The effect of degeneracy on hot electron energy relaxation in a quantum well

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The probability of pairwise electron-electron scattering is calculated for hot electrons in a degenerate two-dimensional gas with electron temperature T_e. The rate of energy loss Q to optical phonons is determined by using the kinetic equation. It is shown that the energy loss rate increases when degeneracy is taken into account.

1. INTRODUCTION

The relaxation of highly-excited (hot) electrons with some temperature T_e is associated with the emission of optical phonons (O-scattering). Under typical experimental conditions[1–4] the free electron density n is so high that in the passive region $\varepsilon < \hbar\Omega_0$ ($\hbar\Omega_0$ is the optical phonon energy) electron-electron (ee) scattering dominates, which gives rise to an equilibrium electron distribution with electron temperature T_e. This will happen if the energy relaxation time for ee-scattering turns out to be shorter than other energy relaxation times (e.g., for acoustic phonons or impurities). As the density n is further increased, degeneracy ensues; electron-electron scattering tends to bring about an equilibrium distribution and emission of optical phonons above the threshold $\varepsilon = \hbar\Omega_0$. In the active region $\varepsilon > \hbar\Omega_0$ the electron distribution function is depleted because of O-scattering. The magnitude of this depletion is related directly to the rate of energy loss Q to optical phonons, since Q is the integral of the distribution function in the active region.

The problem of calculating Q was solved in Ref. 5 for a nondegenerate gas situated in the lowest subband of a quantum well for $T_e \ll \hbar\Omega_0$. A critical concentration N_c^+ was found above which electron scattering dominates; in the active region the distribution function above the threshold is Maxwellian, and the rate of energy loss equal.

$$Q_0 = \frac{N\hbar\Omega_0}{\tau_O}\exp\left(-\frac{\hbar\Omega_0}{T_e}\right). \tag{1}$$

We will refer to this as the "maximal" rate of energy loss. The meaning of Eq. (1) is simple: the electron gas has a density N; a fraction $\exp(-\hbar\Omega_0/T_e)$ of these electrons are found above threshold, and will emit phonons of energy $\hbar\Omega_0$ in a time τ_0. The deficit of electrons due to O-scattering is effectively "filled in" thanks to ee-scattering, so that the "bottleneck" is essentially phonon emission. Formula (1) was obtained by Stratton.[6]

If the electron density is subcritical, the loss power decreases. The density N_c^+ equals

$$N_c^+ = \frac{m\hbar\Omega_0}{\pi^{3/2}\hbar^2}\varkappa\left(\frac{T_e}{\hbar\Omega_0}\right)^{1/2}, \quad \varkappa = \frac{\hbar}{\varepsilon_B\tau_O}; \tag{2}$$

here $\varepsilon_B = me^4/2\hbar^2$ is the Bohr energy, and m and e are the effective mass and charge of an electron (taking into account the dielectric permittivity).[5]

We note at this point that the Fermi energy equals

$$\frac{\varepsilon}{T_e} = \varkappa\left(\frac{\hbar\Omega_0}{\pi T_e}\right)^{1/2}\frac{N}{N_c^+},$$

so that the gas is already degenerate at $N = N_c^+$ [5] ($\varkappa \sim 1$[7]; this also requires that $\hbar\Omega_0 \gg T_e$). Therefore, the maximal rate of energy loss cannot be achieved for a nondegenerate gas.

Equation (1) is usually used in experimental papers; in these papers, instead of a τ_0 given by

$$\frac{1}{\tau_O} = \frac{1}{\tau_{DO}} + \frac{1}{\tau_{PO}} \approx \frac{1}{\tau_{PO}} = 2\alpha\Omega_0 \approx \frac{1}{0.11\,\text{psec}} \quad \text{(GaAs)}$$

measurements yield quantity larger than 0.11 psec by 5 to 60 times.[1–4] Here DO and PO refer to the deformation and piezoelectric interactions; the latter dominates in the GaAs samples used experimentally, α denotes the Fröhlich coupling constant. The authors of Refs. 1–4 suggested that degeneracy of the gas was one possible cause of the decrease in the relaxation rate, since after phonon emission an electron should drop below the Fermi "line".

The goal of the present paper is to calculate the rate of energy loss Q to optical phonons in a degenerate electron gas located in the lowest level of a quantum well. It is shown that including dengeneracy increases the rate of energy loss.

2. ELECTRON-ELECTRON SCATTERING PROBABILITY

Let us assume that we can describe most of the electrons by a Fermi distribution $f_{T_e}(\varepsilon)$ and that these electrons are located below threshold, so the $\hbar\Omega_0 \gg \max(\mu, T_e)$ where μ is the chemical potential of the gas. In the case of strong degeneracy, we have $\mu = \varepsilon_F$. We will investigate the scattering of a probe electron $\varepsilon \to \varepsilon'$, where $\varepsilon, \varepsilon' \gg \max(\mu, T_e)$ for the bulk of the electrons. Our calculation of the probability of ee-scattering in no way differs from the same calculations for a nondegenerate gas[5]; however, in averaging over the equilibrium electrons it is necessary to insert a factor $(1 - f_{T_e})$. As a result, we obtain for the probability of ee-scattering from an energy ε to an energy ε'

$$W(\varepsilon, \varepsilon') = \frac{2e^4}{\hbar T_e}\left(\frac{\mu}{T_e}\right)^{1/2} K_\beta(y, y'),$$

$$y = \frac{\varepsilon}{T_e}, \quad y' = \frac{\varepsilon'}{T_e}, \quad \beta = \frac{\mu}{T_e}, \tag{3}$$

$$K_\beta(y, y') = \left(\frac{y}{\beta}\right)^{1/2} \int\limits_{(y^{1/2}-y'^{1/2})^2}^{(y^{1/2}+y'^{1/2})^2} \frac{dt}{t^{3/2}[4yy' - (y + y' - t)^2]^{1/2}}$$

$$\times \int\limits_0^\infty d\xi \left\{\left[\exp\left(\xi^2 - \beta + \frac{(\omega + t)^2}{4t}\right) + 1\right]\right.$$

$$\left.\times\left[\exp\left(\beta - \xi^2 - \frac{(\omega - t)^2}{4t}\right) + 1\right]\right\}^{-1};$$

$$y, y' \gg 1, \quad y - \beta \gg 1.$$

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We denote by $\omega = (\varepsilon' - \varepsilon)/T_e$ the dimensionless energy transfer. Let us analyze the case $|y - y'| \ll y, y'$, in which the upper and lower limits in t can be set to zero and ∞, and expand the square root, assuming that a characteristic value of y is such that $\omega^2/y \ll t \ll y$. Introducing the variable $\zeta = t^{-1}$, we have for the scattering kernel (3), depending only on y and $\omega = y' - y$:

$$K_\beta(\omega) = \frac{e^{-\omega/2}}{4\beta^{1/2}} \int_0^\infty \int_0^\infty \frac{d\xi d\zeta}{\operatorname{ch}\psi + \operatorname{ch}(\omega/2)}, \quad \psi(\xi, \zeta) = \beta - \xi^2 - \omega^2\zeta/4 - 1/4\zeta. \tag{4}$$

It is easy to show that the kernel (4) satisfies the principle of detailed balance, i.e., replacing ω by $-\omega$ and at the same time multiplying by e_ω leaves the integral unchanged.

The kernel (4) can be further simplified if we assume that $\omega \ll \beta \ll y, y'$, although formally the function to be integrated depends only on $y' - y$. The function under the integral sign is not small near the extremum $\psi(\xi_0, \zeta) = 0$. Assuming that the integral is concentrated far from the points $\zeta_{1,2}$, where $\zeta_{1,2}$ are toots of the equation $\psi(0,\zeta) = 0$ (this follows from $\beta \gg \max(1, \omega)$), let us expand in the neighborhood of this extremum:

$$\psi(\xi, \zeta) = \psi(\xi_0 + \xi', \zeta) = -2\xi_0\xi', \quad \xi_0 \gg 1, \quad \xi = \xi_0 + \xi'. \tag{5}$$

Then we can take the integral over ξ' for fixed ζ. We obtain

$$K_\beta(\omega) = \frac{|\omega| e^{-\omega/2}}{8\beta^{1/2} \operatorname{sh}(|\omega|/2)} \int_{\zeta_1}^{\zeta_2} \frac{d\zeta}{(\beta - \omega^2\zeta/4 - 1/4\zeta)^{1/2}}, \tag{6}$$

and we require only the asymptotic form of the remaining elliptic integral for $\beta \gg 1$. Calculating it, we can verify that the integral is concentrated around $\zeta \sim \zeta_2$; hence the assumption $\xi_0 \gg 1$ was correct. We can also see that $\omega^2/y \ll \zeta_2^{-1} \ll y$ holds; therefore the transition (3) $\to$ (4) is also correct. Finally we obtain

$$K_\beta(\omega) = \frac{e^{-\omega/2}}{|\omega| \operatorname{sh}(|\omega|/2)}. \tag{7}$$

The kernel (7) has a singularity $|\omega|^{-2}$ as $\omega \to 0$, as in the nondegenerate case. This is a general property of *ee*-scattering in a 2*D* gas. For large transfers $|\omega| \gg 1$ the kernel falls off exponentially for $\omega > 0$ and is proportional to $|\omega|^{-1}$ for $\omega < 0$. However, for $|\omega| \gtrsim \beta$ Eq. (7) becomes inapplicable, and the character of the power-law decrease changes since the degeneracy of the gas no longer enters in. In the nondegenerate gas $K(\omega) \sim |\omega|^{-3/2}$, $\omega < 0$, $|\omega| \gg 1$.[5] All of this behavior of $K(\omega)$ is implied by formula (4).

The quadratic singularity in $W(\varepsilon, \varepsilon')$ implies that the diffusion approximation is untenable, and so in order to include *ee*-scattering we must solve a kinetic integral equation.[5] In the corresponding collision integral the kernel is integrated over all possible energy transfers, for example

$$\int_0^\infty d\varepsilon' W(\varepsilon, \varepsilon').$$

Therefore it is possible to use the simple formula (7) if $\ln \beta \gg 1$; the answer so obtained is logarithmically accurate. In the opposite case it is necessary to use the kernel (4).

3. KINETIC EQUATION AND RATE OF ENERGY LOSS

The equation for the distribution function $f(\varepsilon)$ in the vicinity of threshold has the form

$$\int_0^\infty d\varepsilon' g[W(\varepsilon', \varepsilon) f(\varepsilon') - W(\varepsilon, \varepsilon') f(\varepsilon)] - f(\varepsilon)[1 - f_{Te}(\varepsilon - \hbar\Omega_0)]/\tau_0(\varepsilon) = 0, \tag{8}$$

where g is the density of states. The integral describes *ee*-scattering, while for $y' \approx y \approx \hbar\Omega_0/T_e$ the probability W is correctly given by expressions (3), (4). The term $\tau_0(\varepsilon)$ describes optical phonon emission, and the factor $(1 - f_{T_e})$ is introduced to take degeneracy into account. We define a new function φ and a new argument t according to the equations

$$f(\varepsilon) = f(\varepsilon = \hbar\Omega_0)[e^{-t} - \varphi(t)], \quad t = (\varepsilon - \hbar\Omega_0)/T.$$

Then Eq. (8) can be written in the following fashion:

$$\tilde{\lambda}\hat{K}[\varphi] = \theta(t)(e^{-t} - \varphi)(e^{\beta - t} + 1)^{-1}, \tag{9}$$

where a second parameter of the problem (besides β) is

$$\tilde{\lambda} = \frac{4}{\pi\varkappa}\left(\frac{\mu}{\hbar\Omega_0}\right)^{1/2}, \tag{10}$$

θ is the Heaviside function, while the integral operator $\hat{K}$ is by definition

$$\hat{K}[\varphi] = \int_{-\infty}^\infty dt' [K(t' - t)\varphi(t) - K(t - t')\varphi(t')].$$

The parameter $\tilde{\lambda}$ is convenient to use in the case of strong degeneracy. In the opposite case, when $\mu \ll T_e$, we can return to the parameter $\lambda = N/N_c^+$.[5] The relation between these two parameters is

$$\lambda/\tilde{\lambda} = 2^{-3}\pi^{3/2}\beta^{1/2}. \tag{11}$$

In order to investigate Eq. (9), we can use the Wiener-Hopf method.[5,7] If we introduce the functions

$$\varphi^\pm(t) = \theta(\pm t)\varphi(t)$$

and perform the Fourier transforms

$$\bar{\varphi}^\pm(k) = \int dt\, e^{-ikt}\varphi^\pm(t),$$

Eq. (9) takes the form

$$-\tilde{\lambda}\bar{K}(k)[\bar{\varphi}^+(k) + \bar{\varphi}^-(k)] = F(k - i) - \Xi[\bar{\varphi}^+, k],$$

$$F(k) = \int_0^\infty \frac{dt\, e^{-ikt}}{e^{\beta - t} + 1}, \quad \Xi[\varphi, k] = \frac{1}{2\pi}\int_C dk' \bar{\varphi}^+(k) F(k - k'), \tag{12}$$

$$C: \ \operatorname{Im} k < \operatorname{Im} k' < 1;$$

$$\bar{K}(k) = \int_{-\infty}^\infty dt\, K(t)(e^{-ikt} - 1). \tag{13}$$

To calculate the rate of energy loss we must evaluate the integral

$$Q = \int_{\hbar\Omega_0}^\infty d\varepsilon\, gf(\varepsilon)\frac{\hbar\Omega_0}{\tau_0} = Q_0\Phi_\beta(\tilde{\lambda}), \tag{14}$$

$$\Phi_\beta(\tilde{\lambda}) = \frac{1}{\beta}\int_0^\infty dt \frac{e^{-t} - \varphi(t)}{e^{-t} + e^{-\beta}}. \tag{15}$$

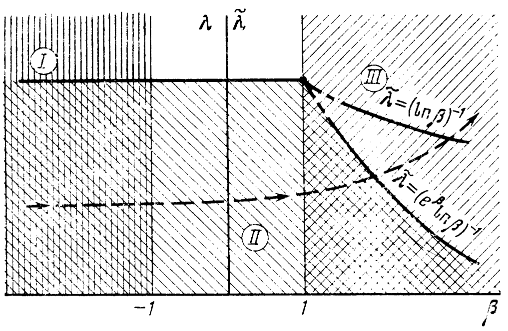

FIG. 1. Regions of values of β and $\tilde{\lambda}$ corresponding to the various energy loss regions: I—nondegenerate gas,[5] II—weak *ee*-scattering, III—strong degeneracy. The dashed line illustrates the dependence of $\lambda(N)$, $\tilde{\lambda}(N)$ on $\beta(N)$; the direction of increasing density N is shown by the arrow.

Expression (15) can also be cast in the form

$$\Phi_\beta(\tilde{\lambda})=\frac{e^\beta}{\beta}\,\Xi[\bar{\varphi}^+,0]. \tag{16}$$

Equation (12) cannot be solved analytically in the general case. Therefore we will investigate a number of limiting cases which, however, exhaust the problem. An additional complication arises because the integral (13) for the "transform" of the kernel also cannot be calculated analytically from the function (4).

Let us investigate the region $\lambda,\tilde{\lambda}, > 0$, illustrated in Fig. 1. The subregion labelled with the number I corresponds to a nondegenerate gas and has been studied in Ref. 5. The solution to Eq. (12) will be carried out in the limits $\lambda \ll 1$, $\tilde{\lambda}e^\beta \ln\beta \ll 1$—i.e, the case of weak *ee*-scattering (II)—and in the case of strong degeneracy $\ln\beta \gg 1$ (III).

Weak ee-scattering

Let us investigate the situation when $\tilde{\lambda}$ is sufficiently small. Then it is convenient to express the function in question as follows:

$$\varphi^+(k)=(ik+1)^{-1}+\tilde{\lambda}e^\beta\bar{\varphi}^+(k),$$

and to rewrite the equation in the following way:

$$e^\beta\Xi[\bar{\varphi}^+,\ k]-\bar{K}(k)\bar{\varphi}^-(k)=\bar{K}(k)(ik+1)^{-1}, \tag{17}$$

where the function Ξ is analytic in the half-plane $\operatorname{Im} k<1$, while function $\bar{\varphi}-$ is analytic in the half-plane $\operatorname{Im} k>0$; the function $\bar{K}(k)$ and the whole of Eq. (17) are analytic in the strip $0<\operatorname{Im} k<1$. The distribution function can be found from (17) in the form of a quadrature[7]; here we present directly the expression for the rate of energy loss (16):

$$\Phi_\beta(\tilde{\lambda})=\tilde{\lambda}e^\beta a,$$
$$a=K_-(i)/K_+(0),\quad K_-(k)/K_+(k)=\bar{K}(k), \tag{18}$$

where K_- and K_+^{-1} are respectively analytic for $\operatorname{Im} k>0$ and $\operatorname{Im} k<1$. The quantity a can be expressed in the form of a definite integral:

$$a=\exp\left\{\frac{2}{\pi}\int_0^\infty\frac{dx}{x^2+1}\ln\left[\int_{-\infty}^{\infty}dt\left[e^{t/2}\cos\left(\frac{xt}{2}\right)-1\right]K(t)\right]\right\}. \tag{19}$$

Analysis of Eqs. (18), (19) show that the conditions for applicability of Eq. (17) take the form

$$\tilde{\lambda}ce^\beta\ll 1,\quad c=\max(1,\ \ln\beta),\quad \beta\gg 1;\ \lambda\ll 1,\ \beta\leqslant 1. \tag{20}$$

Strong degeneracy (ln β ≫ 1)

In this case, the integral (13) can be evaluated with logarithmic accuracy for $|k|\sim 1$, $\bar{K}=\ln\beta$. Equation (9) is reduced to an algebraic equation, from which we obtain the distribution function

$$f(\varepsilon)=f_{Te}(\varepsilon),\quad \varepsilon<\hbar\Omega_0,$$
$$f(\varepsilon)=A(A+1)^{-1}f_{Te}(\varepsilon),\quad A=\tilde{\lambda}\ln\beta\,(e^{\beta-t}+1),\quad 1\leqslant t\leqslant\beta. \tag{21}$$

A calculation of the rate of energy loss then yields

$$\Phi_\beta(\tilde{\lambda})=\beta^{-1}\ln[1+\tilde{\lambda}\ln\beta\, e^\beta(1+\tilde{\lambda}\ln\beta)^{-1}]. \tag{22}$$

4. DISCUSSION

Let us discuss the characteristic scale of variation of the distribution function and rate of energy loss. The probability W_{ee}, over and above its dependence on degeneracy, has the quadratic singularity characteristic of the $2D$ gas. This implies that small energy transfers $|\omega|\ll T_e$ do not dominate, and that in our estimates we need only include the transfers $|\omega|\gtrsim T_e$.

Among the energy transfers $|\omega|\gtrsim T_e$ we can distinguish two groups, separated by the condition $|\omega|\simeq\varepsilon_F$. The large transfers $|\omega|\gg\varepsilon_F$ are determined by the probability for the nondegenerate case, which has a power-law dependence $|\omega|^{-3/2}$. This asymptotic form also was a characteristic of the electron-electron scattering kernel given in Ref. 5, where it was explained that all the scales of variation for the distribution function, including the depth of depletion into the passive region, coincided with T_e. This implied that out of all the energy transfers, only the tranfer $|\omega|\sim T_e$ was effective. In summation, for small transfers $|\omega|\ll T_e$ the collision integral $W_{ee}\propto|\omega|^{-2}$ is concentrated at the upper limit $|\omega|\sim T_e$ while for large transfers $|\omega|\gg\varepsilon_F$, $W_{ee}\propto|\omega|^{-3/2}$ is concentrated at the lower limit $|\omega|\sim\varepsilon_F$. But, what happens for the intermediate transfers $T_e\ll|\omega|\ll\varepsilon_F$? Let us calculate the corresponding time using the probability (3), (7);

$$\frac{1}{\tau_F}=\int_{\varepsilon-\varepsilon_F}^{\varepsilon-T_e}d\varepsilon'\,gW_{ee}(\varepsilon,\varepsilon')=\frac{\tilde{\lambda}}{\tau_0}\int_{T_e}^{\varepsilon_F}\frac{d\omega}{\omega}=\frac{\tilde{\lambda}}{\tau_0}\ln\beta.$$

It is clear that the entire region $T_e\ll|\omega|\ll\varepsilon_F$ gives a logarithmically uniform contribution to τ_F. If $\ln\beta\gg 1$, the situation is greatly simplified; namely, the large transfers operate in the vicinity of ε_F over an interval of order ε_F, while the small ones act in the vicinity of T_e over an interval of order T_e, and no logarithm is included. Therefore the intermediate transfers dominate. Consequently, electrons in the active region (i.e., above threshold) which leave the passive region originate within a depth $T_e\ll|\omega|\ll\varepsilon_F$.

Over this depth, the distribution function $f(\varepsilon)$ is necessarily close to the equilibrium function f_{Te}. The incoming term from *ee*-scattering is determined by the known function $f_{Te}(\varepsilon)$ and also gives rise to an equilibrium function in the active region. Such a function will also arise as long as the efflux from the active region is determined by *ee*-scattering

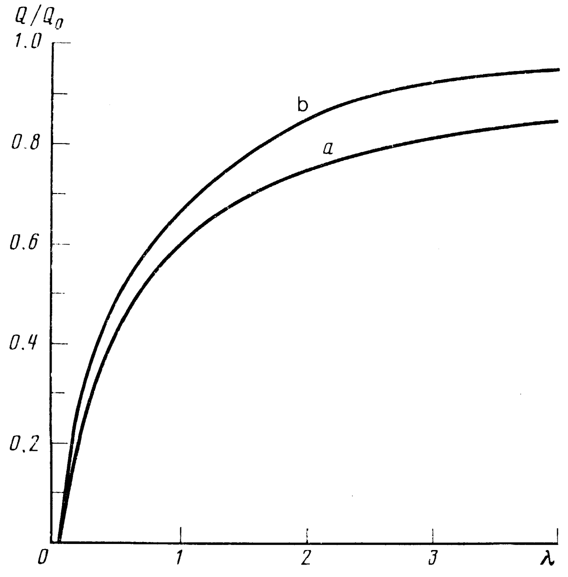

FIG. 2. Ratio of the loss power Q to the maximum Q_0 (1) as a function of $\lambda = N/N_c^+$: a) $\Phi(\lambda)$—result of Ref. 5, b) dependence of Φ_β on λ according to Eqs. (18), (19), (22) using the relation between the parameters β, $\tilde{\lambda}$ and λ through the density N. In the calculation it was assumed that $\hbar\Omega_0/T_e = 10$, $\varkappa = \hbar/\varepsilon_B\tau_0 = .88$ (GaAs).

and not by phonon emission. A competition between these two terms of efflux arises at a depth δ which can be found from the relation

$$\frac{1}{\tau_F} = \frac{1}{\tau_0}[1 - f_{Te}(\varepsilon - \hbar\Omega_0)], \quad \varepsilon = \hbar\Omega_0 + \delta T_e,$$

while in the phonon emission time we also take into account the Pauli principle. It is found that if this equation can be satisfied, i.e., if $0 < \delta < \beta$, we obtain the expression $\delta = \ln(\tilde{\lambda} e^\beta \ln\beta)$ for δ.

In fact, in the case of weak *ee*-scattering the distribution is depleted in the vicinity of threshold compared to $f_{Te}(\varepsilon)$. The characteristic scale of variation of the distribution coincides with T_e. Electrons which are ejected into the active region by *ee*-scattering immediately emit optical phonons. In the case of strong degeneracy the characteristic energy transfers for *ee*-scattering occupy the band $T_e \ll |\omega| \ll \varepsilon_F$. The depletion of the distribution is small at threshold; it starts at a depth $\delta = \beta - \ln[(\tilde{\lambda} \ln\beta)^{-1} - 1]$ if $\tilde{\lambda} \ln\beta \ll 1$ or $\beta \sim \delta$ if $\tilde{\lambda} \ln\beta \sim 1$, or $\delta \sim \lambda^2$ if $\tilde{\lambda} \ln\beta > 1$.[5] The scale over which the depletion occurs equals T_e.

The solution for the distribution function in the case of the strong degeneracy (21) contains a discontinuity for $\varepsilon = \hbar\Omega_0$. If $\tilde{\lambda} e^\beta \ln\beta \gg 1$, this discontinuity is small in amplitude; it arises because of removal in the kernel of energy transfers $|\omega| \ll T_e$. Therefore the discontinuity is actually washed out over the scale $|\varepsilon - \hbar\Omega_0| \ll T_e$. This has no effect on calculation of the rate of energy loss.

Let us now clarify the dependence of the rate of energy loss on the density of the gas. Degeneracy is approached when $\beta \sim 1$; therefore, $\lambda, \tilde{\lambda} \ll 1$. The parametric dependence of $\lambda(N)$, $\tilde{\lambda}(N)$ on $\beta(N)$ is illustrated schematically by the dotted line in Fig. 1, which passes through the regions under investigation. Therefore Eqs. (18), (19), and (22), which have a region of overlap, describe the rate of energy loss for any N, so long as $T_e, \varepsilon_F \ll \hbar\Omega_0$. In Fig. 2 we show the results of a numerical calculation of $\Phi(\lambda)$, $\Phi(\tilde{\lambda})$. It is clear that degeneracy enhances the rate of energy loss. It is easy to prove that this is a consequence of the condition $\beta/\lambda \sim x(\hbar\Omega_0/T_e)^{1/2} \gg 1$. The enhancement of the relaxation for $\tilde{\lambda} < 1$ can be explained qualitatively as follows: for emission of an average electron to pass along the energy axis from ε_F to $\hbar\Omega_0 + \delta T_e$, i.e., a smaller distance than for the nondegenerate case.

The quantities Φ^{-1}, Φ_β^{-1} characterize the increase in the effective time for optical phonon emission for the experimental situation described by Eq. (1). However, numerically Φ^{-1}, Φ_β^{-1} are not large. The fact is that relaxation by optical phonons plays a dominant role in the balance equation for $T_e > 30$ K (GaAs). Therefore $\hbar\Omega_0/T_e < 14$ and all the values of T_e, ε_F, $\hbar\Omega_0$ are numerically quite close.

In Ref. 2 measurements were carried out for $N = 3.9 \times 10^{11}$ cm^{-2}, $\varepsilon_F = 165$ K and $T_e = 65$–165 K. Using the formulas of the present paper and Ref. 5, we obtain a slowing down which does not exceed 2. In Ref. 3, the measurements were carried out for $N_1 = 1.3 \times 10^{13}$ cm^{-2}, $N_2 = 1.5 \times 10^{11}$ cm^{-2} and $N_3 = 2.3 \times 10^9$ cm^{-2}. We investigate the cases with $N_{2,3}$, since for the case N_1 we have $\varepsilon_F > \hbar\Omega_0$. In the case N_2 we have $\varepsilon_F \sim T_e \sim 70$ K and a slowing down $\Phi_\beta^{-1} \approx 2$. In the case of N_3 the slowing down is large, $\Phi^{-1} \approx 70$. The authors of Ref. 3 found a slowing down equal to 60 for the cases $N_{1,2}$, while in the case of N_3 they were unable to measure any slowing down at all.

Thus, degeneracy apparently must be excluded from the possible reasons for slowing down of the energy relaxation for optical phonons.[1–4]

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