Bremsstrahlung from relativistic electrons and positrons traveling nearly parallel to crystal axes and planes

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(Submitted 14 March 1986; resubmitted 24 November 1986)
Zh. Eksp. Teor. Fiz. **92**, 1946–1958 (June 1987)

A consistent quantum-mechanical calculation of the bremsstrahlung intensity due to relativistic electrons and positrons in an oriented crystal is presented. Analytic results are obtained for relatively hard frequencies, exceeding the characteristic emission frequencies due to the motion of particles in the continuous potential of the crystal planes and axes. It is shown that the main contribution to the probability of the process is then due to intermediate virtual states of the particle that are close to the real states of motion well above the barrier. The final formulas can be used to explain existing experimental data on the orientation dependence of the high-frequency part of the emission spectrum of electrons in a crystal.

1. INTRODUCTION

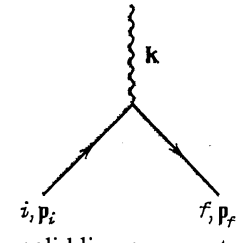
Orientation-dependent phenomena accompanying the passage of fast charged particles travelling through crystals at a small angle to the principal crystallographic directions have been under intensive investigation during the last decade.[1,2] Because the usual Born series diverges as the angle between the particle momentum and one of the atomic planes or chains is reduced, the methodological basis for studies of orientation phenomena is the Lindhard[3] "continuous potential approximation." The effect of the atomic planes or axes along which the particle moves is replaced in this approach with an effective potential distribution and then the particle wave function is factored out into a wave function describing the longitudinal motion (relativistic plane wae) and one describing the transverse motion, which is a solution of the Schrödinger equation with a relativistic mass.[4] The specific features of this range of phenomena is thus seen to require a transformation to the Furry representation, which gives rise to considerable computational difficulties when higher-order diagrams are used evan at the tree level. These diagrams have to be resorted to because of the publication of a series of exerimental data[5–8] indicating the existence of orientational effects in the high-frequency part of the emission spectrum, which is largely due to the bremsstrahlung mechanism.

The dipole impulse approximation was used in Ref. 9 to derive a formula for the spectral and wavelength density of bremsstrahlung radiation emitted by a fast electro in an axial channel. Strictly speaking, this formula is invalid in the energy range ($E \gtrsim 1$ GeV) in which the experiments were carried out. On the other hand, the bremsstrahlung from a relativistic charged particle in an oriented crystal can be calculated rigorously in a fairly general case. This is, in fact, the purpose of the present paper. We shall show later that, under certain definite conditions, the principal contribution to the probability of the process is provided by intermediate virtual states that are close to the real states of motion well above the barrier, so that the calculations can be completed in an explicit form. The final solutions enable us to explain existing experimental data and to propose further experiments on the stability of particle motion in a crystal channel.

2. FORMULATION OF THE PROBLEM: MATRIX ELEMENT OF THE PROCESS

Let us suppose that the difference between the true potential and the "continuous" potential is a perturbation V which is responsible for the bremsstrahlung.

First-order perturbation theory in the electromagnetic interaction constant shows that the emission of a photon by a particle traveling in an oriented crystal is described by the diagram



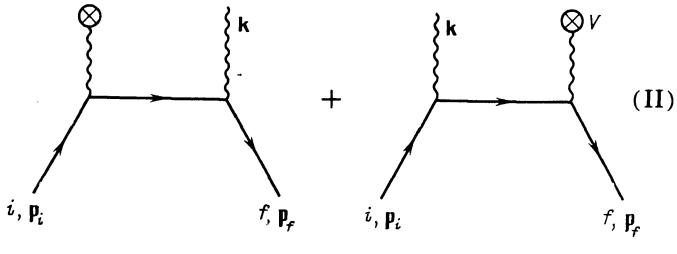
(I)

The solid line represents a particle in the continuous potential of the axis (plane) with initial longitudinal momentum $\mathbf{p}_i$ (or final momentum $\mathbf{p}_j$) in a state of transverse motion i (correspondingly, f); the wavy line is a photon with wave vector $\mathbf{k}$. This diagram determines the rate of spontaneous emission as a result of $i \to f$ transitions between states of transverse motion.[1,2] Its contribution predominates at relatively low frequencies of the order of[1)]

$$\omega_{ch} \sim 4\pi\gamma^2 T^{-1},$$

where γ is the Lorentz factor and T the time of flight of the particle near the axis or plane (in the dipole case) or the time in which the transverse velocity of the particle changes by an amount of the order of $1/\gamma$ (in the nondipole case).

On the other hand, at frequencies much greater than ω_{ch}, the dominant diagrams can be of first order in the perturbation V, i.e.,



(II)

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Since these diagrams are nonresonant (the crossed circle represents interaction with the external field). Throughout this frequency range, the emission spectrum of the channeled particle is thus described by the sum of diagrams (I) and (II).

We shall examine the high-frequency part of the spectrum, $\omega \gg \omega_{ch}$, where the contribution of diagram (I) to the total emission intensity can be neglected. The very possibility of this separation of the spectrum into a low-frequency "peak" and the high-frequency "background" indicates that we are dealing with a moderately nondipole case, i.e., $E \lesssim 10E^*$, where $E^* = m^2/\bar{\varepsilon}$ and $\bar{\varepsilon}$ is the average kinetic energy of transverse motion of the particle. Estimates show that, for energies exceeding $10E^*$, frequencies comparable with the total energy of the particle appear in the spontaneous emission spectrum described by (I). It is clear that, in this case, there is no physical justification for assuming that the contribution of diagram (I) is small in comparison with diagram (II).

We shall therefore assume that

$$E \lesssim 10E^*, \tag{1}$$

$$\omega \gg \omega_{ch}. \tag{2}$$

Moreover, we shall assume that $\omega \ll E$. It will be clear from the ensuing discussion that this limitation is not fundamental, but it does result in considerable simplification of very unwieldy formulas, and enables us to look upon the electron as a spinless particle.

To be specific, let us consider an axial channel. In view of the foregoing, the basis wave functions can be written in the form

$$\psi_{i(f)}(\mathbf{r}) = L^{-1/2} \exp(ip_{i(f)}z)\varphi_{i(f)}(\boldsymbol{\rho}),$$

where z is the longitudinal (i.e., measured along the axis) and $\boldsymbol{\rho}$ the transverse coordinate of the particle; L is the normalizing length. The form of the transition current vector $\mathbf{j}_{fi}$ corresponding to (II) is typical for bremsstrahlung problems:

$$\begin{aligned}\mathbf{e}\mathbf{j}_{fi} = -\frac{i}{(E_iE_f)^{1/2}} \int d^3\mathbf{r}_1 \int d^3\mathbf{r}_2\, \psi_i(\mathbf{r}_1) \\ \times \Big[V(\mathbf{r}_1) G(E_i; \mathbf{r}_1, \mathbf{r}_2) \Big(\mathbf{e} \frac{\overleftrightarrow{\partial}}{\partial \mathbf{r}_2} \Big) \exp(-i\mathbf{k}\cdot\mathbf{r}_2) \\ + \exp(-i\mathbf{k}\cdot\mathbf{r}_1) \Big(\mathbf{e} \frac{\overleftrightarrow{\partial}}{\partial \mathbf{r}_1} \Big) G(E_f; \mathbf{r}_1, \mathbf{r}_2) V(\mathbf{r}_2) \Big] \psi_f^*(\mathbf{r}_2),\end{aligned} \tag{3}$$

where $\mathbf{e}$ is the polarization vector of the escaping photon, $E_{i(f)}$ is the particle energy at the beginning (end) of the process, and $G(\mathbf{E}; \mathbf{r}_1, \mathbf{r}_2)$ is the propagator of a scalar particle of energy E in the oriented crystal.

Even at this stage of the calculation, we encounter a significant difficulty, namely, in contrast to the free particle, this propagator does not have a simple analytic expression. The simplest way to proceed is then as follows. We shall use the following representation of the propagator of a particle in an external field:[10]

$$G(E; \mathbf{r}_1, \mathbf{r}_2) = \sum_{\nu} \frac{\psi_\nu(\mathbf{r}_2)\psi_\nu^*(\mathbf{r}_1)}{E - E_\nu},$$

where the sum is evaluated over intermediate states and extends only over positive-frequency solutions of the Klein-Gordon equation because resonance denominators are absent from the sum over the negative-frequency solutions. For a particle in an oriented crystal, the total energy can be approximately represented by the sum of two energies, namely, the longitudinal energy $E_\nu^{\parallel} = (p_\nu^2 + m^2)^{1/2}$ and the transverse energy ε_ν, i.e., $E_\nu = E_\nu^{\parallel} + \varepsilon_\nu$, where p_ν is the longitudinal momentum, m is the particle mass, and, usually, $\varepsilon_\nu \ll m \ll p_\nu$.

Next, we recall that the effective angles of emission by an ultrarelativistic particle are of the order of $1/\gamma$ and we resolve the total photon momentum $\mathbf{k}$ into the longitudinal $\mathbf{k}_{\parallel}$ and the transverse $\mathbf{k}_\perp$ components (relative to the axis), where $\mathbf{k}_{\parallel} \simeq \omega(1 - \theta^2/2)$, $|\mathbf{k}_\perp| \simeq \omega\theta$, and ω is the frequency and $\theta \ll 1$ the angle of emission of the photon. If we then use the conservation laws, we find that the energy denominators for the first and second terms in (3) assume the form

$$\begin{aligned}E_i - E_\nu &\approx {}^1/_2\omega(\theta^2 + m^2/p_f(p_f + k_{\parallel})) - (\varepsilon_\nu - \varepsilon_f), \\ E_f - E_\nu &\approx (\varepsilon_i - \varepsilon_\nu) - {}^1/_2\omega(\theta^2 + m^2/p_i(p_i - k_{\parallel})).\end{aligned}$$

Any possible further simplification is intimately related to the condition given by (2).

Consider the first term in (3) (the procedure for the second term is virtually the same). Specifically, we have to evaluate the matrix element

$$M = \int d^2\rho\, \varphi_{\nu, p_f + k_{\parallel}}(\boldsymbol{\rho}) \frac{\hat{D}_1}{A - (\varepsilon_\nu - \varepsilon_f)} \exp(-i\mathbf{k}_\perp\cdot\boldsymbol{\rho})\, \dot{\varphi}_{f, p_f}(\boldsymbol{\rho}),$$

where, for brevity, we have substituted $A = \frac{1}{2}\omega[\theta^2 + m^2/p_f(p_f + k_{\parallel})]$ and

$$\hat{D}_1 = \mathbf{e}\cdot\Big\{ \frac{\partial}{\partial\boldsymbol{\rho}} - 2i\mathbf{k}_\perp \frac{p_f + k_{\parallel}}{k_{\parallel}} \Big\}.$$

We also define the operator

$$\begin{aligned}\hat{H}_\nu &= -\frac{1}{2(p_f + k_{\parallel})} \frac{\partial^2}{\partial\boldsymbol{\rho}^2} + U(\boldsymbol{\rho}), \\ \hat{H}_f &= -\frac{1}{2p_f} \Big(\frac{\partial}{\partial\boldsymbol{\rho}} + i\mathbf{k}_\perp \Big)^2 + U(\boldsymbol{\rho}),\end{aligned}$$

where $U(\boldsymbol{\rho})$ is the continuous potential of the axial channel. By definition, the transverse wave function $\varphi_{\nu, p_f + k_{\parallel}}$ and φ_{f, p_f} satisfy the equations

$$\begin{aligned}\varphi_{\nu, p_f + k_{\parallel}} \hat{H}_\nu &= \varphi_{\nu, p_f + k_{\parallel}} \varepsilon_\nu, \\ \hat{H}_f \exp(-i\mathbf{k}_\perp\cdot\boldsymbol{\rho})\, \dot{\varphi}_{f, p_f} &= \varepsilon_f \exp(-i\mathbf{k}_\perp\cdot\boldsymbol{\rho})\, \dot{\varphi}_{f, p_f}.\end{aligned} \tag{4}$$

We shall show that, when $\omega \gg \omega_{ch}$, the required matrix element can be approximately represented by

$$M \approx M_0 = \int d^2\rho\, \varphi_{\nu, p_f + k_{\parallel}} \frac{\hat{D}_1}{A + \hat{D}} \exp(-i\mathbf{k}_\perp\cdot\boldsymbol{\rho})\, \dot{\varphi}_{f, p_f},$$

where $\hat{D} = \hat{H}_f - \hat{H}_\nu$.

To prove this we use the expansion of the matrix element M into a series in "powers of smoothness" of the continuous potential $U(\boldsymbol{\rho})$. This expansion can be formally carried out by replacing the argument in the continuous potential with $\alpha\boldsymbol{\rho}$ and then expanding α into a power series for $\alpha \to 0$. The depth of the potential well remains constant in

this procedure, but the variation of α alters the "steepness of the walls," i.e., the gradients of the continuous potential. Accordingly, the derivatives $\partial U/\mu\rho$ are of first order in the parameter α, whereas $\partial^2 U/\mu\rho^2$ and $(\partial U/\partial\rho)^2$ are of second order, and so on.

It is readily shown that, to within $O(\alpha^2)$, the matrix element is given by

$$M=M_1=\int d^2\rho\,\varphi_{\nu,\mathbf{p}_f+\mathbf{k}_\parallel}\frac{\hat{D}_1}{A+\hat{D}-[U,\hat{D}_1]\hat{D}_1^{-1}+[U,\hat{D}](A+\hat{D})^{-1}}\times\exp(-i\mathbf{k}_\perp\cdot\boldsymbol{\rho})\,\varphi^*_{f,\mathbf{p}_f},\tag{5}$$

where $[U,\hat{D}]$ is the commutator of the continuous potential and the corresponding operator, and the order of operations in (5) is not significant. Actually, since the operators $\hat{D}$ and $\hat{D}_1$ are pure differentiations, the result of commuting them with U contains the derivative $\partial U/\partial\rho$, i.e., it is of first order. It follows that, from now on, $[U,\hat{D}_1]$ and $[U,\hat{D}]$ can be treated as c-numbers: their commutator with any other operator is second order. Substituting

$$[U,\hat{D}_1]\hat{D}_1^{-1}-[U,\hat{D}](A+\hat{D})^{-1}=\varkappa$$

and using (4), we can write

$$(M-M_1)=(A-\varepsilon_\nu+\varepsilon_f)^{-1}\int d^2\rho\,\varphi_{\nu,\mathbf{p}_f+\mathbf{k}_\parallel}\times\left(\hat{H}_\nu\frac{\hat{D}_1}{A+\hat{D}-\hat{\varkappa}}+\frac{\hat{D}-\varepsilon_f-\hat{\varkappa}}{A+\hat{D}-\hat{\varkappa}}\hat{D}_1\right)\exp(-i\mathbf{k}_\perp\boldsymbol{\rho})\,\varphi^*_{f,\mathbf{p}_f}.$$

Taking the commutator of $\hat{H}_\nu$ to the right and using (4), we readily verify that

$$M-M_1=O(\alpha^2).$$

So far, our discussion has been purely formal. We must now establish the conditions under which the true values of the continuous potential gradients can be regarded as "small." Taking the Fourier transform of the wave functions in (5), and using the notation of (7), we obtain the following expression for the integrand in the integral with respect to ξ:

$$\int d^2\xi\,(\mathbf{e}_\perp\boldsymbol{\xi})\left[\frac{\omega}{2p_f(p_f+k_\parallel)}(m^2+\xi^2)+i\left(\mathbf{e}_\perp\frac{\partial U}{\partial\boldsymbol{\rho}}\right)\Big/(\mathbf{e}_\perp\boldsymbol{\xi})+2i\left(\boldsymbol{\xi}\cdot\frac{\partial U}{\partial\mathbf{p}}\right)\Big/(m^2+\xi^2)\right]^{-1}\ldots.$$

The most significant region in this integral is $|\xi|\sim m$ for which the main term in brackets is of the order of $\omega m^2/2p_fp_i\sim l_{\text{coh}}^{-1}$ and the correction terms are of the order of $m^{-1}|\partial U/\partial\boldsymbol{\rho}|$. Hence, it is clear that, when

$$m^{-1}|\partial U/\partial\boldsymbol{\rho}|\ll l_{coh}^{-1}\tag{6}$$

the correction terms provide a small contribution to the integral, thus justifying the series expansion in the parameter α and the use of the approximate expression for M_0 instead of M (which corresponds to $\alpha=0$). The significance of (6) is that the increase in the transverse momentum of the particle over the coherent emission length l_{coh} is much smaller than m, which is always the case when the condition (2) is satisfied. At the same time, it was shown earlier that (6) meant that we could replace the difference between the total transverse energies in the resonance denominator of the propagator by the difference between the transverse kinetic energies, i.e., the main contribution to the process was provided by virtual states that were close to the real states of motion well above the barrier. After summation over intermediate states and a number of simple transformations, we obtain

$$\mathbf{e}j_{fi}=\frac{m}{2\pi^2\omega L}(E_iE_f)^{1/2}\int d^2\xi\frac{(\mathbf{e}_\perp\boldsymbol{\xi})}{m^2+\xi^2}\times\int d^2\rho_1\int d^2\rho_2\,\varphi_i(\boldsymbol{\rho}_1)\varphi_f^*(\boldsymbol{\rho}_2)\times\exp\left\{i\boldsymbol{\xi}(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)+i\frac{p_i}{k_\parallel}(\boldsymbol{\rho}_1\mathbf{k}_\perp)-i\frac{p_f}{k_\parallel}(\boldsymbol{\rho}_2\mathbf{k}_\perp)\right\}\times[V_q(\boldsymbol{\rho}_1)-V_q(\boldsymbol{\rho}_2)]_{q=\mathbf{p}_i-\mathbf{p}_f-\mathbf{k}_\parallel},\tag{7}$$

where $\mathbf{e}_\perp$ is the component of the polarization vector in the transverse plane and the subscript q represents (here and in what follows) the corresponding Fourier component:

$$F_q(\boldsymbol{\rho})=\int dz\exp(-iqz)F(\boldsymbol{\rho},z).\tag{8}$$

We note that, in the small-angle approximation, conservation laws yeild the following expression for the quantity q in (7):

$$q=p_i-p_f-k_\parallel\approx{}^1/_2\omega(\theta^2+m^2/E_iE_f)-(\varepsilon_i-\varepsilon_f).\tag{9}$$

3. COHERENT AND INCOHERENT BREMSSTRAHLUNG EMITTED BY A PARTICLE TRAVELING ALONG A CRYSTAL AXIS OR PLANE

Neglecting correlations between the collisions of a particle with different atomic chains, the remainder of our discussion of the axial case can be based on the single-chain approximation.

The wave functions for the transverse motion must be normalized to one particle per atomic chain, and the perturbation can be taken in the form

$$V(\boldsymbol{\rho},z)=\sum_a\Phi(\boldsymbol{\rho}-\boldsymbol{\rho}_a,z-z_a)-U(\boldsymbol{\rho}),\tag{10}$$

where $\Phi(\boldsymbol{\rho},z)$ is the potential of a single atom and $U(\boldsymbol{\rho})$ is the continuous potential of the chain. The sum is evaluated over all the atoms in the given chain, where ρ_a and z_a are the corresponding coordinates of the ath atom.

Since the atoms in the chain execute oscillations, the probability of the process must be averaged over the thermal displacements of the different atoms. This procedure is equivalent to taking into account the creation (annihilation) of phonons.[11] Assuming that the oscillations of the individual atoms occur independently, and that the probability density of thermal displacemetns in each of the coordinates is described by the normal distribution with variance u, we readily obtain the following general formula for average quantities:

$$\lim_{N\to\infty}\left\langle\left|\sum_{a=1}^{N}\int dz\exp(-iqz)F(\boldsymbol{\rho}-\boldsymbol{\rho}_a,z-z_a)\right|^2\right\rangle=N[\langle|F_q(\boldsymbol{\rho}-\boldsymbol{\rho}_a)|^2\rangle_t-\exp(-q^2u^2)\,|\langle F_q(\boldsymbol{\rho}-\boldsymbol{\rho}_a)\rangle_t|^2]+N\frac{2\pi}{d}\exp(-q^2u^2)\,|\langle F_q(\boldsymbol{\rho}-\boldsymbol{\rho}_a)\rangle_t|^2$$

$$\times \sum_{j=-\infty}^{+\infty} \delta\left(q-\frac{2\pi}{d}j\right), \qquad (11)$$

where d is the separation between the atoms in the chain. The symbol $\langle \ldots \rangle_t$ represents averaging over transverse thermal displacements:

$$\langle F(\rho-\rho_a)\rangle_t = (2\pi u^2)^{-1}\int d^2\rho_a \exp(-\rho_a^2/2u^2) F(\rho-\rho_a). \qquad (12)$$

The result of this averaging procedure is that the probability for the process splits into coherent and incoherent terms.

Let us first consider the coherent bremsstrahlung, represented by the last term in (11). The particular feature of the coherent term is that it contains the δ-function. This represents conservation of crystal momentum, which is due to the periodicity of the dielectric properties of the atomic chain. When this is combined with energy conservation, we obtain a correlation between the photon frequency and the angle of emission:

$$\omega = \frac{2}{\gamma^{-2}+\theta^2}\left(\frac{2\pi}{d}j+\varepsilon_i-\varepsilon_f\right), \quad j=1,2,\ldots \qquad (13)$$

provided $\omega \ll E$ and $\theta \ll 1$. Since the characterisitic change in the transverse energy $\Delta\varepsilon_\perp = \varepsilon_i - \varepsilon_f$ is small ($\sim 10^{-2}$–10^{-4}) in comparison with the reciprocal lattice period $2\pi/d$, the spectrum of the coherent bremsstrahlung produced during axial channeling consists of relatively well-separated lines (corresponding to different values of j), each of which has a relatively complicated satellite structure. Since inclusion of the finite level widths and the actual spectral and angular resolution of the measuring equipment smears out the fine structure of the lines, we shall calculate the total intensity of the lines by summing over all the final states of transverse motion i.e., over f, assuming $\Delta\varepsilon_\perp = 0$ in (13).

Summation of the probability for the process over final states in the transverse spectrum can be carried out analytically in this case. However, summation over initial states i must be performed with weights $C_i(z)$ representing the population of the states of transverse motion at a given depth z. This leads to the appearance of the characteristic factor

$$Q(\rho, z) = \sum_i C_i(z)\,|\varphi_i(\rho)|^2 \qquad (14)$$

describing the redistribution of particle flux in the oriented crystal (the sum implies summation over discrete states and integration over the continuous spectrum).

It is important to note that, in general, the quantities $C_i(z)$ are not known in advance and must be determined by solving the corresponding kinetic equations.[12]

The frequency of first-harmonic coherent emission is of order $2\pi\gamma^2/d$, so that the expressions we have obtained are valied only for particle energies $E \lesssim$ (30–40) MeV so long as $\omega \ll E$. At higher energies, only very hard quanta with frequency $\omega \sim E$ can be emitted coherently. We may therefore conclude that, when the coherent term is examined, we can confine our attention to the dipole approximation ($E \ll E^*$), assuming that the transverse momentum $p_\perp$ of the electron is small in comparison with m.

It is readily seen that integration with respect to ξ in (7) results in the modified Bessel function $K_1(m|\rho_1-\rho_2|)$, which, in effect, restricts the range of variation of the variable $\Delta = \rho_1 - \rho_2$ to $|\Delta| \lesssim m^{-1}$. This enables us to put $\varphi_f(\rho_2) \simeq \varphi_f(\rho_1)$ to within terms $O(p_\perp^2/m^2)$ in the dipole approximation. An expansion into a series in terms of Δ can also be made in the function $V_q(\rho_2)$, except that it must be remembered that the Coulomb nature of the field $\Phi(\mathbf{r})$ at short distances from the nucleus produces the characteristic logarithmic divergence in expressions for the radiation intensity, which must be cut off at distances of the order of m^{-1} in calculations with logarithmic accuracy.

The emission probability must also be summed over the two independent values of the polarization vector $\mathbf{e}$. The summation is performed using the formula

$$\sum_{\mathbf{e}} (\mathbf{e}_\perp\xi_\perp)(\mathbf{e}_\perp\xi_2) = (\xi_1\xi_2) - \frac{(\mathbf{k}_\perp\cdot\xi_1)(\mathbf{k}_\perp\cdot\xi_2)}{k^2},$$

where the second term can be neglected in the small-angle approximation. As a result, transforming from the amplitude (7) to the probability, and restoring the necessary factos, we obtain the following expression for the spectral and angular density of coherent bremsstrahlung, integrated over the aximuthal angle of photon emission:[2)]

$$\frac{d\mathscr{E}^{\mathrm{coh}}}{d\omega\, dz\, d\theta} = \frac{4e^2}{(m\gamma d)^2}\frac{\theta(\theta^4+\gamma^{-4})}{(\theta^2+\gamma^{-2})^4}\sum_{g\neq 0}\exp(-g^2u^2)$$
$$\times\,\delta\left(\frac{\omega}{2}(\theta^2+\gamma^{-2})-g\right)\int d^2\rho\, Q(\rho,z)\left|\frac{\partial}{\partial\rho}\langle\Phi_g(\rho-\rho_a)\rangle_t\right|^2, \qquad (15)$$

where e is the electron charge and the sum is evaluated over the reciprocal-lattic vectors $g = 2\pi j/d, j = 1, 2, \ldots$.

It is readily seen that, when we evaluate the spectral and angular density of incoherent bremsstrahlung corresponding to the first term in (11), we can put $q = 0$. Actually, we find from (9) that $q \sim \omega m^2/2E_iE_f$, which, for $E_i \gtrsim$ (30–40) MeV, yields $q \ll a^{-1}$ throughout the frequency range under consideration, where a is the effective size of the atom. Consequently, the Fourier components Φ_q in (11) can be replaced by

$$\tilde{\Phi}(\rho) \equiv \Phi_{q=0} = \int_{-\infty}^{+\infty} dz\, \Phi(\rho, z). \qquad (16)$$

Direct use of (7) yields

$$\frac{d\mathscr{E}^{\mathrm{inc}}}{d\omega\, d\Omega\, dz} = \frac{e^2m^4\gamma^2}{4\pi^4 d}\sum_i C_i(z)\int d^2\rho\int d^2\Delta_1$$
$$\times\int d^2\Delta_2\,\varphi_i(\rho+\Delta_1)\varphi_i^*(\rho+\Delta_2)$$
$$\times\, B(\Delta_1)B^*(\Delta_2)D(\rho,\Delta_1,\Delta_2), \qquad (17)$$

where

$$B(\Delta) = \exp\left[-i(\mathbf{k}_\perp\cdot\Delta)\frac{p_i}{k_\parallel}\right]\int d^2\xi\,\frac{\exp(i\xi\cdot\Delta)}{m^2+\xi^2},$$
$$D(\rho,\Delta_1,\Delta_2) = \langle[\tilde{\Phi}(\rho-\rho_a+\Delta_1)-\tilde{\Phi}(\rho-\rho_a)]$$
$$\times[\tilde{\Phi}(\rho-\rho_a+\Delta_2)-\tilde{\Phi}(\rho-\rho_a)]\rangle_t$$
$$-\langle\tilde{\Phi}(\rho-\rho_a+\Delta_1)-\tilde{\Phi}(\rho-\rho_a)\rangle_t\cdot\langle\tilde{\Phi}(\rho-\rho_a+\Delta_2)-\tilde{\Phi}(\rho-\rho_a)\rangle_t,$$

and $d\Omega$ is the solid-angle element containing the direction of emission of the photon.

If we restrict ourselves to logarithmic accuracy, we can expand $D(\rho\,\Delta_1,\Delta_2)$ into a series in terms of Δ_1, Δ_2, so that, retaining the first nonvanishing terms, we have

$$D(\rho,\Delta_1,\Delta_2) \approx \langle(\Delta_1\cdot\nabla\tilde{\Phi}(\rho-{}_a))\cdot(\Delta_2\cdot\nabla\tilde{\Phi}(\rho-\rho_a))\rangle_{t,m}$$

$$-(\Delta_1\cdot\nabla\langle\Phi(\rho-\rho_a)\rangle_t)\cdot(\Delta_2\cdot\nabla\langle\Phi(\rho-\rho_a)\rangle_t) \quad (18)$$

(the subscript t, m shows that the resulting divergence is cut off at distances $|\rho-\rho_a|\sim m^{-1}$). Integration of (17) with respect to Ω results in the delta-function $\delta(\Delta_1-\Delta_2)$, after which using (18), we obtain

$$\frac{d\mathscr{E}^{inc}}{d\omega\,dz}=\frac{2e^2}{3\pi m^2 d}\int d^2\rho\,Q(\rho,z)\,T(\rho), \quad (19)$$

where

$$T(\rho)=\langle(\nabla\tilde{\Phi}(\rho-\rho_a))^2\rangle_{t,m}-(\nabla\langle\tilde{\Phi}(\rho-\rho_a)\rangle_t)^2. \quad (20)$$

The formula for the spectral density of incoherent bremsstrahlung, given by (19), is identical with the corresponding formula found in Ref. 9 for the dipole case, whereas the spectral and angular density in the nondipole case is described by the very unwieldy expression (17). The reason for this is that the high-frequency bremsstrahlung arises, as already noted, over a portion of the trajectory that is small in comparison with the oscillations, and is concentrated in a cone of radius $1/\gamma$ around the instantaneous direction of the particle velocity. Subequent averaging over different directions of the instantaneous velocity produces a considerable complication of the angular distribution of the bremsstrahlung, which can be reduced to a relatively simple form (see Ref. 9) only in the dipole case, when the characteristic angle of deflection of the particle from the axis is small in comparison with $1/\gamma$. When integration over the photon emission angles is carried out, all directions of the instantaneous particle velocity become equivalent, so that there is no difference between the dipole and nondipole cases as far as the spectral density of the bremsstrahlung is concerned. We not that the dependence of the intensity of incoherent bremsstrahlung on the frequency ω remains the same as in the amorphous target.

The above approach can obviously be extended to the case of a crystal plane. In the "single-plane approximation," the coresponding forumals become

$$\frac{d\mathscr{E}^{inc}}{d\omega\,dz}=\frac{e^2}{3\pi^2m^2\Delta}\int dx\,Q(x,z)\,T(x), \quad (21)$$

$$T(x)=\int_{-\infty}^{+\infty}dq\Big\{\Big\langle\Big|\frac{\partial\Phi_q(x-x_a)}{\partial x}\Big|^2\Big\rangle_{t,m}-\exp(-q^2u^2)\Big|\Big\langle\frac{\partial\Phi_q(x-x_a)}{\partial x}\Big\rangle_t\Big|^2+q^2(\langle|\Phi_q(x-x_a)|^2\rangle_{t,m}-\exp(-q^2u^2)\,|\langle\Phi_q(x-x_a)\rangle_t|^2)\Big\}, \quad (22)$$

$$\frac{d\mathscr{E}^{coh}}{d\omega\,dz}=\frac{\omega e^2}{m^2\gamma^2\Delta^2}\sum_{g_\parallel\neq0}\frac{|S(\mathbf{g})|^2}{g_\parallel^2}\exp(-g^2u^2)\times\Big[1-\frac{\omega}{g_\parallel\gamma^2}\Big(1-\frac{\omega}{2g_\parallel\gamma^2}\Big)\Big]\times\eta\Big(1-\frac{\omega}{2g_\parallel\gamma^2}\Big)\int dx\,Q(x,z)K(x), \quad (23)$$

$$K(x)=\Big|\Big\langle\frac{\partial\Phi_g(x-x_a)}{\partial x}\Big\rangle_t\Big|^2+g_\perp^2|\langle\Phi_g(x-x_a)\rangle_t|^2, \quad (24)$$

$$Q(x,z)=\sum_i C_i(z)\,|\varphi_i(x)|^2. \quad (25)$$

where Δ is the area per atom on the crystal plane, $S(\mathbf{g})$ is a geometric structure factor normalized to the number of atoms per unit cell,[11] $\varphi_1(x)$ is the wave function for the transverse motion, and $\langle\ldots\rangle_t$ represents averaging over the transverse thermal oscillations relative to the plane under consideration. The sum in (23) is evaluated over reciprocal lattice vectors lying in the plane, the z-axis is assumed to lie along the longitudinal particle momentum, and $g_\parallel$ and $g_\perp$ represent the components of the reciprocal-lattice vector, respectively along and across the z-axis; Φ_q is given by (8) and $\eta(y)$ is the Heaviside function ("step").

The spectral and angular density of the radiation in the dipole case is now given by

$$\frac{d\mathscr{E}^{inc}}{d\omega\,d\theta\,dz}=\frac{e^2}{m^2\gamma^2\pi^2\Delta}\frac{\theta(\theta^4+\gamma^{-4})}{(\theta^2+\gamma^{-2})^4}\int dx\,Q(x,z)\,T(x),$$
$$\frac{d\mathscr{E}^{coh}}{d\omega\,d\theta\,dz}=\frac{4e^2}{m^2\gamma^2\Delta^2}\frac{\theta(\theta^4+\gamma^{-4})}{(\theta^2+\gamma^{-2})^4}\sum_{g_\parallel\neq0}|S(\mathbf{g})|^2 \quad (26)$$

$$\times\exp(-g^2u^2)\,\delta\Big(g_\parallel-\frac{\omega}{2}(\theta^2+\gamma^{-2})\Big)\int dx\,Q(x,z)K(x), \quad (27)$$

where the polar angle θ is measured from the z axis. When $Q(x, z) = \text{const}$, the last formula reproduces the corresponding result of the theory of coherent bremsstrahlung.[11]

We also note that, in the derivation of (17)–(27), it was not assumed that the motion of the particles in the channel was quasiperiodic, which means that the results are valid even in the energy range where, according to Ref. 13, the transverse energy is not an adiabatic invariant of the motion.

4. ANALYSIS OF THE RESULTS

Let us first compare the relative spectral intensity of coherent and incoherent bremsstrahlung in an oriented crystal. It is well-known[11] that, in the planar case, when the particle propagates at a small angle to a family of axes, the peak coherent-bremsstrahlung intensity can exceed the background by an order of magnitude or more. On the contrary, well away from the peak, the intensity of coherent bremsstrahlung is small in comparison with the background.[14] This result was obtained without taking into account the redistribution of the particle flux in the channel. A similar relationship is valid for the planar case even when the redistribution of the flux *is* taken into account, because the factor $Q(x, z)$ has approximately the same effect on the spectral density of both coherent and incoherent bremsstrahlung.

The situation is different in the axial case. Assuming, for the purposes of estimates, that $Q(\rho, z) = \text{const}$, and approixmating $\Phi(r)$ by the screened Coulomb potential $\Phi(r) = Ze^2r^{-1}e^{-r/a}$, we find that, at an intensity peak corresponding to one of the terms in (15),

$$\frac{d\mathscr{E}^{coh}}{d\omega\,dz}\Big/\frac{d\mathscr{E}^{inc}}{d\omega\,dz}\approx\frac{3}{2}\frac{\lambda_g}{1-\lambda_g}, \quad (28)$$

where

$$\lambda_g=(-\mathrm{Ei}(-x)\,(1+x)e^x-1)/2\ln(ma)-1,\quad x=u^2(g^2+a^{-2}).$$

For a real crystal, the numerical value of (28) is at most 0.1–0.2 for all the terms in (15). It follows that, in the axial case, coherent bremsstrahlung results merely in a relatively weak (no more than 10–20%) modulation of the incoherent "background."

We now turn to the oreintational dependence of the spectral density of incoherent bremsstrahlung. We note that measurements of $d\mathscr{E}^{inc}/d\omega dz$ are of considerable physical interest because they enable us to judge the stability of particle motion in the crystal channels. Actually, comparison of (19)–(22) with the results reported in Ref. 13 readily shows that the spectral density of incoherent bremsstrahlung is proportional to the square of the anlge of multiple scattering by crystal atoms, averaged over the particle flux (or, in other words, the average increase in the transverse energy $\delta\varepsilon_\perp/\delta z$. This means that $d\mathscr{E}^{inc}/d\omega dz$ carries information about the rate of scattering of particles in an oriented crystal.

It follows from (19)–(22) that the spectral denisity of bremsstrahlung emitted in a crystal depends on the dynamic characteristic of the motion, namely, the distribution of particle flux denisty over the channel cross section, $Q(\rho, z)$, $Q(x, z)$. Different types of orientational effects can be related only to the redistribution of the particle flux density. In particular, when $Q(\rho, z)$ is small for $\rho \lesssim u$ (which occurs, for example, in the case of positron channeling), the bremsstrahlung intensity is found to be suppressed as compared with the amorphous target:

$$\frac{d\mathscr{E}^{inc}}{d\omega\, dz}=\frac{4e^2u^2d}{3m^2}\int\limits_{\rho\gtrsim u}\rho\, d\rho\, Q(\rho, z)\left\{\left(\frac{\partial^2\tilde{\Phi}}{\partial\rho^2}\right)^2+\frac{1}{\rho^2}\left(\frac{\partial\tilde{\Phi}}{\partial\rho}\right)^2\right\}.$$

This is in agreement with the results obtained in Ref. 14 by a classical approach. On the contrary, in the case of electron channeling, the particle flux is distributed over the channel relatively uniformly. In this case, the corresponding integral over the channel cross section in (19) and (21) can be accurately estimated as

$$Q(\rho=0, z)\int d^2\rho\, T(\rho)$$

for axial channeling, or

$$Q(x=0, z)\int dx\, T(x)$$

for planar channeling, since the effective density of scatterers $T(\rho)$, $T(x)$ has a sharp peak near the channel center.

Let us now estimate orientational effects accompanying the motion of electrons in a planar channel. The current density at the center of the channel is approximately given by

$$Q(x=0, z)\approx\int\limits_{-U_0}^{\infty}d\varepsilon\, f(\varepsilon, z)\nu(\varepsilon)v_0^{-1}(\varepsilon), \tag{29}$$

where $v_0(\varepsilon)=[2(\varepsilon+U_0)/E]^{1/2}$ is the transverse velocity of a particle of transverse energy ε at the center of the channel, U_0 is the channel depth, E is the total energy of the particle, $\nu(\varepsilon)$ is the frequency of collisions between the particle and the atomic plane, and $f(\varepsilon, z)$ is the transverse energy distribution normalized to one particle per channel:

$$\int\limits_{-U_0}^{\infty}d\varepsilon\, f(\varepsilon, z)=1.$$

The transformation to the "disoriented" crystal formally corresponds to $\varepsilon/U_0\to\infty$. We then have $\nu(\varepsilon)\to v_0(\varepsilon)d_p^{-1}$, and hence $Q(x=0, z)=1/d_p$, where d_p is the separation between the planes.

For a channel model in the form of an inverted hyperbola, we have

$$\nu(\varepsilon)=\frac{(2U_0)^{1/2}}{d_pE^{1/2}}\begin{cases}[\operatorname{arch}(U_0/|\varepsilon|)^{1/2}]^{-1}, & -U_0\leqslant\varepsilon<0\\ [\operatorname{arsh}(U_0/\varepsilon)^{1/2}]^{-1}, & \varepsilon\geqslant 0\end{cases}. \tag{30}$$

For emission angles Ψ exceeding the Lindhard angle $\Psi_L=(2U_0/E)^{1/2}$, we have $\varepsilon\simeq E\Psi^2/2\gg U_0$, and it then follows from (29) and (30) that the spectral density of incoherent bremsstrahlung, including orientational effects, is

$$I(\Psi)\equiv\frac{d\mathscr{E}^{inc}}{d\omega\, dz}(\Psi)\Big/\frac{d\mathscr{E}^{inc}}{d\omega\, dz}(\Psi\to\infty)\approx 1-\frac{1}{3}\left(\frac{\Psi_L}{\Psi}\right)^2.$$

We note that the correction to unity amounts to about 10% even for $\Psi=2\Psi_L$, and decreases rapidly (as Ψ^{-2}) with increasing Ψ. For $\Psi<\Psi_L$, the intensity $I(\Psi)$ increases with decreasing Ψ, reaching 4–5 in a thin crystal for $\Psi=0$. However, it must be remembered that, for small emission angles $\Psi\lesssim 2\Psi_L-3\Psi_L$, the initial particle distribution $f(\varepsilon, z=0)$ changes very substantially even for small crystal thicknesses $z\gtrsim(3d_p/\Psi_L-4d_p/\Psi_L)$. Substituting

$$f(\varepsilon, z)\approx\begin{cases}1/\varepsilon_{max}, & -U_0\leqslant\varepsilon\leqslant\varepsilon_{max}-U_0\\ 0, & \varepsilon>\varepsilon_{max}-U_0\end{cases} \tag{31}$$

and assuming that ε_{max} is a linear function of the depth z (as it is in the amorphous target), we obtain

$$\varepsilon_{max}(z)\approx\frac{1}{2}E\langle\Delta\theta^2\rangle_{at}, \tag{32}$$

$$\langle\Delta\theta^2\rangle_{at}=\frac{E_s^2}{2E^2}\frac{z}{R} \tag{33}$$

where R is the radiation length and $E_S\simeq 21$ MeV. For large z, we have

$$Q(x=0, z)\approx\frac{1}{d_p}\left[1+\frac{U_0}{3\varepsilon_{max}(z)}\left(A-\ln\frac{\varepsilon_{max}(z)}{U_0}\right)\right]. \tag{34}$$

It follows from this expression that, as z increases, the correction to $1d/_p$ decreases and can become negative for $\varepsilon_{max}(z)\gtrsim U_0e^A$.[3] However, the constant A, determined by particles with relatively low transverse energy ($-U_0\leqslant\varepsilon\lesssim U_0$), has a positive sign and is relatively large [$A=(5$–$10)$, depending on the detailed form of the distribution function $f(\varepsilon, z)$], so that, according to (34), the possible relative reduction in the current density for large z is of order $e^{-A}\ll 1$.

The above estimates do not confirm the possibility of significant suppression of multiple-scattering intensity and incoherent bremsstrahlung in oriented crystals, predicted in Refs. 16 and 17. On the contrary, our results show that, for paticle incidence angles exceeding $2\Psi_L-3\Psi_L$, or for crystal thicknesses exceeding 100–100 μm, the spectral density of incoherent bremsstrahlung is practically orientation-independent to within small corrections.

On the other hand, there are experimental data on orientational effects in incoherent bremsstrahlung for angles Ψ up to $10\Psi_L$ (Refs. 7 and 8). In the light of the formulas obtained above, this result can only be explained by collimation of the photon beam. Actually, as we have already noted, the emission of high-frequency gamma-rays occurs within a cone of angle $1/\gamma$ containing the instantaneous direction of the particle velocity, i.e., the gamma-ray distribution over the emission angles is correlated with the transverse velocity distribution of the particles. At the same time, the angular broadening of the particle beam in the case of axial channeling for angles Ψ exceeding the critical value may be much greater than in amorphous targets. This is due to the multi-

ple scattering of particles by atomic chains,[18] which means that the characteristic "annular" transverse velocity distributions are produced at exit from the crystal. The result is that the fraction of radiation entering a collimator with relatively small angular aperture may be less than that in the case of the "disoriented" crystal of the same thickness.

To estimate the magnitude of the effect, we use the approximate expression for the mean square angle of scattering by chains[18]

$$\langle \Delta\theta^2 \rangle_{\text{chain}} = D_{\text{chain}} z,$$

where

$$D_{\text{chain}} \approx \frac{U_0^2}{E^2} \frac{2a}{d^2} \frac{1}{\Psi},$$

d is the mean separation between the chains, and Ψ is the angle of incidence of a particle to the atomic axes. This estimate is valid for $\Psi \gtrsim 2\Psi_L$ and thicknesses

$$z \lesssim \frac{d^2}{a} \frac{\Psi^3}{\Psi_L^4}.$$

Assuming that beam broadening along one of the directions is due entirely to collisions with individual atoms [see (33)], we find that the relative intensity of bremsstrahlung incident on the collimator of small angular aperture has the following orientation dependence

$$I(\Psi) = \left[\frac{\langle \Delta\theta^2 \rangle_{\text{at}}}{\langle \Delta\theta^2 \rangle_{\text{at}} + \langle \Delta\theta^2 \rangle_{\text{chain}}} \right]^{1/2} = \left[1 + \frac{4Ra}{d^2\Psi} \left(\frac{U_0}{E_s} \right)^2 \right]^{-1/2}$$

This formula is in satisfactory agreement with experimental data for $\Psi \gtrsim 2\Psi_L$ and explains the absence of a thickness dependence.[7] It also shows that, to achieve a reliable interpretation of experimental data, the bremsstrahlung spectral density must be measured for photon collimation angles exceeding $4RaU_0^2/d^2E_S^2 \sim 3$–5 mrad, which is greater than the values usually employed in photon collimation.

5. CONCLUSION

We have obtained explicit analytic expressions for the spectral and spectral-angular density of bremsstrahlung emitted by a relativistic spinless charged particle in an oriented crystal. The results show that the spectral intensity of incoherent bremsstrahlung is not significantly lower than in the amorphous target. Possible orientational effects in the bremsstrahlung spectral intensity are related exclusively to the redistribution of the particle flux over the channel cross section.

We have shown that the high-frequency component of hte bremsstrahlung carries information about the stability of the particle motion in crystal channels. However, studies of these questions will require new experiments that do not employ the collimation of the gamma-ray flux because the spectral and angular distribution of bremsstrahlung may be subject to orientational effects that are unrelated to the flux redistribution. One of these effects is the multiple scattering of particles by atomic chains. Estimates made with allowance for multiple scattering by chains lead to orientational effects that are in agreement with experimental data obtained in the region where the effect is significant.

[1)]Here and in what follows we take $\hbar = c = 1$.
[2)]The difference in the numerical factor as compared with Ref. 9 is related to the definition of Φ_q.
[3)]We note the discrepancy between this result and the results reported in Ref. 15 where the corrections to the root mean square angles of scattering along the channeling plane and at right angles to it have different signs. This assymetry cannot be explained in terms of the redistribution of the particle flux over the channel cross section.

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Translated by S. Chomet