Induced plasma oscillations when resonance ions are coherently perturbed by light

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We study the mechanism for exciting monochromatic Langmuir waves in a plasma by resonant coherent effects of light pressure in a biharmonic field when the beat frequencies of the field are close to the nutation frequency in a strong optical field and to the electron plasma frequency. On the basis of the procedure used here, of separating fast and slow variables, taking into account parametric Rabi resonance and collective motions, we obtain equations for induced plasma oscillations, and we find their main characteristics. We show thus that the manifestations of the effects of the coherent optical perturbations of the translational degrees of freedom of the resonant particles which were considered by Krasnov [Sov. Phys. JETP **62**, 238 (1985)] may be appreciably enhanced in a plasma medium. In particular, they can cause buildup of parametric plasma instabilities.

I. INTRODUCTION

Dubetskiĭ *et al.*[1] were the first to draw attention to the possibility to apply resonance radiation presure (RRP) methods[2,3] for the optical excitation of coherent motions in a system of non-interacting atoms. It was suggested to use for this the Kapitza-Dirac effect and an atomic beam interacting with spatially dispersed standing waves.

It was shown in Ref. 4 that quasi-stationary coherent perturbations of a collisionless gas can be induced by using a biharmonic field under the conditions for Rabi resonance. They are manifested first of all in the form of a travelling wave with a directed velocity which is propagating through the gas:

$$\mathbf{v}=\mathbf{v}_1 \sin(\delta\omega t-\mathbf{QR}+\Phi),$$

where $\delta\omega$ is the mismatch of the field frequencies relative to one another.

We show in the present paper that such perturbations of ions which are in resonance with optical radiation may be appreciably enhanced in a plasma medium if their frequency is close to the frequency of the collective plasma oscillations—the electron Langmuir frequency $\delta\omega\sim\omega_{pe}$. The induced monochromatic oscillations of the macroscopic plasma characteristics which then appear may have a level which is sufficient for the development of parametric instabilities.[5]

In constrast to other methods for exciting plasma oscillations by electromagnetic radiation the mechanism considered here is not connected with any direct action on the electrons, but is caused by the excitation of an alternating ion current at the electron Langmuir frequency with a coherent transfer (due to a recoil effect) of momentum from the biharmonic optical radiation. Participation of the electrons in this process leads to buildup and establishment (in the stability region) of plasma oscillations with an amplitude and phase which are determined by the coherent optical field. The optical radiation is here in a region where the plasma is transparent ($\omega_0\gg\omega_{pe}$), which makes it possible to excite Langmuir oscillations in plasma media of large dimensions. In actual fact one is dealing with a plasma with electron densities $n_e\sim10^{10}$–10^{12} cm^{-3} ($\omega_{pe}\sim6\times10^9\sim6\times10^{10}$ Hz) in a field of biharmonic radiation with an intensity $\gtrsim$ kW/cm^2. For such intensities the oscillations of the electrons and ions at the optical frequency have a small effect on the state of the plasma as compared to the processes considered.

The ion collisions with a frequency $\nu_i\ll\delta\omega$ do not inhibit the main effect of the bare coherent perturbations of the ion motion, and outside the regions of parametric instability they guarantee, together with the electron collisions (or Landau damping in the short-wavelength case), that the induced plasma oscillations are stationary.

2. BASIC EQUATIONS

We consider first of all the propagation of a longitudinal electrostatic wave $\mathscr{E}$ in a plasma with an ion component which is in resonance with an external biharmonic field at an optical frequency which is considerably higher than the electron Langmuir frequency:

$$\mathbf{E}=\sum_{m=0}^{1}\mathbf{E}_m\exp i(\mathbf{k}_m\mathbf{R}-\omega t)+\text{c.c.},\quad \omega_0\neq\omega_1,\quad |E_0|\gg|E_1|. \tag{2.1}$$

We shall describe the motion of the electrons in the plasma by the hydrodynamic conservation laws, neglecting the electron oscillations at the optical frequency:

$$\left(\frac{\partial}{\partial t}+\nu_e+\mathbf{v}_e\nabla\right)\mathbf{v}_e=\frac{e}{m_e}\vec{\mathscr{E}}-\frac{\nabla p_e}{m_en_e},\quad \frac{\partial n_e}{\partial t}=-\nabla(n_e\mathbf{v}_e), \tag{2.2}$$

where n_e, $\mathbf{v}_e$, m_e, p_e, e, ν, and $\mathscr{E}$ are, respectively, the electron density, directed velocity, mass, pressure, electrical charge, collision frequency, and the electrical field strength in the plasma wave. We assume the electrons to be adiabatic so that

$$\nabla p_e=T_e\gamma_e\nabla n_e,\quad \gamma_e=3, \tag{2.3}$$

where T_e is the electron temperature in energy units.

The electric field strength in a longitudinal wave is connected with the electron and ion current density through the Maxwell equation

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$$\partial\vec{\mathscr{E}}/\partial t=4\pi(n_i\mathbf{v}_i-n_e\mathbf{v}_e)e, \quad (2.4)$$

where n_i and $\mathbf{v}_i$ are, respectively, the ion density and directed velocity.

The evolution of the ions is described by the equations for the Wigner density matrix $\hat\rho=\hat\rho(\mathbf{p},\mathbf{R},t)$ which in the resonance approximation can be written, as in Ref. 4, in the form

$$\left(\frac{\partial}{\partial t}+\frac{\mathbf{p}}{M}\frac{\partial}{\partial\mathbf{R}}+e\vec{\mathscr{E}}\frac{\partial}{\partial\mathbf{p}}\right)z=\frac{1}{\mu}(\hat A_0+i\hat\Delta)z+(\hat\Gamma+\hat A_1)z+\hat L_s(z), \quad (2.5)$$

where we have introduced the column vector

$$z=\mathrm{col}\,(\hat\rho_{21}e^{i\omega_0t},\ \hat\rho_{12}e^{-i\omega_0t},\ \hat\rho_{22}-\hat\rho_{11},\ \mathrm{Sp}\,(\hat\rho)),$$

the operators $(\hat A_0/\mu)$, $\hat A_1$, and $\hat\Gamma$ are, respectively, determined by the field and relaxation terms, and their explicit form is given in Ref. 4, $(\hat\Delta/\mu)=\mathrm{diag}\ (1,-1,0,0)\Delta$, $\Delta=\omega_0-\omega_{21}$ is the mismatch of the strong field. The formal small parameter $\mu\ll1$ in (2.5) assumes the following scaling of the Rabi frequency of the strong field and of the mismatches:

$$\frac{\mathbf{dE}_0}{\hbar}=\frac{\Omega_0}{\mu},\qquad \delta\omega=\omega_0-\omega_1=\frac{\Delta_0}{\mu}+\delta_0,$$
$$\Delta=-\frac{\Delta'}{\mu},\qquad |\Delta'|\sim|\Delta_0|\sim|\Omega_0|\sim|\delta_0|, \quad (2.6)$$

which guarantees that the inequalities necessary for the realization of the parametric Rabi-resonance regime considered here are satisfied:

$$|\delta\omega|\sim\left|\frac{\mathbf{dE}_0}{\hbar}\right|\sim|\Delta|\gg\left|\frac{\mathbf{dE}_1}{\hbar}\right|,$$
$$\gamma,\ \gamma_\perp,\ k_0\frac{p_0}{M},\ e\mathscr{E}_{\max}\left|\frac{\partial\hat\rho_{mn}}{\partial p}\hat\rho_{mn}^{-1}\right|,\ \nu_i, \quad (2.7)$$

where γ, $\gamma_\perp$ are the longitudinal and transverse relaxation rates, and ν_i, M, and p_0 are, respectively, the characteristic collision frequency, the mass, and the thermal momentum of the ions. The terms in (2.5) which are new as compared to Ref. 4, $e\mathscr{E}(\partial z/\partial p)$ and $\hat L_s(z)$ take into account the effect of the field of the Langmuir wave and of the collisions on the ion motion. In a weakly ionized plasma where collisions with neutral atoms play the main role, $\hat L_s$ is a linear operator:

$$\hat L_s(z)=\mathrm{col}(0,0,I_s^{(+)}(z_3)+I_s^{(-)}(z_4),\ I_s^{(+)}(z_4)+I_s^{(-)}(z_3)). \quad (2.8)$$

Here z_m is the m-th component of $z\,(m=1-4)$,

$$I_s^{(\pm)}(z_m)={}^1\!/_2\Big(-[\nu_1\pm\nu_2]z_m+\int[K_1(\mathbf{p},\mathbf{p}')\pm K_2(\mathbf{p},\mathbf{p}')]z_m(\mathbf{p}')d^3\mathbf{p}'\Big),$$

where ν_l, $K_l(\mathbf{p},\mathbf{p}')$ are the frequencies and kernels of the collisions in the states $l=1,2$.

Equations (2.2) to (2.5) are closed by the expression for the ion current density

$$\mathbf{J}_i=en_i\mathbf{v}_i=e\int f\frac{\mathbf{p}}{M}d^3\mathbf{p},\qquad f=\mathrm{Sp}(\hat\rho). \quad (2.9)$$

One can take the Landau damping phenomenologically into account, similarly to what is done in Refs. 6, 7 through the substitution

$$\nu_e\to\nu_e+\frac{\pi^{1/2}}{2}\frac{\omega_{pe}}{(Q\lambda_e)^3}\exp\left[-\frac{1}{2Q^2\lambda_e^2}-\frac{3}{2}\right], \quad (2.10)$$

where $\lambda_e=(T_e/m_e\omega_{pe}^2)^{1/2}$ is the Debye radius and Q the wave number of the excited Langmuir oscillations.

3. ION MOTION

The coherent perturbations of the ion component of the plasma by the optical field reach their largest value when the mismatch of the field frequencies is (with a relative accuracy $\sim\mu$) close to the nutation frequency in a strong field:[4]

$$\Delta_0=\pm G,\qquad G=(4|\Omega_0|^2+\Delta'^2)^{1/2}. \quad (3.1)$$

For an effective excitation of a plasma wave it is necessary that the frequency of the optical perturbations be in resonance with the electron Langmuir frequency ω_{pe}, which we scale by analogy with (2.6): $\omega_{pe}=(\omega'_{pe}/\mu)+\delta_e$. We shall therefore assume that the second resonance condition is satisfied:

$$G=\omega_{pe}'=|\Delta_0|. \quad (3.2)$$

The solution of Eqs. (2.5) under the conditions (2.7) contains fast oscillating and slow components. The separation of the fast and slow motions, taking recoil effects into account, can be accomplished by using the methods of the theory of singular perturbations on the basis of the procedure, described in Ref. 4, of expanding in the small parameter μ:

$$z=\hat F^{-1}_{\sigma\to p}(\bar z^{(0)}(\boldsymbol\sigma,\mathbf{R},\mathbf{t})+\mu\bar z^{(1)}(\boldsymbol\sigma,\mathbf{R},\mathbf{t})+\ldots), \quad (3.3)$$

where $\hat P^{-1}_{\sigma\to p}$ is the inverse Fourier transform operator,

$$\hat F^{-1}_{\sigma\to p}(u)=\frac{1}{(2\pi)^3}\int e^{-i\sigma p}u\,d^3\boldsymbol\sigma,$$

which accomplishes the transition from the $\{\boldsymbol\sigma,\mathbf{R}\}$ representation of the density matrix to the initial $\{\mathbf{p},\mathbf{R}\}$ representation, and $\mathbf{t}$ is the time-variable vector which includes both fast and slow variables. In contrast to Ref. 4 it is necessary to include terms of first order in μ, since the effect of the excited Langmuir wave on the high-frequency (hf) and slow components of the density matrix can only be taken into account in higher orders of the perturbation theory. We can construct the expansion in the small parameter in such a way (see Appendix) that the hf components are subject to slowly changing variables satisfying homogeneous differential equations which are fully coupled with one another in view of the resonance conditions (3.1) and (3.2), and which take into account the reaction of the excited hf motions to any order in μ. The main term of the expansion (3.3) then has the form[4]

$$\bar z^{(0)}=\bar\alpha_1\varphi_1+\bar\alpha_2\varphi_2+\bar\alpha_3\varphi_3e^{t_3}+\bar\alpha_4\varphi_4e^{t_4}, \quad (3.4)$$

where $\varphi_i(\boldsymbol\sigma)$ are the eigenvectors of the operator $\hat B=(\hat A_0+i\hat\Delta)$ in the $\{\boldsymbol\sigma\cdot\mathbf{R}\}$ representation, $t_{3,4}=(\pm iGt_1/\mu)$, $t_1=t-\mathbf{k}_0\mathbf{R}/c|k_0|$, $t_1>0$. In the reduced set of basic equations, the slow and fast motions are separated to any order in μ:

$$\frac{\partial\alpha_m}{\partial t}=\hat c_m^{(0)}(\bar\alpha)+\mu\hat c_m^{(1)}(\bar\alpha)+\ldots,\qquad \bar\alpha=(\bar\alpha_1,\bar\alpha_2,\bar\alpha_3,\bar\alpha_4),$$

$$\bar z^{(k)}=\hat z^{(k)}(\alpha,t),\qquad \bar z^{(k)}|_{t=0}=0, \tag{3.5}$$

where $\hat c_m^{(k)}$ and $\hat z^{(k)}$ are linear operators.

The use of specific physical conditions and of uniform expansions leads to the following simplifications. For typical characteristics of the dilute plasma considered here, the ion collision frequency, the photon momentum, and the recoil energy are small, respectively, compared with the radiative relaxation rates, the ion thermal momentum, and $\hbar\gamma$:

$$\nu_i\ll\gamma,\ \gamma_\perp;\quad \hbar k_0\ll p_0,\quad E_R/\hbar=\hbar k_0^2/2M\ll\gamma. \tag{3.6}$$

Estimates of the terms $\propto\mu$ in the complete Eqs. (3.5) show that the reaction of the excited plasma oscillations on the slow motions can be neglected subject to physically obvious limitations on the magnitude of the ion-momentum oscillations and the corresponding Doppler shifts in the electrical hf field $\mathscr{E}$:

$$|\tilde p|\ll p_0,\quad |Q\tilde p/M|,\quad |k_0 p/M|\ll\gamma,\ \gamma_\perp,\ |\Omega_1|, \tag{3.7}$$

where $\tilde p\sim e/\omega_{pe}$, $Q=|\mathbf{k}_0-\mathbf{k}_1|$, $\Omega_1=\mathbf{d}E_1/\hbar$. In the majority of the cases the manifestation of fluctuations in the RRP force

$$\max\left(\nu_i,\frac{1}{\tau}\right)\gg(\hbar k_0/p_0)^2\gamma, \tag{3.8}$$

where τ is the characteristic time of action of the optical fields, is unimportant. The spatially periodic correction to the RRP force $\propto\mu$ due to the smallness of trapped particles is also unimportant, for in the majority of interesting situations

$$(p_0^2/2M)\gg\hbar\Omega_1(k_0p_0/M(\delta\omega))\sim\hbar\Omega_1\mu \tag{3.9}$$

(which for $\Omega_1\sim(k_0p_0/M)$ is equivalent to the inequality $E_R/\hbar(\delta\omega)\leqslant1$).

Taking into account all conditions which we have listed and turning to the $\{\mathbf{p}\cdot\mathbf{R}\}$ representation we have a set of contracted equations (to fix the ideas we put $\Delta_0>0$ in (3.1))[1)]

$$\frac{d\alpha_3}{dt_1}+i\tilde\delta\alpha_3=-\tilde\gamma_+\alpha_3+\frac{iG_1}{2}\alpha_0,$$

$$\frac{d\alpha_4}{dt_1}-i\tilde\delta\alpha_4=-\tilde\gamma_+\alpha_4-\frac{iG_1^*}{2}\alpha_0,$$

$$\frac{d}{dt_1}=\frac{\partial}{\partial t_1}+\frac{\mathbf{p}}{M}\frac{\partial}{\partial\mathbf{R}},$$

$$\frac{d\alpha_1}{dt_1}=-\tilde\gamma_1\alpha_1-\frac{\Delta'}{4|\Omega_0|}i\tilde\gamma_2\alpha_2+i(G_1^*\alpha_3-G_1\alpha_4),$$

$$\alpha_0=\alpha_1+i\frac{\Delta'}{4|\Omega_0|}\alpha_2, \tag{3.10}$$

$$\frac{d\alpha_2}{dt_1}=-\frac{\hbar\mathbf{k}_0\gamma}{2}\frac{\partial}{\partial\mathbf{p}}\left(\alpha_2-\frac{4\Delta' i|\Omega_0|}{G^2}\alpha_0\right)+\frac{2|\Omega_0|\hbar\mathbf{Q}}{G}\frac{\partial}{\partial\mathbf{p}}(G_1\alpha_4-G_1^*\alpha_3)+I_s^{(+)}(\alpha_2)-\frac{4i\Delta'|\Omega_0|}{G^2}I_s^{(-)}(\alpha_0),$$

where we have introduced the variables $\alpha_m=\alpha_m(\mathbf{p},\mathbf{R},t_1)$:

$$\alpha_{1,2}=\hat P^{-1}_{\sigma\to p}\bar\alpha_{1,2},\quad \alpha_{3,4}=\tilde\alpha_{3,4}\exp[\mp i(\delta_0t_1-\mathbf{Q}_1\mathbf{R})],$$
$$\tilde\alpha_{3,4}=\hat P^{-1}_{\sigma\to p}\bar\alpha_{3,4}$$

and modified the relaxation constants, the Rabi frequency, the mismatch, and the wave vectors:

$$\tilde\gamma_+={}^1/_2\gamma_\perp(1+\Delta'^2/G^2)+\gamma|\Omega_0|^2/G^2,\quad \tilde\gamma_1=\gamma_+a_0^{-2}+\gamma\Delta'^2/G^2,$$

$$a_0^{-2}=4|\Omega_0|^2/G^2,\quad \tilde\gamma_2=\tilde\gamma_1+\gamma,$$

$$G_1=((\mathbf{dE}_0)^*(\mathbf{dE}_1)/\hbar|\mathbf{dE}_0|)\left(1-\frac{\Delta'}{G}\right),$$

$$\tilde\delta=\delta_0-\mathbf{Q}_0\mathbf{p}/M,$$

$$\mathbf{Q}_1=(\omega_1/\omega_0)\mathbf{k}_0-\mathbf{k}_1,\quad \mathbf{Q}_0=\mathbf{Q}_1+\mathbf{k}_0\Delta'/G.$$

The ion momentum distribution function (DF) can be written as a sum of slow and hf components:

$$f=\bar f_0+\tilde f,$$

$$\tilde f=\tilde f_0+\mu\tilde f_1+\mu\tilde f_s+\mu\tilde f_{\mathscr{E}}+O\left(\left[\frac{\hbar k_0}{p_0}\right]^2+\mu\frac{\hbar k_0}{p_0}+\mu^2\right),$$

$$\bar f_0=-ia_0^2\alpha_2+O\left(\frac{\hbar k_0}{p_0}\right),\quad \tilde f_0=-\hbar\mathbf{k}_0\left(\frac{\partial\tilde\alpha_3}{\partial p}e^{t_3}+\frac{\partial\tilde\alpha_4}{\partial p}e^{t_4}\right),$$

$$\tilde f_1=i\frac{\hbar\mathbf{k}_0}{MG}\left(\frac{\partial\tilde\alpha_3}{\partial\mathbf{R}}e^{t_3}-\frac{\partial\tilde\alpha_4}{\partial\mathbf{R}}e^{t_4}\right), \tag{3.11}$$

$$\tilde f_s=-\frac{2i}{G}(I_s^{(-)}(\tilde\alpha_3)e^{t_3}-I_s^{(-)}(\tilde\alpha_4)e^{t_4}),$$

$$\tilde f_{\mathscr{E}}=-\frac{\partial f_0}{\partial\mathbf{p}}\left(\frac{e\vec{\mathscr{E}}_0}{iG}e^{t_3}-\frac{e\vec{\mathscr{E}}_0^*}{iG}e^{t_4}\right),$$

where $\vec{\mathscr{E}}_0$ is the amplitude of the electrical field in the plasma wave $\vec{\mathscr{E}}=(\vec{\mathscr{E}}_0e^{t_3}+\text{c.c})$. The hf components of the DF $(\tilde f_0,\tilde f_1),\tilde f_s,\tilde f_{\mathscr{E}}$ connected, respectively with the recoil effect, the difference of the collision frequencies in the ground and the excited states, and with the field of the Langmuir wave give contributions to the hf ion current (see (2.9));

$$\tilde{\mathbf{J}}_i=e\int\frac{\mathbf{p}}{M}\tilde f\,d^3p.$$

We assume in (3.10), (3.11) that the collision integral has the form (2.8). When $t_1\gg\gamma^{-1}$ it follows from Eqs. (3.10), (3.11) (cf. Ref. 4) that the slow part of the DF $\bar f_0$ satisfies the kinetic Boltzmann equation with a force which depends on the momentum and completely determines the hf motion:

$$\frac{d\bar f_0}{dt_1}+\frac{\partial}{\partial\mathbf{p}}(\mathbf{F}\bar f_0)=I_s^{(+)}(\bar f_0)-\frac{\Delta'^2}{G^2}\gamma I_s^{(-)}(\bar y\bar f_0)=\mathrm{St}\{\bar f_0\},$$

$$\alpha_m=M_{m2}(\mathbf{p})\bar f_0,\quad m=3,4, \tag{3.12}$$

where

$$M_{32}=i\Delta'|\Omega_0|L_1G_1\gamma\bar y/2G^2,\quad M_{42}=M_{32}^*,$$

$$\bar y=(\tilde\gamma_1+|G_1|^2\mathscr{L})^{-1},$$

$$L_1=(\tilde\gamma_++i\tilde\delta)^{-1},\quad \mathscr{L}=\tilde\gamma_+/(\tilde\gamma_+^2+\tilde\delta^2).$$

In situations where ion-ion collisions play a large role the kinetic Eq. (3.12) also has a meaning, but with the modified non-linear collision integral ($\mathrm{St}(f_0)$. One shows easily that taking low-frequency electrical fields $\mathscr{E}_s$ (with frequencies $\tilde{\omega} \ll \omega_{pe}$) into account with the moderate intensity $e\mathscr{E}_s(\partial f_0/\partial p)/f_0 \ll \gamma$ is accomplished by modifying the force term in (3.12):

$$F \to F + e\mathscr{E}_s. \tag{3.12a}$$

As the wavelength of the excited Langmuir wave $\lambda_L \sim 1/Q$ must be bounded from below by the Debye radius $\lambda_e \ll \lambda_L$, for typical values of the electron densities and temperature ($n_e \sim 10^{10}$–10^{12} cm^{-3}, $T_e \gtrsim 1$ eV) one must require that $|\mathbf{Q}| = |\mathbf{k}_0 - \mathbf{k}_1| \ll k_0$ ("unidirectional geometry" of the fields) which enables us to write down a simplified expression for the RRP force (Ref. 4 contains the exact expression):

$$\tilde{\mathbf{F}} \approx {}^1/_2\hbar\mathbf{k}_0\gamma\left[1 - \frac{\Delta'^2}{G^2}\gamma\bar{y}\right].$$

4. INDUCED PLASMA OSCILLATIONS

The hf component of the ion current is, after termination of the transient processes ($t_1 \ll \gamma^{-1}$), a functional of the DF$\bar{f}_0$ and satisfies an equation following from (3.11):

$$\frac{\partial \tilde{\mathbf{J}}_i}{\partial t} = \left(\frac{e^2\tilde{\mathscr{E}}}{M} + e\frac{\partial \tilde{\mathbf{u}}_i}{\partial t}\right)\bar{n}, \quad \bar{n} = \int \bar{f}_0\, d^3p,$$

$$\tilde{\mathbf{J}}_i = \left(\frac{e}{M}\right)\int f_0\mathbf{p}\, d^3p, \tag{4.1a}$$

$$\frac{\partial \tilde{\mathbf{u}}_i}{\partial t} = \exp i(\delta\omega t - \mathbf{QR})\left\{i\hbar\mathbf{k}_0\delta\omega \int M_{32}\bar{f}_0\, d^3p + 2\int \mathbf{p}I_s^{(-)}(M_{32}\bar{f}_0)\, d^3p\right\}\frac{1}{M\bar{n}} + \text{c.c.},$$

$$\mathbf{Q} = (\mathbf{k}_0 - \mathbf{k}_1). \tag{4.1b}$$

Therefore, under the initial assumptions made here, the optical fields produce a situation similar to the appearance of an extraneous ion current of density $J_{\text{ext}} = \bar{n}e\tilde{u}_i$ oscillating with a frequency $\delta\omega \sim \omega_{pe}$. For completeness we have included in (4.1) the ion current due to the difference in the collision kernels in the ground and excited states[2)] which is clearly unimportant in a strongly ionized plasma or in the case when the elastic scattering of resonance ions is determined by the polarization interactions with neutral atoms ($I_s^{(-)} = 0$).[9] It can be compared with the contribution connected with recoil (first term in (4.1b)) in the case of a weakly ionized plasma with a high density of neutral particles which are able to undergo charge exchange with excited and unexcited atoms, if $|\nu_2 - \nu_1| \sim |\hbar k_0\omega_{pe}/p_0|$. However, in that case the condition for the excitation of a plasma wave is appreciably worsened due to the large magnitude of the damping since

$$\nu_e \sim (Ms_e/p_0)\nu_i \gtrsim (\omega_{pe}\hbar k_0 Ms_e/p_0^2),$$

where $s_e \sim (T_e/m_e)^{1/2}$. For typical values for a weakly ionized laboratory plasma $s_e \sim 10^8$ cm/s, $\hbar k_0/p_0 \sim 10^{-4}$ we get $\nu_{pe} \sim \gtrsim 0.1\omega_{pe}$.

The largest amplitude of the light-induced extraneous hf current is obtained by choosing the radiation parameters so as to eliminate the "selectivity," in momenta, of the function $M_{32}(\mathbf{p})$ (see (3.12)):

$$|\delta_0| \sim |G_1| > (k_0 p_0/M), \quad |\Delta'| \sim |\Omega_0|. \tag{4.2}$$

We then get from (4.1b) the estimate

$$|J_{\text{ext}}| = \bar{n}e|u_i| \sim \bar{n}e\left(\frac{\hbar k_0}{M}\right) + J'_{\text{ext}},$$

$$J'_{\text{ext}} \lesssim \frac{ep_0}{M}\frac{|\nu_2 - \nu_1|}{\omega_{pe}}\bar{n}. \tag{4.3}$$

We consider the hf perturbations of the macroscopic plasma parameters $\tilde{v}_e$, $\tilde{n}_e$, $\tilde{v}_i$, $\tilde{n}$ relative to the steady-state slow motions $\bar{v}_e$, $\bar{n}_e$, $\bar{v}_i$, $\bar{n}$ for which we assume quasi-neutrality ($\lambda_e \ll l_0$), the absence of current, and quasi-homogeneity:

$$\bar{n}_e \approx \bar{n}, \quad \bar{n}_e\bar{v}_e \approx \bar{n}_i\bar{v}_i, \quad Q \gg (1/l_0), \tag{4.4}$$

where l_0, $1/Q$ are characteristic spatial scales for the slow and hf motions. For instance, for a plasma in a closed tube $\bar{v}_i = 0$ and the DF differs little from a Maxwellian one $((\hbar k_0/p_0)\gamma \ll \nu_i)$ and from (2.2)–(2.4) and (3.12) there follow equations for the densities (cf. Ref. 10)

$$-\frac{m_e}{2M}\langle\nabla(\tilde{v}_e^2)\rangle - \frac{T_i + T_e}{M}\nabla\ln(\bar{n}) + \frac{1}{M}\int Ff_0\, d^3\mathbf{p} - \int I_s^{(-)}\left(\frac{\Delta'^2}{G^2}\gamma\bar{y}f_0\right)\frac{\mathbf{p}}{M}d^3p, \tag{4.5}$$

where the angle brackets indicate averaging over the hf oscillations and T_i is the ion temperature. When (4.2) is satisfied we have thus

$$l_0^{-1} \sim \max\{F, |\langle\nabla m_e\tilde{v}_e^2\rangle|, (T_i k_0|\nu_2 - \nu_1|/\delta_0)\}/(T_e + T_i).$$

Independently of the specific properties of the slow motions, which are determined by the actual physical conditions for satisfying (4.4), and of the inequalities

$$|Q\bar{v}_e|, \; |Q\bar{v}_i| \ll \omega_{pe} \tag{4.6}$$

the set of equations for the induced Langmuir oscillations, which follows from (2.2), (2.3), (4.1) after eliminating the hf field using the Maxwell Eq. (2.4), has the universal form:

$$\frac{\partial^2\tilde{\mathbf{v}}_e}{\partial t^2} + \nu_e\frac{\partial \mathbf{v}_e}{\partial t} + \omega_{pe}^2\tilde{v}_e - s_e^2\nabla(\nabla\tilde{\mathbf{v}}_e) = \omega_{pe}^2\mathbf{v}_i + I_{NL} - \langle I_{NL}\rangle,$$

$$I_{NL} \approx -\frac{\partial}{\partial t}(\tilde{\mathbf{v}}_e\nabla)\tilde{\mathbf{v}}_e - \omega_{pe}^2\frac{\tilde{n}_e}{\bar{n}}\tilde{\mathbf{v}}_e, \quad \frac{\partial n_e}{\partial t} = -\bar{n}\,\mathrm{div}(\tilde{\mathbf{v}}_e), \tag{4.7}$$

$$\frac{\partial^2\tilde{\mathbf{v}}_i}{\partial t^2} + \omega_{pi}^2(\tilde{\mathbf{v}}_i - \tilde{\mathbf{v}}_e) = \frac{\partial^2\tilde{\mathbf{u}}_i}{\partial t^2}, \quad \tilde{\mathbf{v}}_i = \frac{\tilde{\mathbf{J}}_i}{e\bar{n}},$$

where

$$\omega_{pe}^2 = \frac{4\pi e^2}{m_e}\bar{n}, \quad \omega_{pi}^2 = \frac{4\pi e^2}{M}\bar{n}, \quad s_e = \left(\frac{3T_e}{m_e}\right)^{1/2}.$$

The equations obtained correspond to the presence of an induced hf force

$$\tilde{F} \sim M\delta\omega\tilde{u} \sim \hbar k_0\delta\omega,$$

acting selectively solely on the ion component. The electrons are involved in that motion and the more efficiently the closer the frequency of the induced force is to the resonance frequency

$$\omega_c=(\omega_{pe}^2+\omega_{pi}^2+Q^2s_e^2)^{1/2}.$$

Using (3.2) and only the second harmonic of the oscillations we get from (4.4) the explicit expressions:

$$\tilde{\mathbf{v}}_e\approx[\tilde{\mathbf{v}}_{e0}\exp(i\delta\omega t-i\mathbf{QR})+\tilde{\mathbf{v}}_{e0}^{(2)}\exp(2i\delta\omega t-2i\mathbf{QR})]+\text{c.c.},$$

$$\begin{aligned}
&\tilde{\mathbf{v}}_{e0}\approx-i\tilde{\mathbf{u}}_{i0}\omega_{pe}[\nu_e-2(\omega_e-\delta\omega)i]^{-1},\\
&\tilde{\mathbf{v}}_{e0}^{(2)}=\frac{\mathbf{Q}\tilde{\mathbf{v}}_{e0}}{\omega_{pe}}\tilde{\mathbf{v}}_{e0},\qquad \tilde{n}_{e0}=\frac{\mathbf{Q}\mathbf{v}_{e0}}{\omega_{pe}}\bar{n},\\
&n_{e0}^{(2)}=\frac{3}{2}(\mathbf{Q}\tilde{\mathbf{v}}_{e0}/\omega_{pe})^2\bar{n},\\
&\tilde{\mathbf{v}}_{i0}=\tilde{\mathbf{u}}_{i0}\left(1+\frac{m_e}{M}\frac{i\omega_{pe}}{\nu_e-2(\omega_e-\delta\omega)i}\right)\\
&e\vec{\mathscr{E}}_0\approx im_e\tilde{\mathbf{v}}_{e0}\omega_{pe},
\end{aligned}\tag{4.8}$$

where the subscript 0 marks the oscillation amplitudes and $\tilde{\mathbf{u}}_{i0}$ is determined from (4.1b).

The amplitude of the wave of the directed ion velocity exceeds the amplitude of the bare coherent perturbations if $(m_e/M)(\omega_{pe}/\nu_e)>1$. The amplitude of the oscillations in the electron density $\tilde{n}_{e0}$ is always considerably larger than the amplitude of the coherent perturbations of the ion density when there are no collective effects[4] $n_{i0}^{(0)}\sim(Q\tilde{u}_{i0}/\delta\omega)$:

$$(\tilde{n}_e/\tilde{n}_{i0}^{(0)})\sim(\omega_{pe}/\nu_e)\gg1.\tag{4.9}$$

As an example we estimate the maximum amplitude of the electrical field and the electron velocity oscillations in a strongly ionized plasma with $\bar{n}\sim4\times10^{10}$ cm^{-3}, $T_e\sim1$ eV, $\hbar\omega_0\sim2$ eV, $(m_e/M)\sim10^{-4}$: $\nu_e\sim5\times10^5$ s^{-1}, $|\mathscr{E}_0|\sim1$ V/cm, $|\tilde{v}_{e0}|\gtrsim10^5$ cm/s. In a weakly ionized plasma the estimates are impaired by the large frequency of collisions of electrons with neutral particles (for instance, for a neutral particle density $\bar{n}\sim10^{15}$ cm^{-3}, $T_e\sim1$ eV, $\sigma_{en}\sim10^{16}$ cm^2 we have $\nu_e\sim6\times10^6$ s^{-1}).

One can easily extend the consideration given here to the case where the vectors $\mathbf{k}_0$, $\mathbf{k}_1$, $\mathbf{Q}$ are not collinear, when the direction of the oscillations of the coherent optical perturbations $\tilde{u}_i\sim k_0$ is not the same as their direction of propagation $\mathbf{n}_a=(\mathbf{Q}/Q)$. To do this we need use the complete Maxwell equations (instead of (2.4)) and take the magnetic field into account. When $(Qp_0/M)\ll\omega_{pe}$ (see (2.7)) and (3.7) is satisfied, and one can neglect the effect of the Lorentz force on the slow and fast motions, Eqs. (4.1) and (3.12) remain in force and on the left-hand sides of each of Eqs. (4.7) for $\tilde{\mathbf{v}}_e$, $\tilde{\mathbf{v}}_i$ there appear new terms—respectively, c^2 curl curl $\tilde{\mathbf{v}}_e$ and $-c^2(m_e/M)$ curl curl $\tilde{\mathbf{v}}_e$. Therefore, apart from the longitudinal wave there is excited in the plasma a transverse electromagnetic wave propagating in the direction of $\mathbf{Q}$. The amplitude of the oscillations of the electron velocity in it is for $c^2Q^2\gg\omega_{pe}^2$ given by the relation

$$(\tilde{\mathbf{v}}_{e0})_\perp=(\omega_{pe}^2/c^2Q^2)(\tilde{\mathbf{u}}_{i0})_\perp,\qquad |v_{e0}|_\perp\ll|u_{i0}|_\perp,$$

where $(\tilde{\mathbf{u}}_{i0})_\perp$ is the component of the vector $\tilde{\mathbf{u}}_{i0}$ in the direction at right angles to $\mathbf{n}_Q$. For the longitudinal wave Eq. (4.8) is retained in which we must make the substitution $\tilde{\mathbf{u}}_0\to(\tilde{\mathbf{u}}_{i0}\mathbf{n}_Q)\mathbf{n}_Q$.

When the propagation of the optical fields is not collinear, excitation of short-wavelength Langmuir oscillations with $|Q|\gg\delta\omega/c$ and correspondingly an appreciable enhancement of the relative perturbations of the electron density

$$(\tilde{n}_e/\bar{n})\sim(\hbar k_0^2/2M\nu_e)\sin^2(\widehat{\mathbf{k}_0\mathbf{k}_1}/2),$$

is possible, but in this case Landau damping may be large (see (2.10)), so that it is necessary to observe the restriction $Q^{-1}>\lambda_e$.

The most important property of the induced monochromatic Langmuir waves, induced by RRP, consists in the possibility for a given frequency $\delta\omega$ to change their wavelength (there is no dispersion relation in contrast to free waves), phase, and amplitude by varying the parameters and the direction of propagation of the optical fields.

5. PARAMETRIC INSTABILITY

The problem of the stability of the induced plasma oscillations requires a detailed special consideration and is of particular interest from the point of view of studying new possibilities to excite plasma turbulence.[11] In the present paper we restrict ourselves to an analysis of the simplest situations which do not require considerable modifications of the existing theories of parametric plasma instabilities.[5–7]

We consider the weakly selective case $|\delta_0|>k_0p_0/M$ (see (4.2)), when there are no effects due to a non-equilibrium ion DF (of the kind studied in Ref. 12). To describe the low-frequency (lf) perturbations of the ion component $\propto\exp i(\tilde{\omega}t-\tilde{\mathbf{k}}\mathbf{R})$ with $\tilde{\omega}$, $(\tilde{k}p_0/M)\ll\gamma$ we can use directly Eq. (3.12) (taking into account (3.12a)) and the equations for the momenta corresponding to it. Therefore under the named conditions we can in the hydrodynamic approximation easily obtain the well known set of equations (see, e.g., Refs. 6, 10, 11) for the parametrically coupled hf and lf perturbations with coupling coefficients $\propto\tilde{v}_e(t)$. This enables us to use the results of Refs. 6, 7 to estimate the threshold for the parametric instability.

In a non-isothermal plasma $(T_e\gg T_i)$ the lowest threshold is the one for the decay instability of the excited longitudinal wave (accompanied by the emergence of ion sound with a frequency $\omega_A=c_s\tilde{k}$ and a Langmuir wave of frequency $\delta\omega-\omega_A$) determined by the expression

$$(|v_{e0}|/s_e)\sim(\nu_i\nu_e/\omega_A\omega_{pe})^{1/2},$$

where ν_i is the ion collision frequency which for estimates we can normalize to take into account the collisionless damping of ion sound when $T_e\gg T_i$:

$$\nu_i\to\nu_i+(\pi/8)^{1/2}\omega_A(m_e/M)^{1/2}.$$

Hence and also from (4.8), (4.3) we have for the minimum instability threshold at the maximum possible amplitude $\tilde{v}_{e0}$ connected with recoil

$$\hbar k_0/Ms_e\gtrsim8^{1/2}(\nu_i/\omega_A)^{1/2}(\nu_e/\omega_{pe})^{3/2}.\tag{5.1}$$

For instance, assuming the damping of ion sound to be collisionless, we have in a strongly ionized plasma with $n_e\sim4\times10^{10}$ cm^{-3}, $\nu_e\sim5\times10^5$ s^{-1}, $m_e/M\sim10^{-4}$, $T_e\sim1.5$ eV, $T_i\approx0.03$ eV the inequality $(\hbar k/M)>0.5$ cm/s which in the optical range of the spectrum can easily be satisfied. For the case of the aperiodic instability the minimum threshold is higher:

$$\hbar k_0/Ms_e \gtrsim 2^{1/2}(\nu_e/\omega_{pe})^{3/2}, \quad (5.2)$$

but also in principle attainable: $T_e \sim 1.5$ eV, $\nu_e \sim 5\times 10^5\,\text{s}^{-1}$, $\hbar k_0/M > 12$ cm/s (light atoms and short-wavelength transitions). New situations arise for a rather large density of resonant ions when it is necessary to take into account the reaction of the medium on the optical radiation. From the equations for the optical fields we have to zeroth order in μ in the limit $\delta_0 \gg (k_0 p_0/M)$, γ

$$\frac{\partial\Omega_1}{\partial z} = -i\eta_1\Omega_1, \quad \frac{\partial\Omega_0}{\partial z} = \eta_0\Omega_0 i, \quad \frac{\partial\Omega_1'}{\partial z} = -\eta_1' i\Omega_1^*, \quad (5.3)$$

where

$$\eta_1 = \frac{\Delta'}{G}\left(1-\frac{\Delta'}{G}\right)^2 \gamma y \frac{\gamma}{|\delta_0|}\frac{n_i\sigma_0}{2}\,\text{sign}(\delta_0),$$

$$\eta_0 = \frac{\gamma^2\Delta'\bar{y}}{G^2}2\bar{n}_i\sigma_0, \quad \eta_1' = 2\bar{n}_i\sigma_0\Omega_0^2G^{-2}\gamma^2\bar{y}\,|\delta_0|^{-1}\,\text{sign}(\delta_0),$$

$$\sigma_0 = 3\pi/4k_0^2,$$

z is the coordinate in the direction of $\mathbf{Q}$, $\Omega_1' = \mathbf{d}\cdot\mathbf{E}_1'/\hbar$, $\mathbf{E}_1'$ is the amplitude of the field generated at the mirror frequency $\omega_1 = 2\omega_0 - \omega_1$, it follows that when $|\Omega_0|^2 \ll |G|^2$ the following relation is possible between the characteristic spatial scales (l_0 is the plasma dimension in the direction of $\mathbf{Q}$):

$$|\eta_1|^{-1} \ll |\eta_1'|^{-1},\ l_0,\ |\eta_0|^{-1}. \quad (5.4)$$

The main effect of the propagation of optical radiation will then be connected with the phase modulation of the field E_1, so that $\Omega_1 = \Omega_{10}\exp(-i\eta_1 z)$ in a uniform plasma and we must therefore make in Eqs. (4.8) the substitution $Q \to Q + \eta_1$, $\Omega_1 \to \Omega_{10}$. When considering an instability there appears an additional contribution to the parametric coupling between the hf perturbations of the electron velocity and the lf perturbations of the ion density thanks to the dependence of the amplitude of the optical coherent perturbations on the field E_1 which, in turn, is sensitive to the oscillations in the ion density. The basic dispersion relation of the parametrically coupled longitudinal perturbations is then modified (cf. Refs. 10 and 7):

$$\tilde{\omega}^2 - i\nu_i\tilde{\omega} - c_s^2\tilde{k}^2 = \frac{\omega_{pe}^2 m_e}{M}\tilde{k}^2\left\{\tilde{v}_{e0}^*\left(v_{e0} - \frac{\eta_1}{\tilde{k}}\tilde{u}_{i0}\right)\right.$$
$$\times[\omega_+^2 - \omega_e^2 - \nu_e i\omega_+ + s_e^2Q^2 - s_e^2(Q+\eta_1+\tilde{k})^2]^{-1}$$
$$+ v_{e0}\left(v_{e0}^* + \frac{\eta_1}{\tilde{k}}u_{i0}^*\right)[\omega_-^2 + \omega_e^2 + \nu_e i\omega_-$$
$$\left. + s_e^2Q^2 - s_e^2(Q+\eta_1-\tilde{k})^2]^{-1}\right\}, \quad (5.5)$$

$$\omega_+ = \delta\omega + \tilde{\omega}, \quad \omega_- = \delta\omega - \tilde{\omega}, \quad |v_{e0}| \gg |u_{i0}|.$$

Important changes in the spectrum of the small perturbations and in the growth rates arise in the long-wavelength limit $|\eta_1/\tilde{k}| \gg |\tilde{v}_{e0}/\mu_{i0}|$ and can, in particular, lower the thresholds of the above instabilities $\propto (\tilde{k}/\eta_1)^{1/2}$.

In conclusion it is useful to compare the effect considered here with thewell known mechanism, based upon purely plasma non-linearities, for the excitation of plasma waves in a biharmonic field.[13,14] Taking into account the oscillations of electrons at optical frequencies leads to the appearance on the right-hand side of Eq. (4.7) for $\tilde{\mathbf{v}}_e$ of an inducing term at the beat frequency $\delta\omega$: $m_e^{-1}\partial\tilde{F}_e/\partial t$, where

$$\tilde{\mathbf{F}}_e = (ie^2\mathbf{Q}/m_e\omega_1\omega_0)(\mathbf{E}_1\mathbf{E}_0^*)\exp(i\delta\omega t - i\mathbf{QR}) + \text{c.c.},$$

The induced plasma oscillations caused by the coherent perturbations of the ions dominate in the case of an almost unidirectional propagation of optical fields ($cQ \sim \omega_{pe}$) under the condition

$$|\tilde{\mathbf{u}}_{i0}| \gg \xi = |(E_1E_0^*)|e^2/m_e^2\omega_0\omega_1 c. \quad (5.6)$$

For instance, for optical frequencies $\omega_1 \sim \omega_0 \approx 1.5\times10^{15}$ Hz, and for a plasma with $p_0/M \sim 10^4$–10^5 cm/s and $\omega_{pe} \gtrsim 10^{10}$ Hz, the conditions (2.7), (4.2) are satisfied at intensities $I \sim 10^4$–10^5 W/cm^2. Then $\xi \approx 3\times10^{-3}$–$3\times10^{-4}$ cm/s and $|\mu_{i0}| \sim \hbar k/M \approx 3$ cm/s($M = 10$ a.m.u., $\hbar\omega_0 \approx 1$ eV. Hence, at the indicated intensities the effect considered dominates with a large margin, since it operates with non-linearities due to optical resonance with ion quantum transitions.

APPENDIX

Higher orders of perturbation theory

We introduce fast time variables t_3, t_4 (see (3.4)) and $t_2 = \Delta_0 t$ which are connected, respectively, with the spectrum of the limiting problem and with the fast oscillating coefficients of Eq. (2.5), and also a set of slow variables $\tau_m = \mu^m t$. Performing then the appropriate expansions of the derivative[15,16] and using (3.3) we have an equation of the n-th approximation in the $\{\sigma\cdot\mathbf{R}\}$ representation

$$\left(i\Delta_0\frac{\partial}{\partial t_2} + iG\frac{\partial}{\partial t_3} - iG\frac{\partial}{\partial t_4}\right)\bar{z}^{(n)} - \bar{B}\bar{z}^{(n)}$$
$$= \left(\bar{B}_1 - \sum_{l=0}^{n-1}\frac{\partial}{\partial\tau_e}\right)z^{(n-1-l)}, \quad \bar{z}''|_{t=0} = 0, \quad n \geq 1, \quad (A.1)$$

$$\bar{B}_1 = (\bar{\Gamma} + \bar{A}_1(t_2)) + i\hat{I}\left(\text{div}_R\,\text{grad}_\sigma\frac{1}{M} + e\vec{\mathscr{E}}(t_2)\sigma\right) + \bar{L}_\sigma,$$

where $\hat{I}$ is the unit matrix and the bar over the operators indicate that they are written in the $\{\sigma\cdot\mathbf{R}\}$ representation. The right-hand side of (A.1) must be written down taking into account all possible resonance relations corresponding to (3.1), (3.2):[4] $mt_2(t) + t_j(t) = t_i(t)$, $i,j = 3, 4, 0$, $t_0 = 0$. We introduce the operator $\hat{P}(lt_i + mt_j)$ acting on an exponential of the form $\exp(m't_i + l't_j)$ according to the rule

$$\hat{P}(lt_i + mt_j)\exp(l't_i + m't_j) = \delta_{l'l}\delta_{mm'}.$$

When $n = 1$ we get from (3.4) and the condition that (A.1) can be solved the equation

$$\partial\alpha_i/\partial\tau_0 = \langle b_i,\ \hat{P}_i(\bar{B}_1\bar{z}^{(0)}(\alpha, t_3, t_4))\rangle = \hat{c}_i^{(0)}(\alpha), \quad (A.2)$$

where the angle brackets indicate a scalar product, the b_i are the eigenvectors of $\overline{B}^+$ forming with the φ_i a bi-orthonormal system; $\hat{P}_i = \hat{P}(t_i)$ for $i = 3,4$, $\hat{P}_i = \hat{P}(t_0)$ for $i = 1,2$. Since the solution of the inhomogeneous equation is determined accurate to an arbitrary solution of the corresponding homogeneous equation

$$\bar{z}_1^{(1)} = \hat{z}_H(\bar{\alpha}, t_3, t_4, t_2) + \hat{\beta}_1(\alpha)\varphi_1 + \hat{\beta}_2(\bar{\alpha})\varphi_2 + \sum_{m=3}^{4}\hat{\beta}_m(\bar{\alpha})\varphi_m e^{t_m}, \quad (A.3)$$

where the $\hat{\beta}_i$ are arbitrary linear operators which are independent of the time, it follows that the solution is uniquely fixed by the initial conditions. Let similarly at the $(n-1)$st step $(n\geqslant 2)$

$$\partial\bar{\alpha}_i/\partial\tau_k=\hat{c}_i^{(k)}(\bar{\alpha}),\quad 0\leqslant k\leqslant n-2; \tag{A.4}$$

$$\bar{z}^{(k)}=\hat{z}^{(k)}(\bar{\alpha},\ t_2,\ t_3,\ t_4),\quad \hat{z}^{(k)}|_{t_i=0}=0,\quad 0\leqslant k\leqslant n-1.$$

From the solvability condition for the problem of the nth approximation

$$\langle b_i, \hat{P}_i(B_1\bar{z}^{(n-1)})\rangle-\left\langle b_i, \hat{P}_i\frac{\partial\bar{z}^{(0)}}{\partial\tau_{n-1}}\right\rangle -\sum_{l=0}^{n-2}\left\langle b_i, \hat{P}_i\frac{\partial z^{(n-1-l)}}{\partial\tau_e}\right\rangle=0 \tag{A.5}$$

we get, using (A.4)

$$\frac{\partial\bar{\alpha}_i}{\partial\tau_{n-1}}=\hat{c}_i^{(n-1)}(\alpha) =\langle b_i, \hat{P}_iB_1\bar{z}^{(n-1)}\rangle-\sum_{l=0}^{n-2}\left\langle b_i, \hat{P}_i\frac{\partial\bar{z}^{(n-1-l)}}{\partial\tau_e}\right\rangle. \tag{A.6}$$

Here the $\bar{z}^{(n)}$ are found similarly to the $\bar{z}^{(1)}$. Once again, using the representation $\partial/\partial t=\Sigma\mu^m(\partial/\partial\tau_m)$, we get (3.5). To take into account the finite velocity of propagation of the strong field we must use the substitution $t\rightarrow t_1=t-(\mathbf{k}_0/k_0c)\mathbf{R}$.[4]

[1)] We use the set of eigenvectors $\{\varphi_i\}$ from Ref. 4.

[2)] The possibility to excite sound oscillations in a gas in a biharmonic field $\delta\omega\ll\gamma$ taking this effect into account was analyzed in Ref. 8.

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