Thermodynamics of strong wave turbulence

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A model of strong one-dimensional Langmuir turbulence is considered in terms of the weak interaction of Langmuir solitons, plasma waves, and sound waves. With the assumption of an equilibrium distribution of free plasmons, the spectral characteristics of the Langevin source are determined and the Fokker-Planck equation for the distribution function of the Langmuir solitons is derived. For the thermodynamically equilibrium case, at a fixed number of plasmons in the system, the stationary distribution function, the partition function, and the dependences of the numbers of plasmons in the soliton and wave subsystems on the effective temperature are found. The proposed description method can also be applied to other essentially nonlinear quasiconservative systems whose dynamics are determined by the weak interaction of nonlinear structures.

1. INTRODUCTION

At present a large number of different physical phenomena can be described satisfactorily in terms of the weak interaction of linear modes. The theory of weak wave turbulence that describes such an interaction is valid, as is well known, only at relatively low levels of turbulent-energy density. At higher levels a transition occurs to a strong wave turbulence characterized primarily by the presence of developed, spatially localized coherent structures—kinks, solitons, collapsing wave cavities, etc., i.e., nonlinear wave distributions corresponding to a bound state of a large number of spatial harmonics of the fields.

It is natural to expect that when coherent structures are taken into account strong wave turbulence can be interpreted, as before, in terms of the weak interaction of linear and nonlinear modes. Indeed, because of the stability of the coherent structures characterized by a large binding energy of the field "quanta" that form them, and hence by short rearrangement times under weak external perturbations, their parameters vary slowly in comparison with the nonlinear-development times. Therefore, the hope that the interactions are weak is based essentially on the idea that the fact that coherent structures can be distinguished against the background of weak turbulence implies a relatively small energy of interaction of these structures with each other and with the background. These ideas also dictate the language of the description—kinetic equations for weakly interacting linear and nonlinear oscillators.

The basic idea is as follows. Assuming that the system is close to the state of thermodynamic equilibrium and that, consequently, the coefficients of the kinetic equations are determined by the equilibrium distribution function,[1)] we introduce an effective Langevin source. We find the spectral characteristics of the source from the requirement that the distribution of waves be an equilibrium (Rayleigh-Jeans) distribution. In this case the kinetic equation for the solitons is obtained by considering the problem of a probe soliton as a Brownian particle acted upon by the same Langevin source. The stationary solution of this equation is the desired equilibrium distribution of the coherent structures that makes it possible to construct the thermodynamics.

A specific feature of the thermodynamics of strong wave turbulence in the presence of coherent structures is the nonlinear dependence of the energy of the state (soliton) on the number of quanta, and, as a consequence, the possibility of change of the entropy of the soliton subsystem upon change of the number of solitons with no change in the total number of quanta in the soliton subsystem.

In this paper the indicated approach is realized for the example of strong one-dimensional Langmuir turbulence. The existing results of numerical and model experiments on the realization of strong Langmuir turbulence demonstrate that the turbulence parameters approach a quasi-equilibrium regime in the sense that the time-averaged parameters of the turbulence (the spectra, average fields, average intensities, soliton-number density, etc.) depend weakly on the initial conditions and are evidently determined by macroscopic characteristics of the system (the pumping amplitude, the average plasma density, and the electron and ion temperatures).[1,2]

Qualitatively, this can already be understood on the basis of an analysis of the elementary processes of interaction of Langmuir solitons with sound,[3] and with free plasma waves[4] and plasma particles.[5] All these and similar processes correspond in fact to weak interactions and, when included in the picture of the turbulence, lead to corresponding kinetic effects—the retardation of solitons by sound[3] or particles,[5] the isotropization of the ion-acoustic wave spectrum, etc. In order to complete the theory, generally speaking, it is necessary to take into account exchange of plasmons between the soliton and plasma-wave subsystems. This task was partially implemented in Ref. 6. However, the question of the possible stationary amplitude distributions of solitons and "free" plasma waves still remains open. The problem of the determination of these distribution functions is the subject of the present paper. Here also we shall find the total number of solitons in the system without assuming that this number is small.[6]

In the Conclusion we discuss the conditions for applicability of the approach developed and its relation to existing ideas, and also discuss in detail the thermodynamic consequences that stem from the possibility of a gain in entropy

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upon fractionation of the solitons with no change in the total number of plasmons in the soliton subsystem.

2. THE LANGEVIN SOURCE

We shall start from the equations of one-dimensional Langmuir turbulence in the quasi-hydrodynamic approximation,[7] which, in dimensionless variables, have the following form:

$$\begin{gathered} -i\partial u/\partial t+\partial^2 u/\partial x^2-nu=0, \\ \partial n/\partial t+\partial v/\partial x=0, \\ \partial v/\partial t=-\partial n/\partial x-\partial |u|^2/\partial x. \end{gathered} \tag{2.1}$$

Here u is the amplitude of the plasma field, n is the perturbation of the plasma concentration, and v is the hydrodynamic velocity. It is known that this system conserves three integrals of the motion[2)]: the number of quanta

$$I=\int |u|^2\,dx, \tag{2.2}$$

the momentum, and the energy

$$H=\int [\,|\partial u/\partial x|^2+n|u|^2+{}^1/_2(n^2+v^2)\,]dx. \tag{2.3}$$

In the linear approximation the elementary excitations in this system are sound waves (phonons) and "free" plasma waves (plasmons) with the dispersion law

$$\omega_k=k^2, \tag{2.4}$$

where k is the wave number. In (2.3) and everywhere below, the frequency is reckoned from the unperturbed plasma frequency.

In the nonlinear regime the elementary excitations also include solitons:

$$u_c(x,t)=\frac{2^{1/2}}{a\,\mathrm{ch}\,x/a}\exp\left(-\frac{it}{a^2}\right), \tag{2.5}$$

where a is the size of the soliton, and its frequency is

$$\omega_c=-a^{-2}. \tag{2.6}$$

We have given the expressions (2.5) and (2.6), which are valid only for stationary solitons, since, as noted in the Introduction, developed turbulence is accompanied by effective hydrodynamic retardation of the solitons by phonons.

It is obvious that the Hamiltonian of the noninteracting excitations,

$$H_0=H_{p0}+H_{c0}, \tag{2.7}$$

can be written in action-angle variables:

$$H_{p0}=\sum_k \omega_k n_k, \qquad n_k=|u_k|^2, \tag{2.8}$$

$$H_{c0}=\sum_i -J_i^3/48, \qquad J_i=4/a_i, \tag{2.9}$$

where i labels a soliton with number of quanta (2.2) equal to J_i.

The interaction of the plasmons and solitons leads to exchange of quanta between them (exchange of momentum can be disregarded, since the phonon system is responsible for this process). From the total energy (2.3) we separate out the interaction energy:

$$H=H_0+H_{int}, \tag{2.10}$$

which, in accordance with our ideas, we model by means of a Langevin source, starting from the requirement that for free plasmons we obtain the Rayleigh-Jeans distribution. Of course, when the effects of scattering of plasmons by solitons are taken into account, the plasmon dispersion law and, consequently, density of states are changed. We shall neglect these effects.

The simplest source that leads to the Rayleigh-Jeans distribution has, in terms of the interaction energy, the following form:

$$H_{int}=\sum_k (y_k u_k^* \omega_k^{-1/2}+\text{c.c.}). \tag{2.11}$$

Here u_k is the amplitude of the linear "oscillator" with frequency

$$\omega_k=k^2+\mu_p, \tag{2.12}$$

and $y_k(t)$ is a Fourier component of the Langevin force

$$F(x,t)=\sum_k y_k(t)\exp(ikx),$$

with

$$\begin{gathered} \langle y_k(t)y_{k'}^*(t')\rangle=D_k\delta(k-k')\delta(t-t'), \\ \langle y_k(t)y_{k'}(t)\rangle=0. \end{gathered} \tag{2.13}$$

The Fokker-Planck equation is obtained from the equation of motion

$$i\,\partial u_k/\partial t+\omega_k u_k+i\gamma_k u_k+y_k\omega_k^{-1/2}=0, \tag{2.14}$$

where γ_k is the effective damping constant that models the exchange of plasmons between the given state and other states. For the distribution function $f(n_k)$ we have (see, e.g., Ref. 8)

$$\frac{\partial f_k}{\partial t}+\frac{\partial}{\partial n_k}[-\gamma_k n_k f_k]=\frac{\partial}{\partial n_k}\frac{D_k}{|\omega_k|}\frac{\partial f_k}{\partial n_k}.$$

In the stationary state, assuming that the Einstein relation

$$D_k=\gamma_k T \tag{2.16}$$

is fulfilled, we have

$$f_k=Z_k^{-1}\exp(-|\omega_k|n_k/T). \tag{2.17}$$

Taking into account the normalization condition, we have

$$Z_k=T/|\omega_k|. \tag{2.18}$$

From (2.17) and (2.18), for the average number $\langle n_k\rangle$ of plasmons in the state k we indeed have the Rayleigh-Jeans distribution

$$\langle n_k\rangle=T/(k^2+\mu_p). \tag{2.19}$$

In this case the concentration of plasmons is equal to

$$n_p=\int\langle n_k\rangle\frac{dk}{2\pi}=\frac{T}{2\mu_p^{1/2}}. \tag{2.20}$$

The relation (2.20) clarifies the meaning of the parameter μ_p introduced in (2.12): It is the chemical potential of the plasmons, which makes it possible to ensure a finite value for the total number of plasmons in the plasma-wave subsystem and, as we shall see below, in the system as a whole. The introduction of the chemical potential corresponds to the traditional way of taking into account the possible exchange of quanta between subsystems.

3. THE SOLITON DISTRIBUTION FUNCTION

We shall now examine to what distribution the source (2.11) leads in the case of the soliton. If its frequency depends on its amplitude in the same way as for the standing soliton (2.6)–(2.9):

$$\omega_c(J) = -J^2/16 + \mu_c, \tag{3.1}$$

where

$$J = \int |u_c(x,t)|^2 dx$$

is the number of quanta in the soliton, then all the spatial Fourier harmonics oscillate with the same frequency. In (3.1), as in (2.12), we have introduced the soliton chemical potential μ_c. For the solitons in (2.11) it must be assumed that u_c depends only on J and φ, i.e., that the nonlinearity ensures rapid establishment of the soliton spectrum. Thus,

$$H_{int}^c = \omega_c^{-1/2} \sum_k y_k u_k^2(J,\varphi) + \text{c.c.} \tag{3.2}$$

Here, in place of (2.14) we have

$$\frac{\partial J}{\partial t} = -\gamma_c J - \frac{\partial H_{int}^c}{\partial \varphi},$$

$$\frac{\partial \varphi}{\partial t} = \omega_c(J) + \frac{\partial H_{int}}{\partial J}. \tag{3.3}$$

Here

$$\gamma_c = J^{-1} \int \gamma_k |u_k^c|^2 dk, \tag{3.4}$$

if we assume that γ_k depends only on k and does not depend on the frequency of the formation under consideration.

Assuming the condition (2.13) to be fulfilled, for the soliton distribution function $f_c(J)$ we have

$$\frac{\partial f_c}{\partial t} + \frac{\partial}{\partial J}(-\gamma_c J f_c) = \frac{\partial}{\partial J} D_c \frac{\partial f_c}{\partial J}, \tag{3.5}$$

where the soliton diffusion coefficient

$$D_c = \int_0^t \left\langle \left(\frac{\partial H_{int}}{\partial \varphi} \right)^2 \right\rangle d\tau. \tag{3.6}$$

By means of simple calculations this expression can be transformed to the form

$$D_c = \gamma_c J T / |\omega_c|. \tag{3.7}$$

Thus, in analogy with (2.15), we have

$$\frac{\partial f_c}{\partial t} + \frac{\partial}{\partial J}(-\gamma_c J f_c) = \frac{\partial}{\partial J} \frac{\gamma_c J}{|\omega_c|} T \frac{\partial f_c}{\partial J} \tag{3.8}$$

or, for the stationary state,

$$f_c(J) = Z_c^{-1} \exp\left(-T^{-1} \int_0^J |\omega_c(J)|\, dJ \right), \tag{3.9}$$

$$Z_c = \int_0^\infty \exp\left[-\frac{1}{T} \int_0^J |\omega_c(J')|\, dJ' \right]. \tag{3.10}$$

A fundamental feature in (3.9) is the presence of $|\omega_c|$—the absolute value of the frequency; i.e., irrespective of whether the soliton energy increases or decreases with increase of J, rapidly oscillating solitons are improbable. Qualitatively, this is entirely obvious if we note that such solitons interact weakly with the thermostat (for more detail, see Sec. 5). We also draw attention to the fact that (2.18) is a general expression that does not depend on the specific nature of the dependence $\omega_c(J)$ (3.1). Therefore, the subsequent calculations and estimates, given for Langmuir solitons with $\omega_c - \mu_c \sim -J^2$, are only a concrete example.

4. THERMODYNAMICS OF STRONG LANGMUIR TURBULENCE

Having obtained expressions for the distribution functions of free plasmons and solitons, we can now seek the parameters of stationary turbulence—the concentration n_c of plasmons in the solitons, the concentration n_p of free plasmons, and the dependences of the chemical potentials μ_c and μ_p on the temperature T (2.16) and on the "pumping" $n = L^{-1}\int|u|^2 dx$ (the total number of plasmons in the system per unit length). Here L is the length of the system.

First of all we shall discuss the relation between the chemical potentials μ_c and μ_p in the state of equilibrium between the soliton and plasmon subsystems. With increase of the number of quanta in the plasmon subsystem its energy per quantum averaged over the equilibrium distribution function (i.e., the plasmon chemical potential) is positive. Analogously, it follows from the decrease of the energy of the soliton subsystem with increase of the number of quanta in it that $\mu_c < 0$. Since the distributions (2.17) and (3.10) have the form of Gibbs distributions, in the equilibrium state the usual relation[9] $\mu_1/T_1 = \mu_2/T_2$ is fulfilled, and, since $T_p = -T_c = T > 0$, we have

$$\mu_p = -\mu_c \equiv \mu > 0. \tag{4.1}$$

We find the chemical potential $\mu(T)$ from the condition fixing the total number of plasmons in the system:

$$n_p + n_c = n, \tag{4.2}$$

where n_p is determined by the relation (2.20) and n_c, in the approximation of an ideal gas of solitons, can be expressed in terms of the partition function Z_c (3.10). For this we note that in the equilibrium state the probability of realization of N solitons with numbers of quanta $J_1, J_2, \ldots, J_n$ in the intervals $dJ_1, \ldots, dJ_N$ is, by virtue of the statistical independence, equal to

$$f_N(J_1,\ldots,J_N)\, dJ = \frac{(Z_c L)^N}{Z_N^c N!} \prod_{i=1}^N f_c(J_i)\, dJ_i, \quad dJ = \prod_{i=1}^N dJ_i, \tag{4.3}$$

where $N!$ takes account of the fact that the solitons are identical. Hence, the partition function of the soliton distribution is

$$Z_N^c=(LZ_c)^N/N! \qquad (4.4)$$

Minimizing the free energy $\tilde{f}_c = -T\ln Z_N^c$ with respect to N for a fixed number of quanta in the solitons, i.e., assuming that the chemical potential of the gas of solitons is equal to zero, we have, for $N\gg 1$, as usual,[9]

$$N_c/L=Z_c. \qquad (4.5)$$

This implies that the density of solitons is determined by the partition function of one soliton. From (4.5), for the concentration of plasmons in the soliton subsystem in state of equilibrium we obtain

$$n_c=Z_c\bar{J}=-T\partial Z_c/\partial\mu, \qquad (4.6)$$

since

$$\bar{J}=\int_0^\infty Jf_c(J)\,dJ=-T\frac{\partial}{\partial\mu}\ln Z_c. \qquad (4.7)$$

Knowing n_c and n_p (2.20), we write the equation for μ:

$$\tfrac{1}{2}\mu^{-1/2}T-T\partial Z_c/\partial\mu=n. \qquad (4.8)$$

We note that the expression (3.10) of interest for Z_c has at $\omega_c=-J^2/16-\mu$ (here we have taken into account the sign of μ_c (4.1)) a nontrivial dependence on only one parameter

$$\gamma=4\cdot 3^{1/2}\mu^{3/2}/T, \qquad (4.9)$$

$$Z_c=4\cdot 3^{1/2}\mu^{1/2}g(\gamma),\quad g(\gamma)\equiv\int_0^\infty \exp[-\gamma(x^3+x)]\,dx. \qquad (4.10)$$

It can be seen that the function $g(\gamma)$ decreases monotonically and has the following asymptotic forms:

$$g(\gamma)=\begin{cases} c_1\gamma^{-1/3}-c_2\gamma^{1/3}, & \gamma\ll 1\\ 1/\gamma, & \gamma\gg 1\end{cases}. \qquad (4.11)$$

Here $c_1=\Gamma(4/3)\cong 0.893$, and $c_2=(1/3)\Gamma(2/3)\cong 0.451$. With allowance for (4.10), Eq. (4.8) can be conveniently transformed into

$$n=(48T)^{2/3}\gamma^{-1/3}[\tfrac{1}{2}+Q(\gamma)], \qquad (4.12)$$

$$Q(\gamma)=-6\cdot 3^{1/2}\gamma^{2/3}\frac{d}{d\gamma}[\gamma^{1/3}g(\gamma)]=4\cdot 3^{1/2}\begin{cases} c_2\gamma^{1/3}, & \gamma\ll 1\\ \gamma^{-1}, & \gamma\gg 1\end{cases}, \qquad (4.13)$$

which makes it possible to find the dependence of the chemical potential on the temperature:

$$\mu=\frac{T^2}{4n^2}F\left(\frac{n^{3/2}}{T}\right). \qquad (4.14)$$

In this representation the function $F^{1/2}(x)$ is everywhere of order unity and has a characteristic scale in $x=n^{3/2}/T$ that is also of order unity; the asymptotic values of the function both at small and at large values of the argument are equal to unity. By means of this function one can also write the numbers of quanta, namely,

$$n_p=nF^{-1/2}, \qquad (4.15)$$

$$n_c=n(1-F^{-1/2}). \qquad (4.16)$$

Taking (4.12) into account, we obtain from (4.16)

$$n_c=n\begin{cases} x^2/16, & x\ll 1\\ 8\cdot 3^{1/2}c_2/x^{2/3}, & x\gg 1\end{cases}, \qquad (4.17)$$

or, in the variables n and T,

$$n_c=\begin{cases} c_2(48T)^{2/3}, & T\ll n^{3/2}\\ n^4/16T^2, & T\gg n^{3/2}\end{cases}. \qquad (4.18)$$

It follows from this expression that the maximum value $n_c\sim n$ of the plasmon concentration in the solitons is reached at a temperature

$$T^*\sim 0.2n^{3/2}. \qquad (4.19)$$

The dependence of the soliton-number density on the temperature has an analogous appearance. Moreover, at all temperatures it is found that

$$N_c/L=n_c^{1/2}K(x), \qquad (4.20)$$

where $K(x)\sim 1$.

The decrease of the number of solitons at high temperatures is in agreement with the well-known result of Ref. 10 concerning the suppression of the modulation instability in the case of a sufficiently high noise density of the plasma-wave subsystem. Estimates performed with the use of the results of Ref. 10 in the case of a Rayleigh-Jeans distribution of plasmons with μ from (4.14) show that the characteristic temperature threshold for the creation of solitons (the threshold of the modulation instability) is of order $n^{3/2}$.

5. CONCLUSION

The most important feature of the soliton-distribution function (3.9) obtained in the Langevin-source model is its apparent difference from a Gibbs distribution (the temperatures of the subsystems have different signs). In fact, if $H_c\sim -J^3$, then it would appear that in the equilibrium state

$$f_c(J)\sim\exp(-H_c/T_p)\sim\exp(+J^3/T_p),$$

i.e., a configuration with one soliton that has collected a large number of quanta is the most probable. But we obtained $f_c\sim\exp(-J^3/T_p)$, which, in the language of the Gibbs distribution $f_c\sim\exp(H_c/T_c)$, corresponds to negative temperatures $T_c=-T_p$ for the soliton subsystem. Formally, this is a consequence of the essentially nonlinear dependence of the (negative) soliton frequency on the number of quanta in the soliton, as can be clarified as follows.

We shall consider the soliton subsystem separately and shall assume that its equilibrium distribution function corresponds to the maximum of the entropy $S=-\langle\ln f_c\rangle$ for fixed integrals of motion. Then the standard derivation of the distribution function by the method of undetermined multipliers leads us to the usual expression $\ln f_c\sim T_c^{-1}(H_c+\mu_c J)$, where T and μ_c are found from the condition of conservation of the energy and number of quanta in the solitons. Here, since the problem under consideration is essentially nonlinear, we can convince ourselves that it has two different solutions: with $T_{c1}>0$ and with $T_{c2}<0$. The final choice of the value of T_c of interest to us should be made using the entropy-maximum principle. At T_{c1} the entropy is lower than at T_{c2}, since the principal contribution to the integration over J in the expression for the entropy at T_{c1} is made by large values of J, i.e., a configuration with a small number of solitons is the most probable. For $T_c < 0$ the upper limit of integration is unimportant, and, consequently, from the point of view of the entropy-maximum principle, the situation with a negative temperature of the soliton subsystem is preferable.

At first glance, this contradicts the condition for equilibrium of the soliton and plasma subsystems, in which $T_p > 0$. However, the condition $T_1 = T_2$ used in traditional thermodynamics is justified only in the case when the energy of the subsystem depends linearly on the number of particles in all the allowed states, for then the only process that changes the state of the subsystem for a fixed number of particles is heating. In a system in which the energy depends nonlinearly on the number of particles in the bound state, there arises a further channel for variation of the state of the system—the channel arising from the creation (disappearance) of quasi-particles of the soliton type. Allowance for precisely this channel of exchange of energies in our case makes it possible to establish a relation between the temperatures.

In order to find this relation, we shall determine the change ΔS_c of the entropy of the soliton subsystem with allowance for the change of its temperature and of the number of solitons in it upon transfer of an amount of heat ΔQ from the plasmon subsystem to the soliton subsystem. The chemical potential μ_c in the following relations will be assumed to be equal to zero, since here we shall not consider changes of the number of plasmons in the soliton subsystem. Let the parameters N and T_c of the soliton subsystem become $N + \Delta N$ and $T_c + \Delta T_c$ after transfer of a quantity of heat ΔQ. The internal energy of the ΔN newly created solitons is equal to $T_c \Delta N/3$, while the change of the energy of the initial solitons as a result of their change of temperature is equal to $N\Delta T_c/3$. In this case the total change of energy of the soliton subsystem with allowance for the constancy of the number of plasmons $[\Delta(NT_{cc}^{1/3}) = 0]$ is equal to $-2E_N \Delta N/N$ and to ΔQ, from the law of conservation of energy. Here, $E_N = NT_c/3$ is the internal energy of the solitons for $\mu = 0$.

We shall express the change of entropy in terms of ΔN and T_c. Since the free energy of the soliton subsystem (see (4.4) and below) is equal to NT_c, the entropy $S_c = 2N/3$, and consequently, its change $\Delta S_c = 2\Delta N/3 = -\Delta Q/T_c$. Here we have taken into account the relationship between ΔN and ΔQ obtained above. Because of the fact that the total entropy change $\Delta S_p + \Delta S_c = 0$, while the change of the entropy of the plasmon subsystem is $\Delta S_p = -\Delta Q/T_p$, we finally have $T_p = -T_c$.

We emphasize that the entropy gain on account of the formation of new solitons owes its origin to the nonlinear dependence of the soliton energy H_c (J) on the number of quanta in the soliton, and for power dependences of the form $H_c \sim -J^n$ entropy gain occurs for $n > 1$. In particular, in our example (2.9), $n = 3$.

We shall discuss now the condition for applicability of the model considered. First of all we studied the equilibrium spectrum. As for weak turbulence,[11] this is justified if the rate of pumping of energy into the system is small in comparison with the effective frequency of the "collisions" responsible for the establishment of equilibrium. This frequency, evidently, must be compared with the modulation-instability growth constant estimated for a given intensity of the spectrum. Usually, this condition is easy to fulfill, although an exact answer can be given only be experiment (model or numerical).

Another restriction is associated with the assumption that the interaction is weak. The point is that, as follows from (4.20), the ratio of the soliton width ($\sim \bar{J}^{-1}$) to the spacing between the solitons ($\sim Z_c^{-1}$), equal to $K^2(x)$, is of order unity at all temperatures. Generally speaking, the lack of dependence of the duty factor on the parameters is well known[12] from numerical experiments and, in the regime of strong Langmuir turbulence, is consistent with the smallness of the interaction energy in comparison with the self-energy of the solitons. In fact, forced acoustic oscillations excited at the beat frequency $\Omega \sim a^{-2}$ have, according to (2.1), amplitude $n_{\rm int} \sim 1$, and for the interaction energy we have the estimate

$$\int |E|^2 n\, dx \sim n_{int} \bar{J} \sim a^{-1},$$

while the soliton energy is $\sim a^{-3}$. This implies that even for close packing of solitons we have the small parameter $a^2 \ll 1$.

It should be noted that not all the results stemming from the proposed model agree with intuitive ideas. For example, the decrease of the number of solitons at high temperatures is understandable from the point of view of the theory of the modulation instability of the noise spectra of plasmons; the fractionation (coalescence) of solitons has already been obtained in the numerical experiments of Ref. 13 and was used in the theory of Ref. 14; the proportionality of the soliton-number density to the quantity $n_c^{1/2}$ was also observed in the numerical experiments of Ref. 12. Not entirely understandable is the decrease of the number of plasmons in the solitons at low temperatures [see (4.18)]. Formally, this result is evidently connected with the hypothesis of the validity of the Rayleigh-Jeans distribution for free plasmons even at low temperatures. If we assume that free plasmons at low temperatures are "frozen out" (e.g., $n_k \sim \exp[-(\omega_j + \mu)/T]$), then it turns out that the number of solitons approaches a maximum, equal to $n^{1/2}$, as $T \to 0$.

In conclusion, we note that from the standpoint of the general theory of distributed Hamiltonian systems the separating out of coherent states into groups by a method based on the Langevin equation, as shown in this paper, makes it possible to find the equilibrium distributions of the fields even in the case of Hamiltonians that are not bounded from below, and this cannot be done by the known methods.[15]

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[1)]The separating out of the equilibrium solution from all the stationary solutions of the kinetic equation corresponds to the assumption that, because of the coherent structures, the time of establishment of the equilibrium distribution is small in comparison with the characteristic times of the external perturbations.

[2)]As reported to us by A. M. Vinogradov, (2.1) has no other local integrals of motion, i.e., the system (2.1) is evidently nonintegrable.

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