Effects of inclined magnetic fields on accumulation layers in degenerate InAs and semimetallic $Hg_{1-x}Cd_xTe$

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(Submitted 16 August, 1986)
Zh. Eksp. Teor. Fiz. **92**, 1061–1073 (March 1987)

Oscillations of the capacitance C of a surface space charge layer in degenerate n-type InAs and n-type $Hg_{1-x}Cd_xTe$ ($n = 10^{15}$–10^{18} cm^{-3}) were investigated in inclined magnetic fields **B**. In addition to oscillations of $C(B_\perp)$, governed by the component of the field $B_\perp = B_{\cos\alpha}$ normal to the surface and due to magnetic quantization of the orbital motion of electrons in two-dimensional subbands, oscillations of the capacitance $C_\parallel(B)$ were observed also in magnetic fields parallel to the surface and the periods of these oscillations were equal to the periods of the Shubnikov–de Haas oscillations in the original wafers. The phases γ of the $C(B_\perp)$ oscillations in InAs were independent of the orientation of B. In the case of $Hg_{1-x}Cd_xTe$ the phases obeyed $\gamma \propto (\cos\alpha)^{-1}$, but were independent of the number of the two-dimensional subbands and of the surface density of states, which made it possible to attribute the phase shift in $Hg_{1-x}Cd_xTe$ to the characteristics of the spin splitting and to use it to find the g factors. The $C_\parallel(B)$ oscillations, observed in the case of accumulation and weak depletion band bending, were retained for any orientation of **B** and the positions of the oscillation extrema did not change in that range of values of α where these oscillations could be separated from the background of the stronger $C(B_\perp)$ oscillations. The oscillation periods $C_\parallel(B)$ were independent of the surface band bending φ_s, whereas the phases in the intervals of φ_s between the successive edges of the two-dimensional subbands changed by 2π. It is postulated that the $C_\parallel(B)$ oscillations in degenerate semiconductors and semimetals are due to participation of the three-dimensional gas of electrons of the continuum in the screening of the surface field. No $C_\parallel(B)$ oscillations were observed in inversion layers.

1. INTRODUCTION

Experimental investigations of two-dimensional ($2D$) systems in inclined magnetic fields provide, on one hand, the most direct proof of the two-dimensional nature of the motion of carriers near interfaces and, on the other, are among the most widely used methods for the investigation of three-dimensional effects in real quasi-two-dimensional systems. The effects associated with the magnetic field component parallel to a two-dimensional layer have been investigated by the methods of tunnel spectroscopy,[1] cyclotron[2] and combined[3] resonances, and intersubband absorption.[4] However, the main attention has been on inversion or accumulation layers for which the Fermi level lies below the bottom of the conduction band in the bulk of a sample, i.e., below the upper edge of a surface potential well. In the case of accumulation layers on the surfaces of degenerate semiconductors, where we can expect three-dimensional effects associated with electrons in the continuum, the available data are limited to the case of parallel orientation of the magnetic field applied to weakly degenerate semiconductors InAs (Ref. 5), InSb (Ref. 2), and $Hg_{1-x}Cd_xTe$ (Ref. 6). Similar information on more heavily doped materials is lacking, mainly because of methodological difficulties, since even at bulk electron densities of $\sim 10^{15}$ cm^{-3} the surface channel conductance is shunted by the material in the bulk, at least in the case of narrow-gap materials in which the degeneracy effects are the most important.

We investigated the effects of inclined magnetic fields on accumulation layers in heavily doped samples of n-type InAs with $n = 10^{16}$–10^{18} cm^{-3} and of semimetallic n-$Hg_{1-x}Cd_xTe$ with $x < 0.12$ and $n = 10^{15} - 5\times 10^{16}$ cm^{-3} by a method[7] based on determination of the differential capacitance of the space charge region in metal-oxide-semiconductor (MOS) structures in quantizing magnetic fields (this method could be applied to samples with any degree of doping of the substrate). We selected these particular materials because of the high values of the Fermi energy in narrow-gap semiconductors and semimetals at moderate dopant concentrations and also because the occupancy of excited $2D$ subbands in such materials should result in a clearer manifestation of three-dimensional effects. The semiconductors selected, InAs and $Hg_{1-x}Cd_xTe$, are the limiting members of a series of popular narrow-gap materials and their comparative study can give useful information on the effects of nonquadratic dispersion laws.

2. PROPERTIES OF SAMPLES AND EXPERIMENTAL METHOD

The original wafers (with the properties listed in Table I) were used to prepare MOS structures with an anodic oxide ~ 1000 Å thick as the insulator and with a typical area of the gate amounting to $\sim 10^{-3}$ cm^2. Samples were mounted on a rotatable holder, which made it possible to set any orientation with respect to the axis of a superconducting solenoid. The angle α between the direction of the magnetic field and the normal to the interface was found from the emf induced in a flat measuring coil, which was parallel to the interface, when an alternating current was passed through a modulation coil of the solenoid. In other respects the experimental method was similar to that used in Ref. 7. Estimates indicated that the influence of the active component (due to the resistance of the bulk of the substrate) of the complex

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TABLE I.

	Sample no.	x	Bulk density $n\times10^{-16}$, cm^{-3}		Sample no.	x	Bulk density $n\times10^{-16}$, cm^{-3}
	1	0.125	0.35		4	—	2.3
$Hg_{1-x}Cd_xTe$	2	0.12	1.8	InAs	5	—	18
	3	0.11	5		6	—	160

conductivity could be ignored throughout the investigated range of voltages V applied to the gate. This was confirmed experimentally by the fact that the results of the capacitance measurements were independent of the test signal frequency (10^4–10^6 Hz) and by measurements of the Q factor.

3. PARAMETERS OF TWO-DIMENSIONAL SUBBANDS IN INCLINED MAGNETIC FIELDS

In the case of an ideal $2D$ gas the process of magnetic quantization of the orbital motion of electrons is governed solely by the magnetic field component $B_\perp = B\cos\alpha$ applied normal to the surface of a sample. In real structures the wave function is of finite length in the size quantization direction z and the energy spectrum is influenced by the magnetic field component $B_\parallel = B\sin\alpha$ parallel to the two-dimensional layer. Allowing for the spin splitting, governed by the modulus of **B**, we find that the energy levels in the ith subband in an inclined magnetic field are described approximately by[8]

$$E_{Ni}=E_i+(N_i+{}^1/_2)\hbar\omega_{\perp i}\mp|g_i|\mu_B B+{}^1/_2 m_{ci}(\omega_{\parallel i}r_i)^2, \qquad (1)$$

where E_i is the energy at the bottom of the ith $2D$ subband when $B=0$; $\omega_{\perp i} = eB\cos\alpha/m_{ci}c$; $\omega_{\parallel i} = eB\sin\alpha/m_{ci}c$; m_{ci} and g_i are the cyclotron mass and the g factor in the ith subband; $r_i = [(\bar{z}_i)^2 - \overline{(z_i^2)}]^{1/2}$ is the standard deviation of the zth coordinate of electrons in the ith subband. Equation (1) determines the position of the Nth oscillation maximum associated with the passage of the center of the Nth Landau level across the Fermi level E_F and can be written in a form more convenient for analysis:

$$N_i+\frac{1}{2}\mp\frac{|g_i|m_{ci}}{2m_0\cos\alpha}+\frac{eB_\perp \operatorname{tg}^2\alpha}{2c\hbar}\cdot r_i^2=\frac{(E_F-E_i)m_{ci}c}{e\hbar}\frac{1}{B_\perp}, \qquad (2)$$

where m_0 is the mass of a free electron. The nonquadratic nature of the dispersion law, which is important in the case of the investigated materials, may be allowed for if m_{ci} and g_i are considered as functions of the Fermi energy $E_F - E_i$ (Ref. 7). We can easily show that in the two-dimensional case of a nonparabolic band (in the two-band Kane approximation) the density-of-states effective mass $m_{\rho i}$ is equal to the cyclotron mass m_{ci}:

$$m_{\rho i}=m_{ci}=m_{ni}[1+2\beta_i(E_F-E_i)]$$

(m_{ni} is the effective mass at the bottom of the $2D$ subband and $\beta_i = E_{gi}^{-1}$ is the nonparabolicity parameter[7]). Consequently, the coefficient in front of $B_\perp^{-1}$ on the right-hand side of Eq. (2) is independent of the effective mass and amounts to $n_i c\pi\hbar/e$, where n_i is the surface density of carriers in the ith subband. It should be noted that under experimental conditions the gate voltage is kept constant, which is generally not equivalent to the constancy of n_i or of $E_F - E_i$. However, high Landau level numbers (or n_i) such a discrepancy does not result in a significant error.

The existence of a finite component $B_\parallel$ should, in accordance with Eq. (2), be manifested experimentally by a change in the phase factor γ in the dependence of the oscillation number N on $B_\perp^{-1}$: $N+\gamma = n_i c\pi\hbar/eB_\perp$. The changes in γ due to the diamagnetic shift [represented by the fourth term in Eq. (2)] are proportional to $B_\perp$, i.e., they are different for different Landau levels, which is equivalent to a departure from the periodicity of the oscillations of $C(B_\perp^{-1})$ in the case when $\alpha\neq 0$. Observations of a phase shift in oscillations of the conductivity of a two-dimensional gas at GaAs/$Ga_{0.7}Al_{0.3}As$ heterojunctions dependent on an inclined magnetic field were reported recently in Ref. 9. However, this is the only report of this kind so far and in our opinion it requires independent confirmation. This is due to the fact that the conclusion reached in Ref. 9 from comparing the oscillation curves for two orientations of the magnetic field $\alpha = 0°$ and $\alpha = 65°$ is easily shown to be incorrect if the actual values of α differ by just 0.5° from those just stated. Unfortunately, the error in the determination of the orientation of the magnetic field relative to the interface is not stated in Ref. 9.

Neither of the materials we investigated showed any manifestations of the diamagnetic shift in the investigated range of magnetic fields. It is clear from the oscillation curves $C(B_\perp^{-1})$ of sample 1 shown in Fig. 1 that the change in the phase of oscillations in $Hg_{1-x}Cd_xTe$ when α increased was not accompanied by any significant change in the periods, i.e., the phase shift was independent of $B_\perp$. Changes in the phases of the oscillations at a fixed angle of inclination, relative to the values corresponding to $\alpha = 0$, were independent (within the limits of the experimental error) either of the subband number $2D$ or of the surface excess of carriers n_s. In view of the dependence of γ on r_i^2, due to the diamagnetic shift, the changes in γ should be very different for different subbands and should decrease with increasing n_s. Figure 2 gives the surface density of carriers in the $2D$

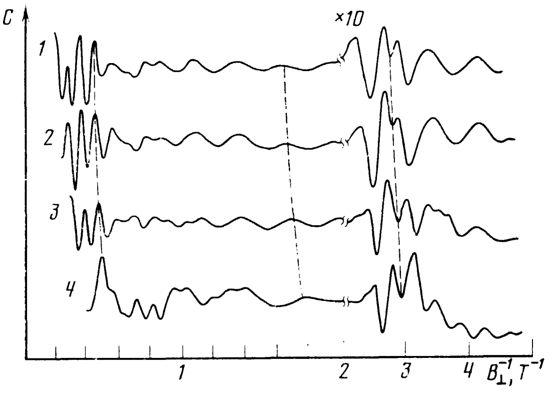

FIG. 1. Capacitance magnetooscillations in inclined magnetic fields plotted for sample 1 ($n_s = 7.5\times10^{11}$ cm^{-2}) using the coordinates C and $B_\perp^{-1}$: 1) $\alpha = 0°$; 2) $\alpha = 26°$; 3) $\alpha = 46°$; 4) $\alpha = 63°$. Note the change in the scale along the B^{-1} axis at $B_\perp^{-1} = 2T^{-1}$.

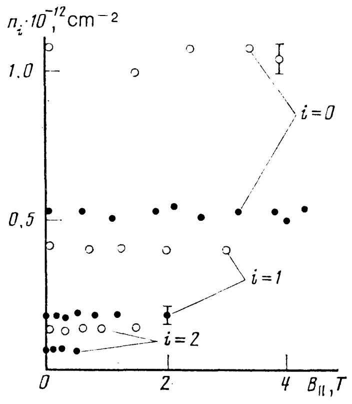

FIG. 2. Carrier densities in the subbands of sample 1 plotted as a function of the magnetic field component parallel to the two-dimensional layer for samples with the following densities n_s (10^{12} cm^{-2}): ●) 0.75; ○) 1.65.

subbands, deduced from the oscillation periods expressed in terms of $B_\perp^{-1}$ for adjacent oscillation extrema, as a function of the component of the magnetic field parallel to the two-dimensional layer. We observed no reduction in n_i on increase in $B_\parallel$ for any of the subbands, which was again an indication of the absence of the diamagnetic shift in the range of $B_\parallel$ investigated.

In the absence of the effects due to the diamagnetic shift, the changes in the oscillation phases due to a change in the magnetic field orientation may be associated with spin splitting of the Landau levels [third term on the left-hand side of Eq. (2)]. The degree of degeneracy g_s of the Landau levels in the investigated InAs and $Hg_{1-x}Cd_xTe$ samples, deduced from the ratio of the surface densities of carriers obtained using the capacitance of an MOS capacitor and surface densities of carriers in the 2*D* subband (found from the magnetooscillation capacitance periods), was 2 for all the subbands. This value presupposes that the capacitance magnetooscillations corresponded to the Landau levels which were unsplit (or weakly split) in respect of the spin so that we should not expect a change in the phases of the oscillations in inclined magnetic fields. This was precisely the behavior which was exhibited by all the InAs samples, and for all angles of inclination for which the capacitance oscillations due to the 2*D* electrons could be recorded the phase factor was close to $-1/2$ for all the subbands, which was to be expected for InAs because the spin splitting in this material is considerably less than the cyclotron splitting.

In the case of $Hg_{1-x}Cd_xTe$ (Fig. 1) the phases of the oscillations changed considerably in inclined magnetic fields for a constant period ΔB^{-1} and, as demonstrated in Fig. 3, the changes in γ at fixed angles of inclination were independent of the subband number 2*D* and of the surface carrier density. According to Eq. (2), this was precisely the behavior one would expect in inclined magnetic fields for the phase shift of the spin-split components of the Landau levels. The spin splitting, deduced from the experimental degree of degeneracy $g_s = 2$ was not manifested in the capacitance magnetooscillations in spite of the large value of the *g* factor of $Hg_{1-x}Cd_xTe$. This discrepancy can be explained by assuming that we observed experimentally only the low-field components of the oscillations (the oscillations in inclined magnetic fields shifted toward lower values of $B_\perp$), corresponding to the high-energy components of the spin-split Landau levels, whereas the high-field components (for example those due to a strong nonthermal broadening of the corresponding levels) were not manifested. This assumption was in agreement with the value of the phase factor $\gamma = -0.8 \pm 0.1$ in the perpendicular orientation of **B**. Quenching of the high-field component is a characteristic feature of narrow-gap and zero-gap materials and, although the reason for it is not clear, the effect has been observed experimentally in studies of three-dimensional oscillation effects and of two-dimensional systems (we have discussed this topic in greater detail in Ref. 7).

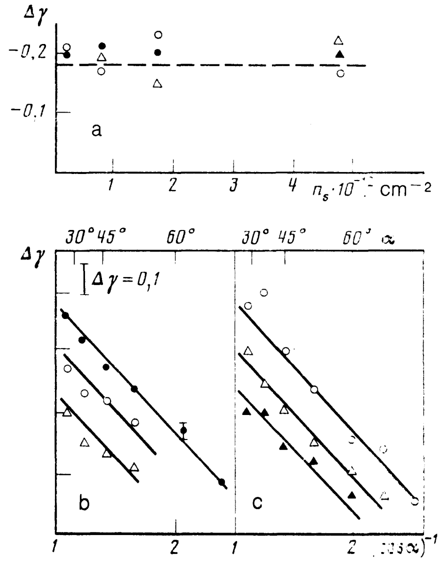

FIG. 3. Changes in the phase factor γ of the oscillations in sample 1 plotted as a function of the surface density of carriers for $\alpha = 46°$ (a) and as a function of inclination of the magnetic field (b,c) for samples with the carrier densities n_s (10^{12} cm^{-2}): b) 0.75; c) 4.75. Notation: ●) $i = 0$; ○) 1; △) 2; ▲) 3.

The hypothesis of the spin nature of the phase shift in inclined fields is in good agreement also with the nature of the experimental dependence of $\Delta\gamma = \gamma(\alpha) - \gamma(0)$ on the angle of inclination of the magnetic field. For all the subbands and throughout the full range of n_s the dependence $\Delta\gamma(\alpha)$ could be rectified using the coordinates $\Delta\gamma$–$(\cos\alpha)^{-1}$ (Figs. 3b and 3c), as predicted by Eq. (2). A study of the angular dependences of the phase shift made it possible to determine the value of the *g* factor (m_{ci} was found from other experiments), which could not be done by conventional methods because of the absence of the second spin components in the oscillations. The slope of the dependence $\Delta\gamma(\cos^{-1}\alpha)$ was the same for all the $Hg_{1-x}Cd_xTe$ samples investigated, amounting to -0.45 ± 0.05, so that the effective *g* factor was $g_i = 0.9 m_0/m_{ci}$. This was the ratio of the *g* factor and the cyclotron mass for bulk parameters of the investigated materials, obtained by us earlier[7] for the two-dimensional gas in accumulation and inversion $Hg_{1-x}Cd_xTe$ layers of semiconductor composition, for which both spin-split components of the capacitance magnetooscillations were recorded.

4. CAPACITANCE OSCILLATIONS IN MAGNETIC FIELDS PARALLEL TO THE SURFACE

Since the capacitance magnetooscillations, due to magnetic quantization of a two-dimensional gas of surface elec-

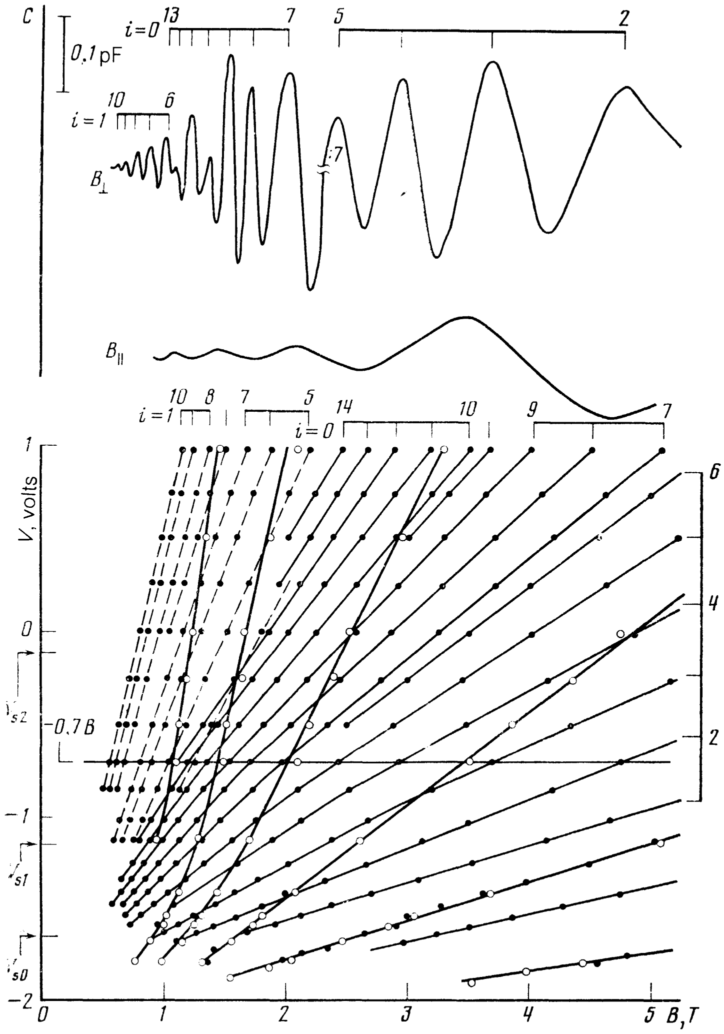

FIG. 4. Capacitance magnetooscillations obtained for $V = -0.7$ V and the positions of the oscillation maxima on the magnetic field scale B exhibited by sample 3, plotted as a function of the gate voltage: ●) $\alpha = 0$; ○) $\alpha = \pi/2$.

trons, are governed by the component **B** perpendicular to the interface, oscillations of this kind should not be observed in the limiting case of the parallel orientation of **B**. However, in all the samples investigated the capacitance of MOS structures did exhibit clear oscillatory behavior also in magnetic fields parallel to the interface [in contrast to the preceding section we shall speak here of the capacitance oscillations as a function of the modulus of the magnetic field $C(B)$]. Figure 4 shows an example of the capacitance magnetooscillations and of the positions of the oscillation maxima on the magnetic field scale plotted for different values of the voltage across a structure in the case of two limiting orientations of the magnetic field $\alpha = 0$ and $\alpha = \pi/2$ applied to sample 3. The oscillations in the $\alpha = \pi/2$ orientation [which will be denoted by $C_{\parallel}(B)$] were observed in a wide range of gate voltages V, beginning from values slightly smaller than the voltage corresponding to the start of the main subband, right up to V corresponding to a surface density of $n_s \sim (3-5)10^{12}\ \text{cm}^{-2}$.

Characteristic features had been observed earlier in parallel magnetic fields when the transverse differential conductance of accumulation layers in InAs (Ref. 5) and in semiconducting $Hg_{1-x}Cd_xTe$ (Ref. 6) were investigated. The observed nonmonotonic behavior of the dependences $d\sigma/dV(B)$ and $d\sigma/dV(n_s)$ were attributed by the authors to hybridization of electric and magnetic surface states in crossed fields and to a change in the carrier mobility in the surface layer because the levels of the hybrid subband were displaced above the Fermi level with increasing magnetic field (or decreasing electric field $\mathbf{E} \propto n_s$).

According to the model used in Refs. 5 and 6, the number of magnetooscillations corresponding to a given surface carrier density n_s cannot exceed the number of filled $2D$ subbands at $B = 0$ and the positions of the oscillations on the magnetic field scale change in inclined magnetic fields, because the energy positions of the hybrid subbands are governed by the component of **B** which is parallel to the surface. Both these predictions are in conflict with the experimental data on the capacitance oscillations found in the present study, so that we cannot interpret the results on the basis of the hybrid subband model. In fact, as can be seen from Fig. 4, throughout the full range of V where oscillations are observed in the $\alpha = \pi/2$ orientation the number of magnetooscillations exceeds the number of filled $2D$ subbands (the voltages V_{si} deduced from an analysis of the oscillations in the $\alpha = 0$ orientation and corresponding to the start of the ith subband are identified by arrows in Fig. 4). Moreover, oscillations in the parallel orientation of **B** occur also for voltages across the structure lower than the start voltage of the main subband V_{s0}, i.e., they occur in the absence of bound states in

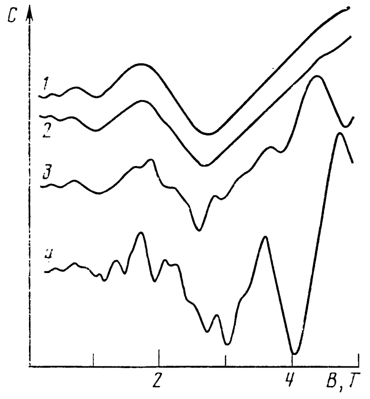

FIG. 5. Capacitance magnetooscillations in inclined magnetic fields of near-parallel orientation applied to sample 1 at the following angles: 1) 90°; 2) 85°; 3) 76°; 4) 69°; $n_s = 0.75\times10^{12}$ cm^{-2}.

the surface layer. The magnetooscillations observed in the range $V < V_{s0}$ occur for an arbitrary orientation of **B** and throughout the full range $\alpha = 0 - \pi/2$ the positions of the oscillation extrema are not affected. The $C_\parallel(B)$ oscillations are observed in inclined fields also for voltages higher than the start of the $2D$ subbands, at least in that range of α and n_s where these oscillations can be separated from the background of the stronger oscillations due to the Landau quantization in the $2D$ subbands (Fig. 5). The changes in the positions of the extrema of the $C_\parallel(B)$ oscillations on the magnetic field scale due to variation of α in the range 50–90° are within the limits of the experimental error, whereas near $\alpha \approx 0$ they are less than 20% (Fig. 6), while according to the hybrid subband model the extrema of the $C_\parallel(B)$ oscillations should undergo a shift as a function of magnetic field proportional to $(\sin\alpha)^{-1}$.

The lack of a dependence on the magnetic field orientation is evidence of the three-dimensional nature of the effects responsible for the capacitance fluctuations in a parallel magnetic field. A characteristic feature of the investigated materials is the degeneracy of the electron gas in the bulk and, consequently, the occurrence of occupied states with energies lying above the surface potential well (bottom of the conduction band in the substrate). Participation of the three-dimensional gas of electrons of the continuum in screening of the surface field affects also the differential capacitance C_s of the space charge region. In particular, in the absence of bound surface states, i.e., for $V < V_{s0}$ but close to the flat-band potential (corresponding to the linear screening condition), the continuum electrons dominate the screening and the capacitance C_s is determined by the Thomas-Fermi length $L_{\rm scr} = (\mu_b\varepsilon/6\pi ne^2)^{1/2}$ (μ_b is the chemical potential, ε is the permittivity, and n is the bulk density of electrons). The experimental values of the capacitance of the investigated MOS structure near the start of the main subband were indeed close to the value $C = C_0C_L/(C_0 + C_L)$, where C_0 is the insulator (oxide) capacitance and $C_s = C_L = \varepsilon/4\pi L_{\rm scr}$. In the linear approximation the screening length is an oscillatory function of the magnetic field **B** for all its orientations[10]; this accounts for the capacitance magnetooscillations described above which are indeed independent of the orientation of **B** in the range of small values of negative band bending $V < V_{s0}$ (the condition for the observation of such oscillations is the smallness of the surface band bending, $|e\varphi_s| < \hbar\omega_c, \mu_b$.

In the range of accumulation voltages $V > V_{s0}$ ($n_s > 0$) the surface field is screened mainly by carriers in the $2D$ subbands, but in this case the $C_\parallel(B)$ capacitance magnetooscillations arise because the continuum electrons participate in the screening process, since the $2D$ electrons cannot make an oscillatory contribution to the screening in magnetic fields parallel to the $2D$ layer in view of the two-dimensional nature of their motion (see Sec. 3). The mechanism of the singularities (responsible for the capacitance oscillations) in the continuum density of states in the surface region of a semiconductor was considered in Ref. 11 for the case of the accumulation-type band bending with the parallel orientation of **B** (in the case of either of the materials investigated the inequality $\lambda_F \gg L_{\rm scr}$ was satisfied, where λ_F is the Fermi wavelength of the continuum electrons).

The positions of the $C_\parallel(B)$ oscillation extrema in the range $V > V_{s0}$ depends strongly on the surface density of states: an increase in n_s shifts the oscillations toward higher values of B and in a wide range of n_s the shift is proportional to n_s (Fig. 7). It is quite clear from Fig. 8, which shows the positions of the extrema using the coordinate B^{-1} and n_s, that for all values of n_s the oscillations are periodic in the reciprocal of the magnetic field[1)] and the oscillation periods $\Delta_\parallel$ remain constant within the limits of the experimental

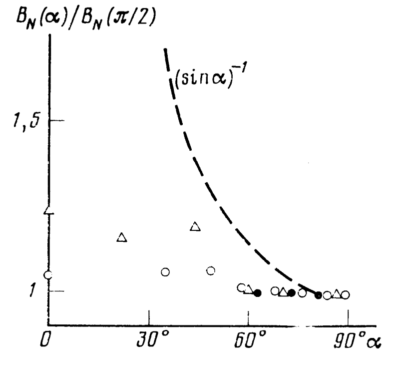

FIG. 6. Magnetic field B_N corresponding to the last observed capacitance maximum in an inclined field (in units of B_N for the parallel orientation of the field) as a function of the angle of inclination of the field applied to sample 1 with the following carrier densities n_s (10^{12} cm^{-2}): ○) 0.02; △) 0.15; ●) 0.75.

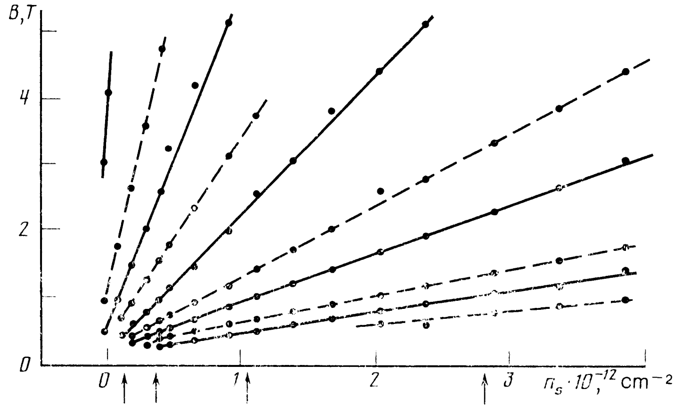

FIG. 7. Positions of capacitance magnetooscillation extrema (the continuous lines represent maxima of C and the dashed lines represent minima of C) obtained for $\alpha = \pi/2$ as a function of n_s in sample 1. The arrows correspond to the starts of filling of the subbands.

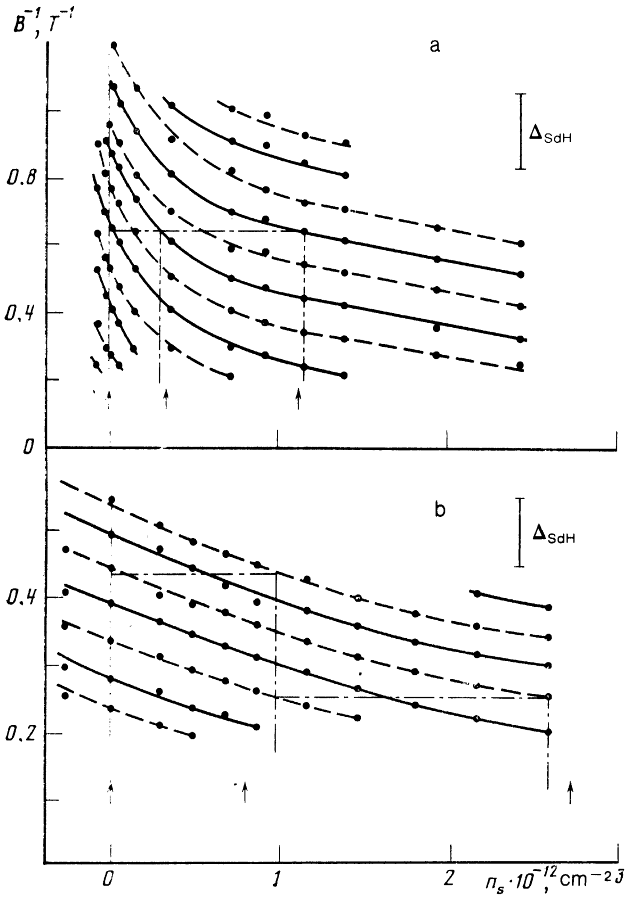

FIG. 8. Extrema of the capacitance oscillations in parallel magnetic fields plotted using the coordinates B^{-1} and n_s for samples 3 (a) and 5 (b).

error when n_s is varied. Throughout the investigated range of bulk densities $n = 10^{15}$–10^{18} cm^{-3} the periods of the oscillations in InAs and $Hg_{1-x}Cd_xTe$ are close to the periods of the Shubnikov–de Haas oscillations Δ_{SdH}, measured for the original wafers (Fig. 8), and the experimental dependence $\Delta_{\parallel}(n)$ is described well by the relationship $\Delta_{\parallel} = \Delta_{SdH} = 2e/c\hbar(3\pi^2 n)^{2/3}$ (Fig. 9). In our opinion this result is a convincing confirmation of the mechanism which attributes the capacitance magnetooscillations in degenerate semiconductors in a parallel magnetic field to the participation of the continuum electrons in the screening process. The cyclotron effective masses $m_{c\parallel}$, found from the temperature dependences of the amplitudes of the $C_{\parallel}(B)$ oscillations, also agree well with the cyclotron masses m_c of the bulk carriers deduced from an analysis of the temperature dependences of the Shubnikov–de Haas oscillations (Fig. 9).

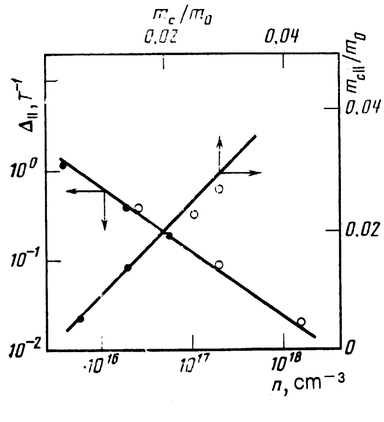

FIG. 9. Dependences of the capacitance oscillation periods (expressed in terms of the reciprocal magnetic field) on the electron density in the substrate in the case when $\alpha = \pi/2$ and the correlation between the cyclotron masses $m_{c\parallel}$ deduced from the $C_{\parallel}(B)$ oscillations and the masses m_c deduced from the Shubnikov–de Haas oscillations: ●) $Hg_{1-x}Cd_xTe$: ○) InAs.

The shift of the $C_{\parallel}(B^{-1})$ oscillation curves due to a change in the surface carrier density n_s for a constant period $\Delta_{\parallel}$ corresponds to a change in the oscillation phase, which is illustrated well by the curves in Fig. 8. The phases $\delta = 2\pi\gamma$ (γ is the phase factor of the oscillations)[2)], determined from the dependences of the oscillation numbers on the reciprocal of the magnetic field, are plotted in Fig. 10a as continuous (not modulo 2π) functions of the surface density of carriers (the experimental points in the range $n_s < 0$ correspond to oscillations obtained when $V < V_{s0}$). The arrows in this figure identify the initial densities n_{si} corresponding to the onset of filling of the $2D$ subbands.[3)] In the case of samples with different dopant concentrations the initial carrier densities n_{si} are quite different,[4)] but a characteristic feature of the $\delta(n_s)$ experimental curves exhibited by all the samples is the increase in the phase by 2π in the ranges of n_s between the successive starts of filling of the two-dimensional subbands.

In our opinion, this behavior may be attributed to the fact that a change in the surface band bending (as n_s increases) alters the wave functions of the bound states and, consequently, the phases of the wave functions of the continuous spectrum which are orthogonal to the former functions.[5)] The phases of the wave functions of the continuum states are known to change by π as a result of formation of a new bound state,[13] which may correspond to a change by 2π

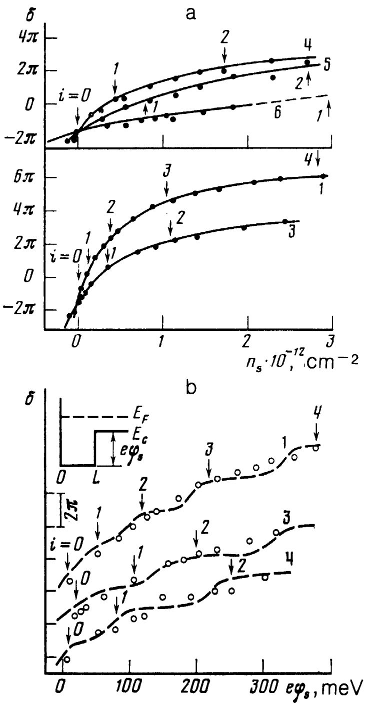

FIG. 10. Phase of the $C_{\parallel}(B)$ oscillations as a function of the surface density of carriers (a) and of the surface band bending (b). The numbers without arrows denote the numbers of samples. The dashed curves are calculated in accordance with Eq. (3) for a rectangular potential well shown as the inset.

in the measured macroscopic quantity, which is the phase of the capacitance oscillations. From the point of view of this mechanism it would be of interest to consider the dependences of the phases of the capacitance magnetooscillations on the depth of the surface potential well, shown for some of the samples in Fig. 10b. The surface band bending was calculated from the experimental values of the carrier density in the $2D$ subbands employing the semiclassical approximation.[12] The same figure gives the calculated values of the doubled phase of the wave function of the continuum electrons with an energy $E = E_F$ when the surface potential is approximated by an asymmetric rectangular well of depth equal to the surface band bending $e\varphi_s$:

$$\delta=2\{\mathrm{arcctg}\,[(k_s/k_b)\,\mathrm{ctg}\,(k_sL)]-k_bL\}, \tag{3}$$

where

$$k_b=(2m\mu_b/\hbar^2)^{1/2}, \quad k_s=[2m(\mu_b+e\varphi_s)/\hbar^2]^{1/2}.$$

The width of a rectangular well L was selected for each sample to ensure that the formation of bound states in the well occurred at the same value of $e\varphi_s$ for which the experimental results indicated the start of filling of the $2D$ subbands (this condition was satisfied by similar values of L for different values of i). It should be pointed out that the nonquadratic nature of the dispersion law could be allowed for quite simply in Eq. (3), but one can show that a reduction in the effective width of the surface potential well when φ_s increases largely cancels the nonquadratic effect and in the limit of strong band bending and strong nonparabolicity $e\varphi_s \gg E_g$ (in the case of $Hg_{1-x}Cd_xTe$ this inequality is obeyed in a wide range of $e\varphi_s$) the cancellation is practically complete. We can see from Fig. 10 that the phase shifts of the $C_\parallel(B)$ magnetooscillations caused by a change in the depth of the surface potential well agree satisfactorily with the phase shifts of the wave function of the continuum states.

It therefore follows that our experimental results—the independence of the positions of the $C_\parallel(B)$ oscillations of the **B** orientation, identity of the periods and temperature dependences of the amplitudes of the capacitance oscillations and of the Shubnikov–de Haas oscillations in the initial wafers, and characteristic features of the behavior of the oscillation phases as the surface density of states n_s increases—can be explained satisfactorily by a mechanism attributing the capacitance magnetooscillations, in magnetic fields parallel to the surface, to the screening of the degenerate electron gas of the continuum electrons. An indirect confirmation of this model is provided also by investigations of inversion layers. The capacitance magnetooscillations were not observed for any one of our p-type $Hg_{1-x}Cd_xTe$ samples or for lightly doped n-type $Hg_{1-x}Cd_xTe$ samples with different compositions ($x = 0.09$–0.25) when the field **B** was applied parallel to the surface (as expected on the basis of the mechanism discussed above in the absence of the electron gas degeneracy in the substrate), whereas the oscillations associated with the Landau quantization of the $2D$ subband were exhibited by these samples as clearly as in the case of degenerate n-type materials.

We shall conclude this discussion by considering the interpretation of the results reported in Refs. 5 and 6. Generally speaking, the profiles of the $d\sigma/dV$ oscillations observed in Refs. 5 and 6 were different from the behavior $C_\parallel(B)$ typical of the magnetooscillation effects found in the present study. However, the results of Refs. 5 and 6 had certain features in common with the results obtained in the present study for a lightly doped sample 1, particularly the identical nature of the shift of the oscillation maxima as n_s increases (compare Fig. 7 in the present study with Fig. 12 in Ref. 6). Bearing this point in mind and also that the interpretation of the data of Refs. 5 and 6 on the basis of the hybrid subband model is not free of serious objections (some of them were mentioned by the authors in Refs. 5 and 14), there are grounds for interpreting the results of Refs. 5 and 6 on the basis of the way the continuum electrons participate in the screening. First of all, one should note that the results of Refs. 5 and 6 were obtained for accumulation layers on degenerate samples of InAs, InSb, and $Hg_{1-x}Cd_xTe$ with electron densities $\sim 2\times 10^{15}$ cm^{-3} close to the bulk values, so that the proposed model can be used to explain the similarity—mentioned in Ref. 6—of the results of these investigations (like the similarity of the results for sample 1 in the present study). As for the way the continuum electrons influence $d\sigma/dV$, a change in the phase of the wave function of these states due to a change in n_s and B and, consequently, a change in the surface density of the continuum carriers (deficit of the continuum electrons[12]) may alter the screening of the scattering centers at the interface and, consequently, may modify the mobility of electrons responsible for transport in the surface channel. It should be pointed out that the singularities in the dependences of $d\sigma/dV$ on n_s associated with this effect may occur also for $B = 0$, as confirmed in Ref. 6. The proposed mechanism may account also for the results of an investigation of the temperature dependences on the amplitudes of the $d\sigma/dV$ oscillations reported for InAs (Ref. 15) namely that the cyclotron mass deduced from an analysis of the data in Ref. 15 is close to its bulk value. However, it should be pointed out that the hypothesis that the capacitance magnetooscillations we have investigated and the $d\sigma/dV$ oscillations reported in Refs. 5 and 6 in the case of parallel magnetic fields are the same would require further study. One difficulty is that the $d\sigma/dV$ oscillations on substrates with different degrees of degeneracy and the evolution of these oscillations when the magnetic field deviates from the parallel orientation have been studied very little. A decisive proof in support of the hybrid subband model would have been detection of the $d\sigma/dV$ oscillations in inversion layers or on nondegenerate n-type substrates, because in the absence of the electron gas degeneracy in the bulk there should be no effects associated with the continuum electrons, whereas hybridization of electric and magnetic states should occur in this case also.

[1)]Sample 1 exhibited deviations from the periodicity of the extrema in the range of magnetic fields close to the extreme quantum limit.

[2)]The values of γ for sample 1 were determined in high-Landau levels.

[3)]The $i = 4$ subband of sample 1 was not manifested experimentally in the $C_\perp$ oscillations and the value of n_{s4} given in Fig. 10a was found by a semiclassical calculation[12] which for bands with $i < 4$ gave values of n_{si} that agreed with the experimental values within the limits of experimental error.

[4)]The increase in n_{si} on increase in the dopant concentration is mainly due to the nonparabolicity of the dispersion laws of the investigated materials.

[5)]In terms of Ref. 11, the change in the oscillation phase corresponds to a change in the energy position of the singularities in the density of states of a continuous spectrum when there is a change in the surface potential.

[1]D. C. Tsui, Solid State Commun. **9**, 1789 (1971).
[2]J. H. Crasemann, U. Merkt, and J. P. Kotthaus, Phys. Rev. B **28**, 2271 (1983).
[3]W. Beinvogl and J. F. Koch, Phys. Rev. Lett. **40**, 1736 (1978).
[4]W. Beinvogl, A. Kamgar, and J. F. Koch, Phys. Rev. B **14**, 4274 (1976).
[5]R. E. Doezema, M. Nealon, and S. Whitmore, Phys. Rev. Lett. **45**, 1593 (1980).
[6]W. Zhao, F. Koch, J. Ziegler, and H. Maier, Phys. Rev. B **31**, 2416 (1985).
[7]V. F. Radantsev, T. I. Deryabina, L. P. Zverev, G. I. Kulaev, and S. S. Khomutova, Zh. Eksp. Teor. Fiz. **88**, 2088 (1985) [Sov. Phys. JETP **61**, 1234 (1985)].
[8]F. Stern and W. E. Howard, Phys. Rev. **163**, 816 (1967).
[9]E. J. Pakulis, Phys. Rev. B **31**, 6807 (1985).
[10]F. A. Buot, Phys. Rev. B **14**, 977 (1976).
[11]D. A. Romanov and L. D. Shvartsman, Pis'ma Zh. Eksp. Teor. Fiz. **42**, 9 (1985) [JETP Lett. **42**, 10 (1985)].
[12]V. F. Radantsev, T. I. Deryabina, L. P. Zverev, G. I. Kulaev, and S. S. Khomutova, Zh. Eksp. Teor. Fiz. **91**, 1016 (1986) [Sov. Phys. JETP **64**, 598 (1986)].
[13]L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, Pergamon Press, Oxford (1974).
[14]M. Nealon, S. Whitmore, R. R. Bourassa, and R. E. Doezema, Surf. Sci. **113**, 282 (1982).
[15]M. Nealon and R. E. Doezema, Solid State Commun. **50**, 661 (1984).

Translated by A. Tybulewicz