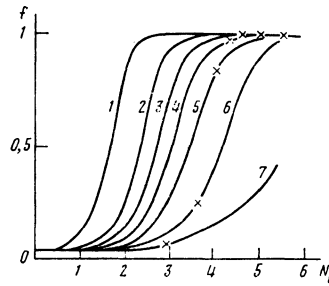


FIG. 3. Relative populations of the light-hole Landau levels for  $p_z = 0$ ,  $\nu_{ac} = 4 \times 10^{10} \text{ sec}^{-1}$ , and different values of the parameter  $\nu_l$ : 1) 0.13; 2) 0.25; 3) 0.33; 4) 0.4; 5) 0.5; 6) 0.67; 7) 1. The crosses correspond to the limit of the passive region for  $H = 20 \text{ kOe}$ . The unit population is the value in the absence of the tunneling effect.

semiclassical approximation we have used is no longer valid. It is clear from Fig. 2 that if  $\nu_l < 1$ , then the lifetime  $\tau$  decreases as the light-hole Landau level increases number and the rate of decrease is faster when  $\nu_l$  decreases. This favors a population inversion. If  $\nu_l > 1$ , the lifetime first falls with an increase in  $N_l$  and then (in the range  $N_l > \nu_l$ ) rises relatively slowly.

The lifetime  $\tau$  should be compared with the characteristic time  $\nu_{ac}^{-1}$  for the scattering by acoustic phonons, which also result in conversion of light holes into heavy ones. According to the estimates of Ref. 2, we have  $\nu_{ac} \approx 4 \times 10^{10} \text{ sec}^{-1}$  at 20 K. The dashed line in Fig. 2 corresponds to the value  $\ln(\Omega_l/2\pi\nu_{ac}) = 3.4$  when  $H = 20 \text{ kOe}$  and  $m_l = 0.046m_0$ . Below this line the tunneling process plays the dominant role in the emptying of the light-hole Landau levels in the passive region.

In connection with an estimate of the influence of tunneling, we plotted in Fig. 3 the dependence on  $N_l$  of the quantity

$$f = \nu_{ac} \tau / (1 + \nu_{ac} \tau), \quad (25)$$

which represents the relative population of the light-hole Landau levels when  $p_z = 0$ .

It is assumed that the population of these levels is uniform ("wide source") in the passive region (this population is due to the emission of optical phonons by heavy holes). The strong rise of the population in Fig. 3 is due to suppression of the tunneling effect. In these calculations we have ignored the weak energy dependence of the frequency  $\nu_{ac}$ . The various curves in Fig. 3 differ in respect of the main dimensionless parameter  $\nu_l$ , which for  $m_l = 0.046m_0$  can be written in the form  $\nu_l = 0.52E^2/H^3$ , where  $E$  is expressed in kilovolts per centimeter and the field  $H$  is in teslas.

In the experiments described in Ref. 1 the process of stimulated emission was observed for  $\nu_l \approx 0.2 - 0.3$ .

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