Scattering mechanisms for electrons and phonons by Abrikosov vortex filaments in vanadium

N. A. Red'ko and B. K. Chakal'skii

A. F. Ioffe Physico-Technical Institute, USSR Academy of Sciences, and Physics Institute of the Daghestan Affiliate of the USSR Academy of Sciences

(Submitted 17 July 1986)
Zh. Eksp. Teor. Fiz. **92**, 1090–1098 (March 1987)

We have investigated the thermal conductivity of two samples of vanadium in the superconducting and mixed states, along with the electrical resistivity ratios $\rho_{300\,K}/\rho_{4.2\,K}$ for these samples, which equal 14 and 14.4. From comparison of theory and experiment, we have identified the lattice component of the thermal conductivity for the "dirtier" sample in the superconducting state. Analysis of the behavior of the lattice thermal conductivity in a magnetic field for the mixed state near H_{c1} allows us to determine the effective scattering cross-section of phonons by vortex filaments, which turns out to be the same order of magnitude as the scattering cross-section of electronic excitations by vortex filaments.

Investigation of the thermal conductivities of type II superconductors (included among these are the superconductoring elements—i.e., niobium and vanadium—and superconducting alloys) is especially interesting, because with its help we can study a phenomenon which has not been throughly investigated—the interaction of heat carriers (electrons, phonons) with the elementary magnetic-field quanta (Abrikosov vortex filaments) in mixed-state superconductors. We present here a fairly detailed investigation of the thermal conductivity of pure vanadium samples in the mixed state, and an analysis of the interaction of electrons with Abrikosov filaments near the critical magnetic fields H_{c1} and H_{c2},[1] In metals which are in the superconducting state, the heat carriers can be electronic excitations or phonons. The lattice component of the thermal conductivity in a pure metal at temperatures close to T_c (the superconducting transition temperature) is small compared to the electronic component. As the temperature decreases, the electronic component falls sharply because of the exponential decrease in the number of electronic excitations, and the lattice component can exceed it. Thus, for pure vanadium the lattice component of the thermal conductivity exceeds the electronic component for $T < 1.5$ K; this result was also obtained by the authors of Ref. 2. In order to observe the effect of the lattice component of the thermal conductivity in vanadium in the superconducting state at temperatures above 1.5 K, it is necessary to decrease the magnitude of the electronic component of the thermal conductivity down to that of the lattice at these temperatures. We accomplished this by introducing impurities into the sample, using the fact that the electronic thermal conductivity is proportional to the mean free path, while the latter decreases in inverse proportion to the introduced impurity concentration. In our investigations we used a bulk crystal, so as to decrease the scattering of phonons by the sample boundaries.

In Ref. 2, the lattice component of the thermal conductivity of vanadium was extracted in samples of various purities for $T \leqslant 1.8$ K; however, there was no investigation in Ref. 2 of the thermal conductivity of vanadium in the mixed state. In this paper we present measurements of the thermal conductivity of vanadium in a magnetic field for $T < T_c$ on a sample in which the lattice component of the thermal conductivity was comparable to the electronic component for $T < 3$ K; in addition, we will discuss the interaction of phonons with Abrikosov vortex filaments.

The investigation was carried out on two vanadium samples with resistivity ratios $\rho_{300\,K}/\rho_{4.2\,K}$ of 44 and 14.4, and residual resistivities equal to 0.492×10^{-6} and 1.60×10^{-6} Ω-cm, respectively. The first sample was polycrystalline and coarse-grained, with $T_c = 5.38$ K; the half-width of the energy gap Δ_0 at the Fermi surface ($T = 0$ K) equalled 9.5 ± 0.5 K, while the electron mean free path $l = 1.17 \times 10^{-5}$ cm; the sample dimensions were $\phi = 1.4$ mm and $h = 40$ mm. The second sample was a single crystal with $T_c = 5.06$ K, $\Delta_0 = 8.6 \pm 0.5$ K, $l_0 = 3.61 \times 10^{-6}$ cm, and dimensions $4 \times 4 \times 40$ mm^3. The thermal conductivity was measured by driving a steady-state heat current through the sample under study in the temperature interval 1.8 to 15 K; for $T < T_c$ it was measured by the method described in Ref. 1 while the samples were subjected to transverse and longitudinal magnetic fields.

EXPERIMENTAL RESULTS AND DISCUSSION

1. The temperature dependence of the thermal conductivity of the samples under study is presented in Fig. 1; for $T < T_c$ curves 1, 2 correspond to the normal state while curves 1', 2' correspond to the superconducting state. In order to restore the normal state for $T < T_c$, we used a longitudinal magnetic field H which exceeded H_{c2} (the critical magnetic field at which volume superconductivity is destroyed).

Our conclusions about the electronic character of the thermal conductivity of samples 1 and 2 in the normal state are based on the fact that the Lorentz number does not exceed its Sommerfeld value $L_0 = 2.45 \times 10^{-8}$ V^2/K^2. The temperature dependence of the thermal conductivity in the normal state for both samples was the same: $\varkappa^n(T) \propto T$, which indicates that for $T < 10$ K electrons are scattered by impurities and lattice defects. The thermal conductivity in the superconducting state, both for sample 1 and for the pure sample investigated in Ref. 1, is well-described by the Geilikman formula,[3] which takes into account the scattering of electronic excitations by point defects:

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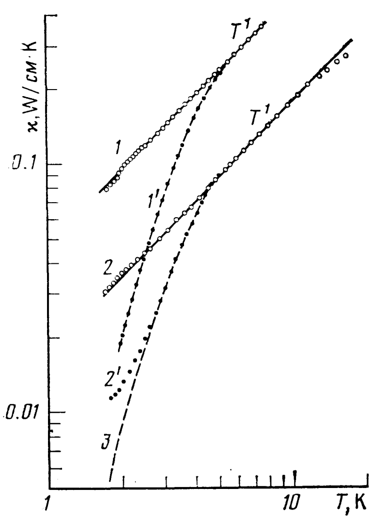

FIG. 1. Dependence of the thermal conductivity of vanadium on temperature for samples 1 and 2. Curves 1, 2 and 1', 2' are for the normal and superconducting states respectively for samples 1 and 2. Curve 3 and the dashed curve 1' are calculations of the electronic thermal conductivity in the superconducting state using Eq. (1).

$$\varkappa^s=\varkappa^n\frac{6}{\pi^2}\left\{\frac{b^2}{e^b+1}+2\sum_{s=1}^{\infty}\frac{(-1)^{s+1}}{s}e^{-sb}+2b\ln(1+e^{-b})\right\},\tag{1}$$

where

$$b=\frac{\Delta(T)}{T}=\frac{f(t)}{t}\frac{\Delta_0}{T_c},\quad t=\frac{T}{T_c}.$$

The temperature dependence of the energy gap $\Delta(T)$ which appears at the Fermi surface in the superconducting state is calculated using $f(t)$ given in Ref. 4. Formula (1) is correct for superconductors with weak electron-phonon coupling; as was shown in Ref. 1, vanadium belongs to this class. The calculated curve (dashed curve 1') is presented in Fig. 1.

In sample 2, along with the electronic component of the thermal conductivity there is present a lattice component for $T<0.6T_c$. It was extracted from experimental data by subtracting off the electronic component, calculated using Eq. (1) for $\Delta_0/T_c=1.692$, and coincides with the experimental value of the thermal conductivity for $T=(0.6\sim 1)T_c$. Below these temperatures we observed a deviation of the experimental values from the calculated ones in the interval 1.8 to 3 K, which increased with decreasing temperature. In Fig. 2 we present the lattice component of the thermal conductivity $\varkappa_l^s$ in the temperature range 1.8 to 3 K as extracted from sample 2. In this figure we show the data on $\varkappa_l^s$ for $T\leqslant 1.8$ K taken from Ref. 2, according to which the lattice thermal conductivity has a maximum in its temperature dependence near $T\approx 1$ K; below this maximum it passes over to a dependence $\varkappa_l^s\propto T^3$. In order to compare the experimental data on the temperature dependence of the lattice thermal conductivity of vanadium to the right of this maximum ($T>1$ K) with theory, we used the formula of Geilikman and Kresin,[5] in which they include the interaction of phonons with electronic excitations of the superconductor. The formula has the following form:

$$\varkappa_l^s=1.13\cdot10^{-6}T^2\left[\int_0^{2b}\frac{x^4e^x\,dx}{(e^x-1)^2[2x-2\ln\{(e^{b+x}+1)(e^b+1)^{-1}\}]}+\int_{2b}^{\infty}\frac{x^4e^x\,dx}{(e^x-1)^2[x+2b-2\ln\{(e^{b+x}+1)(e^{x-b}+1)^{-1}\}]}\right],\tag{2}$$

where $b=\Delta(T)/T$. Calculations carried out using Eq. (2) for the dependence of the lattice thermal conductivity on temperature for $\Delta_0/T=1.692$ satisfactorily describe both our experimental data on $\varkappa_l^s$ and the data of Ref. 2, which are presented in Fig. 2. It should be noted that the value of $\varkappa_l^s$ grows as the temperature decreases, in contrast to the lattice thermal conductivity of a normal metal, where $\varkappa_l^n$ falls as the temperature decreases Eq. (2) with $\Delta=0$ leads to $\varkappa_l^n\propto T^2$). This peculiarity in superconductors is related to an increase in the mean free path of phonons due to the exponential decrease in the number of electronic excitations as the temperature decreases.

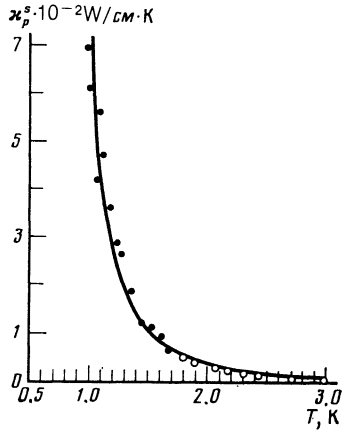

FIG. 2. Temperature dependence of the lattice component of the thermal conductivity of vanadium, separated out from the superconducting state: ○ indicates the result of the present paper, ● indicates data from Ref. 2. The continuous curve is a calculation using Eq. (2) for $\Delta_0/T_c=1.692$.

2. The presence of a lattice component in the thermal conductivity along with the electronic component for sample 2 in the superconducting state when $T<3$ K gives rise to peculiarities in the field dependence of the thermal conductivity in the mixed state compared to the field dependences for sample 1, in which the electronic component dominates in the thermal conductivity.

In Figs. 3 and 4 we present the dependence on magnetic field of the thermal conductivity of samples 1 and 2 both for transverse- and longitudinally-directed fields. When sample 1 is in the superconducting state the electronic component of the thermal conductivity dominates within the temperature interval which we investigated, as it does in a pure sample of vanadium,[1] as we noted earlier. The behavior of the thermal conductivity as a function of magnetic field for sample 1 when $T<T_c$ shown in Fig. 3 can be explained using the same terms as those used to describe a pure sample of vanadium.[1] The thermal conductivity of vanadium in the Meissner-effect region $H<H_{c1}$ (where H_{c1} is the lower critical field), in which the magnetic field does not penetrate the metal, is independent of the magnetic field. For $H>H_{c1}$ as the field increases $\varkappa(H)$ at first decreases sharply owing to the increased electron-excitation scattering by magnetic fila-

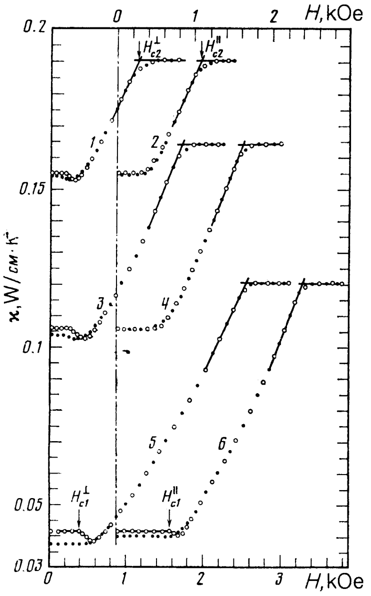

FIG. 3. Behavior of the thermal conductivity of vanadium (sample 1) in a magnetic field: curves 1, 3, 5 are for a transverse field ($\mathbf{H}\perp\nabla T$); curves 2, 4, 6 are for a longitudinal field ($\mathbf{H}\|\nabla T$). Curves 1, 2 are for $T/T_c = 0.76$; 3, 4 are for $T/T_c = 0.635$; 5, 6 are for $T/T_c = 0.454$. O is the forward trace; ● is the reverse trace.

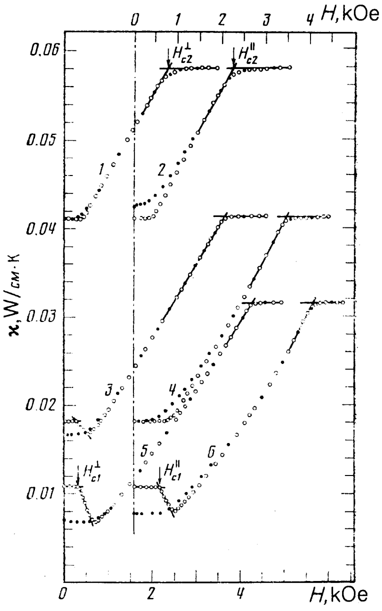

FIG. 4. Behavior of the thermal conductivity of vanadium (sample 2) in a magnetic field: curves 1, 3, 5 are for a transverse field ($\mathbf{H}\perp\nabla T$); curves 2, 4, 6 are for a longitudinal field ($\mathbf{H}\|\nabla T$). Curves 1, 2 are for $T/T_c = 0.68$; curves 3, 4 are for $T/T_c = 0.48$; curves 5, 6 are for $T/T_c = 0.364$. O is the the forward trace, ● the reverse trace.

ments, whose density increases with the field H. After passing through a minimum, $\varkappa(H)$ begins to increase for two reasons: first, because of the increasing mean free path of the electronic excitations, which in turn is due to the fact that at high magnetic filament densities increasing the density further makes the sample more homogeneous on the average; second, because of the fact that as H approaches H_{c2} the number of electronic excitations participating in the thermal balance also increases. For $H > H_{c2}$, the quantity $\varkappa(H)$ does not depend on the field. Increasing the number of defects and impurities in the vanadium samples yields an increase in the thermal resistance of the electronic excitations as the latter scatter off the former; this increased thermal resistance may in fact exceed the thermal resistance due to the magnetic filaments. The thermal resistivities of electronic excitations due to impurities and defects which we identified from our experimental data on the thermal conductivity at $T = 2.43$ K equalled 0.7, 24 and 59 cm- K/W (for pure vanadium[1] and our samples 1 and 2 respectively). At the same time, the thermal resistance of the electronic excitations due to scattering by magnetic filaments at $T = 2.43$ K for sample 1 was the same as for the pure vanadium sample,[1] and equalled 2.1 cm- K/W. These data led us to the conclusion that in the pure sample of vanadium the electronic excitations are predominantly scattered by magnetic filaments, while in sample 1 and particularly in sample 2 the electronic excitations are predominantly scattered by impurities and defects. This scattering also explains the absence of "dips" in the field dependence of $\varkappa(H)$ in sample 2 for temperatures above $T = 3$ K (curves 1, 2 of Fig. 4) where the electronic component of the thermal conductivity exceeds the lattice component. Dips in the $\varkappa(H)$ due to scattering of the electronic excitations by vortex filaments are probably undetectable for temperatures below $T = 3$ K. However, there are dips of considerable magnitude in the field dependence of the thermal conductivity at $T < 3$ K (curves 3, 5, 6 of Fig. 4) where the lattice component of the thermal conductivity is present along with the electronic component; these can be attributed to scattering of phonons by vortex filaments. The behavior of the thermal conductivity with magnetic field in sample 2 for $T < 3$ K comes about in the following way: the thermal conductivity in the Meissner region $H < H_{c1}$ does not depend on H. For $H > H_{c1}$ (i.e., in the mixed state), as the field increases, $\varkappa(H)$ first decreases sharply due to the increased scattering of phonons by vortex filaments which permeate the superconductor. Under these circumstances, the minimal value of the thermal conductivity in a transverse magnetic field coincides with the value of the electronic component of the thermal conductivity in the superconducting state (curve 3 of Fig. 1), which points to the preponderance of scattering of phonons by vortex filaments. Subsequently, once the thermal conductivity $\varkappa(H)$ passes through a minimum it begins to increase, both because the mean free path of electronic excitations increases due to the increased average homogeneity of the sample as the density of magnetic filaments is increased and because as H approaches H_{c2} the number of electronic excitations increases, which causes the electronic component of the thermal conductivity to increase toward that of the normal state at the given temperature (curve 2 of Fig. 2). The resulting thermal resistance due to phonons scattered by vortex filaments in sample 2 for $T = 2.43$ K equals 4.4 cm-K/W, and as the temperature is lowered to 1.84 K it increases to a value of ~ 54 cm K/W.

It should be noted that both sample 1 and sample 2 show substantial hysteresis in the form of $\varkappa(H)$ for the temperature at which the dips are observed. When the course of variation of the magnetic field is reversed—both for the $\mathbf{H}\perp\nabla T$ and $\mathbf{H}\|\nabla T$ orientations of the magnetic field—the curve does not retrace its initial path, but rather stays at the level of its minimum value (Figs. 3, 4). This experimental fact allows us to conclude that the magnitude of the magnetic field trapped in the sample equals $H_{\rm rem} \approx H_{\rm min}$. As a result of this, the heat carriers (electronic excitations in sample 1, phonons in sample 2) are scattered by magnetic filaments even in external magnetic fields less that $H_{\min}$.

When sample 2 is placed in a longitudinal magnetic field at $T > 3$ K, at which temperature the phonon component of the thermal conductivity is small compared to the electronic component, the magnitude of the total thermal conductivity when traced in reverse exceeds its value when traced forward when the magnetic field is close to H_{c1} (see curve 2 of Fig. 4). This is apparently related to the fact that electronic excitations within pinned nonequilibrium Abrikosov vortex filaments begin to play a significant role in the thermal conductivity of the single-crystal sample, because the coherence length ($\xi = 450$ Å; Ref. 6) is comparable to the mean free path of the electrons ($l_0 = 3.6\times 10^{-6}$ cm). At temperatures below $T = 3$ K, a lattice component along with the electronic component to the thermal conductivity is also present in this sample. Therefore, the influence on the overall thermal conductivity of the electronic excitations within the pinned magnetic filament when the field is traced backwards ($\mathbf{H}\|\nabla T$) near H_{c1} is decreased due to the presence of the phonon component, which includes additional (compared to the forward trace) scattering by the pinned vortex filaments (curves 5, 6 of Fig. 4).

In a transverse magnetic field the curves obtained by tracing the field backwards (to the minimum value of the thermal conductivity $\varkappa(H_{\min})$) for sample 2 in the temperature range investigated practically coincide with the forward-traced curves (curves 1, 3, 5 of Fig. 4). This is indicative of the weak influence of the normal excitations contained in the volume of the pinned vortex filaments on the metal's thermal conductivity as a whole, because the directions of the heat current and the magnetic filaments are mutually orthogonal.

3. At the present time no unique theory exists describing the behavior of the thermal conductivity of superconductors in the mixed state; however, there are theories describing $\varkappa(H)$ near H_{c1} and H_{c2}. An analysis of the thermal conductivity of vanadium near H_{c1} for a pure sample for $T > 2$ K, in which the electronic excitations are responsible for the heat transfer ($l_0/\xi \gg 1$), was presented by us[1] using the theory of Cleary. The effective scattering width of electronic excitations by Abrikosov vortex filaments is found to equal 6×10^{-7} cm.

Because of the absence of a theory describing the behavior of the lattice thermal conductivity near H_{c1}, we present an estimate of the scattering cross-section of phonons by vortex filaments based on experimental data obtained for the thermal conductivity of sample 2. In our analysis of the function $\varkappa(H)$, we will neglect scattering of electronic excitations by the vortex filaments because their contribution, as was pointed out earlier, is indeed negligible. Therefore the change in the overall thermal conductivity in a magnetic field near H_{c1} originates as a result of phonon scattering by Abrikosov vortex filaments, and equals

$$\Delta\varkappa(H) = \varkappa_l^s(H) - \varkappa_l^s, \tag{3}$$

where $\varkappa_l^s(H)$, $\varkappa_l^s$ are the lattice component thermal conductivities in a magnetic field H and for $H = 0$, respectively. For the lattice thermal conductivity near H_{c1} we can categorize the scattering in terms of two mechanisms—scattering by vortex filaments and scattering by all other contributions. Then the frequency for scattering by phonons can be cast in the form

$$1/\tau_l(H) = 1/\tau_v + 1/\tau_0, \tag{4}$$

where the scattering frequency of phonons by vortex filaments $1/\tau_v$ can be expressed in terms of the scattering cross-section σ, the density of vortex filaments N and the velocity of sound v in vanadium

$$1/\tau_v = N\sigma v. \tag{5}$$

Using (4), (5), we substitute the value of the scattering relaxation time $\tau_1(H)$ into Eq. (3):

$$\Delta\varkappa(H) = \varkappa_l^s(H) - \varkappa_l^s = \frac{1}{3}Cv^2\tau_p(H) - \frac{1}{3}Cv^2\tau_0 = \varkappa_l^s\left(\frac{1}{1+N\sigma v\tau_0} - 1\right), \tag{6}$$

where $C = \alpha T^3 = 2.6\times 10^{-5}$ J/cm^3-K is the specific heat of the vanadium lattice[6] for $T = 1.84$ K. From expression (6), using $\tau_0 = 3\varkappa_1^s/Cv^2$, we find the scattering cross-section

$$\sigma = \frac{Cv}{3N\varkappa_l^s}\left(\frac{\varkappa_l^s}{\Delta\varkappa(H) + \varkappa_l^s} - 1\right). \tag{7}$$

We determine the magnitude of the cross-section for phonon scattering by Abrikosov vortex filaments by substituting the experimental data for sample 2 at $T = 1.84$ K presented in Fig. 4 (curves 5, 6) into formula (7): for $\mathbf{H}\perp\nabla T$, $\varkappa_1^s = 0.00492$ W/cm-K, $\Delta\varkappa(H) = -0.00345$ W/cm-K, $\Delta B = 140$ gauss. The density of vortex filaments is determined from the dependence $N = \Delta B/\Phi_0$ where $\Phi_0 = 2\times 10^{-7}$ gauss-cm^2 is a quantum of magnetic flux. Averaging the velocity of sound in vanadium[7] gives $v = 3.2\times 10^5$ cm/sec. The magnitude of the scattering cross section for phonons by Abrikosov vortex filaments for $\mathbf{H}\perp\nabla T$ and $\mathbf{H}\|\nabla T$ equal $\sigma_\perp = 9.2\times 10^{-7}$ cm and $\sigma_\| = 4.5\times 10^{-7}$ cm respectively.

Let us compare our experimental estimate of the cross-section for phonon scattering by vortex filaments with theoretical estimates. In doing this we will suppose that the mean free path for phonons is less than twice the coherence length. Then

$$l_\varphi = l_\varphi^n/a, \tag{8}$$

where l_φ^n is the phonon mean free path in a normal metal, $a = N\xi^2$ is the fraction of normal component where N is the density of vortex filaments. As is well-known,[8]

$$l_\varphi^n \sim 1/\Gamma \sim v_F/\omega = \hbar v_F/\hbar\omega \approx \hbar v_F/k_0 T, \tag{9}$$

where Γ is the amplification coefficient for sound waves, v_F is the Fermi velocity of electrons, ω is the frequency of the amplified phonon. As a result of this conversion we obtain

$$l_\varphi \approx \hbar v_F/k_0 T N\xi^2. \tag{10}$$

Because $1/l_\varphi = N\sigma$,

$$\sigma = 1/Nl_\varphi \approx k_0 T\xi^2/\hbar v_F \approx 3\cdot 10^{-7}\ \text{cm}.$$

This estimate of the magnitude of the scattering cross-section of phonons by vortex filaments is in fairly good agreement with the experimental data presented above.

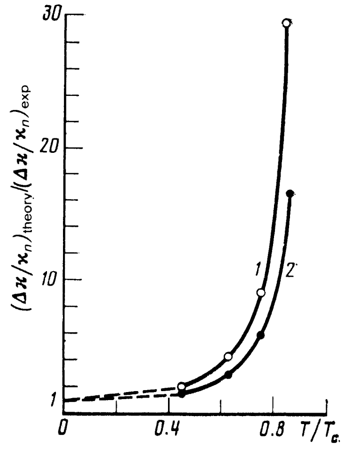

FIG. 5. Agreement of experimental values of $\Delta\varkappa/\varkappa^n$ for sample 1 with calculations which were carried out according to the theory in Ref. 10 for $\mu = 0.2$ at various temperatures: curve 1 is $\mathbf{H}\|\nabla T$; curve 2 is $\mathbf{H}\perp\nabla T$.

4. We now present an analysis of the thermal conductivity of vanadium near H_{c2}, where the primary contribution to the thermal conductivity is due to the electronic excitations for the temperature range 1.8 K $< T < T_c$ under investigation. In the mixed state near H_{c2}, the vortex filaments no longer are isolated but overlap.[9] The behavior of the thermal conductivity in this case is determined both by impurity scattering of the electronic excitations and by scattering due to spatial inhomogeneities in the order parameter Ψ. The function $\varkappa(H)$ near H_{c2} in the theory of Houghton and Maki,[10] including both these scattering mechanisms, depends on the dimensionless parameter

$$\mu=2\pi^{1/2}k_c l(\Psi/\hbar k_c v_F)^2,$$

where $k_c = (2eH/c\hbar)^{1/2}$ is the inverse lattice vector of the lattice of magnetic Abrikosov filaments, while

$$\Psi^2=\frac{1}{2\pi N}\frac{H_{c2}-H}{1.16(2\varkappa_2^2-1)+D}\left(H_{c2}-\frac{t}{2}\frac{dH_{c2}}{dt}\right), \qquad (11)$$

$\varkappa_2$ is the second generalized parameter of the Ginzburg-Landau theory, introduced by Maki,[11] D is the demagnetization factor. When the conditions $\mu < 1$ and $T = T/T_c \to 0$ are fulfilled, the variation in the thermal conductivity near H_{c2} ($H \ll H_{c2}$) for transverse and longitudinal magnetic fields is described by the relations

$$\begin{aligned}\Delta\varkappa^{\perp}/\varkappa^n &= \left(\frac{\varkappa^n-\varkappa(H)}{\varkappa^n}\right)_{\perp} = 3\mu\left[\mu^2 I_1+\left(\frac{\pi}{4}-\mu\right)\right],\\ \Delta\varkappa^{\|}/\varkappa^n &= \left(\frac{\varkappa^n-\varkappa(H)}{\varkappa^n}\right)_{\|} = 6\mu\left[(1-\mu^2)I_1-\left(\frac{\pi}{4}-\mu\right)\right],\end{aligned} \qquad (12)$$

where

$$I_1=\int_0^{\pi/2}\cos\theta\, d\theta/(\cos\theta+\mu)$$

is a tabulated integral.

In Figure 5 we show the relations between the theoretical estimates and the experimental values of these quantities for various T from sample 1. It is clear that these relations go to unity as $t\to 0$, i.e., in the approximation that the conditions for applicability of the Houghton-Maki theory hold; this indicates that theory and experiment are in adequate agreement as $t\to 0$. An analogous result was obtained from analysis of the thermal conductivity near H_{c2} for sample 2 as well.

In conclusion, we note that we have succeeded in observing for the first time the dependence of the thermal conductivity of vanadium with a large concentration of impurities on magnetic field near H_{c1}, where it is related to scattering of phonons by Abrikosov vortex filaments, and we have estimated the magnitude of the scattering cross-section of phonons by the vortices. Our preliminary reports of all this are contained in Ref. 12.

The experimentally observed difference in behavior of the thermal conductivity in a longitudinal magnetic field for forward and reverse traces of the field above $\varkappa$ ($H_{\min}$) can be related to the sizable contribution to the thermal conductivity from electronic excitations within pinned vortex filaments for samples of vanadium in which $l_0 \ll \xi$.

We are grateful to Yu. M. Gal'perin for many discussions, and for an estimate of the mean free path for phonons by vortex filaments, and also to V. M. Azhazha for supplying the vanadium samples.

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Translated by F. J. Crowne