Mesoscopic fluctuations in a superconductor-normal metal-superconductor junction

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Disordered-SNS-junction properties determined by mesoscopic fluctuations are considered. The rms deviations of the superconducting current and of the resistance of the SNS junction from their mean values with respect to realizations of a random impurity potential are calculated. It is shown that mesoscopic fluctuations can determine the superconducting current through the SNS junction if the average current is suppressed. This suppression occurs in the presence of a strong magnetic field or in the case when the normal metal is ferromagnetic. By varying the magnetic field that penetrates into the sample it is possible to reach a condition in which the SNS junction has several stationary states at a given current. The mesoscopic fluctuations of the SNS-junction resistance are discussed. It is shown that they determine the dependence of this resistance on the phase difference of the superconducting order parameter at the junction. The feasibility of experimentally observing the indicated phenomena is discussed.

I. INTRODUCTION AND PRINCIPAL RESULTS

It is known that in a disordered normal metal the phase of the electron wave function remains coherent over distances L_φ much larger than the mean free path l with respect to elastic collisions with impurities. The reason is that L_φ is determined by inelastic collisions of the electrons with phonons or with one another. Inelastic scattering at low temperatures, on the other hand, is much more effective than elastic electron-impurity scattering. Within the framework of classical kinetics, which is usually employed when l is much longer than the electron wavelength h/p_F, this large-scale phase coherence does not come into play. As a result, for example, the known Drude formula for the residual conductivity contains only l. On the other hand, weak-localization effects[1,2] are wholly determined by this phase memory.

This phase coherence determines also the properties of a disordered superconductor-normal metal-superconductor (SNS) junction. Consider the SNS junction shown in Fig. 1, whose order-parameter phases on the opposite superconducting banks are equal to χ_1 and χ_2. The sensitivity of the SNS-junction properties to the phase difference is due to Andreev reflection of the electron from the NS boundary.[3] In this reflection the electron is transformed into a hole and acquires a phase ($-\chi_{1,2}$). The hole, reflected from the NS boundary, is transformed into an electron. This is indeed the cause of the Josphson effect in an NSN junction.[4]

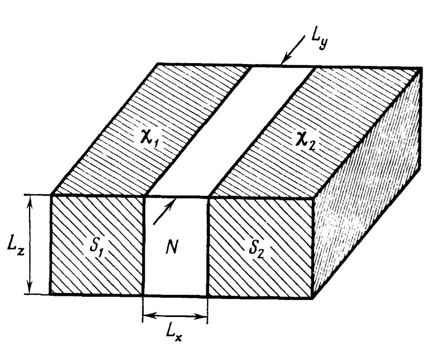

FIG. 1. SNS junction with normal-region dimensions $L_x \times L_y \times L_z$ and with order-parameter phases χ_1 and χ_2 on the opposite banks.

The existence of another phenomenon sensitive to changes of φ was demonstrated in Ref. 5. Namely, the SNS junction conductivity is also an oscillating function of φ with period π. The subject of Ref. 5 was the conductivity of the normal region of the SNS, averaged over realizations of a random potential, i.e., over a random disposition of the impurities. It was made clear recently that the properties of an actual sample can differ noticeably from their mean values[6–8] (see also Ref. 11 and the citations therein). For example, the conductance (the inverse of the resistance) of the sample fluctuations from sample to sample at a temperature $T=0$ by an amount on the order of e^2/h.[9,10]

We consider in the present paper the SNS-junction properties determined by a specific realization of a random potential in the N region (mesoscopic effects). We do not take into account interactions between the electrons in a normal metal.

The superconducting current of an SNS junction averaged over the samples equals at $T \ll \Delta$ [$\Delta\exp(i\chi_{1,2})$ is the superconducting order parameter]

$$\langle J_s\rangle=\langle J_c\rangle \sin\varphi, \tag{1}$$

$$\langle J_c\rangle=\frac{\Delta}{e}\langle G\rangle\begin{cases}1, & L_x\ll\xi_T,\\ \exp(-L_x/\xi_T), & L_x\gg\xi_T.\end{cases}$$

Here J_c is the critical current of the junction, $\langle \dots \rangle$ denotes averaging over the realizations of the random potential,

$$\xi_T=(2\pi)^{-1}(Dh/T)^{1/2}, \tag{2}$$

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D is the electron diffusion coefficient in the normal metal, and L_x is the distance between the superconductors of the SNS junction. We assume that the region is a rectangular parallelepiped with volume $V = L_x \times L_y \times L_z$ (see Fig. 1).

We calculate in the present paper the mean squared deviations of the superconducting current and the conductivity of the SNS junction from their mean values. It turns out that at $T \ll hD/2\pi L_x^2$, i.e., at $\xi_T \gg L_x$, we have for $\langle(\delta J_c)^2\rangle = \langle J_c^2\rangle - \langle J_c\rangle^2$

$$\langle(\delta J_c)^2\rangle = \left(\frac{eE_c}{h}\right)^2 \begin{cases} 15\pi^{-1}\zeta(5)L_yL_z/L_x^2, & L_x \ll L_y, L_z, \quad (3a) \\ 12\zeta(3), & L_x \gg L_y, L_z. \quad (3b) \end{cases}$$

If, however, $T \gg E_c$, i.e., $L_x \ll \xi_T$, we get

$$\langle(\delta J_c)^2\rangle = \left(\frac{4eT}{h}\right)^2 \exp\left(-\frac{2L_x}{\xi_T}\right) \times \begin{cases} (2\pi)^{5/2}L_x/\xi_T, & \xi_T \gg L_y, L_z, \\ 2\pi V/\xi_T^3, & \xi_T \ll L_y, L_z. \end{cases} \quad (4)$$

Here, $\zeta(x)$ is the Riemann zeta function: $\zeta(3) \approx 1.20$, $\zeta(5) \approx 1.04$.

It can be seen from (3) and (4) that if the transverse dimensions L_y and L_x of the normal region are small compared with its length L_z, the fluctuations of the critical current are determind only by the temperature and by the quantity $E_c = h/\tau$, where τ is the characteristic time of diffusion of the electron through the N region.

It follows from (1), (3), and (4) that the fluctuations δJ_c of the critical current are small compared with $\langle J_c\rangle$ relative to the parameter δ/Δ, where $\delta = (\nu V)^{-1}$ is the distance between the energy levels and ν is the density of states in the normal metal.

Equation (1), however, is valid in the absence of a magnetic field also if the N region does not contain localized spins that form a frozen structure (ferromagnetism or spin glass). It is known that these factors, by violating the invariance to time reversal, lead to an exponential decrease of $\langle J_c\rangle$. In this case ξ_T^{-1} in (1) is replaced by a quantity of the order of

$$\xi^{-1} = (\xi_T^{-2} + L_I^{-2} + L_H^{-2})^{1/2},$$

where $L_H = (hc/2eH)^{1/2}$ is the magnetic length and L_I is the length of coherence loss due to exchange interaction. For example, if the normal metal is ferromagnetic we have $L_I \sim (hD/T_C)^{1/2}$), where T_C is the Curie temperature.

A remarkable property of the fluctuations of J_c is that at small L_H or L_I ($\xi \ll \xi_T$) the value of $\langle(\delta J_c)^2\rangle$ is smaller by only a factor of 2 than (3) or (4), whereas $\langle J_c\rangle$ decreases exponentially. A situation can therefore be reached in which $\langle J_c\rangle^2 \ll \langle J_c^2\rangle$ and the entire critical current through the SNS junction is determined by the mesoscopic contribution. It is just this situation $\xi \ll \xi_T$ that we shall consider in the study of the critical current.

In principle, we can study the correlator of the superfluid currents at different values of the phase difference $\langle J_s(\varphi)J_s(\varphi')\rangle$ For example, in the case of a narrow junction $L_x \gg L_y, L_z, \min\{L_I, L_H^2/L_y\}$ we have

$$\langle J_s(\varphi)J_s(\varphi')\rangle = \left(\frac{e}{h}\right)^2 \times \begin{cases} 16(2\pi)^{5/2}\dfrac{L_x}{\xi_T}E_c^2\exp\left(-\dfrac{2L_x}{\xi_T}\right)\cos(\varphi-\varphi'), & T \gg E_c, \\ -E_c^2\dfrac{d^2}{d(\varphi-\varphi')^2}\displaystyle\int\dfrac{x^4\,\mathrm{sh}\,x\,dx}{\mathrm{ch}\,x-\cos(\varphi-\varphi')}, & T \ll E_c, \end{cases} \quad (5)$$

where $J_s(\varphi)$ is a periodic function with period 2π. In the interval $[0,2\pi]$ the function $J_s(\varphi)$ is random and different in different samples. It is important that at $L_x \gg \min\{L_I, L_H^2/L_y\}$ the value of $\langle J_s(\varphi)J_s(\varphi')\rangle$ depends only on $\varphi - \varphi'$. Therefore under these conditions, first, the extrema of $J_s(\varphi)$ in a given sample can have arbitrary probability of being located at any point in the interval $[0,2\pi]$. Second, $J_s(\varphi)$ has no parity whatever, in view of disruption of the T-invariance of the SNS junction by the magnetic field or by the magnetic structure.

In a study [10] of the dependence of the conductance G on the magnetic field H, an ergodicity hypothesis was advanced, according to which averaging over the samples (i.e., over the realization of a random potential, is equivalent to averaging over H, i.e.,

$$\langle G(H)G(H+\Delta H)\rangle = \lim_{H_0\to\infty}\frac{1}{H_0}\int_0^{H_0} G(H)G(H+\Delta H)\,dH. \quad (6)$$

This hypothesis was proved in Ref. 12 for metallic samples of sufficient size (such that fluctuations of the impurity density could be neglected).

Obviously, there is no relation similar to (6) for $J_s(\varphi)$. The point is that the nontrivial interval of variation of $\varphi - \varphi'$ is confined to $|\varphi - \varphi'| \leqslant 4\pi$. Obviously, using ergodicity with respect to the magnetic field, it is possible to replace in (5) the averaging over an ensemble of samples by averaging in the given sample over the field H directed along the z axis. By this procedure, investigation of $J_s(\varphi)$ in magnetic fields from zero to H is equivalent to study of H/H_c of the samples. Here $H_c = \Phi_0/L_yL_z$, and $\Phi_0 = hc/e$ is the magnetic-flux quantum.

At high temperatures $T \gg E_c$, the $J_s(\varphi)$ plot is, according to (5), a sinusoid with a random phase φ_0 determined by the disposition of the impurities in the sample:

$$J_s(\varphi) = J_c\sin(\varphi+\varphi_0). \quad (7)$$

The higher harmonics are exponentially small. This means that in the interval $[0,2\pi]$ the function $J_s(\varphi)$ has two zeros, one minimum, and one maximum.

At low temperatures $T \ll E_c$, the $J_s(\varphi)$ dependence contains also higher harmonics:

$$J(\varphi) = \sum_{m=-\infty}^{\infty} J_m e^{im\varphi}. \quad (8)$$

Calculations show that

$$\langle J_mJ_{m'}\rangle = \delta_{m,-m'}\left(\frac{e}{h}E_c\right)^2 \begin{cases} 24/m^3, & L_x \gg L_y, L_z, \\ 15/\pi m^5, & L_x \ll L_y, L_z. \end{cases} \quad (9)$$

It can be seen from (8) that the probability of having several minima of $J_s(\varphi)$ on the interval $[0,2\pi]$ is not exponentially small. By varying the magnetic field that passes through the given samples, it is possible to reach a situation wherein the equation $J_s(\varphi) = J$ has several solutions, so that the SNS junction has several stationary states at a given current through the junction. It is interesting that a system with a large number of such bound SNS junctions is equivalent to planar spin glass, in view of the random scatter of φ_0 in (7).

The effect described above can be qualitatively attributed to the fact that in disordered metallic samples of finite size the electronic energy levels are randomly distributed in energy space, and therefore the density of states on the Fermi level fluctuates from sample to sample. The change of the phase difference φ in the SNS junction leads to a change of the boundary conditions on the NS interface and to a random shift of the levels, by about $\delta E \sim E_c \delta\varphi$ (Ref. 13). This is accompanied by a change of the density of states on the Fermi level, and with it of the free energy of the SNS junction. Recognizing that the number of states in an energy interval of the order of E_c fluctuates by a value on the order of unity,[14,15] we arrive at Eq. (3b). At large L_y and L_z, each cube of side $\min\{L_x, \xi_T\}$ makes an independent additive contribution to $\langle(\delta J_c)^2\rangle$ and we obtain Eqs. (3a) and (4).

It was noted in Ref. 5 that at $L_x \gg \xi_T$, when the critical current of the usual Josephson effect is exponentially small, the oscillations of $G(\varphi)$ with change of φ are far from small and attenuate exponentially only over the phase-relaxation length $L_\varphi = (D\tau_\varphi)^{1/2}$, where τ_φ is the time of electron wave-function phase-relaxation loss due to inelastic collisions. In Ref. 5, however, was investigated only the quantity $\langle G(\varphi)\rangle$, whose oscillations had a period π. It is clear from the foregoing that in an actual sample $G(\varphi)$ is a periodic function of φ with period 2π.

A direct analogy exists between this phenomenon and the Aronov-Bohm effect. In Refs. 16 and 17 was measured the dependence of the conductance $G(\Phi)$ of a non-singly-connected conductor as on the magnetic flux Φ passing through an opening. It was observed that $G(\Phi)$ is a periodic function with a period $hc/2e$ in large samples[16] and hc/e in mesoscopic samples.[17] A transition between the two indicated regimes with change of the magnetic field was observed in Ref. 18.

The conductance of the normal region of an SNS junction can be represented in the form

$$G(\varphi) = \langle G\rangle + \sum_{m=-\infty}^{\infty} G_m e^{im\varphi}. \tag{10}$$

In this case $\langle G_m\rangle = 0$ and, as will be shown below, in the absence of an external magnetic field and of a magnetic structure ($\xi_T = \xi$) the following expression holds for the correlators $\langle G_m, G_{m'}\rangle$:

$$\langle G_m G_{m'}\rangle \propto \frac{E_c}{T}\left(\frac{e^2}{h}\right)^2 \exp\left(-2m\frac{L_x}{L_\varphi}\right) R_y R_z [\delta_{m,m'} + \delta_{m,-m'}], \tag{11}$$

where

$$R_{y,z} = \max\{1, L_{y,z}/(mL_xL_\varphi)^{1/2}, L_{y,z}/mL_x\}.$$

If a magnetic field $H > H_c$ is applied, the expression for $\langle G_m G_{m'}\rangle$ will differ from (11) by the absence of the first term in the square bracket, i.e., it will be proportional to $\delta_{m,-m'}$. In the next section we present a brief derivation of the expressions for the mesoscopic fluctuations of the superconducting current and for the conductance.

2. DERIVATION OF PRINCIPAL RESULTS

The superconducting current J_s is connected with the free energy Ω_s of the SNS junction by the relation

$$J_s = \frac{4\pi e}{h}\frac{d}{d\varphi}\Omega_s(\varphi). \tag{12}$$

To calculate the correlator $\langle\Omega_s(\varphi)\Omega_s(\varphi')\rangle$ we must sum the diagrams shown in Fig. 2. Important elements of these

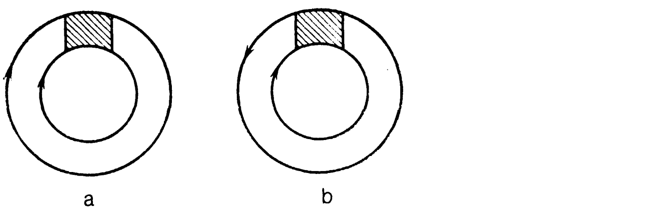

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FIG. 2. Diagrams for the calculation of the mean squared fluctuation of the thermodynamic potential. The diagram representation for the shaded rectangles is shown in Fig. 3.

diagrams are the so-called cooperons $P^C_{\varepsilon,\omega}$ (Fig. 3a) and diffusons $P^D_{\varepsilon,\omega}$ (Fig. 3b):

$$P^C_{\varepsilon,\omega} = \langle G_\varepsilon^R(\mathbf{r},\mathbf{r}') G^A_{\varepsilon+\omega}(\mathbf{r},\mathbf{r}')\rangle, \quad P^D_{\varepsilon,\omega} = \langle G_\varepsilon^R(\mathbf{r},\mathbf{r}') G^A_{\varepsilon+\omega}(\mathbf{r}',\mathbf{r})\rangle, \tag{13}$$

where $G^{R(A)}(\mathbf{r},\mathbf{r}')$ is the exact retarded (advanced) Green's function of the electron in the coordinate representation.

The diffusion P^D and the cooperon P^C are sums of ladder diagrams, and satisfy the equation

$$\left\{-i\omega + D(\partial_{C,D})^2 + \frac{1}{\tau_\varphi}\right\} P^{C,D}_{\varepsilon,\omega}(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}'). \tag{14}$$

Here

$$\partial_C = -i\partial/\partial\mathbf{r} - (2e/c)\mathbf{A}(\mathbf{r}), \quad \partial_D = -i\partial/\partial\mathbf{r}, \tag{15}$$

and $\mathbf{A}(\mathbf{r})$ is the vector potential of the external magnetic field. The triangles in Fig. 3 represent Andreev scattering of an electron from the NS boundary, whereby the energy ε goes over into $-\varepsilon$, and $\varepsilon + \omega$ into $-\varepsilon - \omega$. The difference between the phases of the electron wave functions on opposite sides of the triangle is equal to the phase χ of the order parameter of the superconductor.

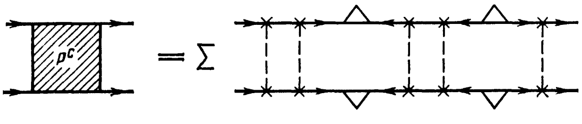
The boundary conditions for the diffuson and cooperon on the NS boundary are similar to those proposed in Ref. 5 and take the form

$$\left(\mathbf{n}\frac{\partial}{\partial\mathbf{r}}\right) P^{C,D}_{\varepsilon,\omega} = -\left(\mathbf{n}\frac{\partial}{\partial\mathbf{r}}\right) P^{C,D}_{-\varepsilon,-\omega}\exp(i\chi_{C,D}),$$

$$P^{C,D}_{\varepsilon,\omega} = P^{C,D}_{-\varepsilon,-\omega}\exp(i\chi_{C,D}), \tag{16}$$

where $\mathbf{n}$ is the normal to the NS boundary, and

$$\chi_C = \chi_{1,2} + \chi'_{1,2}, \quad \chi_D = \chi_{1,2} - \chi'_{1,2}. \tag{17}$$



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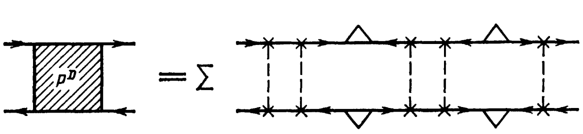

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FIG. 3. Diagram series for the calculation of a cooperon (a) and a diffuson (b) with allowance for Andreev reflection, corresponding to the triangles on the diagrams, of the electrons from the NS interface.

To calculate $\langle J_s(\varphi)/J_s(\varphi')\rangle$ we must know $P^{C,D}$ in the case when one of the electron lines in the ladder diagrams of Figs. 2 and 3 corresponds to the normal region of the SNS junction, with a phase difference equal to φ, and the other electron propagator is calculated at a phase difference equal to φ'. This is the reason why both the cooperon and the diffuson depend on the SNS junction phase difference: the cooperon depends on $\varphi_C=\varphi+\varphi'$ and the diffuson on $\varphi_D=\varphi-\varphi'$. Note that in calculations of the mean values $\langle G(\varphi)\rangle$ and $\langle J_s(\varphi)\rangle$ the P^C and the P^D appear only with identical phases $\varphi=\varphi'$. In this case only the cooperon P^C depends on φ and contributes to $\langle G(\varphi)\rangle$ and $\langle J_s(\varphi)\rangle$.[5,19] The boundary conditions (16) differ from those proposed in Ref. 6 just because $\varphi\neq\varphi'$.

It is simpler to carry out the calculation in the momentum representation, in which the diffuson takes the form

$$P^D_{\varepsilon,\omega}(\mathbf{q})=(-i\omega+Dq^2+1/\tau_\varphi)^{-1}, \qquad (18)$$

where $q^2=q_x^2+q_y^2+q_z^2$, and it follows from the boundary conditions (16) that

$$q_x=\frac{\pi}{L_x}\left(n_x+\frac{\varphi_D}{2\pi}\right),\quad q_y=\frac{\pi n_y}{L_y},\quad q_z=\frac{\pi n_z}{L_z},$$

$$n_{x,y,z}=0,\pm1,\pm2,\ldots. \qquad (19)$$

If there is no magnetic field, the cooperon takes likewise the form (18), and the quantization conditions for the cooperon momentum differ only in that φ_D is replaced by φ_C. For the strong magnetic field $H>H_c$ of interest to us, however, the contribution of cooperon-containing diagrams can be neglected.

An expression for the correlation function of superconducting currents, corresponding to the diagrams of Fig. 2, takes after integration with respect to ε the form

$$\langle J_s(\varphi)J_s(\varphi+\varphi_D)\rangle$$
$$=\frac{4T^3}{3\pi^2V^2}\left(\frac{e}{h}\right)^2\int_{-\infty}^{\infty}d\omega\, f\left(\frac{\omega}{T}\right)\frac{d^2}{d\varphi_D{}^2}\sum_{n_xn_yn_z}(Dq^2-i\omega)^{-2}, \qquad (20)$$

where

$$f(x)=(2\pi^2x-x^3)/(e^x-1).$$

Since the values significant in (20) are $\omega\sim T\gg\tau_\varphi^{-1}$, we have neglected τ_φ^{-1} in (18). Equation (20) leads directly to (3)–(5), (7), and (9).

It makes sense to consider the SNS-junction conductance fluctuations only if $T>E_c$, i.e., at $L_x>\xi_T$, otherwise the superconducting current will not be small compared with the normal one. At temperatures high compared with E_c one can disregard also the contribution made to $G(\varphi)$ by the fluctuation of the density of states. The correlator $\langle\delta G(\varphi)\delta G(\varphi')\rangle$ is thus entirely determined by the sum of diagrams of Fig. 4, which takes, as shown in Refs. 15 and 20, the form

$$\langle\delta G(\varphi)\delta G(\varphi')\rangle$$

$$=\left(\frac{2e^2}{h}E_c\right)^2\int_{-\infty}^{\infty}\frac{d\omega}{3T}\sum_{n_xn_yn_z}\left|Dq^2-i\omega+\frac{1}{\tau_\varphi}\right|^{-2}. \qquad (21)$$

Using the quantization conditions (19) we can write the expression for the correlator of the Fourier coefficients (10) in the form

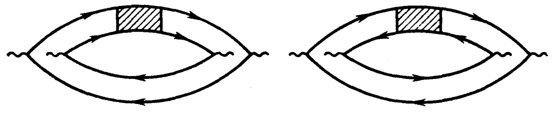

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FIG. 4. Diagram for the calculation of the rms fluctuation of an SNS junction conductance.

$$\langle G_mG_{m'}\rangle=\int_{-\pi}^{\pi}\frac{d\varphi\,d\varphi'}{(2\pi)^2}\exp(im\varphi+im'\varphi')\langle\delta G(\varphi)\delta G(\varphi')\rangle$$
$$=\delta_{m,-m'}\frac{4E_c}{3\pi^2T}\left(\frac{e^2}{h}\right)^2\int_{-\infty}^{\infty}du\int_{-\infty}^{\infty}\frac{d\varphi_D}{2\pi}\sum_{n_yn_z}\frac{\exp(im\varphi_D)}{[(\varphi_D/2\pi)^2+v^2]^2+u^2}, \qquad (22)$$

where

$$u=\omega/\pi^2E_c,\quad v^2=L_x^2(n_y^2/L_y^2+n_z^2/L_z^2+1/\pi^2L_\varphi^2). \qquad (23)$$

This yields readily

$$\langle G_mG_{m'}\rangle=\frac{2E_c}{3T}\left(\frac{e^2}{h}\right)^2\delta_{m,-m'}\sum_{n_yn_z}\frac{1}{v}e^{-2\pi mv}. \qquad (24)$$

To obtain Eq. (11), which is valid in a zero magnetic field and in the absence of a magnetic structure, we must take into account also the contribution of cooperon-containing diagrams. This contribution differs from (24) only in that $\delta_{m,-m'}$ is replaced by $\delta_{m,m'}$, a replacement necessitated by the difference between φ_D and φ_C.

3. FEASIBILITY OF EXPERIMENTAL OBSERVATION

As already noted, observation of mesoscopic effects in a study of the superconducting current of an SNS junction calls for satisfaction of the condition

$$L_x\gg\min\{L_I,\ L_H^2/L_z\}.$$

In addition, as seen from (4) and (5), to keep the mesoscopic critical current from being exponentially small it is necessary to meet the condition $L_x\lesssim\xi_T$. It is necessary also that the external magnetic field be less than the critical magnetic field of the superconductor.

We shall use for the numerical estimates the parameters of the samples investigated in Ref. 17 and 21. Assuming $L_x\approx L_y\approx L_z\approx10^{-4}$ cm and $D\sim10^2$ cm^2/s, we find that the mesoscopic contribution to the superconducting current becomes decisive at $H>H_c\sim10^2$ Oe. (The samples investigated in Refs. 17 and 21 had $L_x\sim10^{-4}$ cm and $L_y\sim L_z\approx4\cdot10^{-6}$ cm. In this case H_c increases to $\sim10^3$ Oe). If $T\lesssim E_c\approx0.1$ K, we obtain $J_c\sim10^{-8}-10^{-9}$ A, which in our opinion can be measured.

Another method of suppressing $\langle J_c\rangle$ is to use a superconductor-ferromagnetic normal metal-superconductor (SFS) junction. The condition under which the mesoscopic contribution determines J_c can be written in this case in the form $T_C>E_c$. The quantity $\langle J_c\rangle$ was studied in Ref. 19, where it was shown that if there is no interaction between the electrons, we have $\langle J_c\rangle\propto\exp(-L_x/L_I)$. Interaction between the electons in the normal region at $L_x<\xi_T$, L_H,

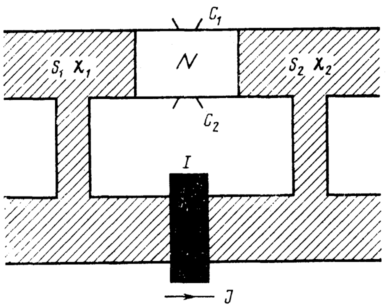

FIG. 5. Experimental set up for the study of the conductance of an SNS-junction on the order-parameter phase difference (I–ordinary Josephson junction carrying a current J) (C_1 and C_2 are point contacts)

however, leads to a value of $\langle J\rangle$ on the order of $\beta eE_c/h$, where $\beta\ll 1$ is a constant determined by the electron-electron interaction. Thus, in terms of the parameter $\beta^2\ll 1$ the quantity $\langle J_c\rangle^2$ is small compared with $\langle J_c^2\rangle$. In addition, as shown in Ref. 19, $\langle J_c\rangle$ is suppressed by the magnetic field and by spin-orbit scattering at $L_x>\min\{L_H,L_{SO}\}$

Methods of experimentally investigating the $G(\varphi)$ dependences were discussed in Ref. 5. For example, a voltage U can be applied to opposite banks of an SNS junction with $L_\varphi\gtrsim L_x\gg\xi_T$. In this case

$$d\varphi/dt=2\pi eU/h \tag{25}$$

and the normal current $J(t)=G[\varphi(t)]U$ flowing through the SNS junction is a periodic function of the time. This phenomenon is described by Eq. (25) only at $eU<E_c$,[5] when the period of the oscillations turns out to be longer than the time required for the electron to diffuse over a distance L_x. At large U, the oscillation amplitude $J(t)$ becomes strongly damped.

It should be noted that the nonstationary Josephson effect also contributes in this case to the current at a frequency $2\pi eU/h$. At $L_x>\xi_T$, however, this contribution is proportional to $\exp(-L_x/\xi_T)$. At the same time, $dG/d\varphi$ attenuates exponentially only over the phase-relaxation length $L_\varphi\gg\xi_T$:

$$dG/d\varphi\propto\exp(-L_x/L_\varphi).$$

The usual Josephson effect can therefore be neglected in the region $L_x>\xi_T$ and the entire current of frequency $2\pi eU/h$ is determined by the dependence of G on $\varphi(t)$.

There is also another method of experimentally investigating the $G(\varphi)$ dependence.[5] The experimental setup is shown in Fig. 5. A superconducting current J is made to flow through the Josephson junction I and produces between the banks of the SNS junction a phase difference equal to $\varphi=\arcsin(J/J_c^0)$, where J_c^0 is the critical current of the Josephson junction. What is to be measured is the resistance of the point junction C_1 to spreading, or the resistance between the two point junctions C_1 and C_2. These resistances are oscillating functions of φ. In analogy with Refs. 8 and 10, the resistance of such a junction has a nonmonotonic dependence also on an external magnetic field.

If the normal metal is a spin glass and the electrons are scattered by localized frozen spins, one can expect an analogy with Ref. 22 low-frequency noise of the critical current, due to the long-time relaxations in the spin structure.

We note in conclusion that samples suitable for the study of the effects considered above were already used in Ref. 21, where the material of the current junctions to the mesoscopic conductors was superconducting.

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