Emission of radiation by high-energy particles in oriented single crystals

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A theory is developed of the emission of a photon by a charged high-energy particle in an oriented crystal. At low angles of incidence the theory describes the emission of radiation in a continuous potential of a crystal axis or plane, but at sufficiently high angles it reduces to the standard theory of coherent bremsstrahlung. A modification of the theory of coherent bremsstrahlung is obtained for a wider range of angles of incidence. The adopted approach makes it possible to find the orientational dependence of the total radiation intensity for any particle energy and, at sufficiently high energies when the synchrotron radiation description becomes valid at low angles of incidence, to calculate any characteristics of the radiation for an arbitrary angle of incidence.

1. INTRODUCTION

The emission of a photon by a charged particle accompanied by creation of an electron-positron pair by a photon in the case when the initial particle moves at a low angle ϑ_0 relative to the direction of an axis or a plane in a single crystal is quite different from the corresponding process in an amorphous medium. Processes of this kind are being actively discussed at present. Recently we developed an approach to the description of the creation of pairs by high-energy photons in single crystals which is valid at any energies and angles of incidence of the photon.[1-3] This approach is based on a general semiclassical formalism[1)] for the description of phenomena in external fields (Chap. 3 in Ref. 4). A similar approach will be applied below to the problem of radiation emission. Several new aspects have to be faced and, in particular, it is necessary to allow for a redistribution of a flux of charged particles in a crystal.

We shall consider qualitative features of the emission of radiation in an oriented single crystal. It is well known (see, for example, Sec. 1 in Ref. 4) that the features of this process depend strongly on the ratio of the characteristic emission angle $\vartheta_\gamma \equiv m/\varepsilon = 1/\gamma$ (m is the electron mass[2)] and ε is the electron energy) to the angle of deflection of a particle on its trajectory $\Theta^2 \sim \langle(\Delta\mathbf{v})^2\rangle = \langle\mathbf{v}^2\rangle - \langle\mathbf{v}^2\rangle^2$, where $\langle\ldots\rangle$ denotes averaging with respect to time. The relevant parameter ρ was introduced in Ref. 7 (see also Refs. 3 and 8):

$$\rho=2\gamma^2\langle(\Delta\mathbf{v})^2\rangle. \tag{1.1}$$

We recall that for the values of the parameter in the range $\rho \ll 1$ the radiation is of dipole nature and it appears in a time of the order of the period of motion, whereas for $\rho \gg 1$ it is of synchrotron nature (for frequencies contributing to the intensity) and it is emitted from a small part of the trajectory.

The motion of a particle and, consequently, the parameter ρ depend on the angle ϑ_0 at which a particle is incident on a crystal compared with the characteristic channeling angle $\vartheta_c \equiv (2V_0/\varepsilon)^{1/2}$, where V_0 is the scale of a continuous potential of an axis or a plane relative to which the angle ϑ_0 is defined. At angles of incidence in the range $\vartheta_0 \lesssim \vartheta_c$, electrons falling on a crystal are captured by channels or low above-barrier states, whereas for $\vartheta_0 \gg \vartheta_c$ the incident particles move well above the barrier. In the latter case we can describe the motion using the approximation of a rectilinear trajectory, for which we find from Eq. (1) the following estimate

$$\rho(\vartheta_0)\approx(2V_0/m\vartheta_0)^2. \tag{1.2}$$

For angles of incidence in the range $\vartheta_0 \lesssim \vartheta_c$ the transverse (relative to an axis or a plane) velocity of a particle is $v_\perp \lesssim \vartheta_c$ and the parameter obeys $\rho \lesssim \rho_c$, where

$$\rho_c=2V_0\varepsilon/m^2. \tag{1.3}$$

It is clear from Eq. (1.2) that the problem has another characteristic angle $\vartheta_v \equiv V_0/m$, where $\rho_c = (2\vartheta_v/\vartheta_c)^2$ (the same characteristic angle occurs also in the problem of pair creation by photons in single crystals[3]).

It is shown in our earlier paper[8] that for a given frequency of motion ω_0 the characteristic frequencies of the emitted photons are described by

$$u\equiv\frac{\omega}{\varepsilon-\omega}\sim\frac{2\omega_0\varepsilon}{m^2}(1+\rho)^{1/2}. \tag{1.4}$$

When a charged particle moves near an axis (plane) of a crystal we can assume approximately that $\omega_0 \sim v_\perp/a_s \sim V_0/ma_s\rho^{1/2}$, where a_s is the size of the region of action of the continuous potential. It then follows from Eq. (1.4) that $u \sim (1+1/\rho)^{1/2}\chi_s$, where the parameter

$$\chi_s=V_0\varepsilon/m^3a_s \tag{1.5}$$

represents quantum effects in the course of emission of radiation. In fact, our estimate (valid up to $\chi_s \sim 1$) shows that the ratio $\omega/\varepsilon \ll 1$ for characteristic emission frequencies, which allows us to ignore the recoil and to use the classical theory of emission of radiation, may be satisfied only if $\chi_s \ll 1$. However, in this case the validity of the classical description of the emission of radiation breaks down for $\rho^{1/2} \lesssim \chi_s$, i.e., for $\vartheta_0 \gtrsim V_0/m\chi_s = ma_s/\gamma \equiv \vartheta_m$. It should be pointed out that a typical value for crystals is $ma_s \sim 10^2 \gg 1$.

At sufficiently high angles of incidence ϑ_0 we can apply the theory of coherent bremsstrahlung—CBS (see Chaps 1 and 2 in Ref. 9 and Chap. 8 in Ref. 10; the criterion of validity of the theory of CBS is discussed in Ref. 11), which represents essentially the Born approximation to the potential of a crystal and its validity requires (in our terms) that the condition of the emission of dipole radiation is satisfied simultaneously with the condition of validity of the rectilinear trajectory approximation.

We shall now consider the dependence of the emission pattern on the particle energy. At low energies (up to several megaelectron-volts in the axial case and up to several tens of megaelectron-volts in the planar case) the radiation is domi-

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nated by transitions between energy levels in the relevant potential well. We shall consider the range of energies when the number of these levels is high and the particle motion can be described classically.

As long as the condition $\rho_c \ll 1$ is satisfied (we then have $\vartheta_v \ll \vartheta_c \ll \vartheta_\gamma \ll \vartheta_m$) the radiation is of dipole nature for all angles of incidence of a particle on a crystal. The parameter χ is small; $\chi_s = \rho_c/2ma_s \ll \rho_c \ll 1$ and $\chi_s/\rho_c^{1/2} \ll 1$, i.e., the classical description of the radiation is valid and, although it does break down for angles $\vartheta_0 \sim \vartheta_m$, at these angles we can already apply the CBS theory which is then valid if $\vartheta_0 \gg \vartheta_c$. At higher energies the condition $\rho_c \sim 1$ begins to apply, i.e., we now have $\vartheta_v \sim \vartheta_c \sim \vartheta_\gamma \ll \vartheta_m$, but all that we have said so far about the nature of the emitted radiation remains valid except that at low angles of incidence $\vartheta_0 \lesssim \vartheta_c$ the radiation is no longer of the dipole nature. Finally, when $\rho_c \gg 1$, which corresponds to $\vartheta_\gamma \ll \vartheta_c \ll \vartheta_v$, at angles of incidence $\vartheta_0 \ll \vartheta_v$, the synchrotron radiation description applies ($\rho \gg 1$) and the CBS theory can be used in the opposite case when $\vartheta_0 \gg \vartheta_v$ [if $\vartheta_0 \sim \vartheta_v$, we find from Eq. (1.2) that $\rho \sim 1$ and the dipole emission condition is no longer obeyed]. At energies of this kind the parameter χ_s can no longer be regarded as small (for example, already for $\chi_s = 0.1$ the classically calculated total intensity in a constant field is approximately 1.5 times greater than the correct result) and it becomes essential to allow exactly for the quantum recoil in the course of emission of radiation.

We shall obtain one further estimate of the length of formation l_f for angles of incidence $\vartheta_0 \lesssim \vartheta_c$. Since $l_f \propto \varepsilon/m^2u$, we find from Eq. (1.4) that if $\rho_c \ll 1$, then $l_f \propto l_0 \rho_c^{1/2} \propto 1/\omega_0$, where $l_0 = a_s/\vartheta_v$ and the length l_f rises proportionally to $\varepsilon^{1/2}$; when the condition $\rho_c \sim 1$ begins to be satisfied, this rise ceases and as long as $u \sim \chi_s$, i.e., up to $\rho_c \sim 10^2$, the length of formation of the radiation remains practically constant: $l_f \sim l_0$; on further increase in the energy we have $u \sim 1$ and $l_f \propto \varepsilon/m^2$; at energies $\varepsilon \sim 10$ TeV or higher this law changes to a slower one: $l_f \propto l_0 \chi_s^{1/3}$. The value of l_0 is greater for lighter crystals and, for example, along the $\langle 111 \rangle$ axis it varies from 5.7×10^{-5} cm for diamond to 2.6×10^{-6} cm for tungsten. When the energy is $\varepsilon \sim 10$ TeV, the length of formation of radiation becomes of the same order of magnitude for different substances: $l_f \sim 10^{-3}$ cm. We shall assume that the thickness of a crystal L satisfies the condition $L \gg l_f$.

We shall assume that our crystal is thin, i.e., we shall ignore the change in the distribution function in the transverse phase space during the passage of a particle through the crystal. This approximation is justified if (for a discussion see Ref. 12) the thickness of the crystal is less than the characteristic dechanneling length and the characteristic energy loss length L_{ch}:

$$L_{ch}^{-1} = I(\varepsilon)/\varepsilon, \tag{1.6}$$

where $I(\varepsilon)$ is the total radiation intensity. An analysis (see Ref. 12 and Sec. 3 below) demonstrates that the range of thicknesses in which a crystal can be regarded as thin and the condition $L \gg l_f$ be still satisfied can be found at any energy. On the other hand, in the range of energies under discussion ($\rho_c \gg 1$), for a thick crystal satisfying the condition $\vartheta_0 \lesssim \vartheta_v$ there is always a specific electromagnetic shower (see Ref. 2). Then the description of the emission of radiation in terms of one particle is no longer satisfactory and it is necessary to solve the appropriate equations of the cascade theory in which the kernels are the expressions obtained below for the description of the emission of radiation and the expressions describing the creation of pairs by a photon obtained in Ref. 3.

In the majority of recent investigations only some aspects of the theory of emission of radiation in oriented single crystals have been discussed. Studies have been made of coherent bremsstrahlung and of the emission of radiation by particles in channels moving above a barrier, with most of the treatments dealing with the range $\rho_c \lesssim 1$. The quantum recoil during the emission of radiation has been usually ignored when dealing with the range $\rho_c \gg 1$.

We shall develop a theory of the emission of radiation by electrons and positrons in thin crystals for the range $\rho_c \gg 1$ ($\varepsilon \gg m^2/V_0$) and we shall allow for the quantum effects. The results obtained will then be valid for any angle of incidence ϑ_0. In the range $\vartheta_0 \ll \vartheta_v$ these results reduce to the theory of emission in axial fields in the synchrotron radiation limit (this range is analyzed in Refs. 1 and 13; see also Refs. 5 and 6). We shall also find the correction $(m\vartheta_0/V_0)^2$ which applies in this range (Sec. 3). For $\vartheta_0 \gtrsim \vartheta_v$, the general formulas yield an expression for the probability representing a modification of the standard theory of CBS, whereas for $\vartheta_0 \gg \vartheta_v$, we obtain the standard theory of CBS rather than its modification (Sec. 4). The general expression obtained for the description of the orientational dependence of the total intensity of the emitted radiation is found to be valid at any energies at which we can apply the semiclassical theory of the emission of radiation (i.e., it is valid when $\rho_c \lesssim 1$).

2. SEMICLASSICAL DESCRIPTION OF THE EMISSION OF A PHOTON BY A CHARGED PARTICLE IN A SINGLE CRYSTAL

The most satisfactory approach to the problem of emission of radiation by relativistic particles is the formalism utilizing the semiclassical operator method developed by two of the present authors (see Ref. 4), because it is valid for all types of external fields, including inhomogeneous and alternating fields. For the majority of quantum numbers of motion (corresponding to the semiclassical condition) this method can be used to go over from the exact quantum expressions by a series of transformations to quantities on a classical trajectory of a particle when the recoil due to the emission of a photon can be allowed for exactly. The general expression for the energy of the emitted radiation is (see § 10 in Ref. 4)

$$dE = e^2 \frac{d^3k}{(2\pi)^2} \left| \int dt\, R(t) e^{ik'x(t)} \right|^2, \tag{2.1}$$

where

$$R(t) = \varphi_f^+ [A(t) + i(\boldsymbol{\sigma}, \mathbf{B}(t))] \varphi_i,$$
$$A = \frac{\mathbf{e}^*\mathbf{p}(t)}{2(\varepsilon\varepsilon')^{1/2}} \left[\left(\frac{\varepsilon'+m}{\varepsilon+m} \right)^{1/2} + \left(\frac{\varepsilon+m}{\varepsilon'+m} \right)^{1/2} \right],$$
$$\mathbf{B} = \frac{1}{2(\varepsilon\varepsilon')^{1/2}} \left[\left(\frac{\varepsilon'+m}{\varepsilon+m} \right)^{1/2} [\mathbf{e}^*\mathbf{p}(t)] - \left(\frac{\varepsilon+m}{\varepsilon'+m} \right)^{1/2} [\mathbf{e}^*(\mathbf{p}(t)-\mathbf{k})] \right], \tag{2.2}$$
$$k'^\mu = k^\mu \varepsilon/\varepsilon', \quad \varepsilon' = \varepsilon - \omega, \quad k^\mu = (\omega, \mathbf{k}),$$

$\mathbf{e}$ is the polarization vector of the emitted photon, and φ_i and φ_f are spinors describing the initial and final spin states of the incident particle. Equations (2.1) and (2.2) describe all the characteristics of the emitted radiation, including the spin and polarization parameters. The formulas for unpolarized electrons, but allowing for the photon polarization, obtained from Eq. (2.1) can be found in Refs. 4 and 8. In this case, integrating over the angles of emission of a photon, we find that the spectral distribution of the emitted energy is

$$dE^{(j)}=\frac{i\alpha\omega\, d\omega}{2\pi}\int\frac{dt\, d\tau}{\tau-i0}e^{-iA_1}T^{(j)}, \tag{2.3}$$

where

$$\alpha=e^2=\frac{1}{137},\quad T^{(0)}=\frac{1}{\gamma^2}+\frac{\varphi(\varepsilon)}{4}(\Delta\mathbf{v}_2-\Delta\mathbf{v}_1)^2,$$

$$T^{(1)}=\left(1+\frac{e^{-iQ}-1}{iQ}\right)\frac{(\mathbf{v}_1\mathbf{a}_\perp)(\mathbf{a}[\mathbf{v}_2\mathbf{v}_0])+(\mathbf{v}_2\mathbf{a}_\perp)(\mathbf{a}[\mathbf{v}_1\mathbf{v}_0])}{\mathbf{a}_\perp^2}+\frac{\mathbf{a}[\mathbf{v}_0(\mathbf{v}_1+\mathbf{v}_2)]}{\tau},$$

$$T^{(2)}=\frac{i\varphi(\varepsilon)}{2}\left\{\mathbf{v}_2[\mathbf{v}_1\mathbf{v}_0]+\frac{(\mathbf{v}_2-\mathbf{v}_1)[\mathbf{v}_0\mathbf{a}]}{\tau}\right\}, \tag{2.4}$$

$$T^{(3)}=\left(1+\frac{e^{-iQ}-1}{iQ}\right)\frac{(\mathbf{v}_1\mathbf{a}_\perp)(\mathbf{v}_2\mathbf{a}_\perp)-(\mathbf{a}[\mathbf{v}_1\mathbf{v}_0])(\mathbf{a}[\mathbf{v}_2\mathbf{v}_0])}{\mathbf{a}_\perp^2}-\frac{1}{\gamma^2}-\frac{\Delta\mathbf{v}_1^2+\Delta\mathbf{v}_2^2}{2},$$

$$A_1=\frac{\omega\varepsilon\tau}{2\varepsilon'}\left[\frac{1}{\gamma^2}+\frac{1}{\tau}\int_{t_1}^{t_2}dt(\Delta\mathbf{v}(t))^2-\left(\frac{1}{\tau}\int_{t_1}^{t_2}dt\,\Delta\mathbf{v}(t)\right)^2\right].$$

Here,

$$\mathbf{v}_{1,2}\equiv\mathbf{v}(t_{1,2}),\quad t_{1,2}=t\mp\tau/2,\quad \varphi(\varepsilon)=\varepsilon/\varepsilon'+\varepsilon'/\varepsilon,$$

$$\mathbf{a}=\int_{t_1}^{t_2}dt\,\mathbf{v}(t),$$

$$\mathbf{a}_\perp=\mathbf{a}-\mathbf{v}_0(\mathbf{a}\mathbf{v}_0)/v_0^2,\quad Q=\omega\varepsilon\mathbf{a}_\perp^2/2\varepsilon'\tau.$$

The particle velocity can be represented in the form $\mathbf{v}(t)=\mathbf{v}_0+\Delta\mathbf{v}(t,\mathbf{v}_0)$, where $\mathbf{v}_0$ is the average velocity. The zero subscript refers to the expression which is summed over the photon polarizations. The polarization vectors are selected as in Ref. 8:

$$\mathbf{e}^{(1)}=[\mathbf{k}\mathbf{v}_0]/|[\mathbf{k}\mathbf{v}_0]|,\quad \mathbf{e}^{(2)}=[\mathbf{k}\mathbf{e}^{(1)}]/\omega.$$

The Stokes parameters $\xi^{(j)}$ are described by $\xi^{(j)}=dE^{(j)}/dE^{(0)}$. It should be noted that the expressions for A_1 and $T^{(0)}$ are not affected by the substitution $\Delta\mathbf{v}\to\Delta\mathbf{v}+\mathbf{b}$, where $\mathbf{b}$ is any vector independent of time.

We shall generally assume that a crystal is thin and the condition $\rho_c\gg1$ is satisfied. In this case the extremely difficult task of averaging Eqs. (2.1) and (2.3)—derived for a given trajectory—over all possible trajectories of a particle in a crystal simplifies radically. In fact, if $\rho_c\gg1$ then in the range where the trajectories are essentially rectilinear ($\vartheta_0\lesssim\vartheta_c$, $v_\perp\sim\vartheta_c$) the mechanism of emission of radiation is of the synchrotron nature and the characteristics of the radiation can be expressed in terms of the local parameters of motion. The averaging procedure can then be carried out simply if we know the distribution function in the transverse phase space $dN(\boldsymbol{\rho},\mathbf{v}_\perp)$, which for a thin crystal is defined directly by the initial conditions of incidence of particles on a crystal. For a given incidence angle ϑ_0 (which is the angle between the momentum of the incident particle and the crystal axis of the plane), we have $dN/N=d^3rF(\mathbf{r},\vartheta_0)/V$, where V is the volume of a crystal and N is the total number of particles. In the axial case the function $F(r,\vartheta_0)$ is of the form

$$F_{ax}(\boldsymbol{\rho},\vartheta_0)=\int\frac{d^2\rho_0}{S(\varepsilon_\perp(\boldsymbol{\rho}_0))}\theta(\varepsilon_\perp(\boldsymbol{\rho}_0)-U(\boldsymbol{\rho})), \tag{2.5}$$

where $U(\boldsymbol{\rho})$ is the continuous axial potential dependent on the transverse coordinate $\boldsymbol{\rho}$ normalized so that $U(\boldsymbol{\rho})=0$ at the boundary of a cell; U_0 is the depth of the potential well;

$$S(\varepsilon_\perp(\boldsymbol{\rho}_0))=\int d^2\rho\,\theta(\varepsilon_\perp(\boldsymbol{\rho}_0)-U(\boldsymbol{\rho})),$$

$$\varepsilon_\perp(\boldsymbol{\rho}_0)={}^1/_2\varepsilon\vartheta_0^2+U(\boldsymbol{\rho}_0);$$

$\theta(x)$ is the Heaviside function: $\theta(x)=0$ when $x<0$ and $\theta(x)=1$ when $x>0$. In the planar case, we have

$$F_{pl}(x,\vartheta_0)=\int\frac{dx_0}{v(\varepsilon_{\perp0},x)}\frac{\theta(\varepsilon_{\perp0}-U(x))}{\int dy\,\theta(\varepsilon_{\perp0}-U(y))/v(\varepsilon_{\perp0},y)}, \tag{2.6}$$

where $U(x)$ is the continuous potential of a plane dependent on the transverse coordinate x, U_0 is the depth of the potential well, $v(\varepsilon_{\perp0},x)=[2(\varepsilon_{\perp0}-U(x))/\varepsilon]^{1/2}$ is the transverse velocity of the particle, and $\varepsilon_{\perp0}=\frac{1}{2}\varepsilon\vartheta_0^2+U(x_0)$. It should be noted that in the case when $\varepsilon_\perp(\boldsymbol{\rho}_0)>U_0$ (above-barrier particles) the distribution (2.5) becomes uniform, whereas the distribution (2.6) becomes uniform only if $\varepsilon_{\perp0}\gg U_0$. We shall represent the crystal potential in the form

$$U(\mathbf{r})=\sum_{\mathbf{q}}G(\mathbf{q})e^{-i\mathbf{q}\mathbf{r}}. \tag{2.7}$$

The quantities $G(q)$ are defined in Ref. 3 [see Eqs. (5.1)–(5.4)]. We shall use a potential which is averaged over thermal vibrations, which in fact excludes from consideration the emission of radiation in accordance with the Bethe-Heitler mechanism. This is again a result which is different for crystals from that for amorphous substances (see, for example, Ref. 9 dealing with the theory of CBS). If $\vartheta_0\ll\vartheta_c$, we find $\Delta\mathbf{v}(t)$ using the rectilinear trajectory approximation:

$$\Delta\mathbf{v}(t)=-\frac{1}{\varepsilon}\sum_{\mathbf{q}}G(\mathbf{q})\exp[-i(q_\parallel t+\mathbf{q}\mathbf{r})]\frac{\mathbf{q}_\perp}{q_\parallel}, \tag{2.8}$$

where $q_\parallel=\mathbf{q}\mathbf{v}_0$, $\mathbf{q}_\perp=\mathbf{q}-\mathbf{v}_0(\mathbf{q}\mathbf{v}_0)$. If $\rho_c\gg1$, there is a range of angles ϑ_0 which satisfies $\vartheta_c\ll\vartheta_0\ll\vartheta_v$, i.e., when both the synchrotron description and the rectilinear trajectory approximation can be applied. However, since the synchrotron approach provides the same description for all angles $\vartheta_0\ll\vartheta_v$, the formulas obtained in this range of ϑ_0 remain valid (as shown in the next section) right up to $\vartheta_0=0$. In the range $\vartheta_0\gtrsim\vartheta_v$, the rectilinear trajectory approximation is known to be valid for $\rho_c\gg1$. Substituting Eq. (2.8) into Eq. (2.3) and summing over the trajectories of particles in a crystal subject to an allowance for the redistribution of the flux [Eqs. (2.5) and (2.6)], we find that simple transformations yield a general expression for the intensity of the radiation valid for any angle of incidence ϑ_0:

$$I\equiv\frac{dE^{(0)}}{dt}=\frac{i\alpha m^2}{2\pi\varepsilon^2}\int\omega\,d\omega\int\frac{d^3r}{V}F(\mathbf{r},\vartheta_0)\int\limits_{-\infty}^{\infty}\frac{d\tau}{\tau-i0}\Big\{1$$

$$-\varphi(\varepsilon)\Big(\sum_q\frac{G(\mathbf{q})\mathbf{q}_\perp}{m}\frac{\sin q_\parallel\tau}{q_\parallel}e^{-i\mathbf{q}\mathbf{r}}\Big)^2\Big\}e^{-iA_2},\qquad(2.9)$$

where

$$A_2=\frac{m^2\omega\tau}{\varepsilon\varepsilon'}\Big[1+\sum_{qq'}\frac{G(\mathbf{q})G(\mathbf{q}')}{m^2}\frac{(\mathbf{q}_\perp\mathbf{q}_\perp')}{q_\parallel q_\parallel'}\Big(\frac{\sin(q_\parallel+q_\parallel')\tau}{(q_\parallel+q_\parallel')\tau}$$

$$-\frac{\sin q_\parallel\tau}{q_\parallel\tau}\frac{\sin q_\parallel'\tau}{q_\parallel'\tau}\Big)e^{-i(\mathbf{q}+\mathbf{q}')\mathbf{r}}\Big].$$

Similar expressions are obtained also for the quantities describing the polarization of the emitted radiation; for example, if $I^{(2)}=dE^{(2)}/dt$, we find that

$$I^{(2)}=\frac{i\alpha}{4\pi\varepsilon^2}\int d\omega\,\omega\varphi(\varepsilon)\int\frac{d^3r}{V}F(\mathbf{r},\vartheta_0)\int\limits_{-\infty}^{\infty}\frac{d\tau}{\tau-i0}$$

$$\times e^{-iA_2}\sum_{qq'}\frac{G(\mathbf{q})G(\mathbf{q}')}{q_\parallel q_\parallel'}[\mathbf{q}\mathbf{q}']\mathbf{v}_0\Big[\sin q_\parallel\tau\Big(\cos q_\parallel'\tau-\frac{\sin q_\parallel'\tau}{q_\parallel'\tau}\Big)$$

$$-\sin q_\parallel'\tau\Big(\cos q_\parallel\tau-\frac{\sin q_\parallel\tau}{q_\parallel\tau}\Big)\Big]e^{-i(\mathbf{q}+\mathbf{q}')\mathbf{r}}.\quad(2.10)$$

In Eqs. (2.9) and (2.10) we have made the substitution $\tau\to2\tau$. We shall assume that the thickness of a crystal is considerably greater than the length of formation of radiation l_f, so that we can consider a description in terms of the intensity.

It is known that there is a relationship between the differential probabilities of the emission of a photon by a charged particle and the creation of a pair by a photon in quantum electrodynamics. The probabilities integrated over the angles of emission of the final particles do not generally exhibit such a relationship. However, we can show that in view of the smallness of all the characteristic angles of the problem, such a relationship applies in our case also to the spectrum of the final particles. Therefore, Eq. (2.9) can be obtained from Eq. (2.13) of Ref. 3 by means of the substitutions $\varepsilon\to-\varepsilon$, $\omega\to-\omega$, $\varepsilon'\to\varepsilon'$, $\varepsilon^2d\varepsilon\to-\omega^2d\omega$ allowing for a redistribution of a flux of charged particles and using the self-evident relationship between the intensity and the probability $dI=\omega dW$. We shall apply this relationship extensively in the treatment given below.

Equation (2.9) describes the spectral properties of the radiation (these properties are obtained by dropping the integral with respect to ω) for the range $\vartheta_0\gg\vartheta_c$ and any value of ρ_c, whereas if $\vartheta_0\lesssim\vartheta_c$, such a description is obtained if the condition $\rho_c\gg1$ is obeyed. However in the case of the total (integral) intensity of the radiation the range of validity of Eq. (2.9) is wider and it is the same as that of the semiclassical approximation. This situation arises because at energies when $\rho_c\lesssim1$ and the synchrotron radiation description ceases to be valid, the classical theory of the emission of radiation can be used for angles $\vartheta_0\lesssim\vartheta_c$ and the total intensity then depends only on the local characteristics of the motion of a particle.

We shall show below that if $\vartheta_0\ll\vartheta_v$, then Eq. (2.9) describes the emission of radiation in the synchrotron limit, whereas for $\vartheta_0\gg\vartheta_v$ we obtain the familiar expressions for coherent bremsstrahlung (CBS). In the intermediate range of angles one should use directly Eq. (2.9).

3. INTENSITY OF RADIATION FOR $\vartheta_0\ll\vartheta_v$. CORRECTIONS TO THE SYNCHROTRON RADIATION LIMIT

If $\vartheta_0\ll\vartheta_v$, the above substitution rules can be used to derive the spectral distribution of the radiation intensity $dI^F(\omega)$ from Eq. (3.6) of Ref. 3:

$$dI^F=\frac{\alpha m^2\omega\,d\omega}{3^{1/2}\pi\varepsilon^2}\int\frac{d^2\rho}{S}\Big\{\int\frac{d^2\rho_0\theta(\varepsilon_\perp(\rho_0)-U(\rho))}{S(\varepsilon_\perp(\rho_0))}$$

$$\times\Big[\varphi(\varepsilon)K_{2/3}(\lambda)-\int\limits_\lambda^\infty dy\,K_{1/3}(y)\Big]-\frac{1}{3}\frac{(\mathbf{b},(\mathbf{n}\nabla)^2\mathbf{b})}{\mathbf{b}^4}\varphi(\varepsilon)$$

$$\times\Big[K_{2/3}(\lambda)-\frac{2}{3\lambda}K_{1/3}(\lambda)\Big]+\frac{\lambda[((\mathbf{n}\nabla)\mathbf{b})^2+3(\mathbf{b},(\mathbf{n}\nabla)^2\mathbf{b})]}{30\mathbf{b}^4}$$

$$\times\Big[K_{1/3}(\lambda)-\frac{4}{3\lambda}K_{2/3}(\lambda)$$

$$+\varphi(\varepsilon)\Big(\frac{4}{\lambda}K_{2/3}(\lambda)-\Big(1+\frac{16}{9\lambda^2}\Big)K_{1/3}(\lambda)\Big)\Big]\Big\},\quad(3.1)$$

where

$$\lambda=2m^2\omega/3\varepsilon\varepsilon'|\mathbf{b}|,\quad\mathbf{b}=\nabla U(\rho)/m,\quad\nabla=\partial/\partial\rho;$$

$K_\nu(\lambda)$ is a modified function of the second kind; $U(\rho)$ is a continuous axial potential, which depends only on the transverse coordinate ρ:

$$U(\rho)=\sum_{q_t}G(\mathbf{q}_t)\exp(-i\mathbf{q}_t\rho),\qquad(3.2)$$

the index t denotes that component of the vector which is perpendicular to the axis: $\rho=\mathbf{r}_t$. (We shall assume from now on that the vector $\mathbf{n}=\mathbf{v}_0/|\mathbf{v}_0|$ does not lie near crystallographic planes. The problem of emission of radiation in the planar regime will be discussed separately.) Since the expression for dI^F is independent of z (which is the coordinate $\mathbf{r}$ along the axis), it follows that $\int d^3r/V\to\int d^2\rho/S$, where S is the cross-section area per axis. The first two terms in Eq. (3.1) independent of $\mathbf{n}=\mathbf{v}_0/|\mathbf{v}_0|$ represent the spectral distributions of the intensity in the synchrotron radiation limit (see, for example, § 10 in Ref. 4) if we allow for a redistribution of the flux of charged particles; the other terms represent a correction proportional to $\sim\vartheta_0^2$ [see Eq. (4.16) of Ref. 8]. Equation (3.1) allows for the fact that in the range $\vartheta_0\sim\vartheta_c$ we have $(m\vartheta_0/V_0)^2\sim1/\rho_c$ and retention of terms of this kind in Eq. (3.1) would represent unnecessary precision, because these terms are dropped in the derivation of Eq. (2.9). Therefore, corrections proportional to $\sim(m\vartheta_0/V_0)^2$ need be included only if $\varepsilon_\perp>U_0$, when we have $F(\mathbf{r},\vartheta_0)=1$. Then, the potential of an axis (for a discussion see Sec. III in Ref. 3) can be regarded as axially symmetric, $U=U(\rho^2)$, and integration with respect to the angles of the vector ρ is carried out in Eq. (3.1):

TABLE I. Parameters of potential and some quantities characterizing emitted radiation.

Crystal	Ось	T, K	u_1, Å	eV	η	a_s, Å	x_0	χ_s (ε=100GeV)	ρ_c (ε=10GeV)	r_γ^{max}
C(d)	111	293	0.040	29	0.025	0.326	5.5	0.13	2.22	168
Si	111	293	0.075	54	0,150	0.299	15.1	0,27	4.14	71
Si	110	293	0.075	70	0,145	0.324	15.8	0.32	5.36	81
Ge	111	293	0.085	91	0.130	0.300	16.3	0.45	6.97	26
Ge	110	280	0.083	110	0.115	0.337	15.8	0.48	8.43	30
Ge	110	100	0,054	114,5	0,063	0.302	19.8	0,56	8.77	30
W	111	293	0.050	417	0.115	0.215	39.7	2.87	31.94	11

4) *Note.* Here, $C(d)$ denotes diamond; u_1 is the amplitude of thermal vibrations; v_0, η, a_s, and x_0 are the parameters of the potential described by Eq. (3.5); χ_s is the parameter representing the magnitude of the quantum effects of Eq. (1.5); ρ_c is the parameter which determines the multipolarity of the radiation of Eq. (1.3); r_γ^{max} is the maximum magnitude of the effect estimated on the basis of Eq. (3.12).

$$dI^F=\frac{\alpha m^2\omega\,d\omega}{3^{1/2}\pi\varepsilon^2}\int_0^{x_0}\frac{dx}{x_0}\left\{\int_0^{x_0}\frac{dy\,\theta(\varepsilon_\perp(y)-U(x))}{y_0(\varepsilon_\perp(y))}\right.$$
$$\times\Big[\varphi(\varepsilon)K_{2/3}(\lambda)-\int_\lambda^\infty dy\,K_{1/3}(y)\Big]-\frac{1}{6}\left(\frac{m\vartheta_0}{V_0}\right)^2\Big[\varphi(\varepsilon)\frac{xg''+2g'}{xg^3}$$
$$\times\Big(K_{2/3}(\lambda)-\frac{2}{3\lambda}K_{1/3}(\lambda)\Big)$$
$$-\frac{1}{20g^4x^2}(2x^2g'^2+g^2+14gg'x+6x^2gg'')$$
$$\times\Big(\lambda K_{1/3}(\lambda)-\frac{4}{3}K_{2/3}(\lambda)$$
$$\left.+\varphi(\varepsilon)\Big(4K_{2/3}(\lambda)-\Big(\lambda+\frac{16}{9\lambda}\Big)K_{1/3}(\lambda)\Big)\Big)\Big]\right\},\qquad(3.3)$$

where we have adopted a new variable $x=\rho^2/a_s^2$; $x_0^{-1}=\pi a_s^2nd$; d is the average distance between the atoms of a chain forming the axis; n is the density of atoms in a crystal;

$$y_0(\varepsilon_\perp(y))=\int_0^{x_0}dx\,\theta(\varepsilon_\perp(y)-U(x)),\qquad \varepsilon_\perp(y)=\frac{\varepsilon\vartheta_0^2}{2}+U(y).$$

The notation $U'(x)=V_0g(x)$ is used in Eq. (3.3), so that

$$\lambda=\frac{m^3\omega a_s}{3\varepsilon\varepsilon' V_0x^{1/2}|g|}=\frac{u}{3\chi_sx^{1/2}|g|}\qquad \chi_s=\frac{V_0\varepsilon}{m^3a_s},\qquad u=\frac{\omega}{\varepsilon'}.\qquad(3.4)$$

The above expression (3.3) represents the spectral distribution of the radiation intensity (after integration with respect to ω, it gives the total radiation intensity) for an arbitrary form of an axially symmetric potential of the selected axis. In specific calculations we shall use a potential found to be fully satisfactory in specific problems of emission of radiation[14] and pair-production[1-3] discussed by us earlier:

$$U(x)=V_0\Big[\ln\Big(1+\frac{1}{x+\eta}\Big)-\ln\Big(1+\frac{1}{x_0+\eta}\Big)\Big],$$
$$g(x)=\frac{1}{(x+\eta)(x+\eta+1)},\qquad(3.5)$$
$$\chi(x)=\frac{eE(x)\gamma}{m^2}=\frac{2V_0\varepsilon}{m^3a_s}x^{1/2}g(x)=2\chi_sx^{1/2}g(x);$$

where, $\chi(x)$ is the local value of the quantum parameter χ (at a distance $x=\rho^2/a_s^2$ from the axis). The parameters of the potential (3.5) are given in Table I.

Equations (3.1) and (3.3) are derived from the intensity equation (2.9) within the range of validity of the latter ($\vartheta_0\gg\vartheta_c$). However, the intensity changes only slightly on further reduction in the angle ϑ_0, as demonstrated by Eq. (3.3). Hence, it follows that Eqs. (3.1) and (3.3) and, therefore, Eq. (2.9) are all valid up to $\vartheta_0=0$.

In this limit in all specific calculations and estimates we shall assume, for the sake of simplicity, that the distribution is uniform along the transverse coordinates (which is true in the case of large angles of incidence $\vartheta_0>\vartheta_c$ and approximately correct in the case of beams with a large angular scatter $\Delta\vartheta_0\sim\vartheta_c$). Numerical calculations were carried out for the potential of Eq. (3.5). The influence of a redistribution of a flux on the orientational dependence will be discussed in Sec. 5.

The results are illustrated in Figs. 1 and 2. Figure 1 gives the reciprocal of the characteristic length of the loss of energy by a particle in the synchrotron radiation limit

$$L_{ch}^{-1}=-\frac{1}{\varepsilon}\frac{d\varepsilon}{dl}=\frac{I^s(\varepsilon)}{\varepsilon}$$

as a function of the electron energy for some semiconductors. We can see that if $\varepsilon\approx1$ TeV, the value of L_{ch}^{-1} reaches its maximum near which the energy dependence of L_{ch}^{-1} is fairly weak and the value of L_{ch} is several times less than the radiation wavelength in the corresponding amorphous substance [for example, in the case of Si (⟨110⟩ axis), we have

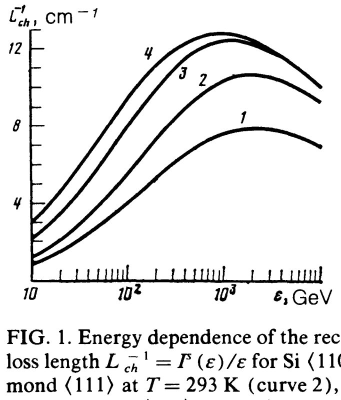

FIG. 1. Energy dependence of the reciprocal of the characteristic energy loss length $L_{ch}^{-1}=I^s(\varepsilon)/\varepsilon$ for Si ⟨110⟩ at $T=293$ K (curve 1), for diamond ⟨111⟩ at $T=293$ K (curve 2), for Ge ⟨110⟩ at $T=280$ K (curve 3), and for Ge ⟨110⟩ at $T=100$ K (curve 4).

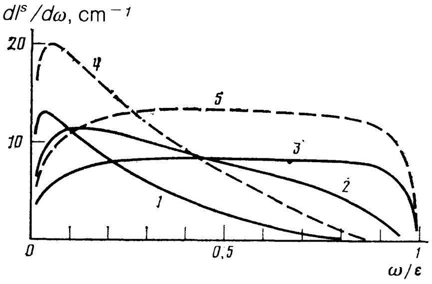

FIG. 2. Spectral dependences of the intensity of the radiation obtained for a given energy in the case of Si ⟨110⟩ at $T = 293$ K: $\varepsilon = 100$ GeV (curve 1), $\varepsilon = 700$ GeV (curve 2), $\varepsilon = 5$ TeV (curve 3). The dashed curves give the corresponding results for Ge ⟨110⟩ at $T = 280$ K: $\varepsilon = 100$ GeV (curve 4) and $\varepsilon = 3$ TeV (curve 5).

$(L_{rad}/L_{ch})_{max} \approx 75$]. Figure 2 shows the spectral distribution of the intensity $dI^F/d\omega$ (in the synchrotron radiation limit) of the emitted photons calculated for different electron energies along the ⟨110⟩ axis in silicon and germanium. We can see that if the energy is $\varepsilon = 100$ GeV, then the value of $dI^F/d\omega$ has a maximum at $\omega/\varepsilon/ \ll 1$ and it falls rapidly on increase in the frequency. However, as the energy is increased, the distribution becomes more uniform over the whole spectrum right up to $\omega \approx \varepsilon$.

The spectral characteristics of the radiation depend very strongly on the relationship between the values of $u = \omega/\varepsilon'$ and χ. If $u \gg \chi_s \eta^{-1/2}$, the emission is exponentially suppressed, so that the emission spectrum consists mainly of the frequencies characterized by $u \lesssim \chi_s \eta^{-1/2}$. If $u \ll \chi_s$, we can obtain an explicit asymptotic expression for the spectral distribution of the radiation intensity, which is particularly convenient for the analysis in the range of high energies when $\chi_s \gg 1$. The range of integration with respect to x in Eq. (3.3) can be split into three parts: 1) $0 \leqslant x \leqslant x_1 \ll 1$; 2) $x_1 \leqslant x \leqslant x_2 \gg 1$; 3) $x_2 \leqslant x \leqslant x_0$, if we assume that $x_0 \gg x_2 \gg 1$, where $\lambda(x_1) \ll 1$ and $\lambda(x_2) \ll 1$. Then, in the ranges 1 and 3 we can replace $g(x)$ with approximate expressions at low and high values of x, and we can then integrate with respect to λ, whereas in the range 2 we can use the expansion of $K_\nu(\lambda)$ valid in the range $\lambda \ll 1$. Fairly cumbersome calculations yield

$$\frac{1}{\omega}\frac{dI^F}{d\omega} = \frac{\alpha m^2}{3^{1/2}\pi x_0 \varepsilon^2}\left[\Phi_1^s - \left(\frac{m\vartheta_0}{V_0}\right)^2 \Phi_2\right], \qquad (3.6)$$

where

$$\Phi_1^s = \Gamma\left(\frac{2}{3}\right)\left(\frac{6\chi_s}{u}\right)^{2/3}\varphi(u)\left[\ln\frac{18\cdot 3^{1/2}\chi_s}{u} - \frac{\pi}{2\cdot 3^{1/2}} - C - \frac{3}{2\varphi(u)} - l_1(\eta)\right],$$

$$\Phi_2 = \frac{4\Gamma(1/3)(1+2\eta)}{405}\left(\frac{6\chi_s}{u}\right)^{4/3}\varphi(u)\left[\ln\frac{18\cdot 3^{1/2}\chi_s}{u} + \frac{\pi}{2\cdot 3^{1/2}} - C + \frac{3}{8} - \frac{21}{4\varphi(u)} - l_2(\eta)\right], \qquad (3.7)$$

where $\varphi(u) = 1 + u + (1+u)^{-1}$, $C = 0.577$ ... is the Euler constant; $l_{1,2}(\eta) = B_{1,2}(0) - B_{1,2}(\eta)$. The functions $B_1(\eta)$ and $B_2(\eta)$ occur in the asymptotic representation of the probability of creation of pairs by a photon in the case when $\chi_s \gg 1$ [see Eqs. (3.12) and (3.13) of Ref. 3].

In the derivation of Eq. (3.6) the integration with respect to x is carried out between infinite limits. Since the main (logarithmic) contribution to Eq. (3.6) comes from the range $x \sim (\chi_s/u)^{2/3}$ (range 3), it is clear that the asymptotic expression (3.6) is valid only if $(\chi_s/u)^{2/3} < x_0$.

We must bear in mind that Eq. (3.6) is not valid at low frequencies in the limit $u \to 0$ (for this reason it cannot be used to calculate the corrections to the total intensity and to the emission probability). The physical reason for the inadmissibility of the synchrotron approach for the description of the low-frequency radiation is that at low frequencies the radiation is formed throughout the particle transit trajectory along the axis. We shall now analyze this topic. The synchrotron radiation approach is valid if the transferred momentum corresponding to an impact parameter $x^{1/2}a_s$ exceeds considerably the mass:

$$\frac{\Delta p_\perp}{m} \approx \frac{2V_0 x^{1/2}}{m a_s}\int_{-\infty}^{\infty}\frac{dt}{(x+t^2\vartheta_0^2/a_s^2)^2} = \frac{\pi V_0}{m x \vartheta_0} \gg 1.$$

Substituting here $x = (\chi_s/u)^{2/3}$, we can find the lower limit to the frequency range in which the synchrotron radiation description is valid [see Eqs. (3.1), (3.3), and (3.6)]:

$$(\chi_s/u)^{4/3}\rho^{-1}(\vartheta_0) \ll 1 \qquad (3.8)$$

or

$$u \gg u_{min}, \quad u_{min} = \chi_s \rho^{-3/4}(\vartheta_0).$$

In connection with this criterion we must draw attention to the anomalously small [by a factor $(\chi_s/u)^{2/3}$] first correction in the expansion of Eq. (3.6). It should be pointed out that if $\chi_s \ll 1$, then the asymptotic expression (3.6) describes the spectrum outside the range of fundamental frequencies [Eq. (3.6) applies when $u \ll \chi_s$, whereas the main contribution to the radiation intensity comes from $u \sim \chi_s$], whereas for $\chi_s \gg 1$ the condition $u \ll \chi_s$ is satisfied by frequencies with $u \sim 1$, which dominate the intensity. On increase in $\chi_s(\varepsilon)$, the fraction contributed by frequencies $\omega \sim \varepsilon$ become significant ($\chi_s \sim 1$) and for $\chi_s \gg 1$ the frequencies $\omega \sim \varepsilon$ dominate the emission. At such high electron energies the spectral intensity of the radiation depends weakly on the frequency of an emitted photon. This follows from Eq. (3.6) and is illustrated in Fig. 2. In the range under discussion the value of L_{ch}^{-1} depends weakly on the energy (Fig. 1), so that the situation is on the whole similar to the properties of bremsstrahlung emitted by single nuclei (Bethe-Heitler mechanism), but the radiation lengths are now several tens of times less than the corresponding wavelengths in an amorphous medium. In the same range of energies an analogous situation occurs in the process of creation of an electron-positron pair by a photon (see Refs. 2 and 3). Hence, it follows that at low angles of incidence of a photon (or an electron) of sufficiently high energy on a single crystal a special electron-photon shower[2] develops (over distances much shorter than in the corresponding amorphous substance) and the evolution of such showers is in many respects different from the Bethe-Heitler mechanism, as shown separately below.

We shall now consider calculation of the total radiation intensity. It is convenient to carry out integration of Eq.

(3.3) by parts with respect to the variable u (and then in some terms the integration by parts with respect to the variable x), so that the final result is

$$I^F \equiv i_1{}^M(\chi_s) - \rho^{-1}(\vartheta_0)\, i_2(\chi_s) = \frac{\alpha m^2}{3^{1/2}\pi x_0}\int_0^\infty \frac{u\,du}{(1+u)^4}\int_0^{x_0} dx$$

$$\times\left\{\int_0^{x_0}\frac{dy\,\theta(\varepsilon_\perp(y)-U(x))}{y_0(\varepsilon_\perp(y))}(4u^2+5u+4)K_{2/3}(\lambda)\right.$$

$$-\rho^{-1}(\vartheta_0)\frac{6}{5}\chi_s\frac{x^{1/2}}{(1+u)^2}K_{1/3}(\lambda)\left[5A(u)+\frac{1+2\eta}{x}B(u)\right.$$

$$\left.\left.+\frac{1}{2}g(x)C(u)\right]\right\}, \qquad (3.9)$$

where

$$A(u)=4u^3-u^2+14u-1, \quad B(u)=-4u^3+27u^2-2u+27,$$
$$C(u)=16u^3-43u^2+38u-43, \quad \rho(\vartheta_0)=(2V_0/m\vartheta_0)^2.$$

As pointed out already, the range of validity of Eq. (3.9) is the same as that of the semiclassical approximation. At low energies (in the classical limit) it is well known that $I(x)\propto\chi^2(x)$ (see, for example, Ref. 4) irrespective of ϑ_0 and this remains valid right up to angles where the emission of frequencies $\omega\sim\gamma^2\vartheta_0/a_s\sim\varepsilon$ becomes significant and the classical theory ceases to be valid. Therefore, at relatively low energies when $\chi_s\ll 1$, we can have $i_2/i_1\ll 1$. In fact, it follows from Eq. (3.9) that if $\chi_s\ll 1$, we find that $i_2/i_1\propto\chi_s\eta^{-1/2}$.

At high energies ($\chi_s\gg 1$), the expression for $L_{ch}{}^{-1}=I^F(\varepsilon)/\varepsilon$ becomes:

$$L_{ch}^{-1}\approx\alpha\frac{V_0}{mx_0a_s}\left[F(\chi_s)-\frac{8}{9}\chi_s\rho^{-1}(\vartheta_0)\right],$$

$$F(\chi_s)=\frac{g_1(\ln\chi_s+g_0)}{\chi_s^{1/3}}, \qquad (3.10)$$

where

$$g_1=(2/3)^6\cdot 6^{2/3}\Gamma(2/3)\approx 0.3925,$$

$$g_0=\frac{\pi}{2\cdot 3^{1/2}}+\frac{5}{2}\ln 3+\ln 2-C-\frac{99}{32}-l_1(\eta)\approx 0.6756-l_1(\eta).$$

It is clear from Eq. (3.10) that if $\chi_s\gg 1$, then the synchrotron radiation description is valid in the range where $\chi_s{}^{4/3}/\rho(\vartheta_0)\ll 1$. This is in agreement with Eq. (3.8) if we bear in mind that for these values of χ_s the main contribution to the intensity comes from $u\sim 1$.

The quantity $F(\chi_s)$ in Eq. (3.10) reaches its maximum at a point $\chi_s{}^0=\exp(3-g_0)$, near which it varies extremely smoothly. Since $\chi_s{}^0\gtrsim e^3\gg 1$, this justifies the use of Eq. (3.10) in the region of the maximum. In view of the smooth variation of $F(\chi_s)$ near the maximum, the energy ε_0 obtained from χ_s^0 represents only an estimate. However, the value of $(L_{ch}^{-1})_{\max}$ given by Eq. (3.10) is quite satisfactory and it is in good agreement with the results of a numerical calculation carried out on the basis of Eq. (3.3):

$$(L_{ch}^{-1})_{max}\approx 0{,}543\frac{\alpha V_0}{mx_0a_s}\exp\left[-\frac{l_1(\eta)}{3}\right]. \qquad (3.11)$$

It is interesting to compare Eq. (3.11) with the maximum probability of creation of pairs by a photon [Eq. (3.14) in Ref. 3]. We have the relationship $W_e^{\max}\approx(L_{ch}{}^{-1})_{\max}\cdot 0{,}72$; however, we must bear in mind that the characteristics of pair creation and emission of radiation have maxima at different energies. At very high energies the values of W_e and L_{ch}^{-1} become very close to one another irrespective of the nature of the medium: $W_eL_{ch}\approx 5\cdot 3^6\Gamma^3(2/3)\ (7\cdot 2^7\pi^2)^{-1}\approx 1{,}023$. Although the product of these quantities is, as in the Bethe-Heitler case, a constant (the constants are different: 1.023 and 0.778), the quantities W_e and L_{ch}^{-1} themselves depend on the energy, in contrast to the Bethe-Heitler case. It should also be pointed out that the factor $\exp[-l_1(\eta)/3]$ depends weakly on the medium and it varies (for the usually investigated substances) in the range 0.90–0.75 as η changes from 0.025 for diamond to 0.15 for silicon.

The maximum excess of L_{rad} above L_{ch} in the corresponding amorphous substance is of considerable interest:

$$r_\gamma^{max}=\left(\frac{L_{rad}}{L_{ch}}\right)_{max}\approx 0.426\frac{V_0ma_sd\exp[-l_1(\eta)/3]}{(Z\alpha)^2\ln(183Z^{-1/3})}$$

$$\sim\frac{ma_s}{3Z\alpha\ln(183Z^{-1/3})}, \qquad (3.12)$$

where the second (simplified) estimate is fairly rough. Hence, we can see that $r_\gamma^{\max}$ increases on reduction in Z and on increase in a_s. The greatest enhancement (among the investigated substances) is achieved for diamond ($r_\gamma^{\max}\sim 160$); the values of $r_\gamma^{\max}$ are listed in Table I.

We shall now consider calculation of the total probability of emission of radiation which determines the total number of emitted photons and we shall do this in the synchrotron radiation approximation. Then, the potential of Eq. (3.5) obtained for the case when $\chi_s\ll 1$ corresponds to the total probability

$$W_1^s(\chi_s\ll 1)=\frac{5\alpha m^2}{2\cdot 3^{1/2}x_0\varepsilon}\int_0^{x_0}dx\,\chi(x)=\frac{10\alpha V_0}{3^{1/2}a_smx_0}$$

$$\times\left\{(1+\eta)^{1/2}\operatorname{arctg}\left[\left(\frac{x_0}{1+\eta}\right)^{1/2}\right]-\eta^{1/2}\operatorname{arctg}\left[\left(\frac{x_0}{\eta}\right)^{1/2}\right]\right\}$$

$$\approx\frac{5\alpha\pi V_0}{3^{1/2}a_smx_0}\left[(1+\eta)^{1/2}-\eta^{1/2}-\frac{2}{\pi x_0{}^{1/2}}\right]. \qquad (3.13)$$

In the other limiting case of $\chi_s\gg 1$ we can integrate Eq. (3.6) bearing in mind that $d\omega=\varepsilon(1+u)^{-2}du$, i.e., we can integrate Φ_1^m with respect to u with the weight $(1+u)^{-2}$, which gives

$$W_1^s(\chi_s\gg 1)=\frac{28\cdot 6^{2/3}\Gamma(2/3)\alpha V_0}{81mx_0a_s\chi_s^{1/3}}[\ln\chi_s+D(\eta)], \qquad (3.14)$$

where

$$D(\eta)=\ln 2+\frac{5}{2}\ln 3+\frac{\pi}{2\cdot 3^{1/2}}-C$$

$$+\frac{3}{28}-l_1(\eta)=3.8765-l_1(\eta).$$

An analysis of the above expressions for the intensity and the total probability shows that a large number of soft photons is emitted and they do not affect significantly the loss of energy by the particle. At high particle energies ($\chi_s\gtrsim 1$) these photons can nevertheless manifest their presence on experi-

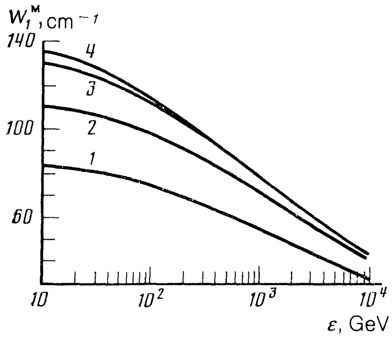

FIG. 3. Energy dependences of the total probability of emission of radiation in Si ⟨110⟩ at $T = 293$ K (curve 1), in diamond ⟨111⟩ at $T = 293$ K (curve 2), in Ge ⟨110⟩ at $T = 280$ K (curve 3), and in Ge ⟨110⟩ at $T = 100$ K (curve 4).

ments. In particular, they can form (as a result of the Bethe-Heitler mechanism) secondary pairs of charged particles, the number of which is determined by the probability described by Eq. (3.14). Figure 3 shows the dependence of the total probability W_1^s on the energy of particles in different substances. We can see that the probability on the left-hand side of the graph exhibits a plateau (when the probability of emission in external field is practically independent of the energy for $\chi \ll 1$). This is followed by a region of logarithmic fall where the falling curve can be approximated satisfactorily by a straight line. Figure 4 shows the probability of the emission of a photon integrated over the frequencies in the range $\omega_0 - \varepsilon$ for different values of ε considered as a function of ω_0:

$$W_1^s(\omega_0) = \int_{\omega_0}^{\varepsilon} d\omega\,(dW/d\omega).$$

The fact that even for $\omega_0 \sim \varepsilon$ this probability obeys $W_1^s(\omega_0\text{-}\varepsilon) \propto L_{ch}^{-1}$ confirms the conclusions reached above on the contribution of soft photons.

We shall conclude this section with a theoretical analysis of the case when the contribution of the range $x \gg 1$ becomes important. In the above approximation we used the potential of Eq. (3.5) characterized by $g(x) \propto x^{-2}$ when $x \gg 1$. We shall first assume that the law describing the fall of the potential in the range $x \gg 1$ is different: $g(x) \propto x^{-(\mu+1)}$,

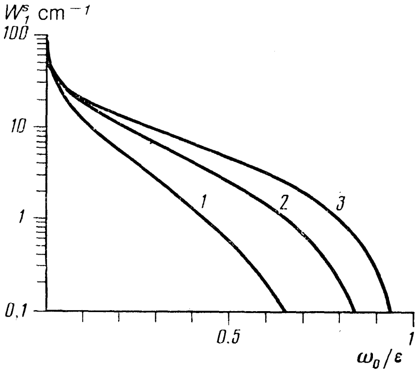

FIG. 4. Dependences of the probabiity of photon emission $W_1^s(\omega_0)$ of energy $\omega \geqslant \omega_0$ on ω_0 for Si ⟨110⟩ at $T = 293$ K: $\varepsilon = 100$ GeV (curve 1), $\varepsilon = 400$ GeV (curve 2), $\varepsilon = 2$ TeV (curve 3).

so that $\chi(x) \propto \chi_s x^{-(\mu+1/2)}$ [see Eq. (3.5)]. If $\chi_s \gg 1$, the range $\chi(x) \gtrsim 1$ lies to the left of the point $x \sim \chi^{2/(2\mu+1)} \equiv x_b$. The contribution to the total intensity from the range $x \sim x_b$ is $I \propto \chi_s^{2/(2\mu+1)}$, whereas the contribution of $x \sim 1$ is $I \propto \chi_s^{2/3}$. Therefore, in the case of the potential which falls faster than x^{-1} ($\mu > 1$), the radiation intensity is governed by the range $x \sim 1$. In this case the characteristics of the radiation behave in the same way as in the synchrotron radiation approach and, in particular, the total intensity obeys $I \propto \chi_s^{2/3}$, where the criterion of validity of the approach is $\vartheta_0 \ll \vartheta_v \chi_s^{-1/3}$. However, if $\mu < 1$, then for $x_s \gg 1$ the contribution of the range $x \sim x_b$ becomes predominant and the criterion of validity of the synchrotron radiation description is then $\vartheta_0 \ll \vartheta_v x_s^{-2\mu/(2\mu+1)}$. As the energy is increased, the value of x_b becomes comparable with x_0 (it becomes a boundary of the area per one axis) and there is a change in the physical situation. If $x_b \gg x_0$ [$\chi(x) \gg 1$ for all values of x], we have $I \propto \chi_s^{2/3}$ and the criterion of validity becomes $\vartheta_0 \gg \vartheta_v \chi_s^{-1/3} x_0^{-(2\mu-1/2)/3}$. For the potential of Eq. (3.5), we have $\mu = 1$ and if $\chi_s \gg 1$, then $I \propto \chi_s^{2/3} \ln \chi_s$. The appearance of the logarithm is due to the fact that the whole range $1 \lesssim x \lesssim x_b$ contributes. In this case the change in the physical situation is that in the total intensity we now have $\ln \chi_s \to 3/2 \ln x_0$. Moroever, if $x_b \gtrsim x_0$, the potential on the axis can no longer be regarded as axially symmetric and instead of Eq. (3.3) we have to use Eq. (3.1). It should be pointed out that for the potential of Eq. (3.5) the value of x_b becomes comparable with x_0 at an energy $\varepsilon \sim$ 10–20 TeV, which depends on the substance.

4. INTENSITY OF RADIATION FOR $\vartheta_0 > \vartheta_v$. MODIFIED THEORY OF COHERENT EMISSION

We shall now consider the range of relatively large angles of incidence of the particles. The general expression (2.9) yields formulas for the spectral distribution and the range of their validity at high values of χ is wider than of the standard theory of coherent bremsstrahlung (CBS).

We shall find a modified expression for the spectrum of the radiation which is emitted by applying the substitution rules to Eq. (4.7) of Ref. 3. We find that

$$dI^{m\ coh} = \frac{\alpha\omega\, d\omega}{4\varepsilon^2} \sum_{\mathbf{q}} |G(\mathbf{q})|^2 \frac{\mathbf{q}_\perp^2}{q_\parallel^2} \left[\varphi(\varepsilon) - \frac{2\omega m^2}{\varepsilon\varepsilon' q_\parallel^2} \left(|q_\parallel| - \frac{\omega m_*^2}{2\varepsilon\varepsilon'} \right) \right] \theta\left(|q_\parallel| - \frac{\omega m_*^2}{2\varepsilon\varepsilon'} \right); \tag{4.1}$$

here,

$$m_*^2 = m^2\left(1 + \frac{\rho}{2}\right),$$

$$\frac{\rho}{2} = \gamma^2[\langle \mathbf{v}^2\rangle - \langle \mathbf{v}\rangle^2] = \sum_{\mathbf{q}, q_\parallel \neq 0} \frac{|G(\mathbf{q})|^2 \mathbf{q}_\perp^2}{m^2 q_\parallel^2}. \tag{4.2}$$

If $\vartheta_0 \gg V_0/m$ ($\rho \ll 1$), then Eq. (4.1) becomes identical with the results of the standard theory of CBS.

It is known[9] that in the standard theory of CBS the angular dependence of the intensity of radiation of a given frequency ω has maxima for angles of incidence

$$\vartheta_0 \sim \frac{l}{2\pi} \frac{\omega m^2}{2\varepsilon\varepsilon'} \sim \frac{u}{\chi_s} \frac{V_0}{m}$$

(where l is the lattice constant). As just pointed out, we should have $\vartheta_0 \gg V_0/m$, so that the standard theory of CBS is valid throughout the region of its maximum only if $\chi_s/u \ll 1$. Hence, it follows that if $\chi_s \gtrsim 1$, then the standard theory of CBS ceases to be valid in the main part of the spectrum near the angular maxima. Similar restrictions apply then to the creation of electron-positron pairs by a photon.[3]

The appearance of m corresponds to the adoption of the effective mass concept in discussing the problem of emission of radiation in the field of a plane electromagnetic wave (see, for example, Ref. 15). In the problem of emission of radiation we must allow for the difference between m and m_* when $\rho/2 \gtrsim 1$, i.e., when the radiation is no longer of the dipole nature. After the integration with respect to ω in Eq. (4.1), we find that the total intensity of the radiation is given by

$$I^{m\,coh} = \frac{\alpha}{4} \sum_{\mathbf{q}} |G(\mathbf{q})|^2 \frac{\mathbf{q}_\perp^2}{q_\parallel^2} F(z), \qquad z = \frac{2\varepsilon |q_\parallel|}{m_*^2}, \quad (4.3)$$

where

$$F(z) = \left[\ln(1+z) - \frac{z(2+3z)}{2(1+z)^2}\right]\left[1 - \frac{8(3+z)}{z^2(2+\rho)}\right] + \frac{z}{(1+z)^2}\left(\frac{8}{2+\rho} + \frac{z(3+2z)}{3(1+z)}\right).$$

When calculations are carried out using Eqs. (4.1)–(4.3), special attention must be given to a satisfactory allowance for the contributions of higher-order planes when the vector $\mathbf{n} = \mathbf{v}_0/|\mathbf{v}_0|$ is accidentally in one of these planes.

At relatively low particle energies and not too large angles of incidence on a crystal, when $z \ll 1$, we find that expansion of Eq. (4.3) as a function of z and retention of the leading term (proportional to z^2) gives a classical expression for the total radiation intensity. The higher terms of the expansion represent the quantum corrections and the values of these corrections can be used to judge the range of validity of the classical expression. In estimating these corrections we shall bear in mind that the terms with additional powers of z are proportional to $|q_\parallel|$ and contain in the sum of Eq. (4.3) the factor $|q_\parallel|$ in the numerator. Consequently, the contribution to the sums is generally made by the higher harmonics [when summation in Eq. (4.3) can be replaced approximately with integration] and the contribution of these to the sum of Eq. (4.3) is truncated at the amplitude of thermal vibrations u_1 ($|\mathbf{q}|u_1 \lesssim 1$), so that these corrections consist effectively of $\chi_u = \chi_s a_s/u_1$ and the relative values of the correction terms are described by the parameter

$$\lambda_u = \frac{\varepsilon}{m^2}\frac{\vartheta_0}{u_1} = \frac{m}{V_0}\vartheta_0\chi_u. \quad (4.4)$$

We shall consider the spectral intensity of the radiation in the extreme quantum limit, when the parameter in question has the value

$$\lambda_m = 2\varepsilon |q_\parallel|_{\min}/m^2 \sim \varepsilon\vartheta_0/m^2 a_s \gg 1.$$

In this case the maximum of the intensity of the coherent radiation is attained at such values of ϑ_0 that the theory of coherent emission becomes invalid. Bearing in mind that if $\lambda_m \gg 1$ and $\vartheta_0 \sim V_0/m$, then $\chi_s \sim \lambda_m \gg 1$, we can use conveniently a modified theory of coherent emission when the spectral distribution becomes

$$\frac{dI^{m\,coh}}{d\omega} \approx \frac{\alpha\omega}{4\varepsilon^2}\left(\frac{\varepsilon}{\varepsilon'} + \frac{\varepsilon'}{\varepsilon}\right) \sum_{\mathbf{q}} |G(\mathbf{q})|^2 \frac{\mathbf{q}_\perp^2}{q_\parallel^2} \theta(u_0 - u), \quad (4.5)$$

where

$$u_0 = 2\varepsilon |q_\parallel|/m^2(1+\rho/2), \quad u = \omega/\varepsilon'.$$

The spectral intensity described by Eq. (4.5) has a sharp maximum near the end of the spectrum at $\omega \approx \varepsilon\lambda_m(1+\lambda_m)^{-1}$ with a relatively small (in terms of λ_m^-) width $\Delta\omega \sim \varepsilon(1+\rho/2)/\lambda_m = m^2(1+\rho/2)/2|q_\parallel|_{\min}$:

$$(dI/d\omega)_{\max} \approx \alpha\rho |q_\parallel|_{\min}/2(2+\rho). \quad (4.6)$$

We can see that both the amplitude of the maximum of the spectral curve and its effective width are independent of the energy. However, in calculating the total radiation intensity we find, as demonstrated by Eq. (4.3), that there is a weak (logarithmic) dependence of the intensity on the particle energy:

$$I \approx {}^1/_8 \alpha\rho m^2 \ln \lambda_m. \quad (4.7)$$

This behavior of the spectral and total intensities of the emitted radiation is typical in the theory of undulator radiation in the extreme quantum case and is in full agreement with the results obtained by us earlier [Eqs. (34)–(38) in Sec. 3 of Ref. 16]. These features of the emission spectrum are associated with the dominant contribution of the higher harmonics to the spectrum of equivalent photons in the case when $\lambda_m \gg 1$ and then its discreteness becomes important. This behavior of the spectrum of equivalent photons differs radically from the corresponding spectrum in the Bethe-Heitler case, when this spectrum is continuous and extends without restriction to frequencies as low as we please to choose.

The spectral curves obtained in Ref. 17 (see Figs. 7 and 8) for various (large) values of λ_m in the case when $\rho \ll 1$ illustrate excellently these features of the spectral distribution of the radiation in the range under discussion. Unfortunately, the published treatment of this range of CBS (see, for example, Ref. 18) has been used to draw an incorrect conclusion that the probabilities of the radiation and pair production in a crystal differ only by a numerical factor from the corresponding probabilities of radiation and pair production in an amorphous medium, whereas the form of the spectrum and the energy dependence of the total radiation intensity are very different in these two cases, as shown above.

5. ORIENTATIONAL DEPENDENCE OF THE RADIATION INTENSITY. DISCUSSION OF RESULTS

The orientational dependence of the spectral distribution and of the total intensity of the radiation is given by Eq. (2.9). The calculation carried out using this formula is a fairly complex computational task. On the other hand, it is relatively simple if we use approximate expressions derived from Eq. (2.9) for the case of low angles of incidence, representing the synchrotron description with a correction [Eqs. (3.1), (3.3), and (3.9)], and for large angles, which is the modified theory of CBS Eqs. (4.1) and (4.3)]. Therefore, the general formula (2.9) should be used only in the intermediate range of angles $\vartheta_0 \sim V_0/m$. Moreover, the interpolation procedure described in Ref. 3 can be used and, in the first approximation, we can employ the above simple formulas to find the orientational dependence of the emitted radiation.

We shall consider in greater detail the orientational dependence of the total radiation intensity. As pointed out already, the range of validity of Eq. (2.9) is then independent of the parameter ρ_c and it is the same as the range of validity of the semiclassical approximation. If $\vartheta_0 \lesssim \vartheta_c$, the orientational dependence for a thin crystal is related to a redistribution of the flux of incident particles, i.e., it is related to a change in the distribution function $F(\mathbf{r},\vartheta_0)$ [see Eq. (2.5)] in the dependence on the angle ϑ_0. This distribution is found to be different for electrons ($-$) and positrons ($+$), so that the emission of the radiation in the range $\vartheta_0 \lesssim \vartheta_c$ depends on the sign of the charge of the particle. If $\vartheta_0 = 0$, we find from Eq. (2.5) that in the case of an arbitrary axially symmetric potential

$$F_{ax}^{(-)}(\rho^2,0)=\ln(x_0/x),$$

$$F_{ax}^{(+)}(\rho^2,0)=\ln[x_0/(x_0-x)], \quad x=\rho^2/a_s^2. \tag{5.1}$$

If $\chi_s \lesssim 1$, the contribution to the total intensity comes from the range $\eta \lesssim x \lesssim 1$, which is characterized by $F_{ax}^{(-)} \gtrsim \ln x_0$ and $F_{ax}^{(+)} \sim x_0^{-1}$, from which it follows that—compared with the case of a uniform distribution over the transverse coordinate—the intensity $I^{(-)}$ is enhanced at $\vartheta_0 = 0$ by a factor of approximately $\ln x_0$ and the intensity $I^{(+)}$ is weakened approximately by a factor x_0. When the angle ϑ_0 is increased, the distribution $F_{ax}^{(-)}(\rho^2,\vartheta_0)$ rapidly becomes uniform, whereas $F_{ax}^{(+)}(\rho^2,\vartheta_0)$ varies quite slowly. This can be deduced also from the results of Ref. 14 [Eqs. (30)–(35)], where it is shown for the case $\chi_s \ll 1$ that $I^{(-)}$ is maximal at $\vartheta_0 = 0$ and an increase in the angle of incidence causes rapid approach to the value I^{as} given by Eq. (33) of Ref. 14, whereas $I^{(+)}$ has a minumum at $\vartheta_0 = 0$ and becomes comparable with I^{as} near $\vartheta_0 = \vartheta_c$. The nature of the distribution $F(\mathbf{r},\vartheta_0)$ becomes important when we discuss the emission of radiation from a beam of electrons characterized by a finite angular width $\Delta\vartheta_0$, since already for $\Delta\vartheta_0 \approx 1/2\vartheta_c$ approximately 80% of the electrons are in subbarrier states and, naturally, are distributed uniformly between the transverse coordinates. Moreover, we should bear in mind that in view of the above-mentioned redistribution of the flux, multiple scattering of electrons is enhanced and that of positrons is weakened in the range $\vartheta_0 < \vartheta_c$, so that we can have a situation in which a crystal is thin for positrons but not for electrons. If $\vartheta_0 \gg \vartheta_c$ and $\chi_s \ll 1$, the radiation intensity has a plateau (uniform distribution) and the orientational dependence is described by formulas from Ref. 14 right up to angles $\vartheta_0 \sim V_0/m\chi_s$, which is followed by a fall of the intensity[3)] described by Eq. (4.3). The intensity of the radiation emitted by electrons and positrons is the same for $\vartheta_0 > \vartheta_c$. The ranges of validity of Eqs. (3.9) and (4.3) for $\chi_s \ll 1$ overlap, so that if $\chi_s \ll 1$ the orientational dependence of the radiation intensity is described completely by these simple formulas (the orientational dependence in the case of planar channeling was considered by us for this situation some time ago[7]). As the parameter χ_s increases, this plateau becomes narrower and it disappears completely when $\chi_u = \chi_s a_s/u_1 \sim 1$. If $\chi_s \gtrsim 1$, the intensity of the radiation emitted by electrons is still maximal at $\vartheta_0 = 0$ and falls monotonically on increase in ϑ_0. In the case of positrons a minimum of the intensity $I^{(+)}$ remains at $\vartheta_0 = 0$; as ϑ_0 is increased, the value of $I^{(+)}$ rises and reaches its maximum at $\vartheta_0 \approx \vartheta_c$ and in the range $\vartheta_0 \gg \vartheta_c$ it is identical with the expression describing the radiation emitted by electrons. It should be pointed out that on increase in χ_s the contribution to the intensity made for $\vartheta_0 < \vartheta_c$ comes from increasing values of x. Consequently, the difference between the intensities of the radiation emitted by electrons and positrons decreases and for sufficiently high values in the range $\chi_s \sim x_0^{3/2} \gg 1$ the difference practically disappears.

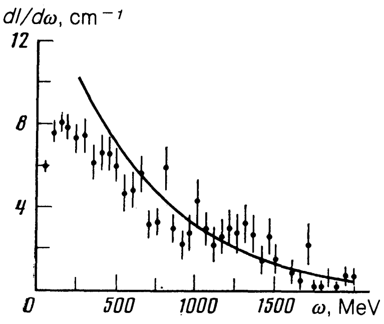

FIG. 5. Spectral dependence of the intensity of the radiation in Si ⟨111⟩ obtained for $\varepsilon = 10$ GeV. The experimental results are taken from Ref. 22.

Experiments were recently carried out on the orientational dependence of the intensity of the radiation emitted by electrons and positrons of $\varepsilon = 150$ GeV energy incident on a Ge crystal near the ⟨110⟩ axis when the crystal temperature was T = 100 K (Refs. 19 and 20). In this situation we found that $\chi_s \sim 1$. We used the above theory to analyze these experimental results.[21] It was found that the theory accounted quite satisfactorily for all the available experimental data.

It should be pointed out also that if $\vartheta_0 \lesssim \vartheta_c$ when $\chi_s \ll 1$ then even in the case of $\rho_c \gg 1$ the maximum of the spectral intensity occurs at frequencies such that $u \sim u_{\min}$, where the bremsstrahlung description becomes invalid. Nevertheless, it can be used at frequencies in the range $\omega \gtrsim \varepsilon\chi_s$. Such a situation occurs, for example, in the experimental results of Ref. 22 when a crystal of silicon was used near the ⟨111⟩ axis and the energy of the incident particles was $\varepsilon = 10$ GeV. Using Eqs. (1.4) and (3.8), we found that the main frequency is $\omega \sim \varepsilon\chi_s \approx 270$ MeV and also that $\omega_{\min} \sim \varepsilon u_{\min} \approx 90$ MeV; $\rho_c \sim 4.1$. Figure 5 shows the results of a calculation of the spectrum carried out for this case in the synchrotron radiation approximation allowing for the effective divergence of the beam and the experimental data of Ref. 22. We can see that the theoretical curve describes quite satisfactorily the experimental data—the photon frequency range $\omega > 300$ MeV.

[1)]In Refs. 5 and 6 we used the initial formulas of the treatment given in Ref. 4 and all the other calculations were carried out numerically.
[2)]We shall use a system of units such that $\hbar = c = 1$.
[3)]We can see from the discussion in Sec. 4 that in fact the parameter in this case is λ_u, defined by Eq. (4.4).

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