Theory of generation of sub-Poisson radiation. Method of rate equations with Langevin sources

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A study is made of the possibility of reducing the quantum noise of laser radiation under conditions of sub-Poisson excitation of atoms. A general relationship is found between the spectrum of fluctuations of the intensity of the radiation emerging from a resonator and the spectral density of the effective-pumping noise. Indications of the sub-Poisson statistics of laser radiation in the case of cw single-mode emission are found for two specific excitation mechanisms.

INTRODUCTION

Lowering of the quantum noise of radiation compared with the shot noise of photodetection is regarded as an important and attainable target.[1–3] After demonstration of antibunching[4] and of the sub-Poisson photon statistics (SPPS)[5] of nonlinear resonance fluorescence of single atoms in a low-density atomic beam, similar positive experimental results have been reported quite recently for radiation form macroscopic sources. The present authors are aware of two such results: a weak SPPS effect in the luminescence of mercury vapor in a Frank-Hertz tube when the shot noise in the exciting electron beam was suppressed[6,7] and manifestation of a "squeezed" state in the spectrum of the intensity fluctuations observed in the case of parametric oscillations in a resonator (Ref. 8)[1)] Four-wave mixing in a resonator or in an optical fiber[10,11] is attracting the greatest attention. However, equally interesting is the idea of observing the SPPS and lowering the spectral density of the intensity fluctuations in the case of "ordinary" cw operation of a laser. A scheme for regular pulsed pumping with complete filling of the upper active level in each pulse is proposed in Ref. 12: in the theoretical limit this should make it possible to reduce to zero the photocurrent noise in detection of radiation from a gas laser. The same approximations were used in Ref. 13 to show that the SPPS and a dip at low frequencies in the spectrum of the intensity fluctuations may occur when cooperative processes are activated in a system of excited atoms (pair deactivation): this ensures the sub-Poisson nature of the effective pumping (nonappearance of resonator photon pairs separated by short time intervals). The results of Refs. 12 and 13 were obtained within the framework of full quantum theory of single-mode emission from a gas laser[14,15] with a refinement[12] which makes it possible to allow correctly for the excitation statistics.

We shall consider the possibility of reducing the quantum noise of radiation obtained as a result of cw single-mode emission from a gas or a solid-state laser in the case of the "repulsive" (sub-Poisson) statistics of the population of the upper active level. We shall use the model of rate equations for the populations of the atomic levels and for the numbers of atoms in the presence of random sources of fluctuations.[16–20] This method is simpler and clearer than the method of quantum master equation for the field. The latter can be derived correctly by a reduction of the initial equation for the density matrix of the whole system only subject to severe restrictions which postulate that the fluctuations of the populations decay much more rapidly than the fluctuations of the field in the resonator (see the Appendix in Ref. 15).

We shall derive in Sec. 2 the general relationships between the spectral density of the photocurrent noise and the spectral density of the photocurrent noise and the spectral densities of the correlation functions of random sources, and we shall use these relationships to analyze two specific mechanisms of sub-Poisson effective pumping: excitation by an electron beam with suppression of the shot noise (Sec. 3) and pair deactivation in the pump channel (Sec. 4).

We shall now identify the main quantities representing the radiation statistics which will be used below. We shall use the variance of the number of photocounts in a time much shorter than the decay time of all the correlations in the radiation:

$$\langle (m-\bar{m})^2\rangle=\bar{m}(1+\xi), \tag{1}$$

where $\bar{m}$ is the average number of photocounts and the parameter ξ represents the deviation of the variance from its value $\bar{m}$ for the Poisson statistics. The parameter ξ may be negative (SPPS) for the states of the field that do not have a classical analog. It is related simply to the spectrum of the photocurrent fluctuations $(\delta I^2)_\omega$ (Ref. 3):

$$\xi=\{[(\delta I^2)_\omega-\bar{I}]/\bar{I}\}_{\omega=0}. \tag{2}$$

Therefore, the parameter $|\xi|$ in the range $\xi<0$ represents the relative depth of a dip in the spectrum at $\omega\approx 0$ below the level $(\bar{I})$ of the shot noise of the photocurrent. The relationship (2) is valid on condition that the correlation function of the fluctuations of the photocurrent $\langle \delta I(0)\delta I(\tau)\rangle$ decays rapidly in the limit $\tau\to\infty$.

2. RELATIONSHIP BETWEEN THE SPECTRUM OF THE INTENSITY FLUCTUATIONS AND THE SPECTRA OF NOISE IN A LASER

We shall consider an effective two-level lasing configuration [assuming that the lower active level 1 is not the ground state and that the pumping $\Lambda(t)$ of the upper level 2 causes slight depletion of the ground state]. We can describe single-mode lasing employing the following variables: Φ is the number of photons in the resonantor; N_2 and N_1 are the populations of the upper and lower active levels; I is the intensity of the radiation emerging from the resonator and

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recorded with a photodetector ($\sec^{-1}$). The rate equations describing the lasing kinetics will be written in the form

$$\dot{N}_2=\Lambda-N_2/\tau_2-{}^1/_2B(N_2-N_1)\Phi+r_2, \quad (3a)$$
$$\dot{N}_1=-N_1/\tau_1+{}^1/_2B(N_2-N_1)\Phi+r_1, \quad (3b)$$
$$\dot{\Phi}=-C\Phi+{}^1/_2B(N_2-N_1)\Phi+\tilde{r}_\Phi-r_I, \quad (3c)$$
$$I=C_t\Phi+r_I. \quad (3d)$$

Here, τ_1 and τ_2 are the lifetimes of electrons at the active levels; $B/2$ is the rate of stimulated transitions (per one photon in a mode); C is a constant describing the resonator losses; C_t is a constant describing the total losses. Assuming that C_t includes the angle factor and the quantum efficiency of the photodetector, and applying the Burgers theorem,[19] we shall identify I with the photocount rate.

The system of equations (3) includes random sources $r_i(t)$ of fluctuations of the quantities N_1, N_2, Φ and I. The statistical properties of these sources follow from the obvious assumptions about the statistics of the radiation: discrete nature of the elementary processes in the active medium, their statistical independence in the case of a large number of atoms and photons, and the relative smallness of the fluctuations. All the contributions on the right-hand sides of the system (3) represent the probabilities (per unit time) of the following random events: changes in the quantities N_1, N_2, Φ and I by ± 1 as a result of pumping (Λ), spontaneous transitions (S), stimulated emission (e), absorption (a), etc. For each of the equations in the system (3), we can derive expressions for arbitrary realizations of the random processes, so that, for example, Eq. (3a) corresponds to

$$\dot{N}_2=\sum_i^{(\Lambda)}\delta(t-t_i)-\sum_j^{(s)}\delta(t-t_j)-\sum_k^{(e)}\delta(t-t_k)+\sum_l^{(a)}\delta(t-t_l)\equiv L_2(t). \quad (4)$$

The moments t_i, t_j, t_k, and t_l at which all these events occur are random. Averaging over any long time interval T brings us back to Eq. (3a). The average numbers given in the sums in Eq. (4) and corresponding to the different processes are, respectively, $\overline{\Lambda}T$, $\overline{N}_2T/\tau_2$, $\frac{1}{2}B\overline{N}_2\Phi T$, $\frac{1}{2}B\overline{N}_1\overline{\Phi}T$.

The noise sources $r_i(t)$ should be defined so that the condition $\langle r_i(t)\rangle=0$ is satisfied and the correlation functions are identical with the coresponding functions deduced from equations of the (4) type:

$$r_2(t)=L_2(t)-\overline{L_2(t)}=L_2(t)-\overline{\Lambda}+\frac{\overline{N}_2}{\tau_2}+{}^1/_2B(\overline{N}_2-\overline{N}_1)\overline{\Phi}. \quad (5)$$

We shall simplify calculations of the correlation matrix $\langle r_i(0)r_j(\tau)\rangle$ by considering first the equality of the lifetime of atoms at active levels

$$\tau_1=\tau_2\equiv\tau_a. \quad (6)$$

Introducing the variables $N(t)$ and $N_+(t)$ (representing the population inversion and the total population of the active levels) and the corresponding noise sources:

$$N(t)={}^1/_2(N_2-N_1),\quad r_N(t)={}^1/_2(r_2-r_1),$$
$$N_+(t)=N_2+N_1,\quad r_{N_+}(t)=r_2+r_1. \quad (7)$$

The equation for $N_+(t)$ is separable when the condition (6) is obeyed. Beginning from a closed system of equations for the variables N, Φ, and I, we obtain a matrix of the noise correlation functions in the form

$$\begin{gathered}\langle r_N(0)r_N(\tau)\rangle={}^1/_4K_\Lambda(\tau)+{}^1/_4(\overline{N}_+/\tau_a+2B\overline{N}_+\overline{\Phi})\delta(\tau),\\ \langle r_N(0)r_\Phi(\tau)\rangle=\langle r_\Phi(0)r_N(\tau)\rangle=-{}^1/_2B\overline{N}_+\overline{\Phi}\delta(\tau),\\ \langle r_\Phi(0)r_\Phi(\tau)\rangle=(C\overline{\Phi}+{}^1/_2B\overline{N}_+\overline{\Phi})\delta(\tau),\\ \langle r_I(0)r_I(\tau)\rangle=\overline{I}\delta(\tau),\\ \langle r_I(0)r_N(\tau)\rangle=\langle r_N(0)r_I(\tau)\rangle=0,\\ \langle r_I(0)r_\Phi(\tau)\rangle=\langle r_\Phi(0)r_I(\tau)\rangle=-\overline{I}\delta(\tau).\end{gathered} \quad (8)$$

Here, $\overline{N}_+$, $\overline{\Phi}$, and $\overline{I}$ are the stationary (steady-state) values. The equations in the system (8) [and later, Eqs. (19) and (26)] are derived by a method borrowed from a theory of random pulse processes[21]: we multiply expressions of Eq. (5) type, average each product of the δ functions in respect of time, and finally average the number of terms of each type in single and double sums (this is explained later).

The function $K_\Lambda(\tau)$ in the first equation of the system (8) reflects the nature of the statistics of elementary events involving excitation of atoms in the active medium; in the case of the Poisson statistics of the pumping, we have $K_\Lambda(\tau)=\overline{\Lambda}\delta(\tau)$. It is worth noting the last correlation function in the system (8). The minus sign is due to "anticorrelation" of the photocounts and photons in the resonator (see Figs. 3c and 3d). This simple but very important (for estimating the reduction in the photodetection noise) type of correlation of laser radiation was first pointed out in Ref. 12.

We shall separate small random deviations in the case of cw operation:

$$N(t)=\overline{N}+\delta N(t),\quad \Phi(t)=\overline{\Phi}+\delta\Phi(t),\quad I(t)=\overline{I}+\delta I(t). \quad (9)$$

After linearization in respect of these deviations, we obtain the following system of equations:

$$\delta\dot{N}(t)=-\frac{n}{\tau_a}\delta N-C\delta\Phi+r_N, \quad (10a)$$
$$\delta\dot{\Phi}(t)=\frac{n-1}{\tau_a}\delta N+\tilde{r}_\Phi-r_I, \quad (10b)$$
$$\delta I(t)=C_t\delta\Phi+r_I, \quad (10c)$$

where $n=\overline{\Lambda}/\Lambda_{\rm th}$ and $\Lambda_{\rm th}=C/B\tau_a$ is the threshold pump power.

The power spectrum of the photocurrent can be deduced from Eq. (10c):

$$(\delta I^2)_\omega=(r_I{}^2)_\omega+2\,\mathrm{Re}\,C_t(\delta\Phi r_I)_\omega+C_t^2(\delta\Phi^2)_\omega, \quad (11)$$

where

$$(\delta X\delta Y)_\omega\equiv\int_{-\infty}^{\infty}\langle\delta X(0)\delta Y(\tau)\rangle e^{i\omega\tau}d\tau. \quad (12)$$

The system (10) yields a new system of equations for the required spectral densities $(\delta X\delta Y)_\omega$; its solution allows us to express $(\delta I^2)_\omega$ in terms of the spectral densities of the fluctuation sources $r_i(t)$:

$$(\delta I^2)_\omega=(r_I{}^2)_\omega+\frac{1}{D(\omega)}\Big\{-2C_t\frac{n}{\tau_a}\Omega_r{}^2(r_I{}^2)_\omega+C_t^2\Big[\Big(\frac{n-1}{\tau_a}\Big)^2(r_N{}^2)_\omega+\Big(\frac{n^2}{\tau_a{}^2}+\omega^2\Big)(r_\Phi{}^2)_\omega+2\frac{n}{\tau_a}\frac{n-1}{\tau_a}(r_Nr_\Phi)_\omega\Big]\Big\}. \quad (13)$$

Here,

$$D(\omega)=(\omega^2-\Omega_r{}^2)^2+\left(\omega\frac{n}{\tau_a}\right)^2,\quad \Omega_r{}^2=(n-1)\frac{C}{\tau_a}. \tag{14}$$

The first term in Eq. (13) corresponds to the shot noise in the photodetector. The minus sign in the second term represents the reduction in the noise mentioned above and due to "anticorrelation" between the shot noise of fluctuations of the photocounts $r_I(t)$ and fluctuations of the number of photons in the resonator.

In the Poisson pumping case we obtain the familiar result: when the lasing threshold is exceeded ($n \gg 1$), the variance of the number of photocounts approaches the variance for the Poisson distribution, which describes the field in a coherent (Glauber) state. In fact, we find from Eqs. (13) and (14) that if $K_\Lambda(\tau) = \bar{\Lambda}\delta(\tau)$, then the variance of the number of photocounts in a time $T \gg \tau_a$, C^{-1} is given by

$$\langle(m-\bar{m})^2\rangle=\bar{m}\left\{1+2\frac{C_t}{C}\frac{1}{n-1}\frac{n}{n-1}\right\}. \tag{15}$$

The result given by Eq. (13) is obtained on the assumption that $\tau_1 = \tau_2$. We shall now obtain the expression for $(\delta I_2)_\omega$ in the limit which corresponds to the greatest reduction in the noise: $\tau_2 \gg \tau_1$. We shall assume that

$$\tau_2^{-1}=0,\quad \tau_1^{-1}=\infty, \tag{16}$$

which gives

$$(\delta I^2)_\omega=(r_I{}^2)_\omega+\frac{C^2C_t}{\tilde{D}(\omega)}\Big\{-2Ck^2(r_I{}^2)_\omega + C_t\left[k^2(r_2{}^2)_\omega+\left(k^2+\frac{\omega^2}{C^2}\right)(r_\Phi{}^2)_\omega+2k^2(r_2r_\Phi)_\omega\right]\Big\}, \tag{17}$$

where

$$k=(\Phi/\bar{N}_2)=B\bar{\Lambda}/C^2,\quad \tilde{D}(\omega)=(\omega^2-kC^2)^2+(\omega kC)^2. \tag{18}$$

The correlation matrix of Langevin sources is defined as before, and it is given by

$$(r_I{}^2)_\omega=\bar{I},\quad (r_2{}^2)_\omega=(K_\Lambda)_\omega+B\bar{N}_2\Phi,$$
$$(r_\Phi{}^2)_\omega=C\Phi+B\bar{N}_2\Phi,\quad (r_2r_\Phi)_\omega=-B\bar{N}_2\Phi. \tag{19}$$

It therefore follows that the spectrum of fluctuations of the photocurrent is expressed in terms of the usual lasing parameters and in terms of the spectrum of fluctuations of the effective pumping $(K_\Lambda)_\omega$ [see Eqs. (8) and (19)].

3. SUB-POISSON PUMPING STATISTICS

We shall consider lasing as a result of excitation of an active medium by sub-Poisson external pumping, for example, by an electron beam under conditions ensuring strong suppression of the shot noise of the beam. It is known that the spectrum of the anode current in the nonsaturation case has a dip at low frequencies against the background of the shot noise and the relative depth of the dip is $(1-\Gamma^2)$, where Γ^2 is the depression coefficient (Γ^2 can be of the order of 10^{-2}) given in Ref. 22: the variance of the number of electrons m recorded in a time T is now $\langle(m-\bar{m})^2\rangle = \bar{m}\Gamma^2 < \bar{m}$. Assuming, for the sake of simplicity, that this dip has a Lorentzian profile, we shall write down the correlation function of the effective pumping in the form

$$K_\Lambda(\tau)=\bar{\Lambda}\delta(\tau)-{}^1/_2\eta\bar{\Lambda}\gamma(1-\Gamma^2)\exp\{-\gamma|\tau|\}, \tag{20}$$

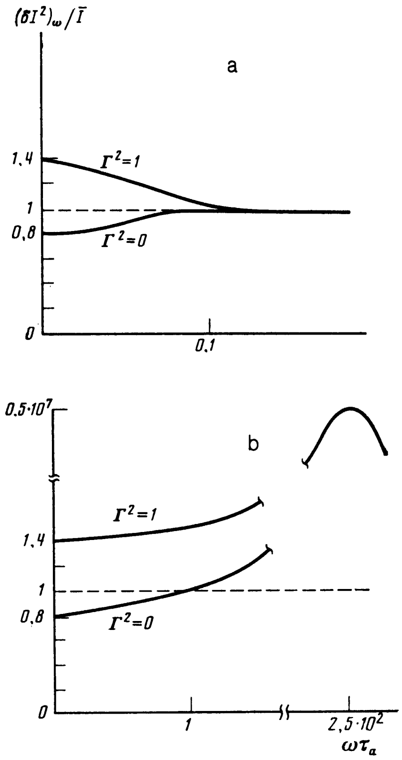

FIG. 1. Normalized spectrum of the photocurrent plotted for a large excess ($n = 7$) above the lasing threshold: a) $\eta = 1$, $C = C_t$, $C\tau_a = 10^{-2}$, $\tau_a = 10^{-8}$ sec, $\gamma = 10^8$ sec^{-1}; b) $\eta = 1$, $C = C_t$, $C\tau_a = 10^4$, $\tau_a = 10^{-3}$ sec, $\gamma = 10^8$ sec^{-1}.

where γ^{-1} is the correlation time of electrons in a beam and η is the efficiency of the excitation of atoms by electron impact ($\bar{\Lambda} \propto \eta$).

The expression (20) taken with Eq. (13) gives the following equation for the spectrum of the photocurrent

$$(\delta I^2)_\omega=\bar{I}\Big\{1+\frac{CC_t}{\tau_a{}^2D(\omega)}\Big[\omega^2\tau_a{}^2(n+1)+2n -\frac{1}{2}\eta(1-\Gamma^2)n(n-1)\frac{\gamma^2}{\omega^2+\gamma^2}\Big]\Big\}. \tag{21}$$

We shall consider two cases typical of lasers: a) $C \ll \tau_a^{-1}$, which is a relationship between the relaxation constants typical of a gas laser (for example, in the case of an He–Ne laser we have $C \sim 10^6$ sec^{-1} and $\tau_a \sim 10^{-8}$ sec—see Ref. 23); b) $C\tau_a \gg 1$, which is typical of solid-state lasers (for a ruby laser we have $C\tau_a \sim 10^4$–10^5—see the Appendix in Ref. 15).

Figure 1 shows the normalized spectrum of the photocurrent for these two cases [subject to Eq. (6)] on the assumption of optimal photodetection conditions [$\eta(1-\Gamma^2) \approx 1$, which corresponds to total suppression of the excitation noise, and $C = C_t$, which corresponds to the creation of a photoelectron by each photon lost by the resonator]. The repulsive correlation of the elementary excitation events creates dips at low frequencies in the spectrum $(\delta I^2)_\omega$ below the level of the spectral density of the shot noise corresponding to the field in a coherent state. The relative depth of this dip under these photodetection conditions is determined by the excess of the pump power above the threshold; when the excess is considerable, the depth of the dip in the photocurent spectrum may reach 0.5. The frequency band in which suppression of the photodetection noise is possible is governed by the ratio of the relaxation constants: in the case of a gas laser the width of the dip is of the order of C, whereas in the case of a solid-state laser it is of the order of τ_a^{-1}. The Lamb and Scully quantum theory of a gas laser is used in Ref. 12 to show that the spectrum of the photocurrent power vanishes at frequencies $\omega \ll C$ (i.e., the photodetection noise vanishes). The factors which reduce this effect include the random nature of the decay of the upper active level 2 ($\tau_2^{-1} \neq 0$); this halves the depth of the dip. In the case of complete suppression of the excitation noise $[\eta(1-\Gamma^2)\approx 1]$ the low-frequency shot component of the photocurrent includes contributions from fluctuations which appear as a result of spontaneous emission of radiation from atoms at the upper active level. We shall give the expression for $(\delta I_2)_\omega$ in the case of Eq. (16) (when the spontaneous radiation noise is suppressed):

$$(\delta I^2)_\omega = \bar{I}\left\{1+\frac{2CC_t}{\tilde{D}(\omega)}\left[\omega^2-(Ck)^2\eta\frac{1-\Gamma^2}{2}\frac{\gamma^2}{\omega^2+\gamma^2}\right]\right\}, \tag{22}$$

where k and $\tilde{D}(\omega)$ are defined in Eq. (14). At frequencies $\omega \approx 0$ when the conditions $\eta \approx 1$, $C = C_t$, $\Gamma^2 \ll 1$ are obeyed, we can expect almost complete suppression of the photocurrent noise.

In the spectrum of the intensity fluctuations of the radiation emitted by a solid-state laser (Fig. 1b) there is a peak[23] at the frequency Ω_r of relaxational oscillations [see Eq. (14)]. To the present authors' knowledge, the possibility of a dip in the photocurrent spectrum in the $C\tau_a \gg 1$ case at frequencies $\omega \lesssim \tau_a^{-1}$ has not been discussed.

The reduction in the photodetection noise associated with suppression of the pump noise reduces the variance of the number of photocounts. We shall give the formulas for the parameter ξ [Eq. (2)] which occurs in the expression for the variance of the number of photocounts recorded during a long observation time $T \gg \tau_a, C^{-1}, \gamma^{-1}$ and we shall do this separately for the cases corresponding to Eqs. (6) and (16) (which we shall denote by ξ_1 and ξ_2, respectively):

$$\xi_1 = 2\frac{C_t}{C}\left\{\frac{1}{n-1}-\eta\frac{1-\Gamma^2}{4}\right\}\frac{n}{n-1}, \tag{23}$$

$$\xi_2 = -\eta\frac{C_t}{C}(1-\Gamma^2). \tag{24}$$

If $\Gamma^2 \ll 1$, $\tau_1 = \tau_2$, the photocount statistics is of the sub-Poisson nature ($\xi_1 < 0$) whtn the threshold is exceeded sufficiently strongly ($n > 5$). In this case the maximum value of $|\xi_1| = 0.5$ (in the $\xi_1 < 0$ case) corresponds to $C = C_t$, $n \gg 1$, $\Gamma^2 = 0$, $\eta = 1$. In an ideal situation ($\tau_2 = \infty$, $\tau_1 = 0$, $C = C_t$, $\Gamma^2 = 0$, $\eta = 1$), we find that $\xi_2 = -1$.

4. SUB-POISSON STATISTICS OF LASER RADIATION DUE TO COOPERATIVE PROCESSES

The sub-Poisson nature of the population of the upper active level can be ensured by cooperative (pair) deactivation of atoms at the beginning of the pumping process.[13,3] By way of example, we shall consider a four-level lasing scheme [active transition 2–1, steady-state Poisson pumping $\Lambda_3(t)$ of the level 3]. We shall assume that these cooperative processes result in the loss of pairs of excited atoms from the level 3. The irreversibility of the summation of excitations is ensured by rapid multi-phonon relaxation. The energy levels and transition scheme corresponding to this model are given in Fig. 2.

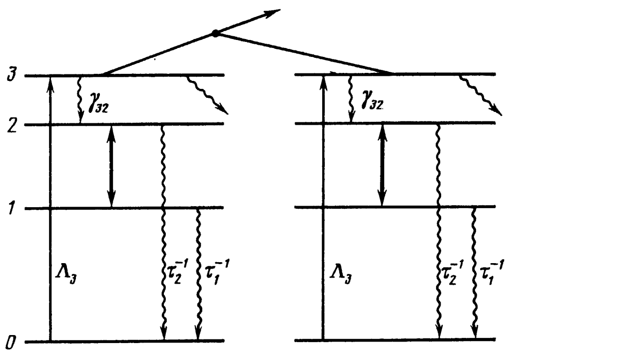

FIG. 2. Schematic representation of the levels and of pair deactivation in the pump channel.

We shall write down the rate equations for the populations of the levels N_2 and N_3:

$$\begin{aligned}\dot{N}_2 &= \gamma_{32}N_3 - N_2/\tau_2 - {}^1/_2 B(N_2-N_1)\Phi + r_2,\\ \dot{N}_3 &= \Lambda_3 - \gamma_3 N_3 - \beta N_3^2 + r_3.\end{aligned} \tag{25}$$

Here, $\beta = \alpha/V$; α (cm^3/sec) is the pair deactivation constant; $\alpha\bar{n}_3 = \beta\bar{N}_3$ is the probability (per second) of the summation of energy per one atom excited to the level 3 in the cw lasing regime; $\bar{n}_3$ is the steady-state concentration of atoms at the level 3; V is the volume of the system. The equations for the variables N_1, Φ, and I remain the same [see Eq. (3)]. The correlation functions of the noise sources in Eq. (25) are found by the method described in Sec. 2:

$$\begin{aligned}\langle r_3(0)r_3(\tau)\rangle &= \bar{\Lambda}_3\delta(\tau) + (\gamma_3+2\alpha\bar{n}_3)\bar{N}_3\delta(\tau),\\ \langle r_3(0)r_2(\tau)\rangle &= \langle r_2(0)r_3(\tau)\rangle = -\gamma_{32}\bar{N}_3\delta(\tau).\end{aligned} \tag{26}$$

Using the linearized equations for the fluctuations δN_3 which follow from Eq. (25), we can show that pair deactivation results in sub-Poisson statistics of the number of atoms at the level 3:

$$\langle (N_3-\bar{N}_3)^2\rangle = \bar{N}_3\left\{1-\frac{\alpha\bar{n}_3}{2(\gamma_3+2\alpha\bar{n}_3)}\right\}. \tag{27}$$

The spectral density of fluctuations of the effective pumping $\Lambda_2(t) = \gamma_{32}N_3 + r_{\Lambda_2}(t)$ of atoms to the active level 2 is found from the system of equations (25):

$$(K_{\Lambda_2})_\omega = \bar{\Lambda}_2\left\{1-\frac{\gamma_{32}\alpha\bar{n}_3}{\omega^2+(\gamma_3+2\alpha\bar{n}_3)^2}\right\}, \tag{28}$$

where $\bar{\Lambda}_2 = \gamma_{32}\bar{N}_3$.

We shall now give the expression for the spectrum of the photocurrent power obtained on recording the laser radiation in the case when $\tau_1 = \tau_3$, derived from Eqs. (8) and (13), using the correlation spectrum of the effective pumping (28)

$$(\delta I^2)_\omega = \bar{I}\left\{1+\frac{C_p C}{\tau_a^2 D(\omega)}\left[\omega^2\tau_a^2(n+1) + 2n - \frac{n(n-1)}{2}\frac{\gamma_{32}\alpha\bar{n}_3}{\omega^2+(\gamma_3+2\alpha\bar{n}_3)^2}\right]\right\}. \tag{29}$$

An analysis of Eq. (29) gives the following result. At low frequencies ($\omega \lesssim C$ for a gas laser and $\omega \lesssim \tau_a^{-1}$ for a solid-state laser) the photocurrent spectrum has a dip of relative depth

$$\xi_1 = 2\frac{C_t}{C}\left[\frac{1}{n-1} - \frac{\gamma_{32}\alpha\bar{n}_3}{4(\gamma_3+2\alpha\bar{n}_3)^2}\right]\frac{n}{n-1}. \qquad (30)$$

The maximum reduction in the photodetection noise ($\xi_1 = -1/16$) is attained under the following conditions: a strong excess above the lasing threshold, $2\alpha\bar{n}_3 = \gamma_3 = \gamma_{32}$, and $C = C_t$. We shall now quote the results of Ref. 24 obtained for cooperative luminescence of Er^{3+} ions in crystals of the fluorite type: $\gamma_3 \sim 10^2 sec^{-1}$, $\alpha \sim 10^{-14}$ cm^3/sec; the condition $2\alpha\bar{n}_3 = \gamma_3$ is reached when $\bar{n}_3 \sim 10^{16}$ cm^{-3}. Such concentrations are possible in many systems at moderate pumping rates.

The noise due to the spontaneous emission of radiation from the upper active level ($\tau_2^{-1} \neq 0$) reduces the depth of the dip in the photocurrent spectrum. For example, when the conditions of Eq. (16) are obeyed, we obtain

$$\xi_2 = -\frac{C_t}{C}\frac{\gamma_{32}\alpha\bar{n}_3}{(\gamma_3+2\alpha\bar{n}_3)^2}, \qquad (31)$$

which under ideal conditions ($C = C_t$, $\gamma_3 = \gamma_{32} = 2\alpha\bar{n}_3$) ensures the minimum value of the parameter in the variance of the number of photocounts: $\xi_2 = -1/8$.

The maximum values of the parameter $|\xi|$ found in this section (in the $\xi < 0$ case) are smaller than those given in Ref. 13, where the pair deactivation effect was considered on the basis of one- and two-particle distribution functions of the excitations at the level 3. Such a quantitative discrepancy can be readily accounted for by the difference between the approaches: the model which can be reduced to the rate equation system (25) involves *a priori* averaging of the probabilities of the cooperative binary processes [term βN_3^2 in the second equation of system (25)] and ignores the spatial aspect. Clearly, terms of the βN_3^2 type in Eq. (25), fully justified in the calculation of steady-state populations (see, for example, Ref. 25), are far too simplified in the case of the correlation characteristics. However, the ability to demonstrate clearly the appearance of cooperative deactivation in SPPS in the case of cw operation of gas and solid-state lasers is the advantage of the rate equations approach. We hope to attract the interest of experimental laser specialists to the problem of generation of light of improved regularity, tackled by means of rate equations.

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