MHD turbulence caused by a comet in the solar wind

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The loading of the solar wind by heavy cometary ions results in the excitation of Alfvén turbulence. As the Alfvén waves cause the distribution function of the comet ions to become more nearly isotropic, they set up a collective mechanism for the entrainment of these ions by the solar wind and thereby strongly influence the motion of the loaded solar wind and the position and structure of the comet shock wave. Like the shock waves of cosmic rays, the cometary shock wave is of dissipative nature. The width of its front is greater by a factor of at least m_i/m_p (the ratio of the masses of the heavy ion and the proton, respectively) than the width of the front of a planetary shock wave. The results derived here agree with observational data from recent fly-bys of comet Jacobini-Zinner and Halley's comet by space vehicles.

INTRODUCTION

One of the most unexpected discoveries during the fly-bys of comet Jacobini-Zinner[1] and Halley's comet[2] was the presence of intense magnetic fluctuations, which are seen in the solar wind even at large distances (on the order of several million kilometers) from the comet. It has been suggested[3] that these fluctuations result from an ion cyclotron instability which is excited when the solar wind is loaded by comet ions. In the present paper we present a detailed nonlinear theory of the phenomenon.

We know that when a comet passes near the sun, it is surrounded by a substantial cloud of gas consisting of neutral molecules (mostly H_2O) which are evaporated from the surface of the core of the comet by solar radiation. At a distance of 1 a.u. from the sun, the gas flux is $Q = 5\cdot 10^{28}$ molecules/s for comet Jacobini-Zinner[4] and $Q = 1.3\cdot 10^{30}$ molecules/s for Halley's comet.[5] Neutral molecules, moving at a typical velocity $v_g = 1$ km/s, are photoionized, and heavy comet ions are produced in the solar wind. The time scale of the photoionization is $\tau = 10^6$ s, so the solar wind is perturbed by the cometary ions even several million kilometers away from the comet.

In the plane perpendicular to the magnetic field, the newly produced ions move in the crossed electric and magnetic fields of the wind, i.e., along cycloids:

$$v_x=u_\perp[1-\cos\omega_{Hi}(t-t_0)],\qquad v_y=u_\perp\sin\omega_{Hi}(t-t_0),\quad (1)$$

where $u_\perp = cE/H$ is the velocity of the solid wind across the magnetic field, t_0 is the time at which the ion is produced, and ω_{Hi} is the ion cyclotron frequency.

If the magnetic field is directed perpendicular to the direction of the solar wind, the comet ions are entrained by the solar wind, while they simultaneously execute cyclotron gyration around magnetic field lines. The energy of the cyclotron gyration of the heavy ions, $\mathscr{E}_c = m_i u^2/2$, is extremely large (for water ions at $u = 400$ km/s, for example, it is $\mathscr{E}_c = 16$ keV), so that the solar wind becomes loaded by the high-energy ions. When this loading becomes significant the pressure of the solar wind is determined specifically by the high-energy component. Usually, however, the magnetic field in the plasma of the solar wind makes an angle with the plasma velocity, and the comet ions are only partially entrained, since their velocity along the direction of the magnetic field remains at a level $v_g \ll u$. The comet ions produce in the plasma of the solar wind a beam of particles which move along the magnetic field at a velocity $-u_\parallel$ and which simultaneously gyrate about the field lines with an energy $\mathscr{E}_c = m_i u_\perp^2/2$. Cyclotron instability of such a beam results in the excitation of Alfvén waves. The pitch-angle diffusion of the resonant ions which is induced by these waves causes the distribution function of these ions to become more nearly isotropic in the coordinate system of the solar wind. In other words, this diffusion sets up a collective mechanism for entrainment along the magnetic field. These processes are analyzed using weak turbulence theory in the present paper. The results show that even for far from the core of the comet [(3–5)$\cdot 10^6$ km for Halley's comet] intense Alfvén waves are excited with a magnetic-fluctuation amplitude comparable to the magnetic field in the solar wind, $H' \sim 1/3$ to $1/5$ of H_0 and with typical frequencies on the order of 10^{-2} Hz. This Alfvén turbulence is by no means simply a by-product of the mass loading of the solar wind. It determines the dynamics of the motion of the wind, the head of the shock front, the position of the front, and its structure, which is quite different from the structure of an ordinary planetary shock wave.

Generally speaking, there are two distinct types of shock waves in space. The first type, and the type which has been studied most thoroughly, is the shock wave of a planetary magnetosphere, which consists of sharp jumps in the properties of the solar wind, with a front width no greater than the proton Larmor radius. The second type of shock wave is a substantially smoother shock wave of a dissipative nature, one example of which is the shock wave of cosmic rays in the interstellar plasma, which was first studied by Axford.[6] In the latter case the cosmic-ray particles which are accelerated at the front of the sharp "subshock" which is present in the shock wave diffuse into the surrounding plasma. Their pressure retards the incoming plasma stream and creates a characteristic diffusion structure, in which a slow increase in the density and magnetic field precedes an abrupt jump.

In a sense there is an extremely profound analogy between a shock wave near a comet and the shock wave of cosmic waves in the interstellar medium—an analogy which stems from the similarity between the loading of the solar

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wind by high-energy comet ions and the diffusive acceleration of cosmic rays by shock waves. In both cases the motion of high-energy particles along magnetic field lines results in the excitation of Alfvén turbulence, which in turn causes a diffusive broadening of the shock front to dimensions comparable to or greater than the Larmor radius of the high-energy ions. As a result of this broadening of the front, the jumps in the values of the properties of the solar wind (the magnetic field and so forth) caused by the shock wave may remain totally unnoticed against the background of the intense fluctuations associated with MHD turbulence.

LINEAR THEORY OF THE CYCLOTRON INSTABILITY

As was mentioned in the Introduction, in the plasma of the solar wind cometary ions form a beam of particles moving along the magnetic field at a velocity $-u_\parallel$ and simultaneously gyrating in Larmor orbits at a velocity $u_\parallel$. The density of this beam increases toward the comet. It can easily be expressed in terms of the relative increase in the mass flux

$$\varphi(x)=\Delta[\rho(x)u(x)]/\rho_\infty u_\infty$$

transported by the solar wind:

$$n_i(x)=(m_p/m_i)n_0^\infty\varphi(x)(1/\sin\alpha).$$

Here n_0^∞ and u_∞ are the unperturbed values of the density and velocity in the solar wind. We have assumed here that comet ions are entrained only in the direction perpendicular to the magnetic field. We have directed the x axis toward the comet, along the wind velocity; the magnetic field vector $\mathbf{H}$ lies in the xy plane, making an angle α with the x axis; and the subscript $\parallel$ here and below means the direction antiparallel to $\mathbf{H}$.

In the limit $x\to-\infty$, the function $\varphi(x)$ tends toward 0, while it increases monotonically toward the comet, to values $\varphi(x)\sim 1$ near the shock wave. The density of the cometary ions remains low in comparison with that of the solar wind, by a factor of $m_p/m_i\sim 1/20$, but the momentum carried by these ions is comparable to that of the unperturbed solar wind. The pressure produced by the ions is substantially greater than both the pressure of the solar wind and the magnetic pressure $H_0^2/8\pi$. A significant energy reservoir is thus formed in the plasma of the solar wind: intense waves can be excited by virtue of this reservoir. The excitation mechanism, first studied in Ref. 7, is the cyclotron instability of electromagnetic waves which occurs as the beam of comet ions interacts with the plasma of the solar wind. The maximum growth rate is associated with propagation of the waves along the magnetic field. The dispersion relation for this case is well known (Ref. 8, for example):

$$c^2k^2-\omega^2+\frac{\omega_p{}^2\omega}{\omega\pm\omega_{Hp}}+\frac{\omega_{pe}{}^2\omega}{\omega\mp\omega_{He}}\mp\omega\frac{\omega_{pe}{}^2}{\omega_{He}}\frac{m_p}{m_i}\frac{\varphi(x)}{\sin\alpha}$$
$$+\omega_p{}^2\left(\frac{m_p}{m_i}\right)^2\frac{\varphi(x)}{\sin\alpha}\left[\frac{\omega+ku_\parallel}{\omega+ku_\parallel\pm\omega_{Hi}}+\frac{1}{2}k^2u_\perp{}^2\frac{1}{(\omega+ku_\parallel\pm\omega_{Hi})^2}\right]=0. \quad (2)$$

In this equation, ω_p, ω_{pe}, ω_{Hp}, and ω_{He} are the plasma and cyclotron frequencies of the protons and electrons, respectively. The dispersion relation (2) has been written for a circularly polarized electromagnetic wave; the upper sign corresponds to a wave polarized in the direction of gyration of electrons in the ion beam which are in resonance according to the anomalous Doppler effect, $\omega+ku_\parallel\approx-\omega_{Hi}$. The lower sign corresponds to a wave with the "ion" polarization, i.e., to the normal Doppler effect, $\omega+ku_\parallel\approx\omega_{Hi}$. The dispersion relation (2) has been analyzed in detail by Winske *et al.*[9] Here we will simply summarize the results of that analysis.

a) The nonresonant case, corresponding to the long-wave limit $k|u_\parallel|\ll\omega_{Hi}$. In this case an aperiodic instability occurs:

$$\omega^2=k^2v_A{}^2\left[1-\frac{u_\parallel{}^2-u_\perp{}^2/2}{v_A{}^2}\frac{\varphi(x)}{\sin\alpha}\right]. \quad (3)$$

This is the ordinary firehose instability, caused by the anisotropy of the pressure of the cometary ions.[10] The condition for its onset is

$$P_\parallel-P_\perp>H_0{}^2/4\pi,$$

where $P_\parallel=n_im_iu_\parallel^2$, $P_\perp=n_im_iu_\perp^2$ for the longitudinal and transverse pressures of the cometary ions.

b) The resonant beam-driven instability, $\omega+ku_\parallel\approx\mp\omega_{Hi}$. Here we assume that the condition $\omega\ll\omega_{Hp}$, holds; i.e., we deal with the excitation of Alfvén waves. We then have

$$\omega=kv_A+\varepsilon,\quad |\varepsilon|\ll\omega,\ \omega_{Hi}.$$

Under the condition $\mathrm{tg}^2\alpha<|\varepsilon|/\omega_{Hi}$, i.e., for quasiparallel propagation of the solar wind, the energy reservoir for the instability is the longitudinal motion of the comet ions. In this case, only the instability corresponding to the anomalous Doppler effect,

$$k(u_\parallel+v_A)=-\omega_{Hi},$$

can occur, and we find from Eq. (2) a growth rate

$$\gamma=\omega_{Hi}\left[\frac{|u_\parallel|}{2v_A}\frac{\varphi(x)}{\sin\alpha}\right]^{1/2}. \quad (4)$$

In the opposite case, $\mathrm{tg}^2\alpha>|\varepsilon|/\omega_{Hi}$, an instability is possible for two polarization directions of the wave, $k(u_\parallel+v_A)=\mp\omega_{Hi}$, and the growth rate $\gamma=\mathrm{Im}\varepsilon$ is given by

$$\gamma=\frac{3^{1/2}}{2^{5/3}}\omega_{Hi}\left[\frac{|u_\parallel|}{v_A}\varphi(x)\frac{\sin\alpha}{\cos^2\alpha}\right]^{1/3}. \quad (4')$$

Since we should have a ratio $|u_\parallel|/v_A\sim 3$–4 for the typical parameter values in the solar wind, the condition $|\varepsilon|\ll\omega_{Hi}$, which we used in deriving (4) and (4'), is violated even for $\varphi(x)<1$. In this case a strong hydrodynamic instability sets in with a growth rate $\gamma\sim\omega$, but it is again of a resonant nature, $ku_\parallel\approx\mp\omega_{Hi}$. For the frequency ω we find the following equation from (3):

$$\left(\frac{\omega}{\omega_{Hi}}\right)^4\pm\frac{\omega}{\omega_{Hi}}\frac{\varphi(x)}{\sin\alpha}-\frac{\varphi(x)\sin\alpha}{2\cos^2\alpha}=0. \quad (5)$$

We thus find

$$\omega=\frac{1+i\sqrt{3}}{2}\omega_{Hi}\left(\frac{\varphi(x)}{\sin\alpha}\right)^{1/3}\quad\text{for}\quad \operatorname{tg}^2\alpha\ll\frac{|\omega|}{\omega_{Hi}}$$

and

$$\omega=\frac{i}{\sqrt{2}}\omega_{Hi}\left(\varphi(x)\frac{\sin\alpha}{\cos^2\alpha}\right)^{1/3}$$

in the opposite limit, $\operatorname{tg}^2\alpha\gg|\omega|/\omega_{Hi}$. In the perturbed solar wind, with $\varphi(x)\sim 1$, the instability growth rate satisfies $\gamma\sim\omega_{Hi}$, and intense Alfvén waves are excited, which causes the distribution function of the comet ions to become more nearly isotropic in the coordinate system of the solar wind, over a time scale $T_{\text{iso}}\sim 10\omega_{Hi}^{-1}$ comparable to the ion cyclotron period. Under these conditions the collective "entrainment" of the comet ions along the magnetic field because of the Alfvén waves becomes as fast as the single-particle entrainment in the perpendicular plane in the crossed E and H fields. The distance over which the collective entrainment of the comet ions occurs.

$$L_{\text{iso}}\approx u_\infty T_{\text{iso}}\sim 10u_\infty/\omega_{Hi}\sim 10^{10}\ \text{cm},$$

is much shorter than the length scale $L\approx(5\text{–}3)10^{11}$ cm over which the motion of the solar wind is perturbed (Fig. 1). Accordingly, in the discussion below we will assume that intense Alfvén waves are excited in the loaded solar wind and that the distribution function of the heavy ions is nearly isotropic in the coordinate system of the solar wind

SOLUTION OF THE QUASILINEAR EQUATION FOR COMET IONS; GAP AT $V_\parallel=0$

In accordance with the results of the preceding section, we assume that the comet ions are only slightly anisotropic. In other words, we seek their distribution function in the form

$$f=f_0(u(x),v)+f_1(u(x),v,\theta),\quad f_1\ll f_0,\tag{6}$$

where v is the absolute value of the velocity in the coordinate system of the solar wind, and θ is the pitch angle ($\sin\theta=v_\perp/v$).

In the steady state, a kinetic equation for f incorporating the pitch-angle diffusion caused by the Alfvén waves and also the continuous production of new ions by photoionization of the gas flow can be written

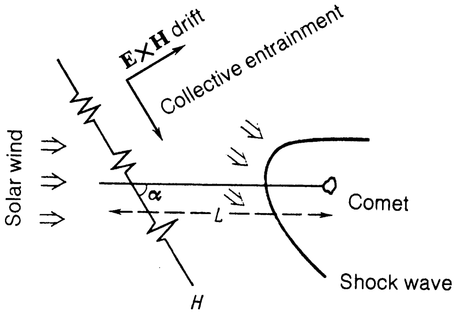

FIG. 1. Motion of the loaded solar wind: cometary shock wave.

$$(u(x)-v_\parallel\cos\alpha)\frac{\partial f}{\partial x}-\frac{v_\perp}{2}\frac{du}{dx}\frac{\partial f}{\partial v_\perp}-\frac{\pi}{2}\frac{e^2}{m_i^2}\frac{\partial}{\partial v_\parallel}\times\left[\sum_{k,\pm}|E_k^\pm|^2\frac{k^2v_\perp^2}{\omega_k^2}\delta(kv_\parallel-\omega_k\pm\omega_{Hi})\frac{\partial f}{\partial v_\parallel}\right]=\frac{Q\exp(-r/v_g\tau)}{4\pi^2r^2v_g\tau}\delta(v_\perp^2-u_\perp^2)\delta(v_\parallel+u_\parallel).\tag{7}$$

The first term on the left side of this equation describes convective drift; the second term is responsible for the adiabatic heating of the cometary ions in the inhomogeneous stream of solar plasma; and, finally, the last term is the quasilinear collision integral corresponding to collisions of the particles with the Alfvén waves (Ref. 11, for example). The superscripts $\pm$ refer to the two polarizations of the electromagnetic wave; the derivative with respect to $v_\parallel$ is calculated along the lines of the pitch-angle diffusion of the resonant particles, which coincide approximately (to within small quantities of order ω/ω_{Hi}) with lines of constant energy, $v_\perp^2+v_\parallel^2=\text{const}$. The term on the right side of Eq. (7) is a source describing the production of new ions with a longitudinal velocity $-u_\parallel$ and a cyclotron-revolution energy $m_iu_\perp^2/2$; Q is the intensity of the gas flow; r is the distance from the comet; and the exponential factor corresponds to the attenuation of the gas flow as a result of photoionization.

Strictly speaking, the pitch-angle diffusion described by the quasilinear collision integral is limited to the region $v_\parallel>0$. Cometary ions are produced with $\theta=\alpha$. Their diffusion into the angular region $\theta<\alpha$ results from the excitation of waves in the normal Doppler effect:

$$\omega-kv_\parallel=\omega_{Hi}.\tag{8}$$

For $\theta>\alpha$, the excitation of waves and the diffusion of particles are associated with a resonance in the anomalous Doppler effect:

$$\omega-kv_\parallel=-\omega_{Hi}.\tag{8'}$$

This diffusion, however, is limited to values $v_\parallel\geqslant v_{\min}$. In a plasma with a large value $\beta\sim 1$ ($\beta=8\pi n_0T/H_0^2$ is the ratio of the gas pressure to the magnetic pressure) we would have $v_{\min}\approx 3v_T$, since waves with $v_\parallel<v_{\min}$ are damped as a result of cyclotron absorption by the thermal plasma. Correspondingly, a gap appears in the diffusion coefficient at $v_\parallel\approx 0$. At $\beta\ll 1$ the boundary of this gap is $v_{\min}=v_A$, since there is no resonant interaction with waves at $v_\parallel<v_A$.

Achterberg[12] has suggested two mechanisms for crossing the gap near $v_\parallel=0$. Since those mechanism are based on a strong nonlinearity, they cannot be described by the quasilinear equation (7). One of these mechanisms is a reversal of the direction in which the ions are moving in the magnetic mirrors which are set up as a result of the long-wave modulation ("long" on the scale of the ion Larmor radius) of the magnetic field by the Alfvén waves. Using the conservation of the magnetic moment, $\mu=m_iv_\perp^2/2H$, we find that a particle which is moving between magnetic mirrors can change direction if its longitudinal velocity $v_\parallel$ satisfies the condition

$$\frac{v_\parallel^2}{v^2}\leqslant\frac{\delta H}{H_0}\approx\frac{1}{2H_0^2}\sum_{k<\omega_{Hi}/u_\infty}|H_k|^2.\tag{9}$$

Here

$$\delta H=\left(H_0^2+\sum_k|H_k|^2\right)^{1/2}-H_0$$

is the modulation of the magnetic field by the waves.

The second mechanism for crossing the gap at $v_\parallel = 0$ is based on the well-known nonlinear Duprel broadening of a wave-particle resonance. According to Ref. 13, the nonlinear broadening of the resonance in velocity in the case of a cyclotron resonance with Alfvén waves is

$$\Delta v_\parallel^{NL} \approx v\left[\frac{|H_k|^2(k=\omega_{Hi}/v_{min})}{H_0^2}\frac{\omega_{Hi}}{v}\right]^{1/3}. \tag{10}$$

The gap at $v_\parallel \approx 0$ is crossed if $\Delta v_\parallel^{HL} = v_{\min}$. Another important point is that in this case, in contrast with (9), the crossing of the gap is associated with the shortest part of the wave spectrum. Both of these mechanisms for the crossing of the gap at $v_\parallel \approx 0$ have recently been observed[14] in a numerical simulation of the process by which resonant ions become more nearly isotropic in the field of Alfvén waves. It has been shown that the conditions for their occurrence agree quite accurately with relations (9) and (10).

A comparison of (9) and (10) for power-law spectra $|H_k|^2 \sim k^{-\nu}$ shows that for $\nu > 2$ the magnetic-mirror mechanism is important, while for $\nu < 2$ the nonlinear broadening of the resonance is important. The quasilinear theory gives rise to a power-law spectrum with $\nu = 2$, which agrees quite well with observations.[1] In this case, conditions (9) and (10) are equivalent to each other. For typical parameter values of the solar wind interacting with a comet, $v_\parallel \approx v_{\min} \approx 3v_{Ti} \approx 8\times 10^6$ cm/s and $v \approx u_\infty \approx 5\times 10^7$ cm/s, the condition for the crossing of the $v_\parallel = 0$ gap can be satisfied easily even if the energy of the magnetic fluctuations is only a few percent of the energy of the unperturbed magnetic field. When the $v_\parallel < 0$ gap is crossed by resonant particles, and instability involving the normal Doppler effect occurs in the region $v_\parallel < 0$ [condition (8) with $v_\parallel < 0$], and the diffusion of the particles in the field of the waves which are excited causes isotropization of the distribution function for the particles. The anisotropic increment in the distribution function persists only by virtue of the convective drift of the resonant particles and the continuous production of new ions. Under the conditions

$$L_{\text{diff}}/L \ll 1, \quad \tau_{\text{diff}}/\tau \ll 1,$$

however, the anisotropy of the distribution of cometary ions remains weak. Under these conditions,

$$\tau_{\text{diff}} \approx \omega_{Hi}^{-1} H_0^2\left[\sum_k |H_k|^2\right]^{-1}, \quad L_{\text{diff}} = u_\infty \tau_{\text{diff}} \tag{11}$$

are the time scale and length scale of the pitch-angle diffusion. The fact that the distribution function of high-energy ions becomes more nearly isotropic in the coordinate system of the solar wind—i.e., that there exists a collective mechanism for the entrainment of comet ions by the solar wind—has been confirmed to high accuracy by *in situ* observations of these ions ahead of the front of the cometary shock wave during the fly-by of comet Jacobini-Zinner.[15]

To find the anisotropic increment, we solve Eq. (7). Substituting the distribution function (6) into it, and separating the terms which depend on θ (as usual, the average over the pitch angle θ is carried out with a weight $\sin\theta$), we find the following equation for f_1:

$$\begin{aligned} &-\frac{e^2}{4m_i^2c^2}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left[\sum_\pm |H_k^\pm|^2(kv\cos\theta = \mp\omega_{Hi}) \times \frac{\sin\theta}{v|\cos\theta|}\frac{\partial f_1}{\partial\theta}\right] \\ &= v\cos\alpha\cos\theta\frac{\partial f_0}{\partial x} + \frac{v}{2}\left(\sin^2\theta - \frac{2}{3}\right)\frac{du}{dx}\frac{\partial f_0}{\partial v} \\ &+ \frac{Q\exp(-r/v_g\tau)}{8\pi^2r^2v_g\tau u^2}\delta(v-u)\left[\delta(\cos\theta - \cos\alpha) - \frac{1}{2}\right]. \end{aligned} \tag{12}$$

Here we have made use of the fact that since the diffusion lines in velocity space of the resonant particles are approximately circles (to within small quantities of order $\omega/kv \sim v_A/u$), the derivatives along the diffusion lines can be written in the form

$$\frac{\partial}{\partial v_\parallel} = -\frac{1}{v\sin\theta}\frac{\partial}{\partial\theta}.$$

We have transformed the δ-function on the right side of (7) in the following way:

$$\delta(v_\perp^2 - u_\perp^2)\delta(v_\parallel + u_\parallel) = (2u^2)^{-1}\delta(v-u)\delta(\cos\theta - \cos\alpha).$$

Integrating (12) over θ, and choosing the integration constant from the condition that $\partial f_1/\partial\theta$ be nonsingular in the limit $\sin\theta \to 0$, we find the following relation for the derivative $\partial f_1/\partial\theta$, which is a measure of the degree of anisotropy of the resonant-ion distribution:

$$\begin{aligned} \frac{\partial f_1}{\partial\theta} = &-\frac{H_0^2}{\omega_{Hi}^2}\left[\sum_\pm |H_k^\pm|^2(kv = \mp\omega_{Hi}\cos^{-1}\theta)\right]^{-1} \\ &\times\Big\{2v^2\sin\theta|\cos\theta|\cos\alpha\frac{\partial f_0}{\partial x} \\ &+ \frac{2}{3}\cos^2\theta\sin\theta\frac{du}{dx}v\frac{\partial f_0}{\partial v} \\ &+ \frac{Q\exp(-r/v_g\tau)}{2\pi^2r^2v_g\tau u}|\cos\theta|\sin^{-1}\theta\delta(v-u) \\ &\times\left[\sigma(\theta-\alpha) + \frac{1}{2}(\cos\theta - 1)\right]\Big\}, \end{aligned} \tag{13}$$

where $\sigma(x)$ is the unit step function [$\sigma(x) = 1$ at $x > 0$, $\sigma(x) = 0$ at $x < 0$]. The anisotropy of the comet ion distribution results in continuous excitation of Alfvén waves in the plasma of the solar wind, at a growth rate[11]

$$\gamma^\pm(k) = -\frac{2\pi^2e^2}{m_i\omega_k}\frac{k}{|k|}\frac{\omega_{Hp}^2}{\omega_p^2}\int v_\perp^2\frac{\partial f_1}{\partial\theta}\left(v_\parallel = \mp\frac{\omega_{Hi}}{k}\right)dv_\perp. \tag{14}$$

To calculate the growth rate of Alfvén waves from this formula with the help of (13), we need to determine the isotropic part of the comet ion distribution $f_0(u,v)$; i.e., we need to determine the motion of the loaded solar wind in the region ahead of the shock wave. We turn now to this problem.

SOLAR WIND AND COMETARY IONS AHEAD OF A SHOCK WAVE; EXCITATION OF ALFVÉN WAVES

We will use the kinetic approximation for the comet ions and hydrodynamics for the motion of the loaded solar wind. This approach was taken previously by Wallis and Ong.[16] In contrast to their treatment,[16] we are ignoring three-dimensional isotropy which the Alfvén turbulence tends to impose the distribution of the comet ions. We find an equation for the comet ion distribution function by averaging Eq. (7) over θ and substituting in $\partial f_1/\partial\theta$ from (13). The spatial diffusion caused by the MHD turbulence becomes important in this equation only near the shock front, where it causes smearing of the front to dimensions comparable to $L_{\rm diff}$ [see (11)]. Spatial diffusion can be ignored in the loaded solar wind ahead of the front, where the length scale of the perturbed zone is substantially greater than $L_{\rm diff}$, so that the equation for f_0 becomes

$$u\frac{\partial f_0}{\partial x}-\frac{1}{3}\frac{du}{dx}v\frac{\partial f_0}{\partial v}=\frac{Q\exp(-r/v_g\tau)}{16\pi^2r^2v_g\tau u^2}\delta(v-u). \qquad (15)$$

The equation for f_0 should be solved simultaneously with the hydrodynamic equations for the solar wind, which model the loading of the wind by the comet ions:

$$\frac{d}{dx}(u\rho)=\frac{Qm_i}{4\pi r^2v_g\tau}\exp\left(-\frac{r}{v_g\tau}\right), \qquad (16)$$

$$\frac{d}{dx}(\rho u^2+P_{xx})=0. \qquad (17)$$

The pressure is due primarily to the cometary ions; by virtue of the conditions $u\gg v_A$, $(T_p/m_p)^{1/2}$, we can ignore the components of the pressure due to the magnetic field and also the plasma of the solar wind. We thus write $P_{xx}=\int m_iv_x^2f_0d\mathbf{v}$. Substituting $v_x=v(\sin\theta\cos\varphi\sin\alpha-\cos\theta\cos\alpha)$ into this expression, and making use of the isotropy of f_0, we find

$$P_{xx}=\iint\frac{m_iv^2}{2}\left[\cos^2\alpha\int_0^\pi\cos^2\theta\sin\theta\,d\theta+\frac{\sin^2\alpha}{2}\int_0^\pi\sin^3\theta\,d\theta\right]f_0v^2\,dv\,d\varphi \qquad (18)$$

$$=\frac{1}{3}\int m_iv^2f_0\,d\mathbf{v}.$$

We can use (16) to express the right side of (15) in terms of the change in the mass flux in the solar wind, ρu. Transforming to the independent variables u, $w=v^3u$ in this equation (lines of $w=$ const are characteristics of the homogeneous equation), and integrating the result over u, we find the following expression for the comet ion distribution function:

$$f_0(w,u)=\frac{1}{4\pi m_i}\int_{u_\infty}^{u}\frac{du'}{u'^3}\frac{d}{du'}(\rho'u')\delta[w^{1/3}/u'^{1/3}-u']. \qquad (19)$$

Substituting f_0 into (18) and integrating over v by making use of the δ-function, we easily find an expression for the pressure created by the comet ions:

$$P_{xx}=\frac{1}{3u^{5/3}}\int_{u_\infty}^{u}\frac{d}{du'}(\rho'u')u'^{8/3}\,du'. \qquad (20)$$

Using this expression, integrating (17) over u, and carrying out some straightforward manipulations, we find a final equation which relates the change in the mass flux in the solar wind, ρu, to the change in the wind velocity:

$$\frac{\rho u}{\rho_\infty u_\infty}=\frac{5}{4}\frac{u_\infty}{u}-\frac{1}{4}\frac{u_\infty^2}{u^2}. \qquad (21)$$

This result is the same as the hydrodynamic result in the case of an adiabatic index $\gamma=5/3$; this value corresponds to the situation in which the comet ion distribution becomes isotropic in three dimensions.[17] A novelty in our approach is the determination of the comet ion distribution function. Using (21) for ρu in (19), we find the final expression for the comet ion distribution function:

$$f_0(u,v)=\frac{15}{64\pi m_i}\frac{\rho_\infty u_\infty^2}{v^{15/4}u^{5/4}}\left(1-\frac{2}{5}\frac{u_\infty}{u^{1/4}v^{3/4}}\right)\times\sigma(v-u)\sigma\left(\frac{u_\infty^{4/3}}{u^{1/3}}-v\right). \qquad (22)$$

The width of the distribution function is nonzero because ions reach each point x flowing with the solar wind which have arisen ahead of this point and therefore have velocities $v>u(x)$. The maximum ion energy increases to a level $(u_\infty/u)^{2/3}$ times the injection energy $m_iu_\infty^2/2$ because of adiabatic heating in the inhomogeneous solar wind.

Finally, integrating Eq. (16) over x, we find a simple relation which determines the increase in the mass flux in the solar wind as a result of the heating of the wind by cometary ions:

$$\rho u=\rho_\infty u_\infty\left[1+\frac{r_0}{r}\Psi\left(\frac{r}{v_g\tau}\right)\right]. \qquad (23)$$

Here we have used the notation

$$r_0=Qm_i/4\pi\rho_\infty u_\infty v_g\tau,\quad \Psi(y)=e^{-y}-y\,\mathrm{Ei}(y), \qquad (24)$$

where

$$\mathrm{Ei}(y)=\int_y^\infty e^{-\xi}\xi^{-1}\,d\xi$$

is the integral exponential function.

It follows from a comparison of (21) and (23) that the solution which has been derived here to describe the progressive slowing the loaded solar wind is applicable under the conditions $u\geqslant 2u_\infty/5$, $\rho u\leqslant 25\rho_\infty u_\infty/16$. Actually, it follows from (21) that under the condition $u<2u_\infty/5$ the slowing of the solar wind as it moves toward the comet does not lead to an increase in the mass flux ρu (Fig. 2 shows ρu as a function of u_∞/u), in contrast to (23). This means that there cannot be a smooth transition in the loaded solar wind from supersonic to subsonic flow, and it occurs at Mach number $M=1$, as we will show below. The dependence of the pressure in the solar wind on the wind velocity can be found easily from (20) and (21):

$$P_{xx}={}^1/_4\rho_\infty u_\infty^2[u_\infty/u-1]. \qquad (25)$$

We can then find the expression

$$c_s^2=\frac{\gamma P_{xx}}{\rho}=\frac{u^2}{3}\frac{u_\infty-u}{u-{}^1/_5u_\infty} \qquad (26)$$

for the sound velocity, and for $u=\frac{4}{5}u_\infty$ we do indeed have $M=u/c_s=1$. Because of the singularity in the smooth solution at $M=1$, there must be a shock wave, at transition to subsonic flow takes place, in the supersonic part of the flow, with $M>1$. According to Ref. 18, this shock wave lies at $M=2$. It follows from a comparison of (26) with Eqs. (21) and (23) that near the front of the $M=2$ shock wave the slowing of the solar wind satisfies $u/u_\infty = 23/35$, the increase in the mass flux carried by the solar wind is $\rho u/\rho_\infty u_\infty = 1.33$, and the position of the front in the bow region lies $r = 1.9r_0 \approx 330\,000$ km from the core of the comet. By way of comparison, the corresponding figures when the isotropization of the ion distribution caused by the interaction with Alfvén waves is not taken into account[19] are $\rho u/\rho_\infty u_\infty = 1.185$ and $r = 2.5r_0$. As a result, this isotropization strongly influences the law of motion of the loaded solar wind ahead of the shock front, and it also strongly influences the position of the front. The position of the shock front has been calculated for parameters appropriate to the solar-wind plasma and Halley's constant, as measured on the Vega-1 space vehicle: $n_0^\infty = 12$ cm^{-3}, $H_0 = 11$ nT, $T = 1.3\cdot 10^5$ K, $Q = 1.3\cdot 10^{30}$ molecules/s, and $v_g\tau = 2\cdot 10^{11}$ cm. The result agrees with observations of the crossing of the front by the vehicle.[20]

As was mentioned above, the presence of strong Alfvén turbulence, with oscillations of the magnetic field at an amplitude $H' \leqslant H_0$, near the shock front smears the front to dimensions comparable to the value $L_{\rm diff}$ found from (11), in total analogy with the shock front of cosmic rays.[6] The width of the front (under the condition $H' < H_0$) is on the order or greater than the Larmor radius of the comet ions, at least m_i/m_p times greater than the width of the jump in a planetary shock wave and on the order of the wavelength scale of the waves in an Alfvén turbulence. This circumstance seriously hinders an identification of the shock front against the background of the intense MHD oscillations, as apparently occurred during the fly-by of comet Jacobini-Zinner.[1] A diffused front structure was also observed during the fly-by of Halley's comet by the Vega-2 space vehicle, for which the magnetometer data indicated crossing of a quasiparallel shock wave.[20]

We turn now to a calculation of the growth rate of the Alfvén waves. We consider a region quite far ahead of the shock front, where the condition $u(x) \approx u_\infty$ holds. In calculating the growth rate from (14) and (13), ignoring the small quantities $\sim \Delta u/u$, we can then discard from $\partial f_0/\partial x$ everything except the term $\sim \delta(v-u)$, and we can omit from the right side of (13) the term $\sim \delta f_0/\partial v$. In this approximation we find

$$\frac{\partial f_1}{\partial \theta} = -\frac{1}{\omega_{Hi}^2}\frac{H_0^2}{|H_k|^2}\frac{Q\exp(-r/v_g\tau)}{4\pi^2 r^2 v_g \tau u}\delta(v-u) \times\left[\frac{\cos\alpha}{2}\sin\theta|\cos\theta| + 2\frac{|\cos\theta|}{\sin\theta}\left(\sigma(\theta-\alpha) + \frac{\cos\theta-1}{2}\right)\right]. \tag{27}$$

For angles $0 < \theta < \alpha$ the derivative is positive: $\partial f_1/\partial\theta > 0$. In this angular range an instability stems from the excitation of waves with $k < 0$, i.e., waves moving toward the comet, through the normal Doppler effect ($\omega - kv\cos\theta = \omega_{Hi}$). The growth rate for these waves is determined by the relation

$$\gamma_L = \Gamma(k)\left[(1-\cos\alpha)\left(1-\frac{\omega_{Hi}^2}{k^2u_\infty^2}\right) + \left(1-\frac{\omega_{Hi}}{ku_\infty}\right)^2\right],$$
$$1 < \frac{|k|u_\infty}{\omega_{Hi}} < \frac{1}{\cos\alpha}, \tag{28}$$

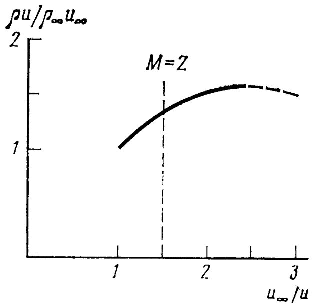

FIG. 2. Mass flux in the loaded solar wind as a function of the relative slowing of the wind, u_∞/u.

where

$$\Gamma(k) = \frac{H_0^2}{k|H_k|^2}\frac{Q\exp(-r/v_g\tau)}{16r^2 v_g n_0^\infty}\frac{m_i}{m_p}\frac{\omega_{Hi}}{\omega_k\tau}\left(1-\frac{\omega_{Hi}^2}{k^2u_\infty^2}\right). \tag{29}$$

In the angular region $\alpha < \theta < \pi/2$ the instability mechanism is a resonance due to the anomalous Doppler effect, which leads to the excitation of waves with $k > 0$ and to a growth rate

$$\gamma_L = \Gamma(k)\left[\cos\alpha\left(1-\frac{\omega_{Hi}^2}{k^2u_\infty^2}\right) + 2 + 2\frac{\omega_{Hi}}{ku_\infty}\right],$$
$$\frac{1}{\cos\alpha} < \frac{ku_\infty}{\omega_{Hi}} < \infty. \tag{30}$$

Finally, at $\theta > \pi/2$ the excitation of waves and the diffusion of particules are due to a resonance involving the Doppler effect. As a result of the instability, waves with $k > 0$ are also excited, at a growth rate

$$\gamma_L = \Gamma(k)\left[\cos\alpha\left(1-\frac{\omega_{Hi}^2}{k^2u_\infty^2}\right) + 2 - 2\frac{\omega_{Hi}}{ku_\infty}\right],$$
$$1 < \frac{ku_\infty}{\omega_{Hi}} < \infty. \tag{31}$$

Consequently, over the entire pitch-angle interval $0 < \theta < \pi$ and instability mechanism operates to excite Alfvén waves and to maintain the isotropy of the distribution function of the resonant particles. The continuous production of new ions in the solar wind, however, creates a small anisotropy in the distribution function, which is the source of the Alfvén-wave instability.

NONLINEAR SATURATION OF THE INSTABILITY; STEADY-STATE SPECTRA; INCLINATION OF THE SOLAR WIND

A process which has been termed "induced scattering of oscillations" in the literature (cf. Ref. 21, for example) results in our case from a resonance between protons of the solar-wind plasma and beats which arise in the nonlinear interaction of Alfvén waves. The condition for the resonance is

$$\omega_k \pm \omega_{k'} = (k \pm k')v. \tag{32}$$

The plus sign in this condition corresponds to absorption of two wave quanta with opposite polarizations, with frequencies ω_k and $\omega_{k'}$, and with wave numbers k and k', by the resonant proton. The minus corresponds to the scattering of a wave quantum ω_k into a quantum $\omega_{k'}$ of the same polarization. In the solar-wind plasma which is interacting with a comet, we would usually have a parameter value

$\beta=8\pi n_0T/H_0^2<1$, so that the proton thermal velocity v_{Tp} would be significantly less than the Alfvén velocity v_A. In this case, only the scattering of wave quanta is important among the resonances (32); for opposite signs of k and k', this scattering can occur on thermal protons with velocities $v_\parallel\lesssim v_{Tp}$, provided only that the condition $||k|-|k'||\sim kv_{Tp}/v_A\ll k$, holds. Absorption, on the other hand, requires superthermal protons with velocities $v_\parallel\sim v_A$, whose number is exponentially small. The growth rate of the induced scattering of Alfvén waves, calculated in Ref. 12, is

$$\gamma_{NL}=-\left(\frac{\pi}{8}\right)^{1/2}\frac{e^2}{m_p^2}\frac{T_p}{m_pc^2}\frac{k}{v_A\omega_{Hp}^2}\left[1+\frac{1+2T_p/T_e}{2(1+T_p/T_e)^2}\right]\times\sum_{k'}\frac{\omega_k-\omega_{k'}}{|k+k'|v_{Tp}}\exp\left[-\frac{(\omega_k-\omega_{k'})^2}{2(k+k')^2v_{Tp}^2}\right]|H_{k'}|^2. \tag{33}$$

Induced scattering which conserves the total number of wave quanta,

$$\frac{d}{dt}\left(\sum_k\frac{|H_k|^2}{\omega_k}\right)=0,$$

gives rise to spectral pumping of wave energy to small wave numbers: $\gamma_{NL}>0$ under the condition $\omega_{k'}>\omega_k$. The resonant absorption by the comet ions of the long ($k<\omega_{Hi}/u_\infty$) Alfvén waves which are excited during this pumping sets the stage for additional acceleration of these ions through stochastic Fermi acceleration. The high-energy ion tails with an energy $\mathscr{E}_i\lesssim500$ keV which were observed during the fly-by of Halley's comet[22] result specifically from this mechanism, but a detailed analysis of this question goes beyond the scope of the present paper.

In the spectral region $k>\omega_{Hi}/u_\infty$ the spectral pumping driven by the induced scattering leads to nonlinear saturation of the cyclotron instability. The limiting amplitudes at which this saturation sets in can be found by comparing the linear growth rate [(28) or (30)] and the nonlinear growth rate, (33). These limiting amplitudes can be found in order of magnitude from the expression

$$\overline{|H_k|^2}=H_0^2\frac{u_\infty^2}{\omega_{Hi}v_{Tp}}\left[\frac{Q}{4\pi r^2v_gn_0^\infty}\frac{1}{\omega_{Hi}\tau}\frac{m_i}{m_p}\right]^{1/2}. \tag{34}$$

An expression analogous to (34), for the limiting amplitudes of Alfvén waves, was derived in Ref. 3. For the properties of the gas and the plasma measured during the fly-by of comet Jacobini-Zinner ($Q=5\cdot10^{28}$ molecules/s $v_g\tau=2\cdot10^{11}$ cm, $T=10^5$ K, $n_0^\infty=5$ cm^{-3}, $u=470$ km/s, and $H=8$ nT), the length scale found from (24) is $r_0=2\cdot10^9$ cm. For the parameters given above for Halley's comet we find $r_0=1.7\cdot10^{10}$ cm. We then see from (34) that with $r\sim(3\text{–}5)r_0$ we have $\Sigma_k|H_k|^2\sim H_0^2$, in both cases. From this result it was concluded in Ref. 3 (for the first time) that extremely strong MHD turbulence prevails near the shock front.

We turn now to a solution of the equations describing the cyclotron excitation of waves by comet ions and their spectral pumping due to induced scattering. In terms of the dimensionless variables

$$\varkappa=ku_\infty/\omega_{Hi},\ \xi=x/r_0,\ |h_\varkappa|^2=|H_k^{(1)}|^2/\overline{|H_k|^2},\quad |b_\varkappa|^2=|H_k^{(2)}|^2/\overline{|H_k|^2}$$

[the superscripts (1) and (2) correspond to waves with $k>0$ and $k<0$, i.e., to waves which are moving away from and toward the comet in the coordinate system of the solar wind], these equations take the form

$$\frac{v_A}{u_\infty}C\left(\cos\alpha-\frac{v_A}{u_\infty}\right)\frac{\partial|h_\varkappa^\pm|^2}{\partial\xi}=\frac{\pi}{\xi^2}\frac{1}{\varkappa^2}e^{-\lambda\xi}\times\left(1-\frac{1}{\varkappa^2}\right)F^\pm(\varkappa)\frac{|h_\varkappa^\pm|^2}{|h_\varkappa^\pm|^2+|b_\varkappa^\mp|^2}-\frac{\eta}{4}\frac{\varkappa}{(\pi\beta)^{1/2}}\times\int\frac{\varkappa-\varkappa'}{|\varkappa+\varkappa'|}\exp\left[-\frac{(\varkappa-\varkappa')^2}{\beta(\varkappa+\varkappa')^2}\right]|h_\varkappa^\pm|^2|b_{\varkappa'}^\pm|^2d\varkappa', \tag{35}$$

$$\frac{v_A}{u_\infty}C\left(\cos\alpha+\frac{v_A}{u_\infty}\right)\frac{\partial|b_\varkappa^\pm|^2}{\partial\xi}=\frac{\pi}{\xi^2}\frac{e^{-\lambda\xi}}{\varkappa^2}\left(1-\frac{1}{\varkappa^2}\right)\Phi^\pm(\varkappa)\frac{|b_\varkappa^\pm|^2}{|b_\varkappa^\pm|^2+|h_\varkappa^\mp|^2}-\frac{\eta}{4}\frac{\varkappa}{(\pi\beta)^{1/2}}\int\frac{\varkappa-\varkappa'}{|\varkappa+\varkappa'|}\exp\left[-\frac{(\varkappa-\varkappa')^2}{\beta(\varkappa+\varkappa')^2}\right]|b_\varkappa^\pm|^2|h_{\varkappa'}^\pm|^2d\varkappa'. \tag{36}$$

Here

$$C=\left(\frac{Q}{4\pi r_0^2v_gn_0^\infty}\frac{m_i}{m_p}\frac{1}{\omega_{Hi}\tau}\right)^{1/2},$$

$$\lambda=r_0/v_g\tau,\qquad \eta=1+\frac{1+2T_p/T_e}{2(1+T_p/T_e)^2},$$

$$F^\pm(\varkappa)=\left[\frac{\cos\alpha}{2}\left(1-\frac{1}{\varkappa^2}\right)\pm\left(1-\frac{1}{\varkappa}\right)\right]\times\sigma(\varkappa-1)+(1\mp1)\sigma\left(\varkappa-\frac{1}{\cos\alpha}\right),$$

$$\Phi^\pm(\varkappa)=-F^\mp(\varkappa).$$

As usual, the signs $\pm$ correspond to waves which are polarized in the ion revolution direction—a resonance with ions involving the normal Doppler effect—and polarized in the opposite direction—a resonance involving the anomalous Doppler effect. In writing Eqs. (35) and (36) we have made use of the fact that the pitch-angle diffusion of the resonant

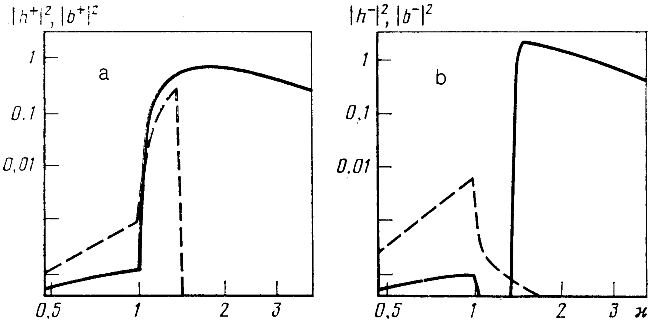

FIG. 3. Spectral energy density of Alfvén waves. a—Left-hand-polarized waves ($|h_\varkappa^+|^2$, $|b_\varkappa^+|^2$); b—right-hand-polarized waves ($|h_\varkappa^-|^2$, $|b_\varkappa^-|^2$). Solid lines) Waves with $k>0$ propagating away from the comet in the coordinate system of the solar wind ($|h_\varkappa^+|^2$, $|h_\varkappa^-|^2$); dashed lines) waves with $k<0$ propagating toward the comet ($|b_\varkappa^+|^2$, $|b_\varkappa^-|^2$). These results were found with the values $\xi=1.5$, $\lambda=0$, $\beta=0.35$, $\alpha=45°$.

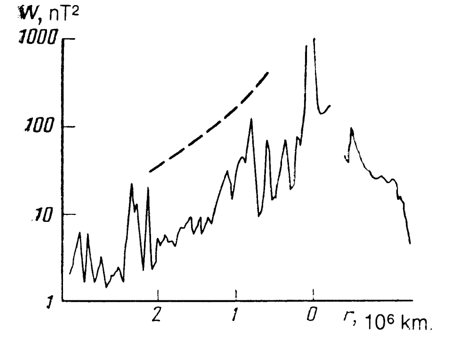

FIG. 4. Energy of magnetic fluctuations, $W = \int df(|H_{xf}|^2 + |H_{yf}|^2 + |H_{zf}|^2)$, in the frequency range $10^{-3} < f < 10^{-2}$ Hz according to measurements by a magnetometer carried on the Vega-1 vehicle, for various distances from Halley's comet. The dashed line is a theoretical prediction based on the property values of the solar wind and of the neutral gas corresponding to this fly-by.

particles is caused simultaneously by waves of the two different polarizations, propagating in opposite directions, i.e., $|h_\varkappa^\pm|^2$ and $|b_\varkappa^\mp|^2$. Figures 3 and 4 show the results of a numerical integration of these equations.

Figure 3 shows the spectral energy density of the waves for typical property values from the fly-by of Halley's comet at a distance $\xi = 1.5$ by a space vehicle. This situation corresponds to the vicinity of the front of the shock wave. This spectral density was found for $\beta = 0.35$, $\lambda = 0$, and $\alpha = 45°$. In the spectrum, in addition to the linearly unstable waves $|h_\varkappa^+|^2$ $(\varkappa > 1)$, $|b_\varkappa^+|^2$ $(1 < |\varkappa| < 1/\cos\alpha)$, and $|h_\varkappa^-|^2 (\varkappa > 1/\cos\alpha)$, there are waves in the long-wave region, $\varkappa < 1$, with a significant amplitude. These are primarily waves which are propagating toward the comet, $|b_\varkappa^+|^2$, $|b_\varkappa^-|^2$, which appear as the result of a nonlinear interaction due to induced scattering. The amplitude of these waves is quite high, $\sim 1/10$, $1/30$ in dimensionless units.

Most of the energy in this spectrum, however, corresponds to linearly unstable waves with $\varkappa \gg 1$. For such waves, the nonlinear interaction is weak in our example, and we find from Eqs. (35) and (36) that their spectral density has the behavior

$$|H_k|^2 \propto 1/k^2 \quad \text{for} \quad ku \gg \omega_{Hi}.$$

The maximum in the spectrum corresponds to waves with $ku_\infty \approx \omega_{Hi}$. These waves are carried along by the solar wind toward the comet, and the Doppler shift causes the frequency of these waves as measured by the space vehicle to be

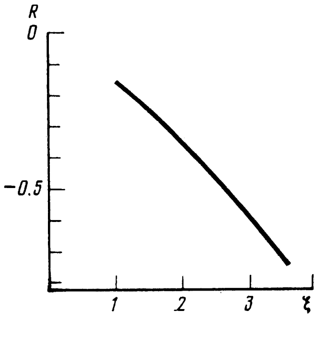

FIG. 5. The function $R(\xi)$ for the values $\lambda = 0$, $\beta = 0.5$, and $\alpha = 45°$.

$$f = \frac{k|u_\parallel|}{2\pi} \approx \frac{\omega_{Hi}}{2\pi}\cos\alpha \sim 10^{-2}\ \text{Hz}$$

for typical cometary conditions.

As we have already mentioned, the resonant absorption of Alfvén waves leads to a tail of high-energy ions with an energy well above the maximum energy of an ion entrained on a cycloid by the solar wind, $\mathscr{E}_{max} = 2m_i u_\infty^2 \sin^2\alpha$. This effect was observed during the fly-bys of both comet Jacobini-Zinner and Halley's comet.

Figure 4 shows the total energy of the MHD waves, $\Sigma_{k,\pm,s=1,2} |H_k^{(s)\pm}|^2$, as a function of the distance from the core of the comet. This plot was constructed from Eqs. (35) and (36) for the property values given above for the plasma and the gas for the fly-by of Halley's comet by Vega-1. Shown for comparison in this figure is the intensity of MHD waves in the frequency interval $10^{-3} < f < 10^{-2}$ Hz according to measurements by a magnetometer during this fly-by.[20]

The slight anisotropy in the comet ion distribution leads to an off-diagonal component of the pressure tensor:

$$P_{yx} = \int m_i v_y v_x f\, dv = {}^1/_2 m_i \sin 2\alpha \int v^2 (\cos^2\theta - {}^1/_2 \sin^2\theta) f_1\, dv. \quad (37)$$

It follows from the equations of motion that the presence of an off diagonal component in the pressure tensor results in a "deflection" of the flux of solar wind, i.e., the appearance of a velocity component perpendicular to the original direction of motion:

$$u_y = P_{yx}/\rho_\infty u_\infty. \quad (38)$$

Integrating by parts over θ in the expression for P_{yx}, substituting in $\partial f_1/\partial\theta$ from (11), and using the dimensionless variables given above, we can write an equation for u_y as follows:

$$u_y = \sin\alpha \cos\alpha C v_{Tp} R(\xi) e^{-\lambda\xi}/\xi^2, \quad (39)$$

where

$$R(\xi) = \int_0^{\pi/2} \frac{\sin\theta\cos^2\theta}{|b_\varkappa^+|^2 + |h_\varkappa^-|^2} (\cos\theta - 1 + 2\sigma(\theta - \alpha) + {}^1/_2 \cos\alpha \sin^2\theta)\, d\theta$$
$$- \int_{\pi/2}^{\pi} \frac{\sin\theta\cos^2\theta}{|h_\varkappa^+|^2 + |b_\varkappa^-|^2} (\cos\theta + 1 + {}^1/_2 \cos\alpha \sin^2\theta)\, d\theta,$$
$$\varkappa = \frac{1}{\cos\theta}.$$

The function $R(\xi)$ is plotted in Fig. 5. Since over most of the ξ region we have $R(\xi) < 0$, the flux of solar wind is deflected downward, with the result that the wings of the shock wave acquire an asymmetry, as shown in Fig. 1. For the property values of comet Jacobini-Zinner near the shock front, the degree of anisotropy is $|u_y|/u \sim 10^{-1}$. Observations confirm this result in both sign and order of magnitude.[4]

Let us summarize the results of this study.

1. The loading of the solar wind by heavy comet ions leads to the excitation of strong MHD turbulence ahead of the shock front, with typical frequencies on the order of 10^{-2}

Hz and with magnitude fluctuations having a relative amplitude on the order of unity.

2. The interaction with the Alfvén turbulence causes the distribution function of the comet ions to become more nearly isotropic in the coordinate system of the solar wind. In other words, the interaction with this turbulence creates an effective mechanism for collective entrainment of the comet ions by the solar wind. This isotropization strongly influences the motion of the loaded solar wind and the position of the shock front. The position of the front is noticeably closer to the core of the comet than it would be according to a model which ignores the excitation of an Alfvén turbulence.

3. The presence of strong Alfvén turbulence near the front of a cometary shock wave results in a substantial increase in the width of the front, to dimensions comparable to or greater than the Larmor radius of the heavy ions. In other words, the front width is about m_i/m_p times greater than that of a planetary shock wave.

4. The continuous production of new ions in the solar wind maintains the weak anisotropy of the distribution function of these ions and is the reason for the "deflection" of the flow of the solar wind and the anisotropy in the position of the wings of the shock wave—an anisotropy which agrees in sign and order of magnitude with observations.

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