Charge relaxation in an anisotropic medium and in low-dimensional media

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The nature of charge relaxation in a three-dimensional anisotropic medium, and also in two- and one-dimensional cases, differs radically from the Maxwellian law. The process of relaxation in an anisotropic medium creates charges in regions which are initially neutral and the density of such charges changes sign as a function of position, falling off with the distance as r^{-3}. Uniform expansion of the charged region, which maintains a constant charge, occurs in a thin conducting film. In the case of a thin filament the charged region spreads in accordance with a diffusion law. A similar law applies to relaxation of a charge in a conducting channel of a metal-insulator–semiconductor structure.

1. INTRODUCTION

It is well known that relaxation of the density of a charge $\rho(\mathbf{r},t)$ in a homogeneous isotropic medium occurs in accordance with the law

$$\rho(\mathbf{r}, t)=\rho_0(\mathbf{r})\exp(-t/\tau_M), \quad \tau_M=\varepsilon/4\pi\sigma, \tag{1}$$

where τ_M is the Maxwellian relaxation time, ε is the permittivity, and σ is the electrical conductivity. We shall consider charge relaxation in a homogeneous anisotropic medium and also in low-dimensional media such as a thin film, a conducting channel in a metal-insulator–semiconductor (MIS) structure, and a thin filament. We shall show that in all these cases the nature of charge relaxation differs radically from the Maxwellian law, Eq. (1). In the case of a three-dimensional anisotropic medium the most important difference is the appearance of charges in regions which are initially neutral. The charge density changes sign as a function of position and decreases quite slowly (as r^{-3}) with the distance.

The case when one of the principal values of the electrical conductivity tensor is negative and the other two are positive is of special interest (a situation of this kind occurs in ruby crystals under photoelectric instability conditions[1,2]). In this case the initial charge inhomogeneity is tranformed into a structure with charge densities of both signs in which the absolute value of the charge density increases with time. Similar results are obtained as a result of growth of a local charge fluctuation in a medium with a negative differential conductance which is created in a strong electric field (for example, under Gunn effect conditions). If we adopt a system of coordinates moving at the average carrier drift velocity, we find that the description of the linear stage of the growth of an initial charge fluctuation reduces to the same problem: one of the principal values of the effective tensor of the electrical conductivity is negative (this is the component of the tensor along the field), whereas the other two are positive.

In the case of two- and one-dimensional conducting media we find that, in contrast to three-dimensional media, there is no characteristic charge relaxation time. In fact, a film of thickness d is characterized by a two-dimensional conductivity $\sigma_s = \sigma d$, which has the dimensions of velocity, and a filament with a cross-sectional area S has a one-dimensional conductivity $\sigma_l = \sigma S$, which has the dimensions of

the diffusion coefficient. Consequently, in the two-dimensional case a charged region expands uniformly, whereas in one-dimensional media the expansion is in accordance with a diffusion law, but the total charge is conserved during such an expansion. [According to Eq. (1) the charge in a three-dimensional body passes entirely from the interior to the boundaries of the conductor.] Spreading of a charge fluctuation in a conducting channel of an MIS structure will be shown to occur also in accordance with a diffusion law.

In all the situations under consideration a redistribution of the charge by ordinary diffusion is unimportant if the Debye radius is considerably less than all the characteristic scales of the problem. We shall assume that this condition is satisfied and ignore the diffusion current in all the cases considered below.

A similar problem of dynamic screening of the electron-electron interaction was solved in Refs. 3–5 for two- and one-dimensional cases in connection with calculation of the density of states in disordered systems. The process of charge relaxation was not discussed.

2. THREE-DIMENSIONAL ANISOTROPIC MEDIUM

In this case the process of charge relaxation is described by the equations

$$\partial\rho/\partial t+\operatorname{div}\mathbf{j}=0, \quad j_i=-\sigma_{ij}\partial\varphi/\partial x_j, \tag{2}$$

$$\operatorname{div}\mathbf{D}=4\pi\rho, \quad D_i=-\varepsilon_{ij}\partial\varphi/\partial x_j, \tag{3}$$

where ρ and $\mathbf{j}$ are the densities of the charge and current; $\mathbf{D}$ is the electric induction; φ is the potential; σ_{ij} and ε_{ij} are the electrical conductivity and permittivity tensors.

The qualitative features of charge relaxation in an anisotropic medium indicated in the Introduction arise because in such a medium (in contrast to the isotropic case) it does not follow from $\operatorname{div}\mathbf{D} = 0$ that $\operatorname{div}\mathbf{j} = 0$. Therefore, even at early times the derivative $\partial\rho/\partial t$ differs from zero in the regions where $\rho = 0$. Therefore, charge appears in all space.

This circumstance is related to another property of an anisotropic medium described in Ref. 6: under conditions of steady-state flow of the current ($\operatorname{div}\mathbf{j} = 0$) a space charge ($\operatorname{div}\mathbf{D}\neq0$) can exist in a homogeneous anisotropic conductor.

We shall now solve the system of equations (2) and (3).

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In view of the linearity of the problem, it is sufficient to study charge relaxation for a point initial distribution $\rho_0(\mathbf{r}) = \delta(\mathbf{r})$. For simplicity we shall first assume that $\varepsilon_{ij} = \delta_{ij}$ and consider the case of uniaxial symmetry when the tensor σ_{ij} has the principal values $\sigma_{xx} = \sigma_{yy} = \sigma_\perp$, $\sigma_{zz} = \sigma_\parallel$. The general case will be described at the end of the present section.

We shall calculate $\partial \rho / \partial t$ at $t = 0$. Assuming that $\varphi = 1/r$, we find from Eq. (2) that

$$\left(\frac{\partial \rho}{\partial t}\right)_{t=0} = -4\pi \frac{2\sigma_\perp + \sigma_\parallel}{3}\delta(\mathbf{r}) + (\sigma_\perp - \sigma_\parallel)\frac{1-3\cos^2\theta}{r^3}, \quad (4)$$

where θ is the angle between the $\mathbf{r}$ vector and the z axis. Therefore, in addition to a reduction in the initial charge at $\mathbf{r} = 0$, charges appear in all space and the sign of these charges depends on the angle θ and on the relationship between $\sigma_\perp$ and $\sigma_\parallel$ (see Figs. 1a and 2a).

The appearance of charges opposite in sign to the initial positive charge is easiest to explain in the case when $\sigma_\perp \ll \sigma_\parallel$. When this condition is obeyed the lines of flow of the current are everywhere parallel to the z axis and directed along this axis if $z > 0$ and opposite to this axis if $z < 0$. Hence, it is clear that a negative charge should accumulate near the $z = 0$ plane, as predicted by Eq. (4). We shall now find the time dependence of the charge density. The solution of Eqs. (2) and (3) gives the following expression for the potential φ:

$$\varphi(\mathbf{r}, t) = 4\pi \int \frac{d^3\mathbf{k}}{(2\pi)^3 k^2}\exp[i\mathbf{kr} - 4\pi t\sigma_{ij}k_i k_j/k^2]. \quad (5)$$

We shall calculate the integral assuming that the vector $\mathbf{r}$ is the polar axis. The integral of the modulus of $\mathbf{k}$ gives $(\pi/r)\delta(\cos\beta)$, where β is the angle between $\mathbf{k}$ and $\mathbf{r}$. After integration with respect to this angle, we obtain
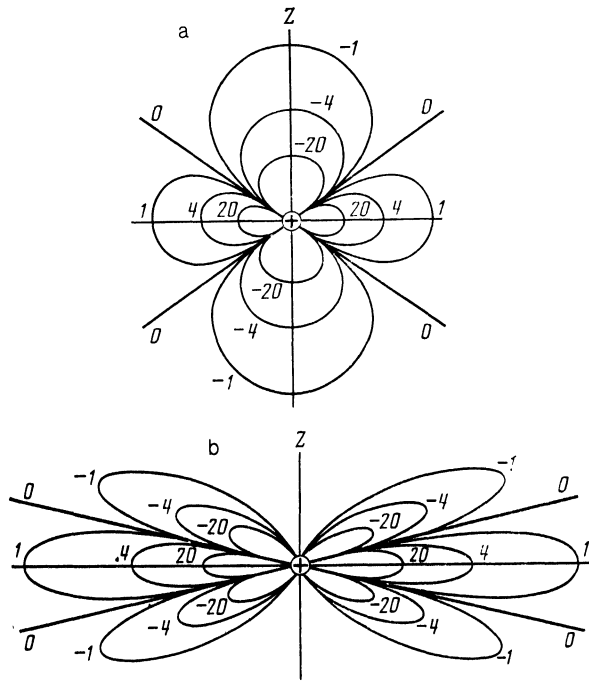

FIG. 1. Distribution of the charge density in the case $\sigma_\perp > \sigma_\parallel$. The figure shows lines of constant charge density. The numbers give the charge density in dimensionless units (which are different for Figs. 1a and 1b): a) for short times; b) for long times ($|\tau| = 10$).
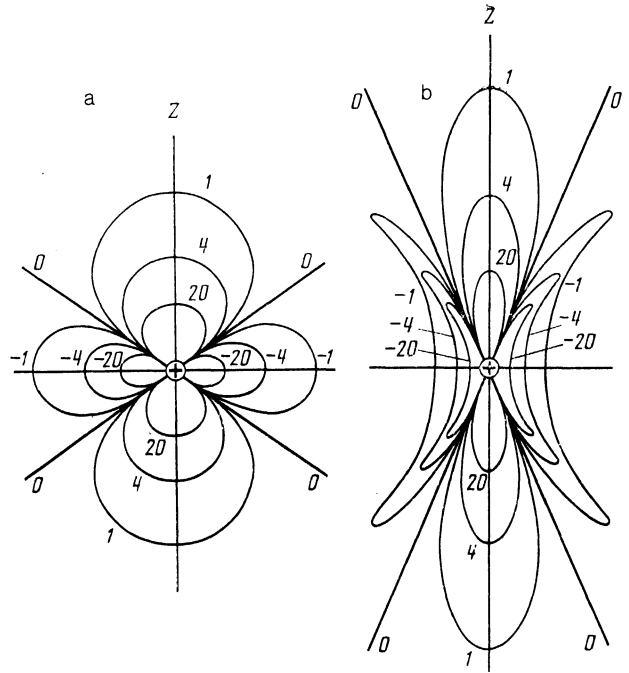

FIG. 2. Same as in Fig. 1, but for $\sigma_\perp < \sigma_\parallel$.

$$\varphi(\mathbf{r}, t) = \frac{1}{r}\int_0^{2\pi}\frac{d\psi}{2\pi}\exp(-4\pi t\sigma_{ij}\varkappa_i \varkappa_j), \quad (6)$$

where $\varkappa$ is a unit vector lying in a plane perpendicular to $\mathbf{r}$ and integration is carried out with respect to the azimuthal angle in this plane.

Under uniaxial symmetry conditions, we have $\sigma_{ij}\varkappa_i \varkappa_j = \sigma_\perp \sin^2\theta_\varkappa + \sigma_\parallel \cos^2\theta_\varkappa$, where $\theta_\varkappa$ is the angle between the vector $\varkappa$ and the z axis. If the azimuthal angle ψ is measured from the line of intersection of a plane passing through the z axis and through the vector $\mathbf{r}$ by a plane perpendicular to $\mathbf{r}$, then $\cos\theta_\varkappa = \sin\theta\cos\psi$. Here, θ is the angle between the vector $\mathbf{r}$ and the z axis.

Integration with respect to the angle ψ gives the following final result

$$\varphi(\mathbf{r}, t) = \frac{1}{r}\exp(-4\pi\sigma_\perp t)f\left(\frac{\tau}{2}\sin^2\theta\right), \quad (7)$$

$$f(u) = e^u I_0(u), \quad \tau = 4\pi t(\sigma_\perp - \sigma_\parallel). \quad (8)$$

Here, I_0 is a modified zeroth-order Bessel function.

We shall also represent the potential in the form of an expansion in terms of Legendre polynomials $P_l(\cos\theta)$ (obviously, this expansion contains only the polynomials with even values of l):

$$\varphi(\mathbf{r}, t) = \frac{\exp(-4\pi\sigma_\perp t)}{r}\sum_{n=0}^{\infty}C_n(\tau)P_{2n}(\cos\theta). \quad (9)$$

The expansion coefficients are described by the expression

$$C_n(\tau) = (4n+1)P_{2n}(0)\int_0^1 d\xi\, P_{2n}(\xi)\exp(\tau\xi^2). \quad (10)$$

We can now calculate the charge density $\rho = -\Delta\varphi/4\pi$. It follows from Eqs. (7)–(10) that

$$\rho(\mathbf{r}, t) = \exp(-4\pi\sigma_\perp t)[C_0(\tau)\delta(\mathbf{r}) + F(\tau, \theta)/4\pi r^3], \quad (11)$$

where

$$C_0(\tau) = \int\limits_0^1 d\xi \exp(\tau\xi^2), \qquad (12)$$

$$F(\tau, \theta) = -\left(\frac{\partial^2}{\partial\theta^2} + \operatorname{ctg}\theta\frac{\partial}{\partial\theta}\right) f\left(\frac{\tau}{2}\sin^2\theta\right). \qquad (13)$$

The function F can be expanded in terms of Legendre polynomials using Eq. (9):

$$F(\tau, \theta) = \sum_{n=1}^{\infty} 2n(2n+1)C_n(\tau)P_{2n}(\cos\theta). \qquad (14)$$

We can thus see that an initial point charge transforms in the course of its relaxation in accordance with Eq. (11). Firstly, there is a nonexponential change in the magnitude of the point charge. It can be seen from Eqs. (11) and (12) that at early times the change is described by Eq. (4) and it is governed by the average electrical conductivity $(2\sigma_\perp + \sigma_\parallel)/3$. After a long time the coefficient in front of the δ function in Eq. (11) becomes $(2\tau)^{-1}\exp(-4\pi\sigma_\parallel t)$ if $\sigma_\perp > \sigma_\parallel$ and $(\pi/4|\tau|)^{1/2}\exp(-4\pi\sigma_\perp t)$ if $\sigma_\perp < \sigma_\parallel$, i.e., the relaxation process is governed by the smallest of the principal values of the tensor $\bar{\sigma}$. It should be pointed out that for $\sigma_\parallel < 0$ and $|\sigma_\parallel| < 2\sigma_\perp$ the magnitude of the point charge varies nonmonotonically: it first decreases and then rises.

Second, throughout the space surrounding the initial point charge there appear charges of density which decreases with distance as r^{-3} and with an angular distribution which is time-dependent and described by the function $F(\tau,\theta)$. The integral of this function with respect to the angles vanishes. For this reason the quantity $C_0(\tau) \times \exp(-4\pi\sigma_\perp t)$ represents the total charge contained in a sphere of arbitrary radius. However, if we wish to consider the total charge in an arbitrary region surrounding the point $\mathbf{r} = 0$, then its magnitude and sign depend on the shape of this region, no matter how far the boundaries of the region are located from the position of the initial point charge. Moreover, the absolute value of the charge in such a nonspherical region increases logarithmically for a similar increase in its dimensions.

For short times $|\tau| \ll 1$ we have $F(\tau,\theta) = -2\tau P_2(\cos\theta)$, which is in agreement with Eq. (4). At late times $|\tau| \gg 1$ the asymptotic form of the function F depends strongly on the relationship between $\sigma_\perp$ and $\sigma_\parallel$. If $\sigma_\perp > \sigma_\parallel$, then throughout the range of angles θ with the exception of small intervals near $\theta = 0$ and $\theta = \pi/2$, it follows from Eqs. (8) and (13) that

$$F(\tau, \theta) = -4\pi^{-1/2}\tau^{3/2}\sin\theta\cos^2\theta\exp(\tau\sin^2\theta),$$
$$\theta, |\theta - \pi/2| \gg \tau^{-1/2}. \qquad (15)$$

Hence, we can see that after a long time the bulk of the charge is located near the $\theta = \pi/2$ plane. In this region we have

$$F(\tau, \theta) = -4\pi^{-1/2}\tau^{7/2}e^\tau(1/2\tau - \alpha^2)e^{-\alpha^2\tau}, \qquad (16)$$

where $\alpha = \pi/2 - \theta$ and its value obeys $|\alpha| \ll 1$.

It is therefore clear that if $\tau \gg 1$, then the positive charge is concentrated near the $\theta = \pi/2$ plane in a narrow region $|\alpha| < (2\tau)^{-1/2}$. Elsewhere in space the charge is negative and its density decreases exponentially for $|\alpha| \gg \tau^{-1/2}$ (Fig. 1b).

If $\sigma_\perp < \sigma_\parallel$, then everywhere with the exception of small angles we have

$$F(\tau, \theta) = -(\pi|\tau|)^{-1/2}\sin^{-3}\theta \quad \text{for} \quad \theta \gg |\tau|^{-1/2}. \qquad (17)$$

However, for small angles $\theta \ll 1$, we find that

$$F(\tau, \theta) = 2|\tau|e^{-\eta}[(1-2\eta)I_0(\eta) + 2\eta I_1(\eta)], \quad \eta = |\tau|\theta^2/2. \qquad (18)$$

The function F vanishes at $\eta \approx 0.8$. Therefore, the positive charge is concentrated in a narrow cone $\theta < 1.3|\tau|^{-1/2}$. Outside this cone the charge is negative (Fig. 2b).

We shall now consider the general case of an anisotropic medium of arbitrary symmetry assuming however that the principal axes of the tensors $\hat{\sigma}$ and $\hat{\varepsilon}$ coincide. We shall introduce a vector $\mathbf{r'}$ in terms of the principal axes, defined by the components

$$x' = x\varepsilon_1^{-1/2}, \quad y' = y\varepsilon_2^{-1/2}, \quad z' = z\varepsilon_3^{-1/2},$$

where ε_1, ε_2, and ε_3 are the principal values of the tensor $\hat{\varepsilon}$. Then, the potential φ is no longer described by Eq. (5), but by

$$\varphi(\mathbf{r'}, t) = \frac{4\pi}{(\varepsilon_1\varepsilon_2\varepsilon_3)^{1/2}}\int\frac{d^3k}{(2\pi)^3 k^2}\exp[i\mathbf{kr'} - 4\pi t\tilde{\sigma}_{ij}k_ik_j/k^2], \qquad (19)$$

where $\tilde{\sigma}_{ij} = \sigma_{in}\varepsilon_{nj}^{-1}$. Calculation of the integral in Eq. (19) gives

$$\varphi(\mathbf{r'}, t) = \frac{\exp(-4\pi At)}{(\varepsilon_1\varepsilon_2\varepsilon_3)^{1/2}r'}I_0[4\pi t(A^2 - B^2)^{1/2}], \qquad (20)$$

where the invariants A and B are defined by the formulas

$$A = \tfrac{1}{2}(\delta_{ij} - n_i'n_j')\tilde{\sigma}_{ij},$$
$$B = (\tfrac{1}{2}\delta_{ij} - n_i'n_j')(\tilde{\sigma}_{ij}\tilde{\sigma}_{kk} - \tilde{\sigma}_{ik}\tilde{\sigma}_{kj}). \qquad (21)$$

Here, $\mathbf{n'}$ is a unit vector along $\mathbf{r'}$. The charge density $\rho(\mathbf{r'},t)$ can be found from Eq. (20): $\rho(\mathbf{r'},t) = -(\Delta'\varphi)/4\pi$, where the differentiation is carried out with respect to the components of the vector $\mathbf{r'}$. We can adopt the coordinates $\mathbf{r}$ by making the following substitutions in Eqs. (20) and (21) ($\mathbf{n}$ is a unit vector along $\mathbf{r}$):

$$n_i'n_j' \to \varepsilon_{ik}^{-1}n_kn_j/\varepsilon_{lk}^{-1}n_kn_l, \quad r' = (\varepsilon_{ik}^{-1}x_ix_k)^{1/2}. \qquad (22)$$

Equations (20)–(22) represent the general solution of the problem formulated above, written in a form which is independent of the choice of coordinate axes. In the case of axial symmetry, we have

$$A = \tilde{\sigma}_\perp - \tfrac{1}{2}(\tilde{\sigma}_\perp - \tilde{\sigma}_\parallel)\sin^2\theta',$$
$$B = \tilde{\sigma}_\perp^2 - \tilde{\sigma}_\perp(\tilde{\sigma}_\perp - \tilde{\sigma}_\parallel)\sin^2\theta', \qquad (23)$$

where $\tilde{\sigma}_\perp = \sigma_\perp/\varepsilon_\perp$, $\tilde{\sigma}_\parallel = \sigma_\parallel/\varepsilon_\parallel$, θ' is the angle between the vector $\mathbf{r'}$ and the z axis, and

$$\sin^2\theta' = \frac{\sin^2\theta}{\sin^2\theta + (\varepsilon_\perp/\varepsilon_\parallel)\cos^2\theta}. \qquad (24)$$

The expressions obtained earlier for the potential φ and the charge density ρ remain valid if we add the factor $(\varepsilon_1^2\varepsilon_\parallel)^{-1/2}$ and make the substitutions $r \to r'$, $\theta \to \theta'$, $\sigma_\perp \to \tilde{\sigma}_\perp$, $\sigma_\parallel \to \tilde{\sigma}_\parallel$. The qualitative features of the relaxation process discussed above are still retained.

In the special case when the principal values of the tensor $\hat{\sigma}$ are proportional to the corresponding principal values of $\hat{\varepsilon}$, we find that $\hat{\sigma}_{ij} \propto \delta_{ij}$ and the process of charge relaxation is described by Eq. (1).

3. TWO-DIMENSIONAL CASE (THIN FILM AND MIS STRUCTURE)

We shall now discuss charge relaxation in a thin film with a two-dimensional conductivity $\sigma_s = \sigma d$, where d is the film thickness and σ is the bulk conductivity, which we shall assume to be isotropic. The ambient medium is an insulator of permittivity ε. The actual value of the permittivity of the film is unimportant. The film thickness d is assumed to be small compared with the characteristic dimensions describing the charge distribution on the film surface.

The nature of relaxation can be understood qualitatively on the basis of the following considerations. At large distances from the initial charge the field and the current density decrease as r^{-2}. Therefore, the total current flowing through a circle of radius r decreases as $1/r$ in the limit $r \rightarrow \infty$. Consequently, the total charge in the film is conserved, so that the process of charge relaxation is accompanied by expansion of the charged region. (This is in contrast to the case of a three-dimensional isotropic medium when the total current across the surface of the sphere is independent of the radius, so that the total charge in the interior decreases with time.) It follows from considerations of dimensionality that the rate of expansion is of the order of σ_s.

The equations describing the spread of the charge in the film are as follows:

$$\frac{\partial \rho_s}{\partial t} - \sigma_s \Delta_2 \varphi = 0, \tag{25}$$

$$\varphi(\mathbf{r}) = \frac{1}{\varepsilon} \int \frac{d^2 r' \, \rho_s(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}, \tag{26}$$

where ρ_s is the surface charge density; $\mathbf{r}$ and $\mathbf{r}'$ are two-dimensional vectors in the plane of the film; Δ_2 is the two-dimensional Laplace operator; $\varphi(\mathbf{r})$ is the potential in the film which is constant across the thickness, subject to the condition given above. The Fourier components ρ_{sk} and φ_k are described by the following expressions which follow from Eqs. (25) and (26):

$$\frac{\partial \rho_{sk}}{\partial t} + \sigma_s k^2 \varphi_k = 0, \qquad \varphi_k = \frac{2\pi}{\varepsilon k} \rho_{sk}. \tag{27}$$

Hence, we find that

$$\frac{\partial^2 \varphi_k}{\partial t^2} + v^2 k^2 \varphi_k = 0, \qquad v = \frac{2\pi \sigma_s}{\varepsilon} \tag{28}$$

or, in coordinate form,

$$\frac{1}{v^2} \frac{\partial^2 \varphi}{\partial t^2} + \Delta_2 \varphi = 0. \tag{29}$$

The same equation is clearly obeyed also by the surface charge density ρ_s.

When the initial charge distribution is described by $\rho_{s0} = \delta(\mathbf{r})$ and $\varphi(\mathbf{r}, 0) = 1/\varepsilon r$, the solution of Eq. (29) is

$$\varphi(\mathbf{r}, t) = \varepsilon^{-1} (r^2 + v^2 t^2)^{-1/2}. \tag{30}$$

The surface charge density ρ_s is then given by

$$\rho_s(\mathbf{r}, t) = \frac{1}{2\pi} \frac{v}{(r^2 + v^2 t^2)^{3/2}}. \tag{31}$$

These expressions are valid if $vt \gg d$. It follows from the above qualitative considerations that the charged region expands at a constant velocity v [Eq. (28)], where $\int d^2 r \rho_s(\mathbf{r}, t) = 1$.

It is interesting to note also that this charge distribution (31) is induced on a perfectly conducting surface by a unit negative point charge traveling away from the surface at a velocity v. Naturally, the results obtained are valid if the velocity v is considerably less than the velocity of light.

We shall now consider another two-dimensional problem of charge relaxation in an MIS structure. We shall assume that a constant voltage applied to this structure is such as to form a thin conducting (for example, inversion) layer of thickness d much less than the insulator thickness a.

Relaxation of a local charge density gradient in a conducting layer is described by Eq. (25). If there is a potential across this layer, we have to allow for the charge induced by such a gradient in the metal. If the surface charge density ρ_s changes little over distances of the order of a, the potential in the layer can be calculated from the formula for a parallel-plate capacitor $\varphi = 4\pi \rho_s a/\varepsilon$, where ε is the permittivity of the insulator. Substituting this expression in Eq. (25), we find that $\rho_s(r, t)$ is described by the following diffusion equation

$$\partial \rho_s / \partial t = D \Delta_2 \rho_s, \qquad D = 4\pi a \sigma_s / \varepsilon. \tag{32}$$

It thus follows that the initial charged region spreads in accordance with a diffusion law and the total charge is conserved.

We can easily show that Eq. (32) is valid if the region $r \ll (Dt)^{1/2} \ln(Dt/a)$, where the bulk of the charge is concentrated. At larger distances the parallel-plate capacitor approximation is invalid and we have $\varphi \propto r^{-3}$ and $\rho_s \propto \sigma_s t a^3 r^{-5}$.

4. ONE-DIMENSIONAL CASE (THIN FILAMENT)

We shall now discuss relaxation of a charge in a conducting filament with a one-dimensional conductivity $\sigma_l = \sigma S$, where S is the cross-sectional area. The filament thickness will be assumed to be small compared with a typical length of the charged region.

The process of charge relaxation is now described by

$$\frac{\partial \rho_l}{\partial t} - \sigma_l \frac{\partial^2 \varphi}{\partial z^2} = 0, \tag{33}$$

where ρ_l is the linear charge density, φ is the potential inside the filament which in the case of a thin filament is practically homogeneous over the cross section, and z is distance along the filament.

Obviously, at large distances from the charged region we have $\varphi = (\varepsilon |z|)^{-1}$, where ε is the permittivity of the medium surrounding the filament. Inside this region, we find from the formula for the filament capacitance that, to within logarithmic terms, we have $\varphi(z) = (1/\varepsilon)\rho_l(z)\Lambda$, where $\Lambda = \ln(b^2/S) \gg 1$ and b is the characteristic size of the distribution $\rho_l(z)$. Substituting this expression in Eq. (33), we obtain a one-dimensional diffusion equation with the diffusion coefficient $D = (2\sigma_l/\varepsilon)\Lambda$. If the initial condition $\rho_{l0}(z) = \delta(z)$ is satisfied, the solution of this equation is

$$\rho_l(z, t) = \frac{1}{(\pi D t)^{1/2}} \exp(-z^2/Dt), \qquad D = \frac{\sigma_l}{\varepsilon} \ln(\sigma_l t/S). \tag{34}$$

The expression for D, valid to lowest order, is obtained by replacing b in the expression for the logarithm with the characteristic scale $(\sigma_l t)^{1/2}$ of the distribution (34).

For large values of z it follows from Eq. (33) that

$$\rho_l(z, t) = 2\sigma_l t / \varepsilon |z|^3. \tag{35}$$

For given values of z and t we find that of the two equations (34) and (35) the one which applies is that which gives a larger value of ρ_l. Therefore, Eq. (34) is valid if $\exp(z^2/Dt) < \ln(\sigma_l t / S)$, i.e., it is valid in the main region where the charge density varies. Moreover, the condition $\sigma_l t \gg S$ must be satisfied.

We can thus see that in the one-dimensional case the process of charge relaxation occurs in accordance with a diffusion law and the effective diffusion coefficient increases slowly with time.

It should be noted that Eqs. (30), (32), and (34) which apply to the two- and one-dimensional cases can also be obtained from expressions for a dynamically screened potential of a point charge derived in the (ω, k) representation in Refs. 3–5.

We shall conclude by stating the conditions under which the neglect of the diffusion current is valid. Such neglect is justified if $l \gg (D_0 t)^{1/2}$ (l is the characteristic scale of the charge distribution, t is the characteristic relaxation time, and D_0 is the diffusion coefficient of carriers). In the three-dimensional case we have $t \propto \sigma^{-1}$ and this condition reduces to the requirement $l \gg r_0$, where r_0 is the screening radius. In the case of a thin film, bearing in mind that $t \propto l/v$ [see Eq. (31)], we have the condition $(ld)^{1/2} \gg r_0$. Finally, if charge relaxation occurs in accordance with a diffusion law, then the effective diffusion coefficient D [Eqs. (32) and (34)] is considerably greater than D_0. Hence, we obtain the condition $(ad)^{1/2} \gg r_0$ for a MIS structure and $d \gg r_0$ for a filament.

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