Photodetection of interference fields in the case of a small number of photons

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A quantum theory of photodetection of interference fields is developed. Among others, this theory deals with the method of two-photon coincidences which can be used to reveal quantum effects. It is shown that an interference pattern obtained by this method depends on the state of atomic systems creating interfering light beams (packets), especially on the correlation between the fields of neighboring atoms: in the absence of a correlation the usual spontaneous radiation is generated, whereas in the case of full correlation the result is in the form of collective radiation; the visibility of the interference patterns observed in these two cases is different. The results are compared with theoretical relationships derived in earlier investigations.

INTRODUCTION

Problems related to interference of extremely weak light beams and to quantum effects which should then be observed are currently attracting much interest. We shall consider the relationship between an interference pattern and the states of atomic systems creating light beams and we shall show that, in particular, interference is different in the case of individual spontaneous emission from atoms in each group and in the case of collective spontaneous emission.

1. INTERFERENCE OF LIGHT BEAMS FROM TWO ATOMS

We shall consider a linearly polarized electromagnetic field; the operator of the vector potential for this field has a single Cartesian component

$$A(\mathbf{r},t)=\sum_\lambda [a_\lambda U_\lambda(\mathbf{r},t)+a_\lambda^+U_\lambda^*(\mathbf{r},t)], \qquad (1)$$

where a_λ and a_λ^+ are annihilation and creation operators of a photon, corresponding to a plane wave

$$U_\lambda(\mathbf{r},t)=(2\pi\hbar c^2/\omega_\lambda V)^{1/2}\exp[i(\mathbf{k}_\lambda\mathbf{r}-\omega_\lambda t)], \quad \omega_\lambda=ck_\lambda \qquad (2)$$

traveling in a quantization region V which is a rectangular parallelepiped for which the wave vectors $\mathbf{k}_\lambda$ are specified by the conditions of periodicity.

We shall assume that this field is created by two groups of identical two-level atoms (the number of atoms in each group is m_1 and m_2) and we shall consider interference between the fields emitted by these groups of atoms. If initially at $t=0$ there is only one excited pth atom, then after $t\gg T$ (T is the spontaneous emission time) the field is in a state $b_p^+|\{0\}\rangle$, where $|\{0\}\rangle$ is the vacuum state of the field, and b_p^+ is the photon creation operator corresponding to a spherical wave diverging from the pth atom. This operator is described by

$$b_p^+=\sum_\lambda \beta_{p,\lambda}a_\lambda^+, \qquad \sum_\lambda |\beta_{p,\lambda}|^2=1, \qquad (3)$$

where $\beta_{p,\lambda}$ are constant coefficients; if the atom emits in free space, then $\beta_{p,\lambda}=\beta_\lambda\exp(-i\mathbf{k}_\lambda\mathbf{r}_p)$. Operators of this type are introduced in Ref. 1. The operator (3) is associated with a complex function

$$W_p=\sum_\lambda \beta_{p,\lambda}U_\lambda, \qquad (4)$$

which represents an analytic signal (see, for example, Ref. 2) for a classical vector potential.

According to the theory of photodetection developed by Glauber[3] (see also Ref. 2), the result of photodetection of a field in a state $|\psi\rangle$ by one and two photodetectors is described by the functions

$$J=\langle\psi|A_+A_-|\psi\rangle, \quad J^{(2)}=\langle\psi|A_+A_+'A_-A_-'|\psi\rangle, \qquad (5)$$

where

$$A_+=\sum_\lambda a_\lambda^+U_\lambda^*, \quad A_-=\sum_\lambda a_\lambda U_\lambda \qquad (6)$$

are operators dependent on $\mathbf{r}$ and t (representing the coordinates of one of the photodetectors and the instant at which it operates), whereas A'_+ and A'_- are the same operators dependent on $\mathbf{r}'$ and t' and representing the second photodetector. The functions of Eq. (5) represent the correlation properties of the field. In the case of a field in the state $b_p^+|\{0\}\rangle$ we have $J=|W_p|^2$ and $J^{(2)}=0$.

The operators of Eq. (3) satisfy the relationships

$$b_pb_p^+-b_p^+b_p=1, \quad b_pb_q^+-b_q^+b_p=R_{pq} \text{ for } p\neq q, \qquad (7)$$

where

$$R_{pq}=R_{qp}^*=\sum_\lambda \beta_{p,\lambda}^*\beta_{q,\lambda} \qquad (8)$$

is the spatial correlation coefficient of the fields emitted by atoms p and q. Bearing in mind that the emitted fields are quasimonochromatic ($\omega_\lambda\approx\omega^0$), we find that the functions W_p are described by the following approximate relationships

$$\int_V |W_p|^2dV=\frac{2\pi\hbar c^2}{\omega^0}, \qquad \int W_p^*W_q\,dV=\frac{2\pi\hbar c^2}{\omega^0}R_{pq}, \qquad (9)$$

which show that the quantity R_{pq} determines also the difference between the energy of the sum field W_p+W_q and the sum of the energies in fields W_p and W_q. If the atoms emit in free space, then $R_{pq}\approx 0$ for $|\mathbf{r}_p-\mathbf{r}_q|\gg\Lambda$ and $R_{pq}\approx 1$ for $|\mathbf{r}_p-\mathbf{r}_q|\ll 1$, where the distance Λ (correlation radius) is of

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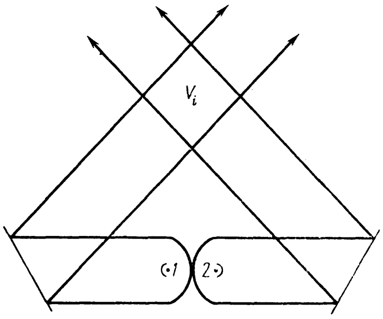

FIG. 1. Interference of one-photon beams (1 and 2 are the centers of spherical mirrors and foci of paraboloidal mirrors).

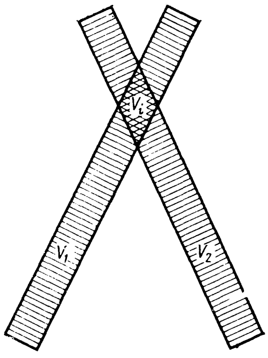

FIG. 2. Interference of cylindrical optical beams with an arbitrary number of photons.

the order of the wavelength $2\pi c/\omega^0$. In the former case the excited atoms emit practically independently of one another: this is the case of generation of the usual spontaneous radiation, which will be discussed in Sec. 2 on the assumption that $R_{pq} = 0$ when $p \neq q$. In the latter case the atoms p and q interact strongly and collective spontaneous radiation (superradiance) is generated. These estimates can be refined,[4] but they are sufficient for our purpose. When the interaction takes place, the quantities $\beta_{p,\lambda}$ change generally differently for different atoms: superradiance will be considered in Sec. 3 on condition that $R_{pq} = 1$ for atoms in the same group and $R_{pq} = 0$ for atoms in different groups. Clearly, these are the limiting cases, but the formulas obtained for the more general conditions are very cumbersome.

Spontaneous radiation emitted by atoms may be fully correlated in an extended region[4] and it is then directional. However, if the radiation emitted by atoms in each group is not directional, it can be converted into a directional wave beam by, for example, a system shown in Fig. 1 and consisting of small spherical and large paraboloid mirrors. Systems of this kind are used widely in antenna technology and they make it possible to convert radiation from atoms in each group (which occupies a volume considerably greater than the wavelength) into a single quasiplane wave. Two such waves intersect in the interference zone V_i.

We shall apply the Eqs. (7) to the interference between fields created by two atoms ($m_1 = m_2 = 1$, $p = 1$ or 2) which at $t = 0$ are in the state

$$|p\rangle=\rho_p|1_p\rangle+\sigma_p|0_p\rangle, \quad |\rho_p|^2+|\sigma_p|^2=1, \tag{10}$$

characterized by constant values of ρ_p and σ_p; here, $|1_p\rangle$ is an excited state of the pth atom and $|0_p\rangle$ is its ground state. Therefore, for $t \gg T$ and $R_{12} = 0$ the field is in a state

$$|\psi\rangle=c_1{}^+c_2{}^+|\{0\}\rangle, \quad c_p{}^+=\rho_p b_p{}^+ + \sigma_p. \tag{11}$$

We then have

$$A_-|\psi\rangle=(\rho_1 W_1 c_2{}^+ + \rho_2 W_2 c_1{}^+)|\{0\}\rangle,$$

$$A_-A_-'|\psi\rangle=\rho_1\rho_2 W_{12}|\{0\}\rangle,$$

where the following notation is used:

$$W_{pq}=W_pW_q'+W_qW_p', \tag{12}$$

so that

$$J=|\rho_1W_1|^2+|\rho_2W_2|^2+2\,\mathrm{Re}\,(\rho_1\sigma_1{}^*\rho_2{}^*\sigma_2W_1W_2{}^*),$$

$$J^{(2)}=|\rho_1\rho_2W_{12}|^2. \tag{13}$$

We shall be interested mainly in the interference of quasimonochromatic and quasiplane waves in the interference zone V_i. We need not then consider a specific optical system and we can discuss (as in Fig. 2) two light beams (packets) with a finite transverse cross section and a finite length (and, consequently, a finite energy) which intersect in the zone V_i. These beams pass through the zone V_i at a velocity c and they interfere for a finite time. If in this zone we have $|W_1| = |W_2|$ and the beams are characterized by wave vectors $\mathbf{k}_1^0$ and $\mathbf{k}_2^0$. then we can substitute in Eq. (13)

$$W_p=J_0{}^{1/2}e^{i\varphi_p}, \quad \varphi_1=\mathbf{k}_1{}^0\mathbf{r}-\omega^0t+\Phi_1, \quad \varphi_2=\mathbf{k}_2{}^0\mathbf{r}-\omega^0t+\Phi_2, \tag{14}$$

where J_0 and Φ_p are slowly varying functions of $\mathbf{r}$ and t. Assuming, moreover, that

$$\rho_p\sigma_p{}^*=|\rho_p\sigma_p|e^{i\psi_p}, \tag{15}$$

we obtain the expressions

$$J=J_0[|\rho_1|^2+|\rho_2|^2+2|\rho_1\sigma_1\rho_2\sigma_2|\cos(\varphi_1+\psi_1-\varphi_2-\psi_2)],$$

$$J^{(2)}=2|\rho_1\rho_2|^2J_0{}^2(1+\cos\Phi),$$

$$\Phi=\varphi_1-\varphi_1'-\varphi_2+\varphi_2'=(\mathbf{k}_1{}^0-\mathbf{k}_2{}^0)(\mathbf{r}-\mathbf{r}'). \tag{16}$$

The first expression shows that the interference recorded by one photodetector occurs only if $\sigma_1 \neq 0$ and $\sigma_2 \neq 0$, i.e., it occurs when the number of photons in each beam is indeterminate and depends on the phases Φ_p and ψ_p. The second expression (for $J^{(2)}$) does not contain these phases and the visibility is always $v = 1$, which is essentially a quantum effect (see Sec. 4). In general, the visibility is defined as follows:

$$v=(J_{max}-J_{min})/(J_{max}+J_{min}).$$

2. INTERFERENCE OF LIGHT BEAMS IN THE CASE OF THE USUAL SPONTANEOUS EMISSION

If light beams (Fig. 2) are created by groups of atoms and these atoms emit independently ($R_{pq} = \delta_{pq}$) beginning with an initial state described by Eq. (10), then the state of the field at $t \gg T$ is given by

$$|\psi\rangle=\prod_p c_p{}^+|\{0\}\rangle, \tag{17}$$

which is a generalization of Eq. (11); the subscript p assumes $m = m_1 + m_2$ values and labels the atoms in the first group (from 1 to m_1) and then in the second group (from m_1+1 to m_1+m_2).

The functions J and $J^{(2)}$ for the state (17) are

$$J=\sum_p |\rho_p|^2|W_p|^2+2\,\mathrm{Re}\sum_{p<q}\rho_p\sigma_p^*\rho_q^*\sigma_q W_p W_q^*,$$
$$J^{(2)}=\sum_{p<q,r<s}\rho_p\rho_q\rho_r^*\rho_s^*(pq;rs)\,W_{pq}W_{rs}^*, \tag{18}$$

where in the case of four different subscripts we have

$$(pq;rs)=\sigma_p^*\sigma_q^*\sigma_r\sigma_s, \tag{19}$$

and when two subscripts are identical (for example, if $p=r$), then the corresponding values of σ vanish (the product $\sigma_p^*\sigma_r$ vanishes); $(pq;pq)=1$. Using Eqs. (14) and (15), we can rewrite Eqs. (18) and obtain

$$J=J_0\Big[\sum_p |\rho_p|^2+2\sum_{p<q}|\rho_p\sigma_p\rho_q\sigma_q|\cos(\varphi_p+\psi_p-\varphi_q-\psi_q)\Big],$$
$$J^{(2)}=2J_0^2\sum_{p<q,r<s}\rho_p\rho_q\rho_r^*\rho_s^*(pq;rs)\times\left(\cos\frac{\Phi_{pq}-\Phi_{rs}}{2}+\cos\frac{\Phi_{pq}+\Phi_{rs}}{2}\right)\times\exp\left(i\frac{\Phi_{pr}+\Phi_{qs}}{2}\right), \tag{20}$$

where

$$\Phi_{pq}=\varphi_p-\varphi_p'-\varphi_q+\varphi_q', \tag{21}$$

which has the value $\Phi_{pq}=0$ if the atoms p and q belong to the same group, but $\Phi_{pq}=\Phi$ if the atom p is in the first group and the atom q is in the second group; in the opposite case we obtain $\Phi_{pq}=-\Phi$. In the process of formation of Φ_{pq} the additional phases Φ_p due to the distribution of atoms in space cancel out and the function $J^{(2)}$ contains the phase shift $\Phi=(\mathbf{k}_1^0-\mathbf{k}_2^0)(\mathbf{r}-\mathbf{r}')$, which is recorded by the method of two-photon coincidences, moreover, this function contains phases ψ_p which depend on how the atoms are excited.

We shall assume that all the phases Φ_p and ψ_p are random and independent. Then, averaging over these phases gives

$$\bar{J}=J_0\sum_p|\rho_p|^2,\quad \overline{J^{(2)}}=2J_0^2\sum_{p<q}|\rho_p\rho_q|^2(1+\cos\Phi_{pq}), \tag{22}$$

and if we assume that

$$|\rho_p|^2=P_1 \text{ when the atom } p \text{ is in the first group},$$
$$|\rho_p|^2=P_2, \text{ when the atom } p \text{ is in the second group}, \tag{23}$$

we find that

$$\bar{J}=J_0(m_1P_1+m_2P_2),$$
$$\overline{J^{(2)}}=2J_0^2[m_1(m_1-1)P_1^2+m_2(m_2-1)P_2^2+m_1m_2P_1P_2(1+\cos\Phi)]. \tag{24}$$

Therefore, there is no interference in the case of one-photon detection and random phases and the oscillatory term disappears. However, if

$$\Phi_p+\psi_p=\chi_1, \text{ when the atom } p \text{ is in the first group},$$
$$\Phi_p+\psi_p=\chi_2, \text{ when the atom } p \text{ is in the second group}, \tag{25}$$

then instead of the first expression in the system (24), we now have

$$J=\bar{J}=J_0\{m_1P_1+m_2P_2+m_1(m_1-1)P_1Q_1+m_2(m_2-1)P_2Q_2+2m_1m_2(P_1P_2Q_1Q_2)^{1/2}\cos[(\mathbf{k}_1^0-\mathbf{k}_2^0)\mathbf{r}+\chi_1-\chi_2]\},\quad Q_l=1-P_l, \tag{26}$$

i.e., interference is observed. The conditions of Eq. (25) are realized when atoms in each group are excited by a laser pulse traveling in the direction of the light beam initiated by such a pulse (Fig. 2). The second expression in Eq. (24) is not affected under the conditions described by Eq. (25). Equation (26) is formally valid also when Φ_p and ψ_p are constant (and different) in each group, but this means that the size of each group is small compared with the wavelength and then the interaction of atoms and radiation becomes collective; the relationships governing this case are different (see Sec. 3).

3. INTERFERENCE OF LIGHT BEAMS IN THE CASE OF COLLECTIVE SPONTANEOUS EMISSION

If the emission of atoms in each group is fully correlated, then the state of the resultant field is described by

$$|\psi\rangle=\frac{(c_1^+)^{m_1}(c_2^+)^{m_2}}{[N_1(m_1)N_2(m_2)]^{1/2}}|\{0\}\rangle \tag{27}$$

or

$$|\psi\rangle=\sum_{n_1,n_2}\gamma_{1,n_1}\gamma_{2,n_2}|n_1,n_2\rangle,\qquad \sum_n|\gamma_{l,n}|^2=1, \tag{28}$$

where the state

$$|n_1,n_2\rangle=\frac{(b_1^+)^{n_1}(b_2^+)^{n_2}}{(n_1!n_2!)^{1/2}}|\{0\}\rangle \tag{29}$$

can be represented as n_1 photons in a cylindrical zone V_1 that moves along the lines of flow at a velocity c, and n_2 is the number of photons in an identical volume V_2, the intersection of which with V_1 gives rise to an additional interference volume V_i (Fig. 2). In the case of uncorrelated spontaneous radiation emitted by excited atoms ($\rho_p=1$, $\sigma_p=0$), discussed above, we now have a different state:

$$|1,\dots,1;1,\dots,1\rangle=\prod_{p=1}^{n_1+n_2}b_p^+|\{0\}\rangle \tag{30}$$

with the same number of photons in each beam. Bearing in mind that because of the interaction of the atoms the functions W_l change (see Sec. 1) and for $m_1\neq m_2$ it is certain that $|W_1|\neq|W_2|$, we find that the state (29) is described by

$$J=J_{01}n_1+J_{02}n_2,$$
$$J^{(2)}=J_{01}^2n_1(n_1-1)+J_{02}^2n_2(n_2-1)+2J_{01}J_{02}n_1n_2(1+\cos\Phi), \tag{31}$$

where $J_{0l}=|W_l|^2$, whereas in the case of the state (30) we have

$$J=J_0(n_1+n_2),$$
$$J^{(2)}=2J_0^2[n_1(n_1-1)+n_2(n_2-1)+n_1n_2(1+\cos\Phi)]. \tag{32}$$

In these formulas the numbers of photons n_1 and n_2 in each beam is fully determinate, and because of complete indeterminacy of the phases one photodetector cannot reveal interference, exactly as in the case of a difference between the states (29) and (30) when $J_{01} = J_{02}$. Under two-photon detection conditions the difference can be revealed even in the classical limit: for example, if $n_1 = n_2 \gg 1$, we find from Eq. (31) that $v = 1/2$, whereas Eq. (32) gives $v = 1/3$. The higher visibility ($v = 1/2$) is clearly due to the greater coherence of the state (29), which appears in the case of superradiance.

Going from Eq. (27) to Eq. (28), we find that the coefficients $\gamma_{l,n}$ become

$$\gamma_{l,n}=\frac{1}{[N_l(m_l)]^{1/2}}\frac{m_l!}{(n!)^{1/2}(m_l-n)!}\rho_l^n\sigma_l^{m_l-n},$$
$$N_l(m_l)=\sum_{n=0}^{m_l}\frac{(m_l!)^2}{n![(m_l-n)!]^2}P_l^nQ_l^{m_l-n}, \tag{33}$$

but it is easy to find J and $J^{(2)}$ for arbitrary values of $\gamma_{l,n}$. If

$$\gamma_{l,n}=\exp\left(-\frac{1}{2}|\alpha_l|^2\right)\frac{\alpha_l^n}{(n!)^{1/2}} \quad (l=1,2) \tag{34}$$

a coherent state closest to a classical field is obtained. For the general state (28), we have

$$J=\langle n_1\rangle|W_1|^2+\langle n_2\rangle|W_2|^2+2\,\mathrm{Re}\,(\alpha_1W_1\alpha_2^*W_2^*),$$
$$J^{(2)}=\langle n_1(n_1-1)\rangle|W_1W_1'|^2+\langle n_2(n_2-1)\rangle|W_2W_2'|^2+\langle n_1\rangle\langle n_2\rangle|W_{12}|^2$$
$$+2\,\mathrm{Re}[\alpha_1^{(2)}W_1W_1'\alpha_2^{(2)*}W_2^*W_2'^*$$
$$+\langle n_1\rangle\alpha_1^{(1)}\alpha_2^*(|W_1|^2W_1'W_2'^*+|W_1'|^2W_1W_2^*)$$
$$+\langle n_2\rangle\alpha_2^{(1)}\alpha_1^*(|W_2|^2W_2'W_1'^*+|W_2'|^2W_2W_1^*)], \tag{35}$$

where

$$\langle n_l^k\rangle=\sum_n n^k|\gamma_{l,n}|^2,\quad \alpha_l=\sum_n (n+1)^{1/2}\gamma_{l,n}^*\gamma_{l,n+1},$$
$$\alpha_l^{(1)}=\frac{1}{\langle n_l\rangle}\sum_n n(n+1)^{1/2}\gamma_{l,n}^*\gamma_{l,n+1}, \tag{36}$$
$$\alpha_l^{(2)}=\sum_n[(n+1)(n+2)]^{1/2}\gamma_{l,n}^*\gamma_{l,n+2},$$

and in the case of the coherent state (34) we have the relationships

$$\langle n_l\rangle=|\alpha_l|^2,\quad \alpha_l^{(1)}=\alpha_l,\quad \alpha_l^{(2)}=\alpha_l^2,$$
$$J=|\alpha_1W_1+\alpha_2W_2|^2,\quad J^{(2)}=JJ', \tag{37}$$

corresponding to a semiclassical theory of photodetection in which the quantum properties of the field are ignored.

The expression (35) for J shows that in the case of an indeterminate number of photons the occurrence of interference can be observed using one photodetector, because $\alpha_1\neq 0$ and $\alpha_2 \neq 0$. The expression for $J^{(2)}$ is very cumbersome, but it can be simplified if we average the quantities α_l, $\alpha_l^{(1)}$, and $\alpha_l^{(2)}$ over random phases. In fact, it follows from Eqs. (15) and (33) that α_l and $\alpha_l^{(1)}$ contain a phase factor exp($i\psi_l$), where $\alpha_l^{(2)}$ contains a factor exp($2i\psi_l$), so that after averaging we obtain the expressions

$$\bar{J}=J_{01}\langle n_1\rangle+J_{02}\langle n_2\rangle,\quad \overline{J^{(2)}}=J_{01}^2\langle n_1(n_1-1)\rangle+J_{02}^2\langle n_2(n_2-1)\rangle$$
$$+2J_{01}J_{02}\langle n_1\rangle\langle n_2\rangle(1+\cos\Phi), \tag{38}$$

with the structure similar to Eq. (31). This is to be expected: in the case of fully determinate numbers n_1 and n_2 the expressions in Eq. (38) should reduce to Eq. (31), and then averaging over the phases is unnecessary.

In the case of the state (27) it follows from Eqs. (33) that

$$\langle n_l\rangle=f(m_l),\quad \langle n_l^2\rangle=f(m_l)f(m_l-1)+f(m_l),$$
$$f(m_l)=m_l^2P_lN_l(m_l-1)/N_l(m_l), \tag{39}$$

which can be used to calculate $\langle n^2\rangle$, $\langle n\rangle$, and $\langle n_l\rangle$ for m_l and P_l.

Assuming that $J_l = J_{0l}\langle n_l\rangle$, we shall rewrite the expressions in Eq. (38) as follows:

$$\bar{J}=J_1+J_2,\quad \overline{J^{(2)}}=(\bar{J})^2(1+\delta^{(2)})+2J_1J_2\cos\Phi, \tag{40}$$

where the parameter

$$\delta^{(2)}=\{J_{01}^2[\langle(\delta n_1)^2\rangle-\langle n_1\rangle]+J_{02}^2[\langle(\delta n_2)^2\rangle-\langle n_2\rangle]\}/(\bar{J})^2,$$
$$\delta n_l=n_l-\langle n_l\rangle \tag{41}$$

is governed by the quantum properties of the optical field. We have $\langle n_l\rangle = m_l^2P_l$ for $m_l^2P_l \ll 1$ and $\langle n_l\rangle = m_l$ for $Q_l = 0$. Therefore, in the simplest case of $P_1 = P_2 = P$ and $m_1 = m_2 = m/2$, we obtain $\delta^{(2)} = -2P(m-1)/m_2$ for $m^2P \ll 1$ and $\delta^{(2)} = -1/m$ for $Q = 0$, so that $\delta^{(2)} \to 0$ in the limit $m \to \infty$; since $d^{(2)} < 1$ corresponds to finite values of m, the quantum effects increase the visibility. The intensity J_l of each beam is proportional to m_l^2 in the case of collective emission. At low values of P_l this is achieved without significant changes in the functions W_l and J_0, whereas at low values of Q_l the function J_{0l} is compressed by a factor m_l (so that the same energy as in the case of an individual emission of radiation is released in a time T/m_l), so that $J_{0l} \propto m_l$ and $J_l \propto m_l^2$.

The visibility of the interference pattern (both J and $J^{(2)}$) is not affected if a part of the radiation reaches the interference zone and the rest escapes to the sides. This follows from the fact that the scattering toward the sides and the reflection and refraction alter only the coefficients $\beta_{p,\lambda}$ which occur in Eqs. (3) and (4).

4. COMPARISON WITH EARLIER INVESTIGATIONS

The expressions in Eqs. (40) and (41) were obtained (for $J_{01} = J_{02} = J_0$) for the first time in our investigation[5] using a simplifying assumption involving replacement of each wave beam with a plane wave (within the limits of the interference zone); a similar replacement is made in the one-mode radiation model in Ref. 4. However, it was noι pointed out in Ref. 5 that these expressions apply to collective emission; in the case of the usual spontaneous emission we have to use Eq. (24) for $J^{(2)}$.

It is worth noting that Eqs. (24), (38), and (40) have the same form

$$\overline{J^{(2)}}=C+2J_1J_2(1+\cos\Phi) \tag{42}$$

and differ only by the term C which is independent of the photodetector coordinates. The term C depends on the quantum state of the field and contains the quantities $m_l(m_l-1)$ and $n_l(n_l-1)$, i.e., it decreases because of the quantum effects, whereas the second term is essentially classical. The physical meaning of this is self-evident (see Ref.

5): the term C is due to the absorption of two photons from the same beam and after the absorption of one photon the state of the beam generally changes and the second photon is absorbed from a beam in the new state. The exception here is the beam in a coherent state, which after the loss of a photon reduces to the previous form; interference between such beams is described by Eq. (40) with $\delta^{(2)}=0$. The second term in Eq. (42) is due to the absorption of photons from different beams: in the case of two one-photon beams (see Sec. 1) only this term is retained, because we then have $C=0$.

Pure states and mixed states with random phases are considered above. In the case of the more general mixed states we shall require additional averaging; the appropriate generalization of Eq. (40) corresponding to $J_{01}=J_{02}$ was given in Ref. 5.

Tatarskiĭ[6] investigated interference patterns corresponding to different states of the field, including the states (29) for $n_1=n_2=1$ and 2; it was pointed out that an increase in the number of photons (from $n=2$ to $n=4$) reduces the visibility of the interference pattern. However, no allowance is made in Ref. 6 for the fact that a complete calculation of this effect (for $J_{01}=J_{02}$ and any value of n) gives Eq. (40) obtained earlier[5] and in this case we have $\delta^{(2)} = -1/n$, so that there is no need for averaging over the phases (see Sec. 3).

Mandel[7] considered the interference of fields generated as a result of spontaneous emission. In fact, Mandel discussed the statistics of photons reaching apertures of photodetectors during a short time interval δt after the arrival of the field at the photodetectors. Therefore, Mandel considered the field near its front and a real photodetector is unsuitable for measurement of such a field (see, for example, p. 276 in Ref. 2), because the field is not narrow-band close to the front. This is important because the functions and J and $J^{(2)}$ represent the results of photodetection only if within the frequency range occupied by the field the spectral sensitivity of the photodetector is constant.[2,3] Nevertheless, many relationships from Mandel's paper[7] are valid in the theory of photodetection[3] and only some of the conclusions have to be corrected. In particular, the terms $m_l(m_l-1)P_lQ_l$ in expressions of type (26) should not be attributed to collective emission: in fact, these expressions are valid when the distances between atoms are large compared with Λ and there is no interaction; in this case the forced phase locking of atoms is due to an exciting pulse. Equation (29) in Ref. 7, derived by cumbersome combinatorics, follows from our Eq. (20) for $J^{(2)}$ on condition that the phases ψ_p are the same in each group, which postulates smallness of the dimensions of the group compared with Λ, when we would have to allow for the collective nature of the emission.

In the review by Paul[8] the simplest Mandel relationships[7] are derived already on the basis of Glauber's theory[3] but the dynamic aspect, which is the difference between individual spontaneous emission and collective emission (Sec. 3), is not discussed.

Equations (24), (31), (32), (38), and (40) for J correspond to Dirac's assertion that different photons do not interfere, but the expressions (16), (20), (26), and (35) for J and all the formulas for $J^{(2)}$ suggest that the reverse is true (see also Refs. 6 and 8).

CONCLUSIONS

In experimental investigations of the interference of weak light beams the most interesting from the physical point of view is the function $J^{(2)}$, measured by the method of two-photon coincidences, because this function manifests the quantum properties of the optical field when the number of photons is small; the physical meaning is given after Eq. (42). An interference pattern obtained by the two-photon coincidence method obeys (for any intensity no matter how low) the classical laws in two cases: when the light beams are in a coherent state (see Secs. 3 and 4) and when the number of independently emitting atoms fluctuates in accordance with the Poisson law [i.e., if in the relationships of Sec. 2, we have $\overline{m_l(m_l-1)}=(\overline{m}_l)^2$; see Refs. 7 and 8]. Therefore, the quantum properties should be observed in experiments on interference between fields emitted by a small number of atoms. Only thought experiments of this kind have been carried out so far.

It should be pointed out that in the experimental part of Ref. 5 the two-photon coincidence method was used to determine for the first time a transient interference pattern formed by two independent laser beams. The nature of this pattern was found to be the same as the interference of a split laser beam and it was independent of the beam intensities right up to values approximately an order of magnitude higher than the intensities in the experiments of Pfleegor and Mandel.[9] This result confirmed that the beams in question were in a coherent state. Although the passage of light through some absorbing and nonlinear media creates nonclassical light beams with photon antibunching, i.e., with a negative value of the parameter $\delta^{(2)}$ (see, for example, the review of Kozierowski[10]), the simplest and most convincing way of observing quantum interference is that described above, i.e., such quantum interference is best observed when the number of atoms is small. However, if a weak intensity is achieved by stopping down, reflection, refraction, or ordinary absorption, the photon statistics and visibility are not affected.

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