Scattering of the wavepacket of an atom by a resonance standing light wave

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(Submitted 7 January 1986)
Zh. Eksp. Teor. Fiz. **92**, 417–431 (February 1987)

We solve the problem of resonance scattering of the wavepacket of an atom by a standing light wave. We show that in the general case, as the result of the scattering, the initial wavepacket of the atom is split into a sum of packets due to the recoil effect. We find that in a strong field the number of packets which are formed after the scattering of a wavepacket which has a size significantly exceeding the wavelength of the light wave is determined by the product of the Rabi frequency and the scattering time. We establish that a wavepacket with a size smaller than the wavelength of the light wave can be split in a strong field into two packets with a difference in momenta proportional to the field gradient of the standing wave. We show that, if the time for the interaction between the atom and the field exceeds the time for the spontaneous relaxation of the upper level, the dynamics of the atom in the field of the standing wave is determined by the action on the atom by force dependent on the coordinate and on the velocity of the atom, and by the diffusion of the atomic momentum.

1. INTRODUCTION

We consider in the present paper the problem of resonance scattering of atoms by a monochromatic standing light wave. We assume that an atomic wavepacket of size Δz, which may be either smaller or larger than the wavelength $\lambda = 2\pi/k$ of the light wave, is incident on a standing light of transverse dimension l. We give the average momentum of the incident packet the magnitude $\mathbf{p}_0$. The mismatch between the frequency ω of the wave and the frequency ω_{ab} of the atomic transition is $\Omega = \omega - \omega_{ab}$. The magnitude of the dipole interaction between the atom and the field of the wave is characterized by the parameter $g = d_{ab}E/\hbar$ where E is the amplitude of the field of the standing wave and d_{ab} the dipole moment matrix element of the atom. The interaction with the field of the standing wave results in a transformation of the initial atomic wavepacket. The aim of the present paper is to find the form of the wavepacket after the interaction between the atom and the wave.

The scattering of atoms by a resonant standing wave has been studied before in Refs. 1 to 8. The strong-field case was considered in Refs. 1 to 4. The concept of a classical potential and a classical force oscillating at the wavelength of the light field were introduced to describe the scattering of the atoms. In Refs. 5 to 8 an analysis was given of scattering in the case of exact resonance between the light wave and the atomic transition ($\Omega = 0$). The form of the momentum distribution caused by the scattering of the atom was found in Ref. 5. In Ref. 6 the scattering features arising under oblique incidence of the atoms on the light wave was considered and the scattering were studied taking into account the effect of the recoil on the change in the kinetic energy of the atom. It was shown in Ref. 7 that the scattering of atoms in their ground state has a deep analogy with the Stern-Gerlach effect. The dependence of the scattering scenario on the size of the wavepacket of the atom was considered. It was shown that, depending on the ratio of the packet width Δz and the wavelength λ of the light field, there are two qualitatively different scattering scenarios. When a broad packet ($\Delta z \gg \lambda$) is scattered, Bragg maxima appear in the momentum distribution of the atom and are caused by the diffraction of the atomic probability density by the periodic light wave. In the case $\Delta z \ll \lambda$ the incident wavepacket is split into two packets similar to the Stern-Gerlach effect.

The experimental observation of the diffraction picture of the scattering was made in Ref. 9.

We find in the present paper the solution of the scattering problem in a general form. In the weak-field case we write down the wavefunction of the scattered wavepacket in second order of perturbation theory in the parameter g. The solution given here is valid for any mismatch and angle of incidence of the atom on the wave. We obtain the solution of the scattering problem, neglecting the kinetic energy of the atom, but for arbitrary mismatch and strength of the light wave. We consider the scattering of a wavepacket of an atom of size either larger or smaller than the wavelength of the light field. When $\Delta z \ll \lambda$ we find the magnitude of the splitting of the wavepacket and the probability for finding the atom in each of the two trajectories for arbitrary mismatches. We show that in the general case of arbitrary mismatch the problem of the motion of an atom in the field of a standing wave cannot be reduced to the classical-mechanics problem of the motion of a particle under the action of a force. The problem of the scattering of an atom is studied in sections 2 to 7 neglecting the radiative relaxation of the upper level. We show that at $\gamma t \gg 1$ the regime of coherent dynamics of the atomic wavepacket existing when $\gamma t \ll 1$ changes into the regime of stochastic motion of the atom. In the latter case the atom is described by a Fokker-Planck equation including a force and a momentum diffusion tensor which depend on the coordinate and the velocity of the atom which were earlier given in Refs. 10 and 11 for arbitrary values of the parameter g.

2. SOLUTION OF THE SCHROEDINGER EQUATION

We consider the interaction of an atom with the electromagnetic field of a standing wave:

$$E(z,\ t) = 4E \cos \omega t \cos(kz + \varphi), \tag{1}$$

where E, ω, k, and φ are the amplitude, frequency, wavevector, and phase of the wave. We assume the wave to be linearly

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polarized such that $E(z,t)$ is the component of the electric field along the y axis. We assume the frequency ω of the field to be close to the frequency ω_{ab} of the atomic transition between the upper (a) and the lower (b) levels. We can assume the atom in that case to be a two-level system with a wavefunction

$$\Psi(\mathbf{r},t)=\begin{pmatrix}\Psi_a(\mathbf{r},t)\\ \Psi_b(\mathbf{r},t)\end{pmatrix}.$$

The Schroedinger equation for the dipole interaction of the atom with the field (1) is

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t)=\left(-\frac{\hbar^2}{2M}\Delta+V(\mathbf{r},t)\right)\Psi(\mathbf{r},t), \tag{2}$$

where

$$V(\mathbf{r},t)=-2V(t)\cos(kz+\varphi),$$

$$V(t)=\hbar g\begin{pmatrix}0 & e^{-i\Omega t}\\ e^{i\Omega t} & 0\end{pmatrix}. \tag{3}$$

Here $\Omega=\omega-\omega_{ab}$ is the mismatch of the wave frequency relative to the frequency of the atomic transition, $g=d_{ab}E/\hbar$ and d_{ab} is the matrix element of the atomic dipole operator which we assume to be a real quantity. In what follows we shall put $\hbar=1$. Changing to the momentum representation

$$\Psi(\mathbf{r},t)=\int d\mathbf{p}\,e^{i\mathbf{pr}}\Psi(\mathbf{p},t),$$

and after that to the interaction representation

$$\Psi(\mathbf{p},t)=e^{-iE(\mathbf{p})t}\Phi(\mathbf{p},t),$$

we have instead of (2)

$$\partial\Phi(p,t)/\partial t=\sum_{\nu}Q(p,p-\nu k,t)\,\Phi(p-\nu k,t),$$

where

$$\Phi(p,t)\equiv\Phi(\mathbf{p},t),\quad p\equiv p_z,\quad \nu=\pm1,$$

$$Q(p,p-\nu k,t)=iV(t)\exp(i\nu\varphi)\exp\{i\omega(p,p-\nu k)t\},$$

$$\omega(p,p-\nu k)=\frac{p^2}{2M}-\frac{(p-\nu k)^2}{2M}=\frac{\nu kp}{M}-\frac{k^2}{2M}. \tag{4}$$

We shall assume that the field is switched on at time $t=0$ and it is switched off at $t=\tau$, i.e., $E=0$ for $t<0$ and $t>\tau$, $E=\text{const}$ for $0<t<\tau$. Integrating the equation obtained from $t=0$ to $t<\tau$ we have

$$\Phi(p,t)=\Phi(p,0)+\sum_{\nu}\int_0^t dt_1\,Q(p,p-\nu k,t_1)\,\Phi(p-\nu k,t_1).$$

Successively substituting the expression for the function $\Phi(p,t)$ into the right-hand side of this equation we get the solution in the form of a series

$$\Phi(p,t)=\Phi(p,0)+\sum_{n=1}^{\infty}\Phi_n(p,t),$$

where

$$\Phi_n(p,t)=\sum_{\nu_1\nu_2\ldots\nu_n}L_{\nu_1\nu_2\ldots\nu_n}(p,t)\,\Phi(p-(\nu_1+\nu_2+\ldots+\nu_n)k,0),$$

$$L_{\nu_1\nu_2\ldots\nu_n}(p,t)=\int_0^t dt_1\int_0^t dt_2\ldots$$

$$\ldots\times\int_0^{t_{n-1}}dt_n\,Q(p,p-\nu_1k,t_1)\,Q(p-\nu_1k,p-(\nu_1+\nu_2)k,t_2)\ldots Q(p-(\nu_1+\nu_2+\ldots+\nu_{n-1})k,p-(\nu_1+\nu_2+\ldots+\nu_n)k,t_n).$$

Changing the order of summation we can write the solution at time $t=\tau$ in the form

$$\Phi(p,\tau)=\sum_{m=-\infty}^{+\infty}D_m(p)\,\Phi(p-mk,0), \tag{5}$$

where

$$D_m(p)=I\delta_{m0}+\sum_{n=1}^{\infty}\sum_{\nu_1\nu_2\ldots\nu_n}\delta_{m,\nu_1+\nu_2+\ldots+\nu_n}L_{\nu_1\nu_2\ldots\nu_n}(p,\tau),$$

$$I=\begin{pmatrix}1&0\\0&1\end{pmatrix} \tag{6}$$

is the unit matrix.

Let at the time when the field is switched on ($t=0$) the wavefunction of the atom be

$$\Psi(\mathbf{r},0)=\int d\mathbf{p}\,e^{i\mathbf{pr}}\Psi(\mathbf{p},0).$$

Then, at time τ,

$$\Psi(\mathbf{r},\tau)=\int d\mathbf{p}\exp\{i\mathbf{pr}-iE(\mathbf{p})\tau\}\Phi(p,\tau).$$

After switching the field off, when $t>\tau$, the atoms are free and their motion is described by the wavefunction

$$\Psi(\mathbf{r},t)=\int d\mathbf{p}\exp\{i\mathbf{pr}-iE(\mathbf{p})t\}\Phi(p,\tau)$$

$$=\sum_{m=-\infty}^{+\infty}\int d\mathbf{p}\,D_m(p)\exp\{i\mathbf{pr}-iE(\mathbf{p})t\}\Phi(p-mk,0). \tag{7}$$

3. PERTURBATION THEORY

The scattering problem is thus reduced according to (5) to finding the matrices $D_m(p)$ which in the general case are given by Eq. (6). It is clear from (3) to (6) that the matrices $D_m(p)$ have the form

$$D_m(p)=\begin{pmatrix}A_m(\Omega) & B_m(-\Omega)\\ B_m(\Omega) & A_m(-\Omega)\end{pmatrix},$$

where the quantities A_m are nonvanishing for $m=0,\pm2,\ldots$, and the B_m for $m=\pm1,\pm3,\ldots$.

Such a structure of the matrices D_m corresponds to the fact that a change in the momentum of the atom by an amount $\hbar k$ always occurs as the result of the internal transition of the atom. Let, for instance, at $t=0$ the atom be in the lower level b:

$$\Phi(p,0)=\begin{pmatrix}0\\ \Phi_b(p,0)\end{pmatrix}.$$

In that case the probability amplitudes with even indices correspond to an even number of transitions of the atom as a result of which the atom turns out to be again in the lower level after scattering turns out. The probability amplitudes with odd indices correspond to an odd number of transitions after which the atom turns out to be in the upper level.

We consider the weak-field case when we can use the

fact that g is small. We shall assume that the recoil energy $R = k^2/2M$ is much smaller than the reciprocal of the time of interaction between the atom and the field $1/\tau$, i.e., $ku\tau \ll 1$, where $u = k/M$ is the atom recoil velocity corresponding to the photon momentum. In that case

$$Q(p, p-\nu k, t) = ig \exp(i\nu\varphi) \begin{pmatrix} 0 & \exp\{-i(\Omega-\nu kv)t\} \\ \exp\{i(\Omega+\nu kv)t\} & 0 \end{pmatrix},$$

where $v = p/M$. Up to order g^2 we have from (5) and (6)

$$B_m(\Omega) = ig \exp(im\varphi) \int_0^\tau dt_1 \exp\{i(\Omega+mkv)t_1\}, \quad m = \pm 1,$$

$$A_m(\Omega) = \delta_{m,0} + \sum_{\nu_1\nu_2} \left\{ \delta_{m,\nu_1+\nu_2} ig \exp(im\varphi) \int_0^\tau dt_1 \exp[i(\nu_1 kv+\Omega)t_1] ig \times \int_0^{t_1} dt_2 \exp[i(\nu_2 kv-\Omega)t_2] \right\}.$$

Hence

$$B_{\pm 1}(\Omega) = g \exp(\pm i\varphi) \frac{\exp\{i(\Omega \pm kv)\tau\}}{(\Omega \pm kv)},$$

$$A_0(\Omega) = 1 - g^2 \left[\frac{2i\Omega t}{\Omega^2 - (kv)^2} + \frac{1 - \exp\{i(\Omega+kv)\tau\}}{(\Omega+kv)^2} + \frac{1 - \exp\{i(\Omega-kv)\tau\}}{(\Omega-kv)^2} \right], \quad (8)$$

$$A_{\pm 2}(\Omega) = \frac{g^2 \exp(\pm ikv\tau)}{\Omega^2 - (kv)^2} \left[(\cos\Omega\tau - \cos kv\tau) + i\left(\sin\Omega\tau - \frac{\Omega}{kv}\sin kv\tau\right)\right] \exp(\pm 2i\varphi). \quad (9)$$

4. CASE OF SMALL KINETIC ENERGY

We consider the case when the kinetic energy of the atom ($-\hbar^2\Delta/2M$) can be dropped in (2). According to (4) and (5) the conditions for this approximation are $pv\tau \ll 1$, $kv\tau \ll 1$, $ku\tau \ll 1$. In that case

$$i\frac{\partial}{\partial t}\begin{pmatrix} \Psi_a(z,t) \\ \Psi_b(z,t) \end{pmatrix} = -2g\cos(kz+\varphi) \begin{pmatrix} 0 & e^{-i\Omega t} \\ e^{i\Omega t} & 0 \end{pmatrix} \begin{pmatrix} \Psi_a(z,t) \\ \Psi_b(z,t) \end{pmatrix}.$$

This equation is considered in Ref. 12 (problem for §40). Its solution for the time τ is

$$\Psi_a(z,\tau) = \frac{\lambda_2 e^{i\lambda_1\tau} - \lambda_1 e^{i\lambda_2\tau}}{\lambda_2 - \lambda_1} \Psi_a(z,0) - \frac{\mu}{\lambda_2-\lambda_1}(e^{i\lambda_1\tau} - e^{i\lambda_2\tau})\Psi_b(z,0),$$

$$\Psi_b(z,\tau) = \frac{\mu}{\lambda_2-\lambda_1}(e^{-i\lambda_1\tau} - e^{-i\lambda_2\tau})\Psi_a(z,0) + \frac{\lambda_2 e^{-i\lambda_1\tau} - \lambda_1 e^{-i\lambda_2\tau}}{\lambda_2-\lambda_1}\Psi_b(z,0), \quad (10)$$

where

$$\lambda_{1,2} = -{}^1/_2[\Omega \pm (\Omega^2 \pm 4\mu^2)^{1/2}], \quad \mu = 2g\cos(kz+\varphi).$$

Changing in (10) to the momentum representation

$$\Psi(p,\tau) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dz\, e^{-ipz}\Psi(z,\tau),$$

and using the fact that in the approximation considered $\Phi(p,\tau) = \Psi(p,\tau)$, we can find $D_m(p)$ in (5) and hence A_m and B_m.

Expression (10) is valid for any field. One can show that for a weak field ($g\tau \ll 1$) the coefficients A_m, B_m obtained using (10) are the same as (8) and (9) if we put in (8) and (9) $kv\tau \ll 1$.

One can simplify Eq. (10) in two important cases: when $\Omega = 0$ and $|\Omega| \gg 2\mu$. We write down the solutions for those cases.

1) $\Omega = 0$. In that case

$$\Psi_a(z,\tau) = \cos\mu\tau\Psi_a(z,0) + i\sin\mu\tau\Psi_b(z,0),$$
$$\Psi_b(z,\tau) = i\sin\mu\tau\Psi_a(z,0) + \cos\mu\tau\Psi_b(z,0).$$

Using the formula

$$e^{\pm i\eta\cos\theta} = \sum_{n=-\infty}^{+\infty} (\pm i)^n J_n(\eta) e^{\pm in\theta},$$

we can rewrite these expressions in the form

$$\Psi_a(z,\tau) = \sum_n (-1)^n \exp\{2in(kz+\varphi)\}[J_{2n}(2g\tau)\Psi_a(z,0) + iJ_{2n+1}(2g\tau)\exp\{i(kz+\varphi)\}\Psi_b(z,0)],$$
$$\Psi_b(z,\tau) = \sum_n (-1)^n \exp\{2in(kz+\varphi)\} \times [iJ_{2n+1}(2g\tau)\exp\{i(kz+\varphi)\}\Psi_a(z,0) + J_{2n}(2g\tau)\Psi_b(z,0)].$$

After changing to the momentum representation it follows from this that

$$A_m = i^m e^{im\varphi} J_m(2g\tau), \quad B_m = i^m e^{im\varphi} J_m(2g\tau).$$

2) $|\Omega| \gg 2\mu$. We can in this case expand (10) in a series in the parameter $\mu/|\Omega|$. Up to order μ^2/Ω^2 we get instead of (10)

$$\Psi_a(z,\tau) = \left[\frac{\mu^2}{\Omega^2}e^{i\lambda_1\tau} + \left(1 - \frac{\mu^2}{\Omega^2}\right)e^{i\lambda_2\tau}\right]\Psi_a(z,0) - \frac{\mu}{\Omega}(e^{i\lambda_1\tau} - e^{i\lambda_2\tau})\Psi_b(z,0),$$
$$\Psi_b(z,\tau) = \frac{\mu}{\Omega}(e^{-i\lambda_1\tau} - e^{-i\lambda_2\tau})\Psi_a(z,0) + \left[\frac{\mu^2}{\Omega^2}e^{-i\lambda_1\tau} + \left(1 - \frac{\mu^2}{\Omega^2}\right)e^{-i\lambda_2\tau}\right]\Psi_b(z,0), \quad (11)$$

where

$$\lambda_1 = -\Omega - \mu^2/\Omega, \quad \lambda_2 = \mu^2/\Omega.$$

In the case considered for even m ($m = 0, \pm 2, \ldots$)

$$A_m(\Omega) = \{{}^1/_2\varkappa i^{m/2}\exp\{i(1+\varkappa)\Omega\tau\}[2J_{m/2} + i(J_{m/2+1} - J_{m/2-1})] + (-i)^{m/2}\exp(-i\varkappa\Omega\tau) \times \left[(1-\varkappa)J_{m/2} + i\frac{\varkappa}{2}(J_{m/2+1} - J_{m/2-1})\right]\} \exp(im\varphi).$$

For odd m ($m = \pm 1, \pm 3, \ldots$)

$$B_m(\Omega)=\{(g/\Omega)[i^{(m+1)/2}\exp\{i(1+\varkappa)\Omega\tau\}(J_{(m+1)/2}-iJ_{(m-1)/2})$$
$$-(-i)^{(m+1)/2}\exp(-i\varkappa\Omega\tau)(J_{(m+1)/2}+iJ_{(m-1)/2})]\}\exp(im\varphi),$$

where

$$J_m=J_m(\varkappa\Omega\tau), \quad \varkappa=2g^2/\Omega^2.$$

5. SCATTERING OF A WAVEPACKET

We now consider the scattering of a wavepacket by the field of the standing wave (1) which is bounded on the x axis such that $E=0$ for $x<0$, $x>l$; $E=$ const for $0\leqslant x\leqslant l$.

Let the center of the atomic packet at $t=0$ be at the point $\mathbf{r}=0$ and have an average momentum $\mathbf{p}_0$; the form of the packet for the upper and lower level are assumed to be the same. In that case the function $\Phi(\mathbf{p},0)$ has the form

$$\Phi(\mathbf{p},0)=cf(\mathbf{p}-\mathbf{p}_0), \quad c=\begin{pmatrix}c_a\\ c_b\end{pmatrix}, \tag{12}$$

where $f(\mathbf{p}-\mathbf{p}_0)$ is a scalar function that has a steep maximum near $\mathbf{p}_0$ and specifies the pulsed form of the packet, and c_a and c_b are the amplitudes for the probability to find the atom in the upper and the lower level, respectively ($|c_a|^2+|c_b|^2=1$). In coordinate space

$$\Psi(\mathbf{r},0)=e^{i\mathbf{p}_0\mathbf{r}}cf(\mathbf{r}),$$

where the function

$$f(\mathbf{r})=\int d\mathbf{q}\,e^{i\mathbf{q}\mathbf{r}}f(\mathbf{q})$$

specifies the spatial form of the packet, $W(\mathbf{r})=|f(\mathbf{r})|^2$ is the probability density for finding the atom at the point $\mathbf{r}$, $\int d\mathbf{r}\,W(\mathbf{r})=1$.

We shall assume that the size of the wavepacket along the x axis is small compared to the size l of the field. This allows us to assume that the field acts on the atom during a time $\tau=l/v_{0x}$, where $v_{0x}=p_{0x}/M$.

When $t\geqslant\tau$ the wavefunction of the scattered atom is described by (7). Substituting (12) into (7) we have

$$\Psi(\mathbf{r},t)=\sum_{m=-\infty}^{+\infty}\exp(i\mathbf{p}_m\mathbf{r})\int d\mathbf{q}\exp\{i\mathbf{q}\mathbf{r}-iE(\mathbf{p}_m+\mathbf{q})t\}$$
$$\times D_m(p_m+q)cf(\mathbf{q}), \tag{13}$$

where $\mathbf{p}_m=\mathbf{p}_0+m\mathbf{k}$, $\mathbf{k}=k\mathbf{e}_z$.

If the function $D_m(p_m+q)$ changes weakly with changing q in the region $\Delta p=1/\Delta z$, where Δz is the characteristic size of the wavepacket along the z axis, we can remove it from under the integral sign. Then

$$\Psi(\mathbf{r},t)=\sum_{m=-\infty}^{+\infty}D_mc\exp\{i\mathbf{p}_m\mathbf{r}-iE(\mathbf{p}_m)t\}f'(\mathbf{r}-\mathbf{v}_mt), \tag{14}$$

where $D_m=D_m(p_{0z}+mk)$,

$$D_mc=\begin{pmatrix}B_m(-\Omega)c_b\\ B_m(\Omega)c_a\end{pmatrix}, \quad m=0,\pm2,\ldots,$$

$$D_mc=\begin{pmatrix}A_m(\Omega)c_a\\ A_m(-\Omega)c_b\end{pmatrix}, \quad m=\pm1,\pm3,\ldots.$$

The function

$$f'(\mathbf{r}-\mathbf{v}_mt)=\int d\mathbf{q}\exp\{i\mathbf{q}(\mathbf{r}-\mathbf{v}_mt)-iE(\mathbf{q})t\}f(\mathbf{q}) \tag{15}$$

describes a wavepacket moving with a velocity $\mathbf{v}_m=\mathbf{v}_0+mu\mathbf{e}_z$ ($\mathbf{v}_0=\mathbf{p}_0/M$, $u=k/M$) and spreading with increasing time. The characteristic time after which the width of the packet is increased by an amount of the order of the initial width Δz by free spreading is $\tau_p=(\Delta z)^2M$.

We consider two qualitatively different regimes for the evolution of the wavepacket when it passes through the standing wave, corresponding to two different ratios of the wavelength λ of the light and the size Δz of the wavepacket.

6. WAVEPACKET MUCH LARGER THAN THE WAVELENGTH OF THE FIELD ($\Delta z\gg\lambda\!\!\!^{-}=\lambda/2\pi$)

According to (14) the interaction of the atom with the field of the standing wave leads to a splitting of the initial packet into a sum of packets with z-components of the velocity equal to $v_m=v_{0z}+mu$. After a time τ_p the centers of two neighboring packets separate by a distance

$$z=u\tau_p=\Delta z(\Delta z/\lambda\!\!\!^{-})\gg\Delta z.$$

This means that neighboring packets do not intersect and, hence, do not interfere with one another. In that case the probability to find the atom in the point $\mathbf{r}$ equals

$$W(\mathbf{r},t)=\sum_{m=-\infty}^{+\infty}\rho_mW'(\mathbf{r}-\mathbf{v}_mt),$$

where

$$W'(\mathbf{r}-\mathbf{v}_mt)=|f'(\mathbf{r}-\mathbf{v}_mt)|^2$$

is the probability density of the packet moving with velocity v_m,

$$\rho_m=|B_m(-\Omega)c_b|^2+|B_m(\Omega)c_a|^2, \quad m=0,\pm2,\ldots,$$
$$\rho_m=|A_m(\Omega)c_a|^2+|A_m(-\Omega)c_b|^2, \quad m=\pm1,\pm3,\ldots.$$

Thus, when $a\gg\lambda\!\!\!^{-}$ the initial wavepacket is split into a sum of packets, each of which moves along its own trajectory (see figure, case a). The probability for finding the atom on the trajectory corresponding to the momentum $\mathbf{p}_m$ is ρ_m. Clearly $\Sigma_m\rho_m=1$.

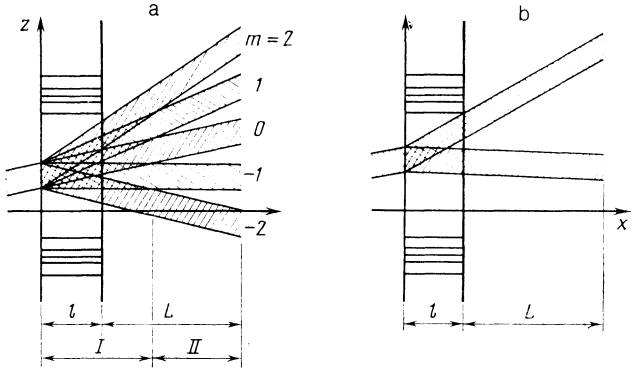

FIG. 1. a) Scattering of a packet of size $a\gg\lambda\!\!\!^{-}$ by a standing light wave of size l along the x axis. The packets $m=0,\pm2,\ldots$ correspond to atoms which after scattering are in the ground state; $m=\pm1,\pm3,\ldots$ to atoms which after scattering are in the upper state. The scattering pattern is independent of the coordinate of the point of flight of the atom in the field. I: Region where the packets interfere, II: region of free flight. b) Scattering of a packet of size $a\ll\lambda\!\!\!^{-}$. The difference of the momenta of the two packets is proportional to the gradient of the field of the wave along the z axis. The packet is not split in the antinodes of the wave and the maximum splitting takes place in the nodes of the standing wave.

As should be the case, the quantities ρ_m turn out to be independent of the phase of the field in which the center of the packet moves, as $\Delta z \gg \lambda\!\!\!^{-}$.

According to (14) the scattering angle of the mth packet is

$$\theta_m = \theta m, \quad \theta = k/p_{0x}.$$

These angles have the meaning of Bragg angles characterizing the scattering by a periodic structure such as the standing wave. The condition $a \gg \lambda\!\!\!^{-}$ means that to the diffraction pattern of the scattering many periods of the periodic structure contribute.

We write down ρ_m for a weak field. The probability for the atom which at $t = 0$ is in the lower state (b) after scattering to travel along a trajectory corresponding to a momentum $p = p_0 + mk$ with $m = \pm 1, \pm 2$, is, according to (8) and (9)

$$\rho_{\pm 1} = g^2 \sin^2 \frac{(\Omega \mp k v_0)\tau}{2} \Big/ \left(\frac{\Omega \mp k v_0}{2}\right)^2,$$

$$\rho_{\pm 2} = \frac{g^4}{(\Omega^2 - k^2 v_0^2)^2}\left[(\cos \Omega\tau - \cos k v_0 \tau)^2 - \left(\sin \Omega\tau - \frac{\Omega}{k v_0} \sin k v_0 \tau\right)\right],$$

where $v_0 = p_0/M$.

The probabilities $\rho_{\pm 1}$ corresponds to atoms which after scattering are in the upper level. It is interesting to note that these probabilities are different. The reason is that the first order of perturbation theory describes an independent interaction of the atom with each of the two counter-moving waves which make up the standing wave. The efficiency of the interaction of the atom with each of the counter-moving waves depends on the value of the Doppler shift $\pm k v_0$. As the Doppler shifts are different, the probabilities $\rho_{\pm 1}$ also differ.

The probabilities $\rho_{\pm 2}$ correspond to atoms which after scattering are in the ground state. They correspond to scattering processes in which the atoms makes a transition into the upper level due to the interaction with one of the counter-moving waves and returns next to the lower level due to the interaction with the other wave. As a result of these processes the atom acquires a momentum $\pm 2k$. We note that the quantities $\rho_{\pm 2}$ are even functions of the velocity v_0. One can also show in the general case that $\rho_m = \rho_{-m}$ for $m = \pm 2, \pm 4, \ldots$, i.e., atoms which end up in the ground state after scattering are scattered symmetrically with respect to the initial momentum p_0.

In the case of a small kinetic energy of the atom (section 4) the probability that an atom which is in the lower level at $t = 0$ moves after scattering along a trajectory corresponding to a Bragg maximum $m = 0, \pm 1, \pm 2, \ldots$ is $\rho_m = J_m^2(2g\tau)$. The number of trajectories appearing after the scattering of the atom is of the order of magnitude $m \sim 2g\tau$, when $g\tau \gg 1$.

7. WAVEPACKET MUCH SMALLER THAN THE WAVELENGTH OF THE FIELD($\Delta z \ll \lambda$)

In this case after a time τ_p the centers of two neighboring packets in (14) diverge by a distance $z = \Delta z(\Delta z/\lambda\!\!\!^{-}) \ll \Delta z$ which is much smaller than the width of the initial packet. This means that the free spreading of the whole packet is appreciably faster than the separation of neighboring packets. The wavepackets in (14), although they move with different velocities, will thus always intersect and strongly interfere.

To find the evolution of a narrow wavepacket in the field (1) we consider the case of small kinetic energy of the atom (section 4). As in (10), the function $\Psi(z,0)$ is nonvanishing in a region $\Delta z \ll \lambda\!\!\!^{-}$ in the expressions for $\lambda_{1,2}$ and μ we may assume that $kz \ll 1$. In the pre-exponential factor we put $kz = 0$ and we expand the argument of the exponential up to terms linear in kz. After that changing to the momentum representation we find from (10)

$$\begin{aligned}
\Phi_a(p,\tau) &= \frac{\Delta+\Omega}{2\Delta}\exp\left(i\frac{\Delta-\Omega}{2}\tau\right)\Phi_a(p+\delta p,0) \\
&+ \frac{\Delta-\Omega}{2\Delta}\exp\left(-i\frac{\Delta+\Omega}{2}\tau\right)\Phi_a(p-\delta p,0) + \frac{\xi}{\Delta}\exp\left(i\frac{\Delta-\Omega}{2}\tau\right) \\
&\times \Phi_b(p+\delta p,0) - \frac{\xi}{\Delta}\exp\left(-i\frac{\Delta+\Omega}{2}\tau\right)\Phi_b(p-\delta p,0), \\
\Phi_b(p,\tau) &= \frac{\xi}{\Delta}\exp\left(i\frac{\Delta+\Omega}{2}\tau\right)\Phi_a(p+\delta p,0) \\
&- \frac{\xi}{\Delta}\exp\left(-i\frac{\Delta-\Phi}{2}\tau\right) \\
&\times \Phi_a(p-\delta p,0) + \frac{\Delta+\Omega}{2\Delta}\exp\left(i\frac{\Delta+\Omega}{2}\tau\right)\Phi_b(p+\delta p,0) \\
&+ \frac{\Delta+\Omega}{2\Delta}\exp\left(-i\frac{\Delta+\Omega}{2}\tau\right)\Phi_b(p-\delta p,0),
\end{aligned} \tag{16}$$

where

$$\Delta = (\Omega^2 + 4\xi^2)^{1/2}, \quad \xi = 2g\cos\varphi, \quad \delta p = k\frac{4g^2\tau \sin 2\varphi}{\Delta}. \tag{17}$$

The expansion procedure used to get (16) is valid when the terms quadratic in kz which were dropped in the exponents are small. This condition restricts the magnitude of g:

$$\frac{8g^2\tau\cos 2\varphi}{\Delta} \ll \left(\frac{\lambda}{\Delta z}\right)^2. \tag{18}$$

According to (7), (12), (15) and (16) the wavefunction of the scattered wavepacket ($t > \tau$) is

$$\begin{aligned}
\Psi(\mathbf{r},t) &= \Psi_+ \exp\{i\mathbf{p}_+\mathbf{r} - iE(\mathbf{p}_+)t\} f'(\mathbf{r} - \mathbf{v}_+ t) \\
&+ \Psi_- \exp\{i\mathbf{p}_-\mathbf{r} - iE(\mathbf{p}_-)t\} f'(\mathbf{r} - \mathbf{v}_- t),
\end{aligned} \tag{19}$$

where $\mathbf{p}_\pm = \mathbf{p}_0 \pm \delta p \mathbf{e}_z$, $v_\pm = p_\pm/M$,

$$\Psi_+ = \begin{pmatrix} \frac{1}{\Delta}\left(\frac{\Delta+\Omega}{2}c_a + \xi c_b\right)\exp\left(i\frac{\Delta-\Omega}{2}\tau\right) \\ \frac{1}{\Delta}\left(\xi c_a + \frac{\Delta-\Omega}{2}c_b\right)\exp\left(i\frac{\Delta+\Omega}{2}\tau\right) \end{pmatrix},$$

$$\Psi_- = \begin{pmatrix} \frac{1}{\Delta}\left(\frac{\Delta-\Omega}{2}c_a - \xi c_b\right)\exp\left(-i\frac{\Delta+\Omega}{2}\tau\right) \\ \frac{1}{\Delta}\left(-\xi c_a + \frac{\Delta+\Omega}{2}c_b\right)\exp\left(-i\frac{\Delta-\Omega}{2}\tau\right) \end{pmatrix}.$$

It follows from (16) to (19) that in the scattering the initial velocity distribution splits into two parts which have average velocities $\mathbf{v}_+$ and $\mathbf{v}_-$ which depend on the phase of the wave in which the atomic packet moves.

After a time τ_p the two parts of the wavepacket which have z-components of velocity $v_+ = v_0 + \delta v$ and $v_- = v_0$

$-\delta v(\delta v = \delta p/M)$ diverge by a distance

$$z = 2\delta v \tau_p = u\tau_p (8g^2\tau \sin 2\varphi)/\Delta.$$

In order that the two parts of the wavepacket not overlap, (i.e., move each along its own trajectory, the condition $z \gg \Delta z$ must be satisfied:

$$\lambda/\Delta z \ll (8g^2\tau \sin 2\varphi)/\Delta. \tag{20}$$

This condition can be satisfied together with condition (18) when $\tan 2\varphi \gg \Delta z/\lambda$. In that case packets with velocities $\mathbf{v}_+, \mathbf{v}_-$ do not interfere and the probability density for finding the atom at the point $\mathbf{r}$ is equal to

$$W(\mathbf{r}, t) = \rho_+ W'(\mathbf{r} - \mathbf{v}_+ t) + \rho_- W'(\mathbf{r} - \mathbf{v}_- t), \tag{21}$$

where

$$\rho_\pm = \frac{1}{2}\left(1 \pm \frac{\Omega(\rho_b - \rho_a) + 4gs\cos\varphi}{[\Omega^2 + (4g\cos\varphi)^2]^{1/2}}\right),$$

$$\rho_a = |c_a|^2, \quad \rho_b = |c_b|^2, \quad s = 2\,\mathrm{Re}\,(c_a c_b^*).$$

We note that as $\Delta z \to 0$ ($kz \to 0$) the light wave field (1) does not change the momentum of the atom. In order to achieve the transition $\Delta z \to 0$ in Eqs. (16) to (21) we must take into account that in that case $\Delta p \to \infty$ where Δp is the momentum width of the wavepacket. In other words, the transition to an infinitely narrow wavepacket must be performed putting $\delta p/\Delta p \to 0$. If we consider an infinitely narrow packet at a node of the field (1) ($\varphi = \pi/2$), we get in (16), as one should expect,

$$\Phi_a(p, \tau) = \Phi_a(p, 0), \quad \Phi_b(p, \tau) = \Phi_b(p, 0),$$

and in (21) we get $w(\mathbf{r},t) = w'(\mathbf{r} - \mathbf{v}t)$ i.e., the state of the atom does not change in a node of the field.

From the equations given here is follows thus that under the conditions (18) and (20) a narrow wavepacket splits during scattering into two packets, analogously to the Stern-Gerlach effect for a spin-$\frac{1}{2}$ particle (see Fig. 1, case b). The probability that the atom after the scattering has a velocity $v_+ = v_0 + \delta v$ is ρ_+ and the probability that it has a velocity $v_- = v_0 - \delta v$ is ρ_-, where $\rho_- + \rho_+ = 1$. The values of the probabilities ρ_+, ρ_- depend on the initial state of the atom.

The simplest cases to analyze are those of small and large mismatches.

In the first case when $\Omega = 0$, we have[8]

$$\rho_\pm = {}^1/_2(1 \pm s), \quad \delta v = 2ug\tau \sin\varphi.$$

When $s = 1$ when $\rho_+ = 1, \rho_- = 0$, after scattering the wavepacket moves along the trajectory corresponding to the velocity v_+. When $s = -1$, when $\rho_+ = 0, \rho_- = 1$ after scattering the packet has the velocity v_-. When $-1 < s < 1$ the initial packet splits into two packets. If we use the analogy with the Stern-Gerlach effect the values $s = \pm 1$ correspond to pure states of the spin particle with spin components $\pm\frac{1}{2}$. The values $-1 < s < 1$ correspond to a mixed state as far as the spin component is concerned for which there occurs a splitting in the trajectory of the spin particle.

In the second case when $|\Omega| \gg g$ we have

$$\rho_\pm = {}^1/_2\left[1 \mp \frac{\Omega}{|\Omega|}(\rho_b - \rho_a)\right], \quad \delta v = 4ug\tau \frac{g}{|\Omega|}\sin 2\varphi.$$

In that case when $\Omega > 0, \rho_+ = \rho_a, \rho_- = \rho_b$, and when $\Omega < 0$, $\rho_+ = \rho_b, \rho_- = \rho_a$. Correspondingly, the atom moves along one trajectory when $\rho_a = 0$ or 1. When an atom is scattered which is in the ground state ($\rho_b = 1$) depending on the sign of the mismatch it can travel along the trajectory corresponding to v_+ or v_-.

We emphasize that just as the doubling of the trajectories of a spin particle in the Stern-Gerlach effect cannot be described in the framework of classical physics, neither can the description of the doubling of the trajectories of an atom when it is scattered by a standing wave be based upon the force concept. The idea of a force may be used only in the particular cases of negligibly small or infinitely large mismatches and only for pure states when the atom moves along a single trajectory. In the general case of the splitting of the wavepacket into two parts the introduction of two forces to describe the motion of parts of the packet along two trajectories is possible only as in auxiliary concept. These two forces do not have the meaning of classical forces as in classical mechanics the force acting on a particle can depend only on the coordinate and the velocity of the particle.

We give numerical estimates for the atom-optical analogy of the Stern-Gerlach effect (see Fig. 1, case b). We shall assume that before the scattering the atom is in the ground state b and we shall put the mismatch Ω equal to zero.

Let $\lambda = 1\,\mu$m, the size of the light wave $l = 100\,\mu$m, the velocity $v_{0x} = 5 \times 10^4$ cm/s. Under those conditions the time of passing through the wave $\tau = l/v_{0x} = 2 \times 10^{-7}$ s. Let $\Delta z = 0.2\lambda = 3.2 \times 10^{-6}$ cm, $M = 20$ amu. Then $u = \hbar k/M = 2$ cm/s, $v = \hbar/M\Delta z = u(\lambda/\Delta z) = 10$ cm/s and the basic conditions are, indeed, satisfied: $Mv^2\tau/2\hbar = 0.2 \ll 1$, $kv\tau = 0.1 \ll 1$, $ku\tau = 2.5 \times 10^{-2} \ll 1$. We put $\varphi = \pi/4$, $g\tau = 20$. Conditions (18), (20) then turn out to be satisfied. At time τ the velocities of the two packets will differ by an amount $2\delta v = 4ug\tau \sin\varphi = 1.7 \times 10^2$ cm/s which is appreciably larger then the velocity $v = 10$ cm/s with which the packets spread. The time t after which the two particles fly apart at a distance Δz is $t = \Delta z/2\delta v = 2 \times 10^{-8}$ s. The time of broadening of the packets to dimensions of the order of the initial width of the packet is $\tau_p = 3.4 \times 10^{-7}$ s. Hence it follows that for the chosen parameters the initial packet splits up into two packets already in the region of the light wave. We assume that the distance to the registration region $L = 100$ cm. After the time it takes the atom to fly this distance $T = L/v_{0x} = 2 \times 10^{-3}$ s the two packets are separated by a distance $2\delta v T = 0.3$ cm and they are broadened only by an amount $vT = 2 \times 10^{-2}$ cm.

The estimates given here show thus that the atom-optical analogy with the Stern-Gerlach effect can be observed under conditions which are typical for experiments on the scattering of atomic beams by a light field.

8. ALLOWANCE FOR RADIATIVE RELAXATION

In the preceding sections we studied the scattering of atoms for interaction times which were small compared to the time of spontaneous relaxation of the levels a, b. We now consider what will be the consequences of taking into account the radiative relaxation. We shall assume that the lower level is the ground state and that the upper level decays into the ground state with a spontaneous relaxation rate 2γ.

To take the relaxation into account we introduce in the usual way the density matrix in the coordinate representation

$$\Psi(\mathbf{r}, t)\Psi^*(\mathbf{r}', t) \to \rho(\mathbf{r}, \mathbf{r}', t).$$

The analysis of the dynamics of the atom in the field of a resonant light wave is carried out using the denisty matrix in the Wigner representation

$$\rho(\mathbf{r}, \mathbf{p}, t) = (2\pi)^{-3} \int d\mathbf{s}\, \rho(\mathbf{r}+\mathbf{s}/2, \mathbf{r}-\mathbf{s}/2, t) e^{-i\mathbf{p}\mathbf{s}}.$$

The equations for the density matrix of an atom in the field of the standing wave (1) have in the Wigner representation the form (see, e.g., Ref. 11)

$$\left(\frac{d}{dt} + 2\gamma\right)\rho_{aa} = -ig(\rho_{ab}{}^{1} - \rho_{ba}{}^{-1})e^{i\psi} - ig(\rho_{ab}{}^{-1} - \rho_{ba}{}^{1})e^{-i\psi},$$

$$\left(\frac{d}{dt} - i\Omega + \gamma\right)\rho_{ab} = ig(\rho_{bb}{}^{-1} - \rho_{aa}{}^{1})e^{i\psi} + ig(\rho_{bb}{}^{1} - \rho_{aa}{}^{-1})e^{-i\psi},$$

$$\frac{d}{dt}\rho_{bb} = -ig(\rho_{ba}{}^{1} - \rho_{ab}{}^{-1})e^{i\psi} - ig(\rho_{ba}{}^{-1} - \rho_{ab}{}^{1})e^{-i\psi} + 2\gamma \int d\mathbf{n}\, \Phi(\mathbf{n}) \rho_{aa}(\mathbf{r}, \mathbf{p}+\mathbf{n}k, t), \tag{22}$$

where

$$\psi = kz + \varphi, \quad d/dt = \partial/\partial t + \mathbf{v}\partial/\partial \mathbf{r}, \quad \mathbf{v} = \mathbf{p}/M,$$

$$\rho_{ij} = \rho_{ij}(\mathbf{r}, \mathbf{p}, t), \quad \rho_{ij}{}^{\pm 1} = \rho_{ij}(\mathbf{r}, \mathbf{p} \pm \mathbf{k}/2, t),$$

$$\int d\mathbf{n}\, \Phi(\mathbf{n}) = \frac{3}{8\pi} \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin^3\theta.$$

The integral term in (22) takes into account the arrival in the level b under the spontaneous emission of a photon, $\mathbf{n}$ is a unit vector in the direction given by the angles θ and φ [the integration in (22) means averaging over the randon recoil momentum $\mathbf{n}k$]; for the off-diagonal elements in (22) we have made the substitution $\rho_{ab} \to \rho_{ab} e^{-i\Omega t}$.

For the determination of the dynamics of the atom in the field (1) the main interest is to find the Wigner function

$$W(\mathbf{r}, \mathbf{p}, t) = \rho_{aa}(\mathbf{r}, \mathbf{p}, t) + \rho_{bb}(\mathbf{r}, \mathbf{p}, t),$$

with the help of which we can find the probability density to find the atom at the point $\mathbf{r}$

$$w(\mathbf{r}, t) = \int d\mathbf{p}\, W(\mathbf{r}, \mathbf{p}, t)$$

and the momentum probability density

$$w(\mathbf{p}, t) = \int d\mathbf{r}\, W(\mathbf{r}, \mathbf{p}, t).$$

Here we restrict ourselves to taking into account the role of the radiative relaxation for small g when Eq. (22) can be solved using perturbation theory methods. When $\tau \ll \gamma^{-1}$ the solution of Eqs. (22) using perturbation theory is, of course, the same as the solution given in section 3. In the intermediate case $\tau \sim \gamma^{-1}$ the solution is extremely complicated. Of physical interest is the case $\tau \gg \gamma^{-1}$ which we consider below. This case was studied experimentally in Ref. 13.

When spontaneous relaxation is taken into account, the characteristic parameter of the problem is the ratio R/γ where $R = k^2/2M$ is the recoil energy. For allowed dipole transitions $R/\gamma \ll 1$. For instance, for the yellow sodium line ($\lambda = 0.6\,\mu$m) $R/\gamma \approx 10^{-3}$.

We consider an arbitrary wavepacket. For the times of interest to us $t \gg \gamma^{-1}$ the width of the packet always satisfies the condition

$$\Delta z \gg u\gamma^{-1} = \lambda(2R/\gamma), \tag{23}$$

where $u = k/M$. Indeed, if this condition is satisfied at $t = 0$ thanks to the free spreading of the wavepacket it will be the better valid for $t \gg \gamma^{-1}$. If, on the other hand, the width of the wavepacket $\Delta z \lesssim u\gamma^{-1}$, by virtue of the uncertainty relation the wavepacket has characteristic velocities $\Delta v \gtrsim \hbar\gamma$ which guarantee for times $t \gg \gamma^{-1}$ the spreading of the packet to a size $\Delta z \gtrsim \hbar t \gg u\gamma^{-1}$.

Taking Eq. (23) into account we can transform Eqs. (22) for times $t \gg \gamma^{-1}$ into a Fokker-Planck equation for the classical coordinate and momentum distribution function. To do this we take the sum of the first and the third Eqs. (22) and expands the elements of the density matrix up to second order in k:

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}}\right) w = 2kg \sin\psi \frac{\partial}{\partial p}(\rho_{ab} + \rho_{ba}) + k^2\gamma \sum_{i=x,y,z} \alpha_{ii} \frac{\partial^2}{\partial p_i^2} \rho_{aa}. \tag{24}$$

Here $w = \rho_{aa} + \rho_{bb}$ has the meaning of a classical distribution function, $p = p_z$, $\alpha_{xx} = 2/5$, $\alpha_{yy} = 2/10$, $\alpha_{zz} = 2/5$. We further restrict ourselves to obtain the Fokker-Planck equation to second order in g. To do this we evaluate the element ρ_{ab} from (22) to first order in g and the element ρ_{aa} to second order in g. As the element ρ_{ab} occurs in (24) in first order in k and the element ρ_{aa} in second order in k we write them down, respectively, to first and zeroth order in k. Using Eq. (23) we have

$$\rho_{ab} = -\frac{ge^{-i\psi}}{\Omega_+ + i\gamma}\left(w + \frac{1}{2}k\frac{\partial w}{\partial p}\right) - \frac{ge^{i\psi}}{\Omega_- + i\gamma}\left(w - \frac{1}{2}k\frac{\partial w}{\partial p}\right),$$

$$\rho_{aa} = \frac{g^2}{\gamma^2}(L_- + L_+)w + \frac{g^2}{\gamma^2}\frac{\cos 2\psi}{\gamma^2 + k^2v^2} \times [\gamma^2(L_- + L_+) + kv(\Omega_- L_- - \Omega_+ L_+)]w + \frac{g^2}{\gamma^2}\frac{\sin 2\psi}{\gamma^2 + k^2v^2}[kv\gamma(L_- + L_+) - \gamma(\Omega_- L_- - \Omega_+ L_+)]w, \tag{25}$$

where

$$\Omega_\pm = \Omega \pm kv, \quad L_\pm = \gamma^2/(\gamma^2 + \Omega_\pm^2), \quad v = v_z.$$

Substituting (25) into (24) we finally get the Fokker-Planck equation to second order in g:

$$\frac{\partial w}{\partial t} + \mathbf{v}\frac{\partial w}{\partial \mathbf{r}} = -\frac{\partial}{\partial \mathbf{p}}(\mathbf{F}w) + \sum_{i=x,y,z} \frac{\partial^2}{\partial p_i^2}(D_{ii}w). \tag{26}$$

In dimensional units the coefficients of Eq. (26) have the form

$$\mathbf{F} = 2\hbar\mathbf{k}\gamma G(L_- - L_+)\sin^2\psi + \hbar\mathbf{k}G(\Omega_- L_- + \Omega_+ L_+)\sin 2\psi,$$

$$D_{ii} = {}^1/_2\hbar^2k^2\gamma d_{ii},$$

$$d_{ii} = \alpha_{ii}G(L_- + L_+) + \alpha_{ii}\frac{G}{\gamma^2 + k^2v^2} \times [\gamma^2(L_- + L_+) + kv(\Omega_- L_- + \Omega_+ L_+)]\cos 2\psi + \alpha_{ii}\frac{G}{\gamma^2 + k^2v^2}[\gamma(\Omega_+ L_+ - \Omega_- L_-) + kv\gamma(L_- + L_+)]\sin 2\psi + 2\delta_{zi}G(L_- + L_+)\sin^2\psi + \delta_{zi}G\gamma^{-1}(\Omega_- L_- - \Omega_+ L_+)\sin 2\psi. \tag{27}$$

where we have introduced the saturation parameter $G = 2g^2/\gamma^2$.

The interaction between the atom and the field of the standing wave thus leads for $t \gg \gamma^{-1}$ to the appearance of the force **F**. Moreover, the momentum distribution of the atom broadens diffusively in accordance with the value of the diffusion tensor D_{ii}.

When $v = 0$ and $|\Omega| \gg \gamma$ the force **F** is the same as the force written down in Ref. 14 and in Eq. (3) of Ref. 15:

$$\mathbf{F} = 4\hbar\mathbf{k}(g^2/\Omega)\sin 2\psi.$$

Under the same conditions the diffusion tensor is

$$D_{ii} = 4\hbar^2 k^2 \gamma (g^2/\Omega^2)(\alpha_{ii}\cos^2\psi + \delta_{zi}\sin^2\psi).$$

We note that the perturbation theory method in principle enables us to find the coefficients in the Fokker-Planck equation to any order in g and thus to obtain **F** and D_{ii} for any value of g. One can show that the results of calculating **F** and D_{ii} on the basis of pertrubation theory are the same as the quantities **F** and D_{ii} determined in Refs. 10 and 11.

9. CONCLUSION

There exist thus in the problem of the scattering of an atom by the field of a standing wave two qualitatively different regions of evolution of the wavepacket of the atom; $\tau \ll \gamma^{-1}$ and $\tau \gg \gamma^{-1}$.

In the case $\gamma\tau \ll 1$ the character of the scattering of the atom depends on the ratio of the width Δz of the wavepacket of the atom and the wavelength $\lambda\!\!\!^{-}$ of the field. When $\Delta z \gg \lambda\!\!\!^{-}$ the incident packet is split, in general, into a large number of scattered packets. When $\Delta z \ll \lambda\!\!\!^{-}$ two scattered packets may be formed.

In the opposite case $\gamma\tau \gg 1$ the scattering reduces to a stochastic motion described by a Fokker-Planck equation. In that case the dynamics of the atom is determined by the force **F** and the value of the diffusion tensor D_{ii}.

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Translated by D. ter Haar