Characteristics of the resistance of degenerate ferromagnetic semiconductors near the Curie point

É. L. Nagaev

(Submitted 24 March 1986; resubmitted 11 September 1986)
Zh. Eksp. Teor. Fiz. **92**, 569–579 (February 1987)

Degenerate ferromagnetic semiconductors exhibit singularities of the electrical resistivity ρ near the Curie point T_c (peak of ρ or an insulator-metal transition), which cannot be explained by critical scattering of carriers on magnetization fluctuations. However, such singularities can be explained by an inhomogeneous magnetoelectric effect due to the electric field or randomly distributed ionized donors. A strong fall of the effective permittivity ε_{eff} of a crystal near T_c due to the magnetoelectric effect enhances the scattering of carriers by donors and reduces the number of electrons in the current states, which gives rise to a peak of ρ. This reduction in ε_{eff} may be so large that the Mott condition for the delocalization of the donor electrons is no longer satisfied near T_c and these electrons become localized. The magnetoelectric response functions are found for ferromagnetic semiconductors and metals near T_c.

INTRODUCTION

Almost all degenerate n-type ferromagnetic semiconductors exhibit singularities of the resistivity ρ near the Curie point T_c. However, the temperature dependence of the resistivity of these materials is quite different from that exhibited by ferromagnetic metals. Instead of a kink of $\rho(T)$ typical of metals, n-type semiconductors exhibit a peak of ρ in the region of T_c and the relative amplitude of this peak rises on reduction in the impurity concentration n. When n is sufficiently low, the amplitude of the peak may be several orders of magnitude higher than $\rho(\infty)$. At even lower values of n, ferromagnetic semiconductors of the EUO type with oxygen vacancies behave as degenerate materials only below T_c; above T_c they act as insulators. The abrupt change in the resistivity at a metal-insulator transition, which occurs slightly below T_c has the record value of 17 orders of magnitude. This transition is observed only in a narrow range of carrier densities $(1–2)\times 10^{19}\,\text{cm}^{-3}$. At lower values of n the ferromagnetic semiconductor EuO is nondegenerate. All the n-type ferromagnetic semiconductors also exhibit a giant negative magnetoresistance near T_c (see Ref. 1).

Generally speaking, p-type ferromagnetic semiconductors exhibit no singularities of ρ near T_c. The hole mobility is normally several orders of magnitude higher than the electron mobility, the magnetoresistance of p-type samples is small, and sometimes, as in the case of nonmagnetic semiconductors, it is positive (for example, this is true of p-type $CdCr_2Se_4$). Moreover, in the case of some p-type materials a peak of ρ is replaced by a shallow minimum at temperatures comparable with T_c (see, for example, Ref. 2).

These major differences between the behavior of n- and p-type semiconductors have a simple physical explanation. In practically all the magnetic semiconductors the magnetic ions with partly filled d or f shells are cations, whereas anions are nonmagnetic. The conduction electrons usually move between cations and the holes between anions. Since holes and magnetic cations are spatially separated, the exchange interaction between them is several orders of magnitude weaker than between the conduction electrons and cations. Consequently, the scattering of holes by magnetization fluctuations is very weak and it is masked by other (nonmagnetic) scattering mechanisms.[1)] If a p-type ferromagnetic semiconductor exhibits a minimum of ρ, this may be explained by the competition between two nonmagnetic scattering mechanisms with opposite temperature dependences (for example, impurity and phonon mechanisms, the former of which becomes weaker on increase in temperature and the latter is enhanced). The correlation between T_c and the temperature of a minimum of ρ is then only apparent.

On the other hand, there is no doubt about the magnetic origin of the peak of ρ in the case of degenerate n-type semiconductors. However, it cannot be explained by critical scattering of electrons on magnetization fluctuations. Firstly, at moderately high values of n the amplitude of this peak is far too high and, secondly, lowering of n transforms this peak into a resistivity discontinuity corresponding to an insulator-metal transition. However, the appearance of a gap in the electron spectrum as a result of such a transition cannot be due to critical scattering.

The same conclusion follows from a direct analysis of the critical scattering of carriers in semiconductors. The nature of the peak of ρ exhibited by semiconductors and characterized by an amplitude which increases on reduction in the Fermi momentum was first determined in Ref. 4 on the basis of a calculation carried in the Born approximation for the s-f exchange. The Born approximation has been used in the subsequent numerous investigations of this topic and the treatment of Ref. 4 has been refined by replacing the Ornstein-Zernike spin correlation function with more rigorous scaling expressions. An analysis of the damping of the electron states, carried out to the fourth order (inclusive) of perturbation theory in respect of the s-f exchange, shows that in the case of n-type semiconductors it has a giant magnitude even far from T_c, which demonstrates that the Born approximation is totally invalid near T_c (Ref. 1). Conseqently, the results of the theories predicting very high peaks of ρ for semiconductors due to critical scattering are unreliable. In fact, if such peaks had existed, they would have been manifest also in p-type samples: near T_c the magnetic scattering in these semiconductors would have predominated over the nonmagnetic contribution, which is not confirmed by the experimental results.

A method developed in Refs. 1 and 5 has made it possi-

 0038-5646/87/020322-06$04.00

ble to avoid the description of electron states near T_c by perturbation theory in terms of the *s-f* exchange. In this method the interaction of an electron with long-wavelength magnetization fluctuations is included even in the zeroth approximation. It is based on the physical idea that in the case of electrons, characterized by a low kinetic energy compared with the *s-f* exchange energy AS, the direction of spin is not fixed in space. As an electron moves in a crystal, this direction follows that of the local magnetic moment which varies slowly from one atom to another (these are precisely the exact eigenstates of long-wavelength electrons in the case of large-period helicoidal ordering). In the case of such states with a fluctuating direction of the spin the contribution of long-wavelength magnetization fluctuations to the scattering of electrons practically disappears and the electron mobility near T_c becomes comparable with its high-temperature value.[1]

A more self-consistent explanation of the anomalies of ρ near T_c is based on the fact that the influence of defects passed through a maximum near the Curie point because of the magnetoelectric effect. This effect represents the appearance of a magnetization in a crystal under the influence of an electric field or an elecric polarization under the influence of a magnetic field. In homogeneous fields the effect is possible only in magnetic insulators with specific lattice symmetry properties.[6] Physically, such behavior originates from relativistic effects.

In magnetic conductors the appearance of a magnetization under the influence of a homogeneous electric field is impossible because of the screening. However, the magnetoelectric effect should occur in these materials when fields are inhomogeneous.[1] This is due to the fact that an electric field then redistributes the conduction electron density in a crystal and thus alters the intensity of the indirect exchange, reducing it in some parts of a crystal and enhancing it elsewhere. There are corresponding changes in the local magnetization. Since the inhomogeneous magnetoelectric effect is not of relativistic origin, it is much stronger than the homogeneous effects and it can occur in crystals of any symmetry.

The specific influence of the magnetoelectric effect on the resistivity of dengenerate ferromagnetic semiconductors is manifested as follows. Spatial fluctuations of the electric field of randomly distributed donors in a crystal create similar fluctuations of the static magnetization and the effect of the latter fluctuations on the conduction electrons has to be added to the effect of the electric field. The magnetization fluctuations induced by this electric fields are maximal near T_c. Enhancement of the influence of defects near T_c enhances also the impurity scattering of carriers and their partial Anderson localization, which gives rise to a peak of ρ. If the increase in the influence of defects is very great, electrons may be transferred from delocalized to localized states and a crystal may be converted to an insulator.

Essentially the same idea was put forward in the very first theory of the insulator-metal transition in degenerate ferromagnetic semiconductors.[7] The inhomogeneous magnetoelectric effect had been investigated earlier only in the spin-wave region.[1] The method of Ref. 5 makes it possible to calculate this effect also in the most interesting range of temperatures where there are singularities of ρ. The present paper is devoted mainly to this topic. We shall find the magnetoelectric response functions and use them to determine the electron mobility near T_c and we shall then formulate a semiquantitative criterion for the transfer of electrons from delocalized to localized states.

We shall also find the magnetoelectric response functions for *p*-type semiconductors and for metals. In contrast to *n*-type semiconductors, in the latter case the inequality is reversed, i.e., we now have $\mu \gg As$, were μ is the energy of the Fermi carriers. This inequality allows us to describe these carriers by the usual Bloch waves with a fixed direction of the spin almost right up to the Curie point T_c. Although the magnetoelectric effect in *p*-type semiconductors and metals has little influence on the carrier kinetics, it may be of interest for its own sake.

1. INDIRECT EXCHANGE IN FERROMAGNETIC SEMICONDUCTORS EXHIBITING *s-f* EXCHANGE

In this section we shall consider the energy spectrum of the conduction electrons and the indirect exchange via these electrons in semiconductors with a strong *s-f* exchange when the inequality $\mu \ll AS$ is satisfied. The idea of alignment of the electron spin along the direction of the local magnetic moment, which is the basis of the analysis given below, can be expressed mathematically by the variational principle.[1,5] It is assumed that a conduction electron migrating from atom to atom exhibits changes in its spin which are such that the spin is always directed along the net magnetic moment of atoms in a region of radius R centered on that atom where the investigated electron is at a given moment. The radius R is the variational parameter.

In the process of calculations based on such a program the energy of an electron with a fluctuating spin direction is expressed in terms of binary correlation functions of the spins of magnetic atoms (*f* spins in accordance with the terminology of the *s-f* model). In the nearest-neighbor approximation, this expression has the following from for a simple cubic lattice if $W \gg AS$ (Refs. 1 and 5):

$$\mathscr{E}_{\mathbf{k}\lambda} \approx -\frac{W}{2}\gamma_{\mathbf{k}} + \frac{W\gamma_{\mathbf{k}}}{8}\left[1 - \frac{\langle \mathbf{M_g M_{g+\Delta}}\rangle}{\langle \mathbf{M_g}^2\rangle}\right] - \frac{A\lambda\langle \mathbf{S_g M_g}\rangle}{\langle \mathbf{M_g}^2\rangle^{1/2}}, \qquad (1)$$

$$\mathbf{M_g} = \sum_{g<R} \mathbf{S_g}, \quad \gamma_{\mathbf{k}} = z^{-1}\sum_{\Delta} \exp(i\mathbf{k}\Delta).$$

Here, $\mathbf{k}$ is the analog of the quasimomentum; $\lambda = \pm 1/2$ is the projection of the electron spin along the direction of the local magnetic moment $\mathbf{M_g}$ in a region centered on an atom g; W is the width of the conduction band; A is the *s-f* exchange integral; Δ is the vector linking this atom with its nearest neighbors; $z = 6$ is the number of such neighbors; the angular brackets denote thermodynamic averaging. Equation (1) is derived on the assumption that the angles between the directions of the moments $\mathbf{M_g}$ and $\mathbf{M_{g+\Delta}}$ of the neighbors regions are small, which corresponds to the condition $R \gg a$, where a is the lattice constant.

We shall assume that the influence of the indirect exchange on the magnetic properties of a crystal is much weaker than that of the direct exchange. In the case if a binary correlation function at distances f considerably greater than the direct-exchange radius, we can use the following expressions (a small critical exponent η is ignored):

$$\langle S_0^z S_f^z\rangle = \mathscr{M}^2 + \rho_< S(S+1)\exp(-\xi_< f)/f, \qquad (2a)$$

$$\langle S_0^x S_f^x\rangle = \langle S_0^y S_f^y\rangle = \rho_< S(S+1)/f \text{ for } T<T_c,$$
$$\langle S_0^\alpha S_f^\alpha\rangle = \rho_> S(S+1)\exp(-\xi_> f)/f \text{ for } T>T_c, \quad (2b)$$
$$\alpha = x, y, z, \quad \rho_< = \rho_> = \rho,$$

where S is the f spin and ρ is a characteristic constant of the dimensions of length which— if the direct-exchange radius is $\sim a$—is of the same order of magnitude. The magnetization $\mathcal{M}$ and the reciprocal of the correlation length ξ depend on temperature and on the electron density:

$$\mathcal{M} = c\tau^\beta, \quad \xi = b|\tau|^\nu, \quad (3)$$
$$\tau = 1 - T/T_c(n), \quad T_c(n) = T_c^0 + \delta T_c(n),$$

where T_c^0 is the Curie point of an undoped semiconductor and δT_c is the shift of this point because of the indirect exchange via the conduction electron. Under our conditions we have $b \propto a^{-1}$ and $c \propto S$ and the dependences of these quantities on n are much weaker than $T_c(n)$. Therefore, we shall ignore these dependences and assume that the magnetoelectric effect is entirely due to the dependence of T_c on n.

It follows from the scaling relations that if $\eta = 0$, then we should have $2\beta = \nu$ (Ref. 8). The following equality then applies in the ordered region:

$$\xi_< = g\mathcal{M}^2, \quad g = b/c^2 \propto (aS^2)^{-1}. \quad (4)$$

If τ is not too small, then instead of $\beta = 0.38$ derived for a Heisenberg magnetic material by a series expansion,[8] we can use the value $\beta_{SFF} = 0.5$, which is obtained in the self-consistent field approximation. In the case of $\mathcal{M}$ which occurs in further calculations this approximation for β is justified if $|(\beta_{SFF} - \beta)\ln\tau| \ll 1$, i.e., it is justified if $\tau > 0.01$.

If we calculate the correlation functions in Eq. (1) with the aid of Eqs. (2a) and (4) accurate to terms of the order of $\mathcal{M}^2$ inclusive and if we minimize $\mathscr{E}_{0\lambda}$ with respect to R, we find that if $\xi R \ll 1$ then the following expression describes the energy of an electron which is in the lowest of the spin subbands (characterized by $\lambda = 1/2$ when $A > 0$):

$$\mathscr{E}_k = \mathscr{E}_k^{(0)} + \mathscr{E}_k^{(2)}\mathcal{M}^2 + O(\mathcal{M}^4),$$
$$\mathscr{E}_k^{(0)} = -\frac{3A}{4}\left(\frac{5l}{6R_0}\right)^{1/2}\left[1+\frac{\gamma_k}{4}\right] - \frac{W\gamma_k}{2},$$
$$\mathscr{E}_k^{(2)} = -\frac{11A}{48}\left(\frac{5R_0}{6l}\right)^{1/2}\left[1+\frac{\gamma_k}{4}\right]p, \quad p = 1 - gS(S+1)\rho_<, \quad (5)$$

where the equilibrium value of the radius R at $T = T_c$ is given by the expression

$$R_0 = (Wa^2/3A)^{1/3}(5/6l)^{1/3}, \quad l = 3\rho_< S(S+1). \quad (6)$$

In the adopted order with respect to $\mathcal{M}$ the result given by the system (5) differs from the results in Ref. 5 by the presence of a factor $p \sim 1$ in the expression for $\mathscr{E}_k^{(2)}$. This factor allows in fact for the influence of the temperature-dependent part of fluctuations of the static magnetization on the carrier energy. The factor p includes phenomenological constants and, therefore, the exact expression for this factor cannot be given. However, in any case we can say that it is positive. This follows from the observation that the correlation function $\langle S_0^z S_f^z\rangle$ rises as a result of cooling for all values of **f**.

It is worth noting that the electron energy (5) depends on the magnetization quadratically rather than linearly. This is a consequence of the fact that if $\mathcal{M} = 0$, then the average projection of the electron spin is zero because it becomes aligned to the fluctuating local moment. However, the magnetization is an analog of the external field which polarizes this system and, therefore, an electron acquires a spin projection which is proportional to $\mathcal{M}$.

Using the system (5) we can find the total energy of degenerate electrons and then the shift δT_c. If we assume that the free energy of an undoped crystal is described by a Landau expansion, we can write down the free energy of a degenerate ferromagnetic semiconductor in the form

$$F = \frac{3N}{2}(T - T_c^0)\frac{\mathcal{M}^2}{S(S+1)} + n\mathscr{E}_0^{(2)}\mathcal{M}^2 + \frac{9NT_c^0}{20}\left[1+\frac{1}{2S(S+1)}\right]\mathcal{M}^4/S^2(S+1)^2 + O(n^{5/3}). \quad (7)$$

Renormalization of the term proportional to $\mathcal{M}^4$ by the indirect exchange will be ignored.

It therefore follows from Eq. (7) that

$$\delta T_c = -\frac{2}{3}\mathscr{E}_0^{(2)} S(S+1)\frac{n}{N} \equiv \alpha n. \quad (8)$$

Equations (7) and (8) yield the following expression valid in the adopted approximation ($\beta = 0.5$):

$$\mathcal{M}^2 = K[T_c^0 + \delta T_c - T],$$
$$K = \frac{c^2}{T_c} = \frac{5}{3}\frac{S(S+1)}{T_c}\left[1+\frac{1}{2S(S+1)}\right]^{-1}, \quad (9)$$

which is a special case of Eq. (3).

2. RESPONSE FUNCTIONS OF A DEGENERATE FERROMAGNETIC SEMICONDUCTOR CHARACTERIZED BY $\mu \ll AS$

In describing the influence of an external electric field with the potential $\Phi(\mathbf{q},\omega)$ on the conduction electrons in degenerate ferromagnetic semiconductors we must bear in mind that this field not only induces an internal potential $\varphi(\mathbf{q},\omega)$, but also alters the magnetization $\mathcal{M}(\mathbf{q},\omega)$, ($\mathbf{q}$ is the wave vector and ω is the field frequency). Therefore, in addition to the usual permittivity described by the expression

$$\Phi(\mathbf{q},\omega) = \varepsilon(\mathbf{q},\omega)\varphi(\mathbf{q},\omega), \quad (10)$$

we have to introduce also a magnetoelectric response function. In view of the quadratic dependence of the electron energy on the magnetization near T_c given by Eq. (5), it is convenient to define this function as the coefficient of proportionality between $\mathcal{M}^2$ and Φ:

$$\mathcal{M}^2(\mathbf{q},\omega) = \zeta(\mathbf{q},\omega)\Phi(\mathbf{q},\omega). \quad (11)$$

We shall calculate the magnetoelectric response functions only in the static case when $q \ll \min\{R_0^{-1}n^{-1/2}\}$, where R_0 is the radius of the local ordering region given by Eq. (6). Then, we can employ the semiclassical approximation. According to Eq. (5), an electron experiences an effective potential

$$e\tilde{\varphi}(\mathbf{r}) = e\varphi(\mathbf{r}) + \mathscr{E}_0^{(2)}[\mathcal{M}^2(\mathbf{r}) - \mathcal{M}_0^2], \quad (12)$$

where $\mathcal{M}_0 = \mathcal{M}(n_0)$ and n_0 is the average density of the conduction electrons. Using the quadratic approximation for the electron dispersion law, we obtain the following con-

dition for the constancy of the electrochemical potential of electrons in a crystal

$$\mu(\mathbf{r})=[6\pi^2 n(\mathbf{r})]^{2/3}/2m^*=\mu_0-e\varphi(\mathbf{r})-\mathscr{E}_0^{(2)}[\mathscr{M}^2(\mathbf{r})-\mathscr{M}_0^2],$$

$$m^*\approx Wa^2/6, \quad \mu_0=\mu(n_0) \quad (\hbar=1). \tag{13}$$

Linearization of Eq. (13) and application of Eqs. (8) and (9), yields the following relationships between the Fourier components of $\mathscr{M}^2$, n, and φ:

$$(\mathscr{M}^2)_{\mathbf{q}}=K\alpha n_{\mathbf{q}}, \tag{14}$$

$$n_{\mathbf{q}}=-3en_0\varphi_{\mathbf{q}}/2\mu_0(1-\Gamma), \tag{15}$$

$$\Gamma=-3\mathscr{E}_0^{(2)}pn_0K\alpha/2\mu_0. \tag{16}$$

On the other hand, using Eq. (10) and the Poisson equation, we obtain

$$n_{\mathbf{q}}=-q^2[\varepsilon(\mathbf{q})-\varepsilon_p]\varphi_{\mathbf{q}}/4\pi e, \tag{17}$$

where ε_p is the permittivity of an undoped crystal. A comparison of Eqs. (15) and (17) shows that the permittivity of a degenerate ferromagnetic semiconductor is given by

$$\varepsilon(\mathbf{q})=\varepsilon_p(1+\varkappa^2/q^2), \quad \varkappa^2=\varkappa_0^2(1-\Gamma)^{-1}, \tag{18}$$

$$\varkappa_0^2=6\pi e^2 n_0/\varepsilon_p\mu_0, \tag{d}$$

where $\varkappa_0^2$ is the reciprocal of the screening length for a nonmagnetic semiconductor with the same values of m^* and n_0.

We can find the magnetoelectric response function from Eqs. (14), (17), and (10):

$$\zeta(\mathbf{q})=-\frac{\alpha Kq^2}{4\pi e^2}\left[1-\frac{\varepsilon_p}{\varepsilon(\mathbf{q})}\right]. \tag{19}$$

Finally, introducing the effective permittivity $\tilde{\varepsilon}(\mathbf{q})$, relating the effective potential $\tilde{\varphi}(\mathbf{q})$ of Eq. (12) to the external field by a relationship of the Eq. (10) type, we find from Eqs. (14)–(19) that this permittivity is described by

$$\tilde{\varepsilon}(\mathbf{q})=\varepsilon_p(1-\Gamma)+\varkappa^2/q^2. \tag{20}$$

If $T>T_c$, a similar calculation can be carried out expressing the electron energy of Eq. (1) in terms $\xi_>$ of Eq. (2b). The resultant expression has the structure of Eq. (5) with $p=1$:

$$\mathscr{E}_{\mathbf{k}}=\mathscr{E}_{\mathbf{k}}^{(0)}+\mathscr{E}_{\mathbf{k}}^{(2)}(\xi l), \quad l=3\rho S(S+1). \tag{21}$$

The self-consistency off the approach is retained if $\xi\equiv\xi_>$ is described using the critical exponent $\nu=1$ instead of 0.7:

$$\xi l=B(T-T_c), \quad B=bl/T_c. \tag{22}$$

The influence of the field on the short-range order can be described by a two-index magnetoelectric function which is the coefficient in the linear realtionship between $\langle \mathbf{S}_{\mathbf{p}}\mathbf{S}_{-\mathbf{p}}\rangle$ and $\Phi(\mathbf{q})$. However, in the case under discussion, when the correlation functions can be described by Eq. (2b) and their behavior when $r\gg R_0$ and $r\gg k_F^{-1}$ is unimportant, we can introduce a slowly varying function $\xi(\mathbf{r})$ and relate its Fourier components of the field by

$$\xi(\mathbf{q})=\eta(\mathbf{q})\Phi(\mathbf{q}). \tag{23}$$

Repeating the operations described above and using Eqs. (20)–(23), we find that Eqs. (19) and (20) are valid as before. The quantity Γ which occurs in these formalus and is defined by Eq. (16) should be modified by replacing K with B, and in $\mathscr{E}_0^{(2)}$ we have to substitute $p=1$. The condition of continuity of $\varepsilon(\mathbf{q})$ at the Curie point T_c is then given by the relationship between B and K. The quantity $\eta(\mathbf{q})$ is described by Eq. (19) with $p=1$.

We can check whether these results are reasonable by comparing them with those obtained in the spin-wave approximation in Ref. 1. Employing the diagram method for the thermal Green functions, an expression for Γ was found in Ref. 1 subject to the condition $T_c/S\ll T\ll T_c$ and extrapolation of this expression to T_c yielded an estimate differing from Eqs. (16) and (5)–(9) only by an additional factor $\sim(AS/W)^{1/6}$. For all the physically reasonable values of AS/W this factor is ~ 1, which shows that the approach adopted above is reasonable.

3. RESPONSE FUNCTIONS OF A FERROMAGNETIC METAL AND OF A *p*-TYPE SEMICONDUCTOR

For the sake of comparison, we shall now calculate the response functions near T_c in the opposite limit $\mu\gg AS$ which corresponds to a metal or to a degenerate p-type semiconductor. We shall consider only an ordered state. In this limit we can use perturbation theory in respect of AS/W (see Introduction). This means that the projection of the electron spin σ along the direction of the magnetic moment $\mathscr{M}$ of a crystal is a good quantum number. In the first approximation the electron energy depends only on the long-range order. The dependence on the short-range order appears in the second approximation. However, in view of almost equal occupancy of the electron subbands characterized by $\sigma=\pm 1/2$, the total conduction electron energy depends on the magnetization in the second order in $A\mathscr{M}/W$. Therefore, the contributions to its temperature dependence made by the long- and short-range magnetic orders are comparable. The short-range order can be allowed for semiphenomenologically in the same way as is done in the preceding section. However, for the sake of simplicity we shall consider here only the long-range order bearing in mind that this does not alter the order of magnitude of the magnetoelectric effect. We shall therefore describe the electron energy by the expression

$$\mathscr{E}_{\mathbf{k}\sigma}=k^2/2m^*-A\sigma\mathscr{M}, \tag{24}$$

i.e., we shall write down the condition of constancy of the electrochemical potential for electrons with both spin projections in the form

$$\mu_\sigma(\mathbf{r})=[6\pi^2 n_\sigma(\mathbf{r})]^{2/3}/2m^*$$

$$=\mu_{0\sigma}-A\sigma[\mathscr{M}_0-\mathscr{M}(\mathbf{r})]-e\varphi(\mathbf{r})=\mu_{-\sigma}(\mathbf{r}). \tag{25}$$

The average values of the Fermi energies $\mu_{0\sigma}=\mu(n_{0\sigma})$ are found from the conditions

$$\mu_{0\uparrow}-\mu_{0\downarrow}=A\mathscr{M}_0, \quad n_{0\uparrow}+n_{0\downarrow}=n_0, \tag{26}$$

which in the first order in $\mathscr{M}_0$ give

$$n_{0\sigma}=(n_0/2)[1+3A\mathscr{M}_0\sigma], \quad \mu_{0\sigma}=\mu_0(1+A\mathscr{M}_0\sigma), \tag{27}$$

$$\mu_0=(3\pi^2 n_0)^{2/3}/2m^*. \tag{28}$$

Linearization of Eq. (25) subject to Eq. (27) gives an equation relating the Fourier component of the electron density

$n_q = n_{q\uparrow} + n_{q\downarrow}$ to the electrostatic potential φ_q of the medium:

$$n_q = -3\varphi_q e n_0/2\mu_0 + 3A^2 n_0 \mathcal{M}_0 \mathcal{M}_q/8\mu_0^2. \quad (29)$$

The relationship between the Fourier components of the magnetization and density can be established from Eq. (9) bearing in mind that the results of the RKKY theory are valid in the limit under consideration; according to these results (see, for example, Ref. 1), we have

$$\delta T_c = \gamma n^{1/3}, \quad \gamma = A^2 S(S+1) n_0^{2/3}/8\mu_0 N \quad (30)$$

[it follows from Eq. (28) that λ is independent of n_0]. Linearization of Eqs. (9) and (30) gives

$$\mathcal{M}_q = \mathcal{M}_0 \delta T_{c0} n_q/6\Theta n_0, \quad \Theta = T_c^0 + \delta T_{c0} - T,$$

$$\delta T_{c0} = \delta T_c(n_0), \quad \mathcal{M}_0 = K^{1/2}\Theta^{1/2}. \quad (31)$$

Using Eqs. (29)–(31), we obtain the expression of Eq. (15) type with a feedback function Γ given by the expression

$$\Gamma = \frac{A^2 K \delta T_{c0}}{16\mu_0^2} \sim \left(\frac{AS}{\mu_0}\right)^2 \frac{\delta T_{c0}}{T_c}. \quad (32)$$

The permittivity of the system is described by Eqs. (18) and (32). In this case the magnetoelectric response function can be described conveniently by

$$\lambda(\mathbf{q}) = \mathcal{M}_q/\varphi_q = \xi(\mathbf{q})/2\mathcal{M}_0. \quad (33)$$

Using Eqs. (30), (31), (17), and (10), we obtain

$$\lambda(\mathbf{q}) = -\frac{K^{1/2}\delta T_{c0} q^2}{24\pi\Theta^{1/2} n_0 e}\left[1 - \frac{\varepsilon_p}{\varepsilon(\mathbf{q})}\right]. \quad (34)$$

In contrast to $\xi(\mathbf{q})$ of Eq. (11), the response function $\lambda(\mathbf{q})$ diverges at T_c. This divergence is formal: it reflects that, according to Eq. (9), the region of linear dependence of $\mathcal{M}(\mathbf{r})$ on $\delta n(\mathbf{r})$ located near T_c is very narrow. It vanishes in the limit $T \to T_c$.

In the disordered state the simplified approach adopted in Sec. 2 is invalid, because if $\mu \gg AS$, the electron spins are not aligned along slowly varying directions of the moment of a crystal. Therefore, the response function should be calculated allowing for more distant correlations, i.e., the relevant theory is strongly nonlocal.

4. SINGULARITIES OF THE RESISTIVITY OF DEGENERATE FERROMAGNETIC SEMICONDUCTORS EXHIBITING A STRONG *s-f* EXCHANGE

The results of Sec. 2 allow us to account for the singularities of the resistivity exhibited by degenerate ferromagnetic semiconductors in the case when $\mu < AS$: this applies to both a peak of the resistivity near T_c and to a metal-insulator transition that occurs at lower electron densities. The magnetoelectric effect responsible for these singularities is manifested by a reduction in the quantity $\varepsilon_{\text{eff}} = \varepsilon_p(1 - \Gamma)$ near T_c which, according to Eq. (20), plays formally the role of the permittivity of a crystal. Therefore, an analysis of the dependence of the resistivity ρ on both T and n must begin with an analysis of the corresponding dependences of Γ.

At $T = 0$ there is no magnetoelectric effect, since the magnetization of a crystal is then maximal for any strength of the exchange and the modification of the intensity of the indirect exchange by an electric field cannot influence the magnetization. The magnetoelectric effect disappears also in the limit $T \to \infty$, because in this limit even the long-range order is destroyed and it is not restored by enhancement of the strength of the indirect exchange because of a finite increase in the electron density. Hence, it is clear that the feedback function Γ passes through a maximum near T_c.

When we consider the dependence of Γ on the electron density, we find from Eq. (16) that if $\mu \ll AS$, then Γ increases on increase in the density n proportionally to $n^{1/3}$. On the other hand, if $\mu \gg AS$, it follows from Eq. (32) that this qunatity decreases on increase in n as n^{-1}. Therefore, $\Gamma(T_c)$ considered as a function of n should have a maximum at $\mu \sim AS$. Extrapolation to such values n in Eqs. (16) and (32) provides an estimate of the maximum value of Γ, which is a quantity of the order of $\delta T_c/T_c$ [this is accurate to within a factor $\sim (W/AS)^{1/3}$]. This result is obtained on the assumption that $T_c < T_c$ and it shows that even on this assumption the quantity Γ may be comparable with unity. Therefore, in the vicinity of T_c the effective permittivity ε_{eff} can be several times smaller than ε_p attained at $T = 0$ or at $T \to \infty$ (the case when $\varepsilon_{\text{eff}} < 1$ is discussed in Ref. 1).

We shall now consider how precisely the reduction in ε_{eff} near T_c gives rise to a singularity of ρ. At relativity high electron densities n a peak of ρ can be explained by a simultaneous reduction in the mobility and density of carriers in the current-carrying states. In degenerate semiconductors the mobility is determined by the scattering of carriers on randomly distributed ionized donors the electric field of which is screened by carriers. Bearing in mind that in the unrenormalized field of defects $\Phi(\mathbf{q})$ the scattering potential deduced from Eq. (20) is $\Phi(\mathbf{q})/\tilde{\varepsilon}(\mathbf{q})$, we find that the relaxation time τ_k obtained in the Born approximation is described by a modified Brooks-Herring formula:

$$\tau_k^{-1} = \frac{2\pi e^2 n m^*}{k^3 \varepsilon_{eff}^2}\left\{\ln(1+\tilde{\eta}) - \frac{\tilde{\eta}}{1+\tilde{\eta}}\right\}, \quad \tilde{\eta} = \frac{4k^2\varepsilon_{eff}}{\varkappa^2}. \quad (35)$$

It is clear from Eq. (35) that the minimum of ε_{eff} near T_c gives rise to a mobility minimum in the same range of temperatures.

On the other hand, the random distribution of the impurities gives rise to density-of-states tails in the energy spectrum of a degenerate semiconductor. Electrons in these states do not carry the current. The density of levels in the tails decreases exponentially on increase in the separation from the bottom of the conduction band $\mathscr{E}_c$ and the exponential law is a generalization of the familair relationship for nonmagnetic semiconductors (see, for example, Ref. 1):

$$g(\mathscr{E}) \sim \exp\{-(\mathscr{E} - \mathscr{E}_c)^2/\mathscr{E}_f^2\}, \quad \mathscr{E}_f \sim \frac{e^2}{\varepsilon_{eff}} n^{1/3}\left(\frac{n^{-1/3}\varepsilon_{eff}}{me^2}\right)^{4/13}. \quad (36)$$

Therefore, the number of the current-free states passes through a maximum at T_c. Since these states lie below the current-carrying states, it follows that for a given total electron density in the conduction band the number of electrons in the current-carrying states passes through a minimum at T_c. In other words, at T_c the gap between the mobility edge and the Fermi level is minimal.

As n descreases, the number of conduction electrons in the current-carrying states decreases and finally the Fermi level shifts below the mobitlity edge, i.e., a crystal becomes an insulator. In nonmagnetic semiconductors the type of

conduction is independent of temperature. In ferromagnetic semiconductors the temperature dependence of ε_{eff} may give rise to a situation in which the Fermi level at $T = 0$ lies above the mobility edge, but drops below it on increase in T. This implies a transition from the metallic to the insulating state.

A metal-insulator transition may result not only from the temperature-dependent Anderson localization described above, but also from the Mott localization of carriers.

Roughly speaking the transfer of electrons from delocalized to localized states which then occurs can be explained as follows. Electron delocalization at $T = 0$ is due to the fact that the electrostatic potential of a donor, which is screened by delocalized electrons, is insufficiently strong to capture an electron. However, at finite temperatures an electron is attracted to a donor also by the exchange forces. The origin of these forces is as follows: since near a donor the electron density is higher, it follows that the local magnetization $\mathscr{M}_L$ is also higher compared with the average value $\mathscr{M}_0$. Consequently, the electrostatic potential must be enhanced by the additional exchange potential $-A(\mathscr{M}_L - \mathscr{M}_0)/2$. The difference $\mathscr{M}_L - \mathscr{M}_0$ rises on increase in T. Therefore, beginning from a certain temperature the total potential of the electrostatic and exchange forces is sufficient for the localization of an elelctron at a donor. A criterion of the metal-insulator transition in a ferromagnetic semiconductor may be expressed in the form of a relationship which is a natural generalization of the usual Mott criterion:

$$\tilde{a}_B n^{1/3} \approx 0.25, \quad \tilde{a}_B = \varepsilon_{eff}/me^2. \tag{37}$$

This analysis refines the earlier theory of such a transition given in Ref. 1. In particular, it follows from Eq. (37) that the maximum electron density at which an insulator-metal transition is still possible is 8 times higher than the density n_m beginning from which electron delocalization occurs at $T = 0$. This result is readily obtained if, in accordance with Eq. (16), we make the substitution $\Gamma(T_c) \propto n^{1/3}$. The same dependence of Γ on n is obtained also in the spin-wave range.[1] Consequently, the carrier density is related to the temperature of a metal-insulator transition by

$$(n_m/n)^{1/3} = 1/2 + [1/4 - \Gamma(n_m, T)]^{1/2}. \tag{38}$$

According to Eq. (38), the highest temperature at which such a transition is possible, if it is lower than T_c, is found from the condition $\Gamma(n_m, T) = 0.25$ and hence we can deduce the above conclusion on the maximum carrier density n_L. If $\Gamma(n_m, T_c) < 0.25$, then n_L is even less. This result shows that a metal-insulator transition is possible only in a relatively narrow range of carrier densities and it is in qualitative agreement with the experimental data on degenerate EuO in which this transition is observed at $n \sim (1\text{–}2) \times 10^{19}$ cm^{-3}, but disappears already for $n = 3 \times 10^{19}$ cm^{-3} (Ref. 9).

[1)]Gurevich (see, for example, Ref. 3) has stated that doping of $CdCr_2Se_4$ with an acceptor or a donor impurity gives rise to Cr^{4+} or Cr^{2+} ions, respectivley, instead of the regular Cr^{3+} ions. However, the model of Gurevich is clearly unsatisfactory. First of all, as explained in detail in Ref. 1, even if carriers move between Cr ions, we should not speak of the coexistence of Cr ions of different valences in a crystal, but of the existence of all the Cr ions in a crystal in a state of mixed valence. Secondly, the ideas of Gurevich according to which holes in $CdCr_2Se_4$ move between magnetic Cr ions are in conflict with the observation that the hole mobility is very high and practically insensitive to the magnetization fluctuations.

[1]E. L. Nagaev, Physics of Magnetic Semiconductors, Mir, Moscow (1983).
[2]E. M. Zub, Author's Abstract of Thesis for Candidate's Degree [in Russian], Institute of Solid-State Physics and Semiconductors, Academy of Sciences of the Belorussian SSR, Minsk (1986).
[3]A. I. Baĭramov, A. G. Gurevich, I. M. Émiryan, and N. N. Parfenova, Fiz. Tverd. Tela (Leningrad) **19**, 2668 (1977) [Sov. Phys. Solid State **19**, 1563 (1977).
[4]P. G. de Gennes and J. Friedel, J. Phys. Chem. Solids **4**, 71 (1958).
[5]É. L. Nagaev, Zh. Eksp. Teor. Fiz. **90**, 652 (1986) [sov. Phys. JETP **63**, 379 (1986)].
[6]L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media*, 2nd ed., Pergamon Press, Oxford (1984).
[7]E. L. Nagaev and A. P. Grigin, Phys. Lett. A **38**, 469 (1972).
[8]Shang-Keng Ma, *Modern Theory of Critical Phenomena*, W. A. Benjamin, Reading, Mass. (1976).
[9]Y. Shapira, S. Foner, and T. B. Reed, Phys. Rev. B **8**, 2299 (1973).

Translated by A. Tybulewicz