Boundary conditions and acoustic coefficients for an He II-vapor interface

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A complete set of boundary conditions for a superfluid-vapor interface is phenomenologically derived. The simplification of the boundary conditions for the calculation of the limiting values of the acoustic coefficients (i.e., the reflection, transmission, and conversion coefficients of the first and second sound) are considered for low frequencies. All the acoustic coefficients are calculated for the case of oblique incidence of the sound on the interface at arbitrary angle.

1. INTRODUCTION

Knowledge of the boundary conditions on the interface between a superfluid liquid and its vapor is needed for the analysis of a large number of phenomena, such as propagation of surface waves and of third and fifth sound, and also the reflection, refraction, and conversion of the sounds. The acoustic coefficients (the reflection, transmission, and conversion coefficients of first and second sound) on an He II-vapor interface were obtained for nondissipative boundary conditions by Chernikova.[1] The experiments of Hunter and Osborne on the reflection of second sound[2] and subsequent experimental investigations of the acoustic coefficients[3–6] have demonstrated, however, that such an approach is insufficient. Boundary conditions that take into account dissipative processes on an He II-vapor interface were deduced both a phenomenological[7–9] and by a microscopic description of the evaporation process.[2,10–13] Although various studies devoted to a phenomenological analysis did indeed take into account various significant factors, such as dissipative mass and heat fluxes through the interface,[7,8] dissipative fluxes in both media,[9] and the hydrodynamics of the surface variables,[14] no simultaneous account of all these factors was taken in the analysis. This gap is filled in Sec. 2 of the present paper, where a complete set of boundary conditions for the He II-vapor interface is derived phenomenologically.

The complete set of boundary conditions contains a large number of kinetic coefficients, not all of which are essential when some specific problem is solved. It is shown in Sec. 3 that the equations for the calculation of the limiting values of the acoustic coefficients at zero frequency contain only one independent combination of kinetic coefficients (i.e., one kinetic coefficient). This simple circumstance escaped the attention of Wiechert and Buchholz, the authors of the most comprehensive paper devoted to the calculation of the acoustic coefficient on an He II-vapor interface,[15] since they used in their numerical solution of the obtained equations boundary conditions with three different kinetic coefficients. The boundary conditions obtained in the present paper are used in Sec. 4 to calculate the acoustic coefficients for oblique incidence of sound on the interface at arbitrary angle. Since the answers contain only one unknown, the kinetic coefficient that determines the evaporation rate, the results extend substantially the possibilities of experimentally verifying the predictions of the theory.

2. PHENOMENOLOGICAL DERIVATION OF THE BOUNDARY CONDITIONS

The interface is defined as a region with zero surface mass density. We introduce on the surface the coordinates ξ^α ($\alpha = 1,2$), i.e., we assume the positions of the surface points are specified by the function $\mathbf{r}(\xi^\alpha, t)$. Since the coordinates ξ^α are curvilinear, a distinction must be made between superscripts and subscripts, which denote respectively contravariant and covariant components of vectors (tensors). We shall use also the following notation: ∇_α—covariant differentiation; $\boldsymbol{\nu}$—unit normal to the surface directed from the vapor; $\mathbf{w} = (\partial\mathbf{r}/\partial t)^\alpha_\xi$—surface velocity;

$$g_{\alpha\beta}=(\partial\mathbf{r}/\partial\xi^\alpha)(\partial\mathbf{r}/\partial\xi^\beta)$$

is the metric tensor of the surface;

$$K_{\alpha\beta}=\boldsymbol{\nu}(\partial^2\mathbf{r}/\partial\xi^\alpha\partial\xi^\beta)$$

is the surface curvature tensor;

$$\frac{\partial}{\partial t}\equiv\left(\frac{\partial}{\partial t}\right)_{\xi^\alpha}-w^\alpha\left(\frac{\partial}{\partial\xi^\alpha}\right)_t$$

is the invariant operation of differentiation of surface quantities with respect to time: $K = g^{\alpha\beta}K_{\alpha\beta}$ is the total curvature.

We denote by the indices ν and t the normal and tangential components of three-dimensional vectors: $\mathbf{a} = a^\nu \boldsymbol{\nu} + \mathbf{a}^t$, $\mathbf{a}^t\boldsymbol{\nu} = 0$. We shall omit the index t in expressions for components of tangential vectors in terms of the surface coordinates ξ^α: $\mathbf{a}^t = a^\alpha(\partial\mathbf{r}/\partial\xi^\alpha)$.

The invariant form of the surface conservation law, which is valid for any choice of coordinates ξ^α, was obtained in Ref. 16. In particular, the energy conservation law is

$$\frac{\partial}{\partial t}E_S+\nabla_\alpha Q_S{}^\alpha+(Q_V{}^\nu-E_V w^\nu)-(Q_L{}^\nu-E_L w^\nu)-E_S K w^\nu=0. \quad (1)$$

The subscripts V, L, and S label here and below quantities pertaining to vapor, liquid, and the interface; E is the energy density (in particular, E_S is the surface density per unit area), and Q is the energy flux. The last term in (1) is connected with the local change of the surface area.

We can write down analogously a mass-conservation law

$$\nabla_\alpha j_S{}^\alpha+(j_V{}^\nu-\rho_V w^\nu)-(j_L{}^\nu-\rho_L w^\nu)=0, \quad (2)$$

in which account is taken of the zero surface mass; the mo-

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mentum conservation law is

$$\frac{\partial}{\partial t} j_S{}^k+\nabla_\alpha \Pi_S{}^{k\alpha}+(\Pi_V{}^{k\nu}-j_V{}^k w^\nu)-(\Pi_L{}^{k\nu}-j_L{}^k w^\nu)-j_S{}^k K w^\nu=0 \tag{3}$$

and the entropy production equation is

$$\frac{\partial}{\partial t} S_S+\nabla_\alpha F_S{}^\alpha+(F_V{}^\nu-S_V w^\nu)-(F_L{}^\nu-S_L w^\nu)-S_S K w^\nu=\frac{1}{T_S} R_S \tag{4}$$

with a positive-definite surface dissipative function R_S in the right-hand side. Here ρ is the material denisity, j^k the mass flux (the momentum density), Π^{kl} the momentum flux, S the entropy density, and F the entropy flux.

In Eq. (3) the surface momentum flux tensor is projected on the surface and only the second index labels a quantity in the curvilinear frame. Projecting this equation on the normal and on the tangential plane and recognizing that $\Pi_S^{\nu\alpha}=j_s^\alpha w^\nu$ (Ref. 16), we get

$$\nabla_\alpha(j_S{}^\alpha w^{\nu 2})+\Pi_S{}^{\alpha\beta}K_{\alpha\beta}+(\Pi_V{}^{\nu\nu}-j_V{}^\nu w^\nu)-(\Pi_L{}^{\nu\nu}-j_L{}^\nu w^\nu)=0, \tag{5}$$

$$\left(\frac{\partial}{\partial t} j_S\right)^\alpha-j_{S\beta}K^{\alpha\beta}w^\nu+\nabla_\beta \Pi_S{}^{\alpha\beta}+(\Pi_V{}^{\alpha\nu}-j_V{}^\alpha w^\nu)$$
$$-(\Pi_L{}^{\alpha\nu}-j_L{}^\alpha w^\nu)-j_S{}^\alpha K w^\nu=0. \tag{6}$$

The expressions for the volume fluxes in the superfluid liquid take the form[17]

$$\mathbf{j}_L=\rho_n \mathbf{v}_n+\rho_s \mathbf{v}_s; \quad \mathbf{F}_L=S_L \mathbf{v}_n+\mathbf{q}_L/T_L,$$
$$\Pi_L{}^{kl}=p_L\delta^{kl}+\rho_n v_n{}^k v_n{}^l+\rho_s v_s{}^k v_s{}^l+\tau_L{}^{kl},$$
$$Q_L{}^k=(\mu_L+v_s{}^2/2+h)j_L{}^k+[T_L S_L+\rho_n \mathbf{v}_n(\mathbf{v}_n-\mathbf{v}_s)-\rho_L h]$$
$$\times v_n{}^k+\tau_L{}^{kl}v_n{}^l+q_L{}^k, \tag{7}$$

where $\mathbf{q}_L$ is the dissipative part of the heat flux, τ_L^{kl} is the viscous-stress tensor, μ_L is the chemical potential, and h is the dissipative term in the hydrodynamic equation for the superfluid velocity:

$$\frac{\partial v_s{}^k}{\partial t}+\frac{\partial}{\partial r^k}(\mu_L+v_s{}^2/2+h)=0. \tag{8}$$

The expressions for the bulk fluxes in the vapor are the same as in the normal liquid:

$$\mathbf{j}_V=\rho_V \mathbf{v}_V, \quad \mathbf{F}_V=S_V \mathbf{v}_V+\mathbf{q}_V/T_V, \quad \Pi_V{}^{kl}=p_V\delta^{kl}+\rho_V v_V{}^k v_V{}^l+\tau_L{}^{kl},$$
$$Q_V{}^k=(\mu_V+v_V{}^2/2)j_V{}^k+\tau_V{}^{kl}v_V{}^l+q_V{}^k. \tag{9}$$

It is convenient to write down first the thermodynamic identity (the expression for the differential of the surface energy density in a reference frame that moves with velocity $\mathbf{v}_s^t$. In this frame the energy depends not on the velocities but on their invariant difference

$$dE_S{}'=T_S dS_S+(\mathbf{V}-\mathbf{v}_s{}^t)d\mathbf{j}_S. \tag{10}$$

Expression (10) can be regarded as the definition of the normal surface velocity $\mathbf{V}$ (the drift velocity of the surface exitations). Using the Galilean transformation $E_S=E_S'+\mathbf{v}_s^t\cdot\mathbf{j}_S$, we easily obtain the form of the thermodynamic identity in the lab:

$$dE_S=T_S dS_S+\mathbf{V}d\mathbf{j}_S+\mathbf{j}_S d\mathbf{v}_s{}^t. \tag{11}$$

The difference between (11) and the equation used in Ref. 14 is that we do not assume beforehand that the surface normal velocity $\mathbf{V}$ coincides with $\mathbf{v}_n^t$.

Differentiating E_S in (1) with the aid of (11), expressing the time derivatives with the aid of (4), (6), and (8), and using (2), (5), (7), (9), and the fact that the superfluid velocity is potential, we can reduce the expression for $\nabla_\alpha Q_S^\alpha+R_S$ to a form from which follow uniquely the nondissipative parts of the surface momentum flux

$$\Pi_S{}^{\alpha\beta}=-Ag^{\alpha\beta}+\frac{1}{2}[j_S{}^\alpha(V^\beta+v_s{}^\beta)+(V^\alpha+v_s{}^\alpha)j_S{}^\beta]+\tau_S{}^{\alpha\beta} \tag{12}$$

and of the surface entropy flux

$$F_S{}^\alpha=S_S V^\alpha+f^\alpha, \tag{13}$$

as well as the surface energy flux

$$Q_S{}^\alpha=[\mu_L+v_s{}^2/2-(\mathbf{v}_s-\mathbf{v}w^\nu)$$
$$\times(\mathbf{V}+\mathbf{v}w^\nu)+h]j_S{}^\alpha+(\Pi_S{}^{\alpha\beta}+Ag^{\alpha\beta})V_\beta+T_S f^\alpha \tag{14}$$

and the surface dissipative function

$$R_S=-\tau_S{}^{\alpha\beta}(\nabla_\beta V_\alpha-w^\nu K_{\alpha\beta})-f^\alpha\frac{\partial T}{\partial \xi^\alpha}-\tau_V{}^{\nu\alpha}(v_{V\alpha}-V_\alpha)$$
$$-\tau_L{}^{\nu\alpha}(V_\alpha-v_{n\alpha})-(F_V{}^\nu-S_V w^\nu)(T_V-T_S)-(F_L{}^\nu-S_L w^\nu)$$
$$\times\left\{T_S-\left[T_L+\frac{\tau_L{}^{\nu\nu}-\rho_L h}{S_L}+\rho_n(\mathbf{v}_n-\mathbf{V}-\mathbf{v}w^\nu)(\mathbf{v}_n-\mathbf{v}_s)\right]\right\}$$
$$-(j_V{}^\nu-\rho_V w^\nu)\left\{\left[\mu_V+\frac{1}{2}(\mathbf{v}_V-\mathbf{V}-\mathbf{v}w^\nu)^2+\frac{\tau_V{}^{\nu\nu}}{\rho_V}\right]\right.$$
$$\left.-\left[\mu_L+\frac{1}{2}(\mathbf{v}_s-\mathbf{V}-\mathbf{v}w^\nu)^2+h\right]\right\}. \tag{15}$$

Here $A=E_S-T_S S_S-\mathbf{V}\cdot\mathbf{j}_S$ is the surface tension (the surface free energy). We have neglected in (15) the difference between the entropy flux F_L^ν in the superfluid liquid and its nondissipative part. If the finite quantity q_L^ν is to be taken into account, the use of the dissipative function R_S in closed form would make it necessary to take into account terms of second order in the reciprocal of the mean free path, which would be outside the framework of the thermodynamic equations used by us.

The dissipative function (15) determines the form of the boundary conditions. The absence of dissipation on the interface corresponds to vanishing of the second factors in each of the seven terms of (15). If dissipation is taken into account, the boundary conditions can be obtained by starting from the condition that the dissipative function must be non-negative. In the lowest (linear) order in the deviations from equilibrium we have for scalar quantities

$$T_V-T_S=-C_{11}(F_V{}^\nu-S_V w^\nu)-C_{12}(F_L{}^\nu-S_L w^\nu)-C_{13}(j_V{}^\nu-\rho_V w^\nu), \tag{16a}$$

$$T_S-\left(T_L+\frac{\tau_L{}^{\nu\nu}-\rho_L h}{S_L}\right)$$
$$=-C_{12}(F_V{}^\nu-S_V w^\nu)-C_{22}(F_L{}^\nu-S_L w^\nu)-C_{23}(j_V{}^\nu-\rho_V w^\nu), \tag{16b}$$

$$(\mu_V+\tau_V{}^{\nu\nu}/\rho_V)-(\mu_L+h)$$
$$=-C_{13}(F_V{}^\nu-S_V w^\nu)-C_{23}(F_L{}^\nu-S_L w^\nu)-C_{33}(j_V{}^\nu-\rho_V w^\nu). \tag{16c}$$

and for vector quantities

$$\begin{aligned}\tau_V{}^{\nu\alpha}&=-C_{11}'(v_V{}^\alpha-V^\alpha)-C_{12}'(V^\alpha-v_n{}^\alpha),\\ \tau_L{}^{\nu\alpha}&=-C_{12}'(v_V{}^\alpha-V^\alpha)-C_{22}'(V^\alpha-v_n{}^\alpha).\end{aligned} \tag{17}$$

The matrices of the coefficients C and C' are symmetric and positive-definite. We do not include in the boundary conditions (16) and (17) terms connected with the dissipative surface fluxes $\tau_S^{\alpha\beta}$ and f^α. As indicated in Ref. 14, these quantities enter in the surface conservation laws with an extra derivative compared with the dissipative fluxes in both media, so that inclusion of these quantities in the general case is an exaggeration of the accuracy. These terms must be taken into account only at low temperatures, when the excitations in the volume "freeze out" compared with the softer surface excitations. The ensuing changes in the boundary conditions can be easily understood.

It is known that the linearized equations of dissipative hydrodynamics of He II have also a stationary solution in the form of a "boundary thermal wave," in which the deviations of the temperature and of other quantities from their equilibrium values attenuate exponentially with increasing distance from the surface. Putterman[19] is of the opinion that the existence of this solution complicates the problem of the boundary conditions and makes it necessary to invoke the principle of minimum entropy production. Note that the combination $T_L + (\tau_L^{\nu\nu} - \rho_L h)/S_L$ in the left-hand side of (16b) is independent of the thermal-wave amplitude and has on the interface itself the same value as at a certain distance, where the thermal wave had already attenuated, i.e., our macroscopic boundary conditions fortunately do not "sense" such a formation as a boundary thermal wave, which is in no way macroscopic, since its attenuation depth is proportional to the mean free path.

3. SIMPLIFICATIONS OF BOUNDARY CONDITIONS IN THE PROBLEM OF ACOUSTIC COEFFICIENTS

In the calculation of the limiting low-frequency values of the acoustic coefficients on a plane interface, the boundary conditions can be greatly simplified. We must in this case not only use linearized boundary conditions, but also omit from them all terms except those of leading order in the derivatives (transition to the low-temperature limit). The surface mass, momentum, and entropy conservation laws (entropy is also conserved in the linear approximation) take then the very simple forms

$$\rho_V(v_V{}^\nu - w^\nu) = \rho_n(v_n{}^\nu - w^\nu) + \rho_s(v_s{}^\nu - w^\nu), \tag{18}$$

$$p_V = p_L, \tag{19}$$

$$S_V(v_V{}^\nu - w^\nu) = S_L(v_n{}^\nu - w^\nu). \tag{20}$$

The boundary conditions (17) yield in the low-temperature limit

$$v_V{}^\alpha = V^\alpha = v_n{}^\alpha. \tag{21}$$

It must be noted that in the presence of any of the sound waves on the interface, the hydrodynamic equations allow the onset of not only sound waves that propagate away from the boundary, but also of a temperature wave in the vapor and of transverse viscous waves whose amplitude decreases rapidly with increasing distance from the interface. In sound waves, the first-order quantities are the oscillation amplitudes of the pressure δp, of the temperature δT, and of the velocities. For a viscous wave in the vapor and having a frequency and wave-vector component parallel to the surface that are the same as in the incident sound wave (i.e., proportional to one another), the leading quantity is only the amplitude of the v_n^α oscillations. The amplitude of the v_V^ν oscillations is of higher order in frequency:

$$v_V{}^\nu \backsim \omega^{1/2} v_V{}^\alpha, \tag{22}$$

while the pressure and temperature do not change at all in the linear approximation. In exactly the same way, for a viscous transverse wave in a liquid the only quantity of leading order is the amplitude of the v_n^α oscillations. Since the quantities v_V^α and v_n^α do not enter in the remaining linearized boundary conditions, the boundary conditions (21) [or (17)] can be discarded, since they are automatically satisfied by the onset of viscous waves of the required amplitude.

Summation of (16a) and (16b) permits the surface temperature T_S to be excluded from the boundary conditions. We must further recognize that if terms of higher order in the derivatives are neglected, the normal entropy fluxes in the liquid

$$F_L{}^\nu - S_L w^\nu = S_L(v_n{}^\nu - w^\nu)$$

and in the vapor

$$F_V{}^\nu - S_V w^\nu = S_V(v_V{}^\nu - w^\nu)$$

coincide and are proportional to the normal mass flow:

$$j_V{}^\nu - \rho_V w^\nu = \rho_V(v_V{}^\nu - w^\nu).$$

Recognizing also that in the approximation of interest to us the left-hand side of (16c) can be written in the form

$$\mu_V - \mu_L = -\Delta s \delta T_L + \Delta\rho^{-1}\delta p_L - (S_V/\rho_V)(\delta T_V - \delta T_L)$$

and the boundary conditions (16) can be transformed into

$$\delta T_V - \delta T_L = -K_2(v_V{}^\nu - w^\nu), \tag{23}$$

$$\rho_V(v_V{}^\nu - w^\nu) = -K_1\left(-\frac{\Delta s}{\Delta\rho^{-1}}\delta T_L + \delta p_L\right), \tag{24}$$

where

$$K_2 = S_V(C_{11} + C_{12}) + S_L(C_{12} + C_{22}) + \rho_V(C_{13} + C_{23}),$$

$$K_1 = (1 - \rho_V/\rho_L)/[S_V C_{13} + S_L C_{23} + \rho_V C_{33} + K_2 S_V/\rho_V],$$

$$\Delta s = S_V/\rho_V - S_L/\rho_L, \quad \Delta\rho^{-1} = 1/\rho_V - 1/\rho_L.$$

One of these boundary conditions [namely, (24)] has been specially reduced to this form to eliminate from it the vapor temperature T_V. Since the only quantity of leading order in the temperature and in the vapor is the deviation of T_V from equilibrium, and T_V is likewise not contained in the boundary conditions (18)-(20), the use of (24) allows us to study the relations between the sound-wave amplitudes, and disregard the presence of the temperature wave. Its amplitude can be obtained if desired from the boundary condition (23), which turns out to be unnecessary for the calculation of the acoustic coefficients.

We have thus shown that to calculate the amplitudes of the sound waves produced when some arbitrary sound wave is incident on the interface, it is necessary to use the boundary conditions (18)-(20) and (24), and disregard the temperature wave and the viscous waves. Boundary conditions in the form (24) were first obtained by Hunter and Osborne from microscopic considerations.[2] It has heretofore not been realized that from the phenomenological standpoint this boundary condition is the only one compatible with the disregard of the temperature wave in the vapor. Other workers (see Ref. 15 and the citations therein) used boundary conditions of more general form, the use of which is impossible without allowance for the onset of a temperature wave.

The kinetic coefficients are found, generally speaking, to be different in different microscopic models of the evaporation process.[2,10-13] Using the data tabulated in Refs. 8 and 15, however, we find that both the drifting-Knudsen-gas model[12] and the Hunter-Osborne model[2] yield for the coefficient K_1 in (24) the value

$$K_1=B\left(\frac{m}{2\pi k_BT}\right)^{1/2},\quad B=\frac{16}{9}\frac{\alpha}{2-\alpha},$$

where m is the helium-atom mass and α is the coefficient of absorption of the atom by the surface. Experiments on scattering of atomic ^{4}He atoms by a free He II surface yield $1-\alpha\lesssim 10^{-3}$, hence $B\approx 1.77$. The numerical solution obtained in Ref. 13 for the kinetic equation at $\alpha=1$ yields $B=1.66$, or merely 8% less.

Wiechert and Buchholz observed in their numerical calculations[15] that the discrete-Knudsen-gas models and of the Hunter-Osborne model lead to practically the same values of the acoustic coefficients. They did not understand, however, that this was due to equality of a certain kinetic-coefficient combination that entered in the boundary condition (24) in the form of the coefficient K_1.

The condition (24) determines the rate of the evaporation process. Evaporation is possible on the considered interface also in the limit of zero frequency, therefore the dissipative coefficient K_1 will enter in the limiting expressions for the acoustic coefficient, and the coefficient of sound absorption on the interface will have a finite value. On the interface of the vapor and normal liquid (for which one can assume $\rho_S=0$), the boundary conditions (18) and (20) are compatible only if $v_V^v-w^v=0$ and no evaporation is possible in the zero-frequency limit.

The experimental-data analysis proposed in Ref. 6 shows that the expressions obtained in the nondissipative approximation for the acoustic coefficients[1] are not valid anywhere.

4. ACOUSTIC COEFFICIENTS

We shall label quantities pertaining to first- and second-sound waves in a superfluid liquid by the subscripts 1 and 2, and the sound waves in the vapor by 0. We choose the wave amplitude x for first sound to be the amplitude of the pressure oscillations, and for the second sound the temperature-oscillation amplitude:

$$x_a=\begin{cases}\delta p, & a=0,1\\ \delta T, & a=2\end{cases}.$$

The acoustic coefficients D_{ab} are defined as the ratios of amplitude of the outgoing wave x_b to the amplitude of the outgoing wave x_b to the amplitude of the incident wave x_a^0:

$$D_{ab}=x_b/x_a{}^0.$$

The diagonal elements of the matrix D are the reflection coefficients, the elements D_{10} and D_{01} are the transmission coefficients of the first sound, and the remaining are the conversion coefficients.

The acoustic coefficients D_{ab} for normal incidence of sound waves were obtained by Wiechert and Buchholz[15] not only numerically but also analytically. They used the boundary condition (24) of Hunter and Osborne as the most compact among those proposed by the microscopic theories, and neglected the thermal expansion and the thermal conductivity of the two media, and also the fact that the ratio ρ_V/ρ_L is finite. We present below the values of the coefficients D_{ab} at zero frequency for oblique incidence of sound waves, calculated with only the thermal expansion of the liquid neglected. As shown in Sec. 3, the use of the boundary condition (24) makes it automatically unnecessary to take into account the finite thermal conductivities of the two media.

Substituting in (18)–(20) and in (24) the relations between the oscillation amplitudes of the different quantities in plane sound waves, we obtain for sound incident from the vapor

$$D_{00}=D_{01}-1=\frac{R_0-R_1-R_2}{R_0+R_1+R_2},\tag{25}$$

$$D_{02}=\frac{\Delta\rho^{-1}}{\Delta_s}\frac{2r_0}{R_0+R_1+R_2},\tag{26}$$

where

$$R_0=\left(\cos\theta_2+K_1\frac{T(\Delta s)^2}{C_Lc_2\rho_L\,\Delta\rho^{-1}}\right)\frac{\cos\theta_0}{\rho_Vc_0}=\frac{\cos\theta_2\cos\theta_0}{\rho_Vc_0}+r_0,$$

$$R_1=\left(\cos\theta_2+K_1\frac{T(\Delta s)^2}{C_Lc_2\rho_L\,\Delta\rho^{-1}}\right)\frac{\cos\theta_1}{\rho_Vc_1}=\frac{\cos\theta_2\cos\theta_1}{\rho_Vc_1}+r_1,$$

$$R_2=K_1\Delta\rho^{-1}\cos\theta_2,$$

C_L is the heat capacity of the liquid (at constant pressure), c_a $(a=0,1,2)$ is the sound-wave velocity, and θ_a are the incidence angles, i.e., the angles between the wave vectors and the normal, related by Snell's law:

$$\frac{\sin\theta_0}{c_0}=\frac{\sin\theta_1}{c_1}=\frac{\sin\theta_2}{c_2}.\tag{27}$$

Similarly, for incidence of first sound on a liquid we get

$$D_{10}=1+D_{11}=\frac{2R_1}{R_0+R_1+R_2},\tag{28}$$

$$D_{12}=\frac{\Delta\rho^{-1}}{\Delta s}\frac{2r_1}{R_0+R_1+R_2},\tag{29}$$

and for incidence of second sound

$$D_{20}=D_{21}=\frac{\Delta s}{\Delta\rho^{-1}}\frac{2R_2}{R_0+R_1+R_2},\tag{30}$$

$$D_{22}=1-2(r_0+r_1)/(R_0+R_1+R_2).\tag{31}$$

It is convenient to define the absorption coefficients γ_a as the fraction of energy lost when the sound wave collides with the surface. From (25)–(30) we get

$$\gamma_a=4R_2(R_a-r_a)/(R_0+R_1+R_2)^2,\quad a=0,\ 1,\tag{32}$$

$$\gamma_2=4(r_0+r_1)(R_0-r_0+R_1-r_1)/(R_0+R_1+R_2)^2.\tag{33}$$

It is interesting that the ratios of the absorption coefficients turned out to be independent of the dissipative coefficient K_1. Equations (25)–(33) can be applied directly only if incidence of the sound wave on the surface leads to the appearance of three different sound waves outgoing from the surface. This is not the case for all incidence angles.

It is known that at $T>T_0\approx 0.7$ the sound velocities satisfy the equalites

$$c_2<c_0<c_1,$$

and c_0 becomes smaller than c_2 at $T < T_0$. If the angles θ_0 and θ_2 are such that Eq. (27) yields $\sin\theta_1 > 1$, the first-sound wave, produced in the liquid following incidence on the interface of a second-sound wave or of a sound wave from the vapor, should have a wave-vector component perpendicular to the interface and pure-imaginary, corresponding to damping in the interior of the liquid. In this case it is possible to use, as before, expressions (25), (26), (30), and (31) for D_{ab} $(a,b \neq 1)$, in which it is necessary to substitute $i(\sin^2\theta_1 - 1)^{1/2}$ for $\cos\theta_1$.

If a second-sound wave is incident on the interface at $T > T_0$ and $\sin\theta_2 > c/c_0$, only one wave propagating from the interface is produced, viz., a reflected second-sound wave. The reflection is not total, i.e., part of the energy is lost in the interface:

$$\gamma_2=4|r_0+r_1||R_0-r_0+R_1-r_1|/[|R_0+R_1|^2+R_2^2],$$

where $\cos\theta_0$ and $\cos\theta_1$ should be replaced by $i(\sin^2\theta_0 - 1)^{1/2}$ and $i(\sin^2\theta_1 - 1)^{1/2}$, respectively. The reflection of the second wave in the vapor is not total, in exactly the same manner, if $T < T_0$ and $\sin\theta_0 > c_0/c_2$.

We have thus found the values of the acoustic coefficients D_{ab} at zero frequency. None of the reflection coefficients D_{aa} vanishes at any incidence angle. To find the frequency-dependence corrections, it is necessary to use also the boundary conditions (17) and (23), and take into account the onset of the temperature wave and of the viscous waves. Since the oscillation amplitudes of these waves are related by an equation of the form (22), the corrections to D_{ab} are proportional to $\omega^{1/2}$ at low frequencies.

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