

Density of states of a two-dimensional electron gas half-way between Landau levels of metal-oxide-silicon structures

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An analysis of the results of measurements of nonlinear characteristics of metal-oxide-semiconductor structures made from (100) silicon and subjected to a quantizing magnetic field yielded the dependences of the density of states half-way between the Landau levels on the magnetic field and temperature. Nonlinear operation made it possible to determine the distribution of the current in a sample and to find the density of states from measurements carried out at a constant temperature. The density of states varied nonmonotonically with temperature and at low temperatures ($T \sim 0.3$ K) the density of states increased as a result of cooling.

Several experimental investigations have been made of the dependence of the density of states D under the conditions of the quantum Hall effect on the position of the Fermi level, electron mobility, magnetic field, etc.¹⁻⁷ In our opinion the most interesting of these are the densities of states deduced from a study of the components of the conductivity or resistivity tensor. The point is this: all the known (to us) attempts to estimate the degree of macroscopic homogeneity of samples under the conditions of the quantum Hall effect⁸⁻¹¹ have shown that samples with values of the Landau-level filling factor (occupancy) close to an integer are macroscopically homogeneous. It has been found that the diagonal component of the conductivity tensor may vary considerably over macroscopic distances¹⁰ and the distribution of the current in a sample can be strongly inhomogeneous and dependent on the filling factor. Therefore, all the methods in which the density of states is determined by averaging over a large area can give values of D which are not very closely related to the density of states in the current-carrying part of the sample.

The idea of determination of the density of states from the temperature dependence of the conductivity was first put forward and applied to two-dimensional electron systems in Refs. 12 and 13. It is assumed that electron states are localized in the region of the gaps of the energy spectrum, so that the diagonal component of the conductivity tensor exhibits a temperature dependence with an activation energy:

$$\sigma_{xx}(\Delta n) = \sigma_0 \exp(-W/kT) \operatorname{ch}(\Delta \epsilon_F/kT). \quad (1)$$

Here, W is the activation energy for a filling factor which is an integer ($\Delta n = 0$); $\Delta \epsilon_F$ is the deviation of the Fermi energy from a quantity which corresponds to a filling factor which is an integer; $n = N_s (hc/eH)$.

The dependence of the activation energy on Δn , i.e., on the value of $W - \Delta \epsilon_F$ in the case when $\Delta \epsilon_F/kT \approx 1$ was used in Refs. 4 and 5 to determine the density of states $D = (\partial n / \partial \epsilon_F) (eH/hc)$ in GaAs-Al_xGa_{1-x}As heterostructures. A different analysis of the results was employed in Ref. 7. The temperature dependence of

$$\operatorname{Arch} \frac{\sigma_{xx}(\Delta n)}{\sigma_{xx}(\Delta n=0)} = \frac{\Delta \epsilon_F}{kT} \quad (2)$$

was determined for different electron densities in silicon

metal-oxide-semiconductor (MOS) transistors and then the slope of the straight lines $\Delta \epsilon_F(N_s)$ was used to find the density of states $D(\epsilon_F)$.

Both measurement methods are based on several assumptions the validity of which should be verified simultaneously with the measurements of $D(\epsilon_F)$. Among the assumptions common to both methods are the following.

1. A sample is regarded as homogeneous or it is assumed that the distribution of the currents in a sample is not affected by temperature or the filling factor.

2. The activation energy W is independent of the Fermi energy.

3. The chemical potential μ is independent of temperature at a fixed electron density N_s ($\mu = \epsilon_F$). In the former method of analysis of the experimental results it is additionally assumed that both σ_0 and W are independent of temperature.

As already pointed out, the first of these assumptions was undoubtedly in conflict with the results of measurements. Recent reports¹⁴⁻¹⁶ have demonstrated that the width of the Landau levels in a two-dimensional electron gas in MOS structures made of (100) silicon depends on ϵ_F . The Landau level was found to be narrowest¹⁵ when it coincides with the Fermi energy and the widest when the Fermi energy lies in the middle of the energy gap. We can expect the energy W to vary with the width of the level.

The third of the above assumptions is satisfied well for small deviations of the filling factor from an integer because in this case we have

$$\frac{\partial \mu}{\partial T} = \frac{\pi^2 T k^2}{3} \frac{\partial D}{\partial \mu} D^{-1} = 0, \quad \mu = \epsilon_F. \quad (3)$$

Therefore, Eq. (1) can be used to determine the density of states at a point in a sample where the current is concentrated, but measurements must be carried out making sure that the spatial distribution of the currents is constant and employing the results obtained at a constant temperature, and at the same time the fact that W is independent of ϵ_F should be checked. The measurement methods used in Refs. 4, 5, and 7 do not satisfy these requirements.

We shall report a determination of the density of states of an electron gas under the conditions of the quantum Hall effect by a different method satisfying all the conditions listed above.

TABLE I.

Sample No.	$d, \text{\AA}$	$\mu_{max} (4,2 \text{ K})$	$N_S (\mu_{max}), \text{cm}^{-2}$
1	1340	3200	$6.3 \cdot 10^{11}$
2	1310	21 100	$1.0 \cdot 10^{12}$
3	1370	19 700	$1.0 \cdot 10^{12}$

MEASUREMENT METHOD

Our measurements were carried out on silicon MOS transistors using the Corbino geometry. The gate in such transistors was a ring with an outer diameter $2r_2 = 675 \mu$ and an inner diameter $2r_1 = 225 \mu$. The measurements were carried out on three samples with different the mobilities (Table I). The measurements were made in a temperature range from 4.2 to 0.3 K using magnetic fields from 4 to 10.6 T.

In these measurements the drain-source current was constant. Measurements were made of the voltage drop ΔU between the drain and the source at various gate voltages V_g . As found earlier,^{10,17} under the nonlinear conditions corresponding to $\Delta U \gtrsim \nu T$ (where $\nu = keD/C_0$; C_0 is the capacitance of an MOS structure normalized to a unit area) the dependences $\Delta U(V_g)$ are of the form shown in Fig. 1. A Hall-current filament of width $\Delta = \delta r$ is established in the sample and

$$\delta = 2\pi\sigma_0(\nu T/I) \exp(-W/kT).$$

The position of this filament is governed by the gate voltage. When this voltage is $V_g = V_g^0 (\Delta n = 0) = V_g^0$, the filament is located near one of the edges of the electron channel (the

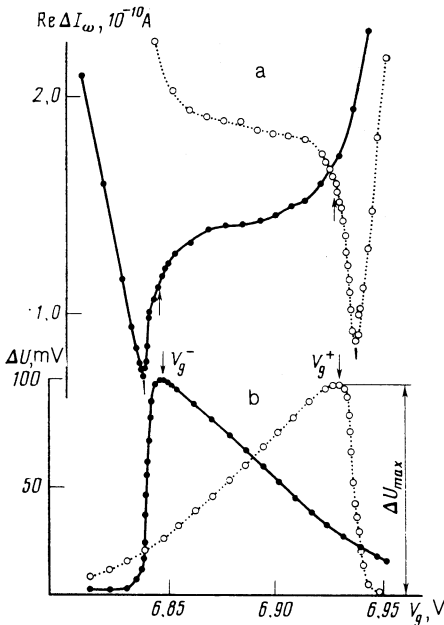


FIG. 1. a) Real part of the current ΔI_ω between the gate and a two-dimensional electron layer: \circ) alternating voltage applied between the gate and external contact with the channel; \bullet) voltage $\Delta V_g(\omega)$ applied between the gate and an internal contact; $n = 4$, $T = 1.6 \text{ K}$, $H = 10.6 \text{ T}$, $I = 5 \times 10^{-10} \text{ A}$, $\omega/2\pi = 10 \text{ kHz}$, $\Delta V_g(\omega) = 5 \times 10^{-4} \text{ V}$, sample No. 3. b) Dependence of the drain-source voltage ΔU on the gate voltage for different ways of application of V_g : \circ) voltage V_g applied relative to the internal contact; \bullet) voltage applied relative to the external contact.

one for which V_g is specified) and as V_g changes, the filament moves to the other edge.^{17,18} The maximum of the drain-source voltage is reached when the filament is located at the radial position $r_0 = (r_1 r_2)^{1/2}$. This may be checked experimentally by a technique proposed in Ref. 10. If in addition to a constant gate voltage V_g we apply an alternating voltage $\Delta V_g(\omega)$ and measure the active or reactive part of the alternating current ΔI_ω , we can obtain information on the position of the Hall current filament. Examples of results of such an experimental determination are given in Fig. 1a. The left-hand minimum of $\text{Re} \Delta I_\omega$ corresponds to the position of the filament at the inner edge of the electron channel. A change in the measuring system can give the corresponding curve for which the minimum representing the position of the Hall current filament is at the outer edge of the sample. The distance from the edge of the sample (points r_1 or r_2) to the current filament Δr corresponding to $V_g = V_g^+$ or V_g^- can be estimated roughly from the ratio of the signals at the points V_g^+ , V_g^- , and V_g^0 , which is $1 - \exp(-\Delta r/\lambda)$. Here, $\lambda = (2\sigma_{xx} \omega^{-1} C_0^{-1})^{1/2}$ is the distance in which an alternating current flows from the gate to a two-dimensional electron layer.¹⁰ Such an estimate shows that the filament corresponding to $\Delta U = \Delta U_{max}$ is separated from the inner and outer radii by a distance exceeding 50μ .

In the subsequent experiments we selected the current I in such a way that the width of the Hall current filament was within the range $5 \mu \lesssim \Delta \lesssim 50 \mu$. We could regard the distribution of the currents in a sample to be constant within this width.

The maximum drain-source voltage ΔU_{max} (Fig. 1) was determined by the current I (Ref. 17):

$$\Delta U_{max} = 2\nu T \text{ Arsh} \left(\frac{\delta^{-1}}{2} \ln \frac{r_2}{r_1} \right). \quad (4)$$

One could easily see that having determined the dependence $\Delta U_{max}(I)$ at a fixed temperature, we could find the density of states since

$$I = \frac{4\pi}{\ln(r_2/r_1)} \sigma_0 \exp(-W/kT) \nu T \text{ sh} \frac{\Delta U_{max}}{2\nu T}, \quad (5)$$

and for a correct value of the density of the states the quantity $\sinh(\Delta U_{max}/2\nu T)$ will be proportional to current I . Therefore, an analysis of the experimental result was reduced to a procedure in which a selection of the parameter ν made it possible to fit experimental points on a linear dependence of $\sinh(\Delta U_{max}/2\nu T)$ on the value of the current I . Since in the experiments the absolute error in the determina-

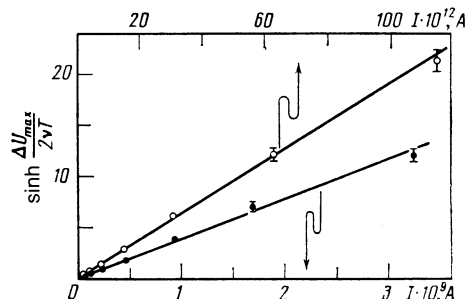


FIG. 2. Dependences of $\sinh(\Delta U_{max}/2\nu T)$ on the current through a sample: \circ) $T = 0.33 \text{ K}$; \bullet) $T = 0.88 \text{ K}$; $n = 8$, $H = 10.5 \text{ T}$, sample No. 1.

TABLE II.

Sample No. 1, $H=10.5$ T, $n=4$					Sample No. 2, $H=10.5$ T, $n=4$				
T , K	ν , mV/K	Dis, %	σ_T^{-1} , Ω	$\Delta\varepsilon_{max}$, K	T , K	ν , mV/K	Dis, %	σ_T^{-1} , Ω	$\Delta\varepsilon_{max}$, K
0.33	48	0.74	$3.0 \cdot 10^{12}$	2.0	1.3	4.9	0.53	$2.7 \cdot 10^{11}$	6.1
0.46	43	1.6	$5.0 \cdot 10^{11}$	2.4	1.9	5.0	3.0	$1.3 \cdot 10^{10}$	6.8
0.89	28	3.5	$1.7 \cdot 10^{10}$	3.9	2.6	4.6	3.8	$2.8 \cdot 10^8$	13.3
1.9	15	2.6	$7.1 \cdot 10^7$	7.0	3.0	4.6	3.9	$4.6 \cdot 10^7$	16.5
3.0	13	2.7	$3.6 \cdot 10^6$	8.2	3.5	5.3	2.0	$1.3 \cdot 10^7$	13.8
4.2	12	0.27	$6.5 \cdot 10^5$	9.5	4.2	6.6	3.2	$2.7 \cdot 10^6$	15.5

tion of ΔU_{max} remained constant when the current I was varied and since in the case of values of the hyperbolic sine exceeding unity this ensured constancy of the relative error $\sinh(\Delta U_{max}/2\nu T)$, the criterion which should give the best value of ν was the minimum of the relative dispersion:

$$\text{Dis} = N - \left(\sum_1^N I_i / \text{sh} \frac{\Delta U_{max}^i}{2\nu T} \right)^2 \left\{ \sum_1^N \left(I_i / \text{sh} \frac{\Delta U_{max}^i}{2\nu T} \right)^2 \right\}^{-1},$$

where N is the number of the experimental points, whereas I_i and ΔU_{max}^i are the values of the current and the maximum drain-source potential difference for the i th curve. Such an analysis of the experimental results was justified if the density of states and the activation energy W remained constant when $\Delta\varepsilon_F \ll \Delta U_{max}/2\nu$. Therefore, the range of currents I was limited at each temperature to those values for which the relative dispersion was comparable with the dispersion set by the experimental error in the determination of ΔU_{max} .

It should be stressed particularly that the measured density of states was $\partial N_S / (\partial\varepsilon_F)$, whereas usually the density of states was understood to be the value $\partial N_S / \partial\varepsilon|_{\varepsilon=\varepsilon_F}$. The difference between these definitions is important because a gap is excited in the energy spectrum of electrons due to the electron-electron interaction.

EXPERIMENTAL RESULTS

The measurements were made on all the samples for the Landau level filling factor was $n = 4$. The temperature dependence of the density of states for a sample with a low mobility was determined additionally for $n = 8$. Figure 2 demonstrates the extent of which the experimental points could be fitted to a linear relationship between I and $\sinh(\Delta U_{max}/2\nu T)$. By way of example, we listed in Table II the

results of an analysis of two series of experimental records. It is clear from this table (and it is supported by all ten series of experimental records) that the range of energies close to $\Delta\varepsilon_F = 0$, where the density of states remained constant to within the experimental error, became wider on increase in temperature. Therefore, our experiments confirmed the existence of a background density of states^{4,5,7} in the vicinity of $\Delta\varepsilon_F = 0$, but in contrast to Refs. 4, 5, and 7 the range of energies where we found $D = \text{const}$ depended on temperature.

The main result of the present investigation was the discovery of a temperature dependence of the density of states half-way between the Landau levels (Table II and Figs. 3 and 4). (In earlier investigations^{4,5,7} such a dependence could not be detected because the assumption of the constancy of the density of states at all temperatures was an essential feature of the analysis of the experimental data.) Figure 5 shows the temperature dependence of the conductivity in the case when $\Delta\varepsilon_F = 0$. This quantity, defined by $\sigma_T = \sigma_0 \exp(-W/kT)$, can be obtained by analysis of the dependence $\Delta U_{max}(I)$ using the coefficient of proportionality between $\sinh(\Delta U_{max}/2\nu T)$ and the current I (see, for example, Table II). As pointed out already, in contrast to the usual measurements of the conductivity in the linear regime, we ensured that the distribution of the currents in the sample remained fixed. Nevertheless, the dependence of $\ln \sigma_T$ on $1/T$ was the same as those known from earlier measurements. Usually the high-temperature part of the temperature dependence $\ln \sigma_T$ is attributed to thermal activation, and the low-temperature part to electron jumps (of variable length) between localized states (variable-range hopping). We checked that at temperatures from 0.3 to 4.2 K the proportionality of $\sinh(\Delta U_{max}/2\nu T)$ to the current I was maintained within the limits of the experimental error (see, for example, Fig. 2). This was in conflict with the assumption

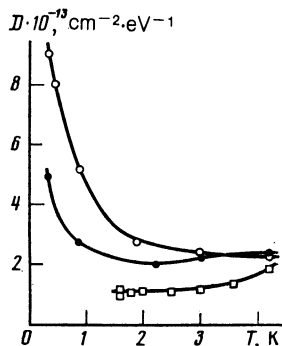


FIG. 3. Temperature dependences of the density of states: \circ) $n = 4$, \bullet) $n = 8$ for $H = 10.5$ T, sample No. 1; \square) $n = 4$, $H = 8.3$ T, sample No. 3.

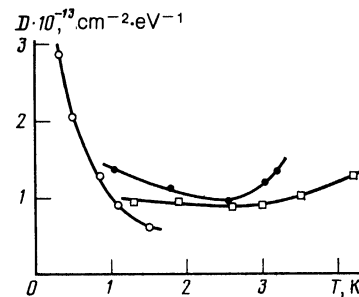


FIG. 4. Temperature dependences of the density of states in sample No. 2: \circ) $H = 4.6$ T; \bullet) $H = 6.7$ T; \square) $H = 10.5$ T; $n = 4$.

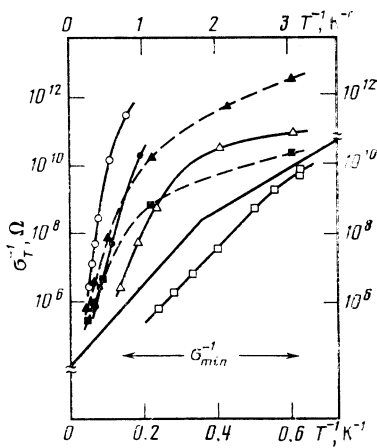


FIG. 5. Dependences of σ_T^{-1} on the reciprocal of temperature: \blacktriangle) $n = 4$, \blacksquare) $n = 8$, $H = 10.5$ T, sample No. 1; $n = 4$, \circ) $H = 10.5$ T, \bullet) $H = 6.7$ T, \triangle) $H = 4.6$ T, sample No. 2 (left and upper scales); \square) $H = 8.3$ T, $n = 4$, sample No. 3 (right and lower scales).

that at low temperatures there was a transition to a second conduction mechanism, so that the experimental results were analyzed in the same way throughout the full temperature range when the value of $D(T)$ was determined. It was found that $D(T)$ was nonmonotonic. In the case of high-mobility samples Nos. 2 and 3, we were able to observe a fall of the density of states as a result of cooling when measurements were carried out in a strong magnetic field, a minimum of the density of states on the temperature scale was observed in a field ~ 6.7 T, and an increase in the density of states as a result of cooling occurred when measurements were made in a relatively weak magnetic field (Fig. 4). [At a fixed value of the magnetic field we were unable to determine the density of states throughout the accessible temperature range. This was due to the fact that the measurement method required setting of a constant current through a sample,

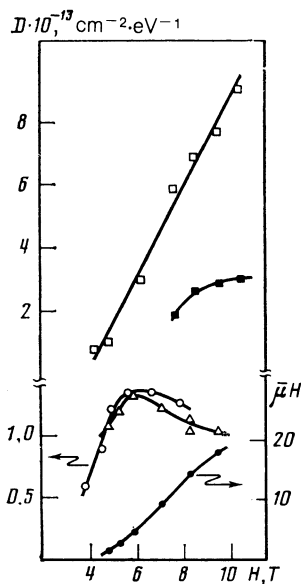


FIG. 6. Dependences of the density of states on the magnetic field: \circ) $T = 1.07$ K, $n = 4$, sample No. 2; \triangle) $T = 1.6$ K, $n = 4$, sample No. 3; $n = 4$, \square) $T = 0.33$ K, \blacksquare) $T = 2.0$ K, sample No. 1. The black dots (\bullet) represent the values of $\bar{\mu}H$ for sample No. 3, where $\bar{\mu}$ is the mobility averaged over the sample in $H = 0$ at $T = 4.2$ K.

which was possible only if $\sigma_T \gtrsim 10^{-12} \Omega^{-1}$. On the other hand, temperatures too high and magnetic fields too weak were excluded by the requirement that the experimental $\Delta U(V_g)$ curve should correspond to the calculations.] In the case of a sample with a low mobility we observed a monotonic rise of $D(T)$ as a result of cooling when the filling factor was $n = 4$ and a nonmonotonic dependence in the case when $n = 8$ (Fig. 3).

The dependence of the density of states on the magnetic field is plotted in Fig. 6. This dependence is also nonmonotonic and we can see from Fig. 6 that the density of states is not a monotonic function of $\bar{\mu}H$ either ($\bar{\mu}$ is the mobility averaged over the sample and deduced from the conductivity in zero magnetic field). It is worth noting the points corresponding to the minimum magnetic fields when $\bar{\mu}H \sim 1$ and it is difficult to expect large changes in the conductivity because of a change in $\Delta\epsilon_F$. However, under real experimental conditions the value of σ_{xx} changed by a factor of 13 when $\Delta\epsilon_F$ was increased from 0 to 5.2 K, which corresponded to $2\Delta\epsilon_F/\hbar\omega_c = 0.31$.

DISCUSSION

The density of states half-way between the Landau levels was determined earlier for GaAs-Al_xGa_{1-x}As heterostructures^{4,5} and silicon MOS transistors.⁷ A comparison of our data with the results of earlier investigations was difficult because of the temperature dependence $D(T)$, although the order of magnitude of the density of states was the same for comparable samples.

It should be pointed out that several circumstances suggest internal inconsistencies in these investigations. The temperature dependence of the resistivity was obtained in Ref. 7 for a fixed value of $\Delta\epsilon_F$ given by an expression similar to Eq. (1), whereas the dependence of ρ_{xx} on $\Delta\epsilon_F$ at a fixed temperature did not obey such a formula. (This can be checked readily by examining Fig. 1 in Ref. 7.) The activation energies obtained from an analysis of the experimental results were unexpectedly large. For $n = 4$ and silicon MOS structures it was found that the energy W could reach 96% of $\hbar\omega_c/2$ (Ref. 19), in spite of the spin and valley splitting. The value of σ_0 found by extrapolation of the experimental dependences $\ln \sigma_{xx}$ to $T^{-1} \rightarrow 0$ differed by several orders of magnitude for different samples, was different for different magnetic fields, and very far from the expected value of the minimum metallic conductivity.¹⁹

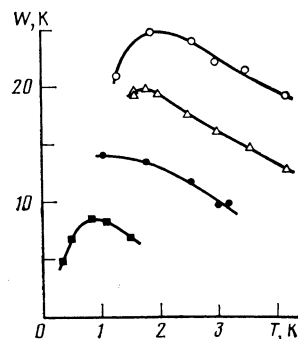


FIG. 7. Temperature dependences of the activation energy: \circ) $H = 10.5$ T, \bullet) $H = 6.7$ T, \blacksquare) $H = 4.6$ T, $n = 4$, sample No. 2; \triangle) $H = 8.3$ T, $n = 4$, sample No. 3.

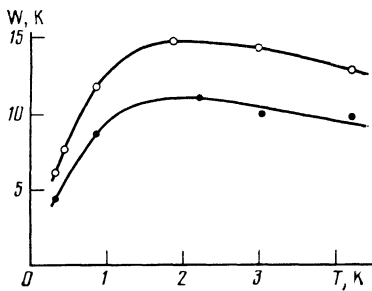


FIG. 8. Temperature dependences of the activation energy for sample No. 1 in a field $H = 10.5$ T: \circ) $n = 4$; \bullet) $n = 8$.

In Refs. 14–16 it was shown experimentally that the Landau level width depends on the position of the Fermi level relative to it. The experimental results reported above, like the results of Ref. 20, demonstrated that the width of the Landau level could depend on temperature for a constant ε_F . It seems that a reduction in the density of states as a result of cooling at relatively high temperatures is due to weakening of some scattering mechanism because of cooling. (The scattering by phonons would be a suitable candidate for such a mechanism if the frequency of the electron-phonon collisions had not been too low.²¹) In our opinion the only way of explaining the rise of D on further cooling is to allow for the temperature dependence of the screening of the fluctuation potential.

Within the framework of this approach the activation energy W is a quantity which depends on temperature. It can be found from the slope of the straight line joining the experimental point on the $\ln \sigma_T^{-1}(1/T)$ plot with the point $\ln \sigma_{\min}^{-1}$ on the ordinate. The activation energy determined in this way is given in Figs. 7 and 8. It is clear from these figures that $W(T)$ has a maximum, which is in full agreement with the expected behavior of the density of states $D(T)$. The hypothesis of a transition to variable-range hopping in silicon MOS structures is no longer needed. Bending of the dependence $\ln \sigma_T^{-1}(1/T)$ at high values of $1/T$ can be explained by the temperature dependence of the activation energy and the general nature of changes in all the experimentally determined parameters becomes self-consistent.

An analysis of the experimental results and all the subsequent conclusions reached in the present paper are based on Eq. (1). (A similar relationship is also used in an analysis of the experimental results in the earlier investigations.^{4,5,7}) Essentially, the conclusions reached in the present study are valid subject to an even weaker hypothesis, namely the assumption that the conductivity σ_{xx} is a function of the ratio of $\Delta\varepsilon_F/kT$. Then, curves of the type shown in Fig. 2 can be regarded as an experimental confirmation of Eq. (1). However, the dependence of the conductivity on the ratio $\Delta\varepsilon_F/kT$ has not yet been confirmed experimentally and model

representations of two mobility edges at neighboring Landau levels have not yet been rigorously justified. Moreover, there is an alternative model^{16,22,23} in which it is assumed that extended and localized states coexist at the same energy, but at different points of a sample.

Future experimental investigations of the density of states under magnetic quantization conditions should include development of an independent method for the determination of $D(H, T)$ valid under conditions of existence of a Hall current filament when distributions of the fields and currents are known.

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