Kinetics of quasi-two-dimensional electrons in the one-dimensional potential of a superlattice located in a quantizing magnetic field

Yu. B. Grebenshchikov, F. R. Ulinich, and N. A. Usov

I. V. Kurchatov Institute of Atomic Energy

(Submitted 25 September 1986; resubmitted 11 December 1986)
Zh. Eksp. Teor. Fiz. **92**, 2170–2186 (June 1987)

The dissipative and Hall components of the electric current due to the motion of quasi-two-dimensional electrons in a quantizing magnetic field and in the smooth potential of a one-dimensional superlattice are calculated as functions of the Landau-level population for a broad range of electric field strengths by means of the quantum-kinetic-equation method. The effect of the electric field and lattice temperature on the parameters of the plateau in the dependence of the Hall conductivity on the occupancy of the Landau level is determined, together with the current-voltage characteristic of the system. It is demonstrated that the interesting nonlinear regimes occur in the experimentally accessible region of parameter values. The effect of the electron-electron interaction on the results obtained is discussed, and it is shown that the electron-temperature approximation is inapplicable in a number of important cases.

§ 1. INTRODUCTION

The quantum Hall effect[1] is at present a subject of great interest, but no complete theory has as yet been constructed for this phenomenon. There is the conductivity-quantization problem proper, i.e., the question of why the isolated Landau level, the overwhelming majority of the states of which may be localized, makes the ideal contribution to the Hall current even though it is occupied. It is precisely this problem that has been given the main attention in the last few years, and it has been explained qualitatively[2–5] and, under certain assumptions about the form of the single-electron spectrum, quantitatively.[6–8] At the same time there is a second aspect to the problem—the question of the nature of, and the mechanisms underlying, the conductivity of a partially occupied Landau level. A clear understanding of the processes that occur here is necessary for the determination of the plateau parameters in the quantum Hall effect, the dependence of these parameters on the amount of impurities in the sample, the temperature, the electric field intensity, etc. The possible thermal and other corrections to the ideal value of the Hall resistance in the region of the plateau determine the accuracy and reproducibility of the quantum resistance standard and the accuracy of measurement of the fundamental constant $e^2/\hbar$ (Ref. 9).

The basic difficulty in the problem of computing the conductivity of a partially occupied Landau level lies in the macroscopic degeneracy of the single-electron states of the level in the absence of perturbations. Apparently, this difficulty was first fully recognized by Baskin *et al.*,[10] who have shown that the use of the so-called self-consistent Born approximation in earlier papers devoted to the computation of the conductivity tensor for a two-dimensional electron gas located in a quantizing magnetic field (see a review of these papers in Ref. 11) cannot be rigorously justified. It has become clear that we should, in constructing a consistent theory, make allowance from the very beginning for the lifting of the degeneracy of the states of the ideal Landau level by a perturbing potential. One of the first attempts in this direction was made by the present authors in Ref. 12 (see also Ref. 13). We constructed a theory of hopping conductivity in the Landau level's impurity band formed by those states of this level which are split off by the Coulomb impurity centers randomly distributed in the insulator in the vicinity of the inversion layer. But such a theory can qualitatively describe the situation only in the limit of low or high occupancy of the Landau level, i.e., in the $\nu \ll 1$ or $1-\nu \ll 1$ limit, and not too high impurity-center concentrations. For the case of arbitrary level of occupancy of the Landau level the analysis of the experimental data is often carried out with the use (see, for example, Ref. 14) of the smooth-potential model proposed in Refs. 3–5, 15. Specifically, it is assumed that the self-consistent potential acting on the electrons in the inversion layer and due, as a rule, to the fluctuations of the density of the charged impurity centers in the insulator can be split up into a long-wave regular part, which can, in a sense, be taken into account in the equations of motion of the electron exactly, and a scattering short-range component. An electron, drifting along smooth quasiclassical trajectories, hops from one to another as a result of collisions with the short-range scattering centers, the phonons, and other electrons. But the topological structure of the two-dimensional network of quasiclassical trajectories in an arbitrary smooth potential remains fairly complicated, and consistent computations have thus far not been carried out in this model.

Recently, Aĭzin and Volkov[16,17] showed that the degeneracy of the states of the Landau level can also be lifted by the periodic potential of a one-dimensional superlattice, and computed in the linear response approximation the components of the system's conductivity tensor for the case $\lambda \gtrsim A$, where $\lambda = (c\hbar/eB)^2$ is the magnetic length and A is the superlattice constant. Such a method of lifting the degeneracy of the Landau level seems to us to be fruitful. Since a bare single-electron spectrum arises in the problem, we can construct a theory in terms of the quantum kinetic equation. This, in principle, allows us to carry over the results of the fairly complete three-dimensional theory (see, for example, Ref. 18) to the present case and take into consideration in a unified fashion the phonons, the impurities, the residual electric fields, the interelectron Coulomb interaction, etc. We emphasize that the case of the residual electric fields is of special interest at present in connection with experiments[19,20] on the destruction of the plateau in the quantum Hall effect in strong electric fields.

 0038-5646/87/061223-10$04.00

The experimental study of the properties of one-dimensional superlattices in a quantizing magnetic field is only beginning.[21] In view of this in the present paper we consider the relatively simple case of the one-dimensional smooth ($\lambda < A$) potential with the aid of the quantum-kinetic-equation method. Our analysis of the joint action of the elastic and phonon mechanisms of scattering in the residual electric fields reveals a fair degree of complexity and a wealth of galvanomagnetic phenomena in the system in question, and this, we hope, will stimulate experimental investigations. The case of the two-dimensional smooth periodic potential can also be studied in this formalism, which will be done in a separate paper. In this connection let us note that the two-dimensional quasiclassical trajectories in an arbitrary smooth potential are actually quasi-one-dimensional. Therefore, the study of the phenomena in the one-dimensional potential of a superlattice is, in our opinion, an important first step in the investigation of the general case.

We shall assume that the amplitude V_0 of the superlattice potential is small in comparison with the Landau-level spacing $\hbar\omega_c^*$, and that the scattering impurity centers are short-range centers. It is assumed that the electrons interact only with longitudinal acoustic phonons, since the optical-phonon energy satisfies $\hbar\omega_0 > V_0$. The phonons are considered to be an equilibrium subsystem. (As has been shown in experiments,[19,20] lattice overheating does not occur in electric fields with intensities of up to $F \approx 100$ V/cm.) We shall study the dynamics of the electrons of a given, partially occupied Landau level, neglecting the electron transitions from the level in question to the other Landau levels.

In § 2 we construct and solve the quantum kinetic equation with allowance for the scattering of the electrons by the impurities and phonons. We consider the regimes of weak, medium (warming), and strong electric fields. In § 3 we compute the dissipative and Hall currents as functions of the field intensity F and occupancy of the Landau level. In § 4 we carry out estimates, which show that the occupancy of the upper (initially empty) Landau levels is exponentially small in a wide range of electric field intensities. There we also discuss the effect of the interelectron collisions on the results obtained.

§ 2. SOLUTION OF THE QUANTUM KINETIC EQUATION

The Hamiltonian of the system under consideration consists of the unperturbed Hamiltonian

$$\hat{H}_0 = \hat{H}_{0e} + \hat{H}_{oph} = \left(\mathbf{p} + \frac{e}{c}\mathbf{A}\right)^2 \Big/ 2m^* + V_0(x) + eFx + \sum_{\mathbf{q}} \hbar\omega_{\mathbf{q}} \left(\hat{b}_{\mathbf{q}}^+ \hat{b}_{\mathbf{q}} + \frac{1}{2}\right) \tag{1}$$

($V_0(x)$ is a one-dimensional superlattice potential with period A and amplitude V_0, F is the external electric field intensity, and $\mathbf{A} = (0, Bx, 0)$ is the vector potential of a homogeneous magnetic field B) and perturbation operators. These operators are the electron-impurity interaction operator $\hat{V}_1$, averaged over the z motion of the electrons in the ground subband of the dimensional quantization with the Stern-Howard wave function[22] $\Psi_0 = (b^3/2)^{1/2} z \exp\{-bz/2\}$:

$$\hat{V}_1(\mathbf{r}) = V_1 \sum_{j=1}^{N_{im}} \varphi(\mathbf{r} - \mathbf{R}_j) \tag{2}$$

(here b^{-1} is the effective thickness of the layer; $\mathbf{r}$ and $\mathbf{R}_j$ are two-dimensional vectors in the plane of the electron layer) and the operator describing the deformation interaction of the electrons with the longitudinal mode of the three-dimensional acoustic phonons:

$$\hat{H}_{e-ph} = -i\varkappa_0 \sum_{\mathbf{q}} \left(\frac{\hbar q}{2s\rho\Omega}\right)^{1/2} \exp\{i\mathbf{q}_\perp \mathbf{r} + iq_z z\} (\hat{b}_{\mathbf{q}} - \hat{b}_{-\mathbf{q}}^+), \tag{3}$$

where $\mathbf{q} = (\mathbf{q}_\perp, q_z)$ is the wave vector of the phonons, s is the velocity of sound, $\rho\Omega$ is the crystal mass, and $\varkappa_0$ is the electron-phonon interaction constant.

The quantum kinetic equation can be constructed by means of the well-known procedure (see, for example, Ref. 18) for determining the diagonal elements of the density matrix of the system described by the Hamiltonian (1)–(3) in the basis of the wave functions of the unperturned single-electron Hamiltonian $\hat{H}_{0e}$. On account of the one-dimensional nature of the interactions entering into $\hat{H}_{0e}$, the wave functions of this Hamiltonian are characterized by the Landau quantum number N and the x coordinate X of the center of the oscillator. In the lowest order in the interaction with the scatterers the collision integral in the quantum kinetic equation is equal to the sum of the electron-phonon and electron-impurity collision integrals ($\mathrm{St}_{\mathrm{ph}}$ and $\mathrm{St}_{\mathrm{im}}$, respectively):

$$\partial f_N(X)/\partial t = \mathrm{St}_{im}[f_N(X)] + \mathrm{St}_{ph}[f_N(X)]. \tag{4}$$

The distribution function $f_N(X)$ has the meaning of the probability for occupation of the single-electron states of the Hamiltonian $\hat{H}_{0e}$.

Further, we consider only the case of sufficiently slowly varying superlattice potentials, i.e., the case in which $\lambda < A$, so that the distribution function $f(X)$ is a smooth function of the quasicontinuous quantum number X. Since in an electron-phonon collision the characteristic electron displacement $\Delta X_{\mathrm{ph}} \ll \lambda$, the corresponding collision integral can be written in the Fokker-Planck approximation:

$$\mathrm{St}_{ph}[f] = \frac{1}{\tau_{ph}} u(E) \frac{\hbar s}{\lambda} \frac{d}{dE}\left\{K(E)\left[T\frac{df}{dE} + f(1-f)\right]\right\}. \tag{5}$$

In (5) we went over to the energy representation of the distribution function $f(X) = f(E)$ by expressing the coordinate X in terms of the energy with the aid of the dispersion law, which for a slowly varying potential in first order perturbation theory in terms of the small parameter $V_0/\hbar\omega_c^* \ll 1$ is given by the expression

$$E_N(X) = V_0(X) + eFX + \mathrm{const}. \tag{6}$$

Furthermore, in (5) we define the bare electron-phonon scattering time, $\tau_{\mathrm{ph}} = 2\pi^2\hbar\rho s^2\lambda^3/\varkappa_0^2$, and the modulus of the local electron drift velocity normalized to the velocity of sound, $u(E) = |\lambda^2 (dE(X)/dX)\hbar s|$. The dimensionless coefficient $K(E)$ can be expressed in terms of the probabilities for local electron hops along the "slope" of the superlattice potential (the rate of similar jumps between slopes is negligible):

$$K(E)=u^{-2}\int_0^\infty dtt^3\exp\{-t^2/2\}J[t(1-u^{-2})^{1/2}],\quad u(E)>1,$$
$$K(E)=0,\quad u(E)\leqslant 1,\tag{7}$$
$$J(t)=\int_0^1 dx\exp\{t^2x^2/2\}(1-x^2)^{-1/2}[1+x^2(t/\lambda b)^2]^{-3}.$$

In deriving (5) we assumed the phonon system to be an equilibrium one, and used for the phonon occupation numbers the limiting value $N(\hbar\omega)=T/\hbar\omega$, assuming the lattice temperature T to be high. These assumptions are not needed for the purpose of writing down the electron-phonon collision integral in the form (5), and do not lead to any qualitative changes in the results obtained below. Of importance, however, is the fact that the collision integral (5) is nonzero only when the condition $u(E)>1$ is fulfilled. In those energy (coordinate) regions where the local slopes are small, i.e., where $u(E)<1$, the electrons cannot, on the basis of the energy and momentum conservation laws, emit or absorp phonons, and the collision integral (5) is equal to zero. This circumstance can be taken into account by assuming that $K(E)=0$ in these regions. In the vicinity of the points $u(E)=1$ the electron-phonon collision integral cannot be written in the form (5) and has a complicated form, but the total length of these regions is small in comparison with the superlattice constant. In the absence of an external electric field the collision integral (5) describes the relaxation of the nonequilibrium distribution function to the Fermi function $f_0(E)$, which satisfies the integral identically.

The electron-elastic-impurity collision integral in (4) describes the hopping of electrons from one slope of the superlattice potential to other slopes under conditions when the electron energy is conserved. Let us denote in terms of X_i the roots of the equation $E(X)=E$. Then

$$\mathrm{St}_{im}[f(X)]=\frac{1}{\tau_{im}}\sum_i\frac{1}{m_i}w(X_i-X)[f(X_i)-f(X)],$$
$$w(X)=\exp\{-X^2/2\lambda^2\}\varphi^2(X/\lambda).\tag{7}$$

In (8) we have introduced the bare time

$$\tau_{im}=\hbar V_0/V_1^2(n_{im}\pi a^2)(a/\lambda)^2$$

characterizing the scattering of the electrons by short-range impurity centers (with amplitude V_1 and range $a\ll\lambda$) uniformly distributed in the plane of the electron layer with concentration n_{im}, and have explicitly separated out the exponential dependence of the probability for electron scattering by the impurities on the distance $|X_i-X|$ between the initial and final jump points. The dimensionless function $\varphi^2(X/\lambda)\approx 1$, which depends on the shape of the impurity potential and the Landau-level number, is a slowly varying function of its argument. The quantity $m_i=|\lambda(dE(X_i)/dX)/V_0|$ is equal to the normalized electron density of states at the final jump points.

To determine the stationary distribution function, we must solve Eq. (4) with the condition $\partial f/\partial t=0$. It is clear that, in the stationary regime, the distribution function $f(X)$ is a periodic function of the coordinate X, with period equal to the superlattice constant: $f(X+A)=f(X)$. Consequently, it is sufficient to consider the steady-state kinetic equation over a period, which we shall take to be the interval between consecutive maxima of the dispersion curve (6), e.g., between the maxima at the points $X=-A$ and $X=A-A_1$ in Fig. 1. Let us set $E(-A_1)=E_0$; then from (6) it follows that $E(A-A_1)=E_0+\Delta$, where $\Delta=eFA$. Since the inverse function $X=X(E)$ is multiple-valued, let us, going over to the energy variable in the collision integral (8), set ($l=0,\pm 1,\ldots$)

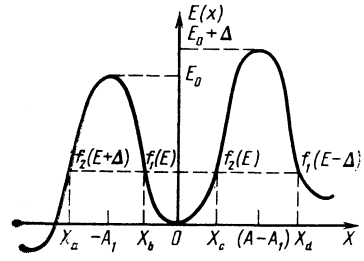

FIG. 1.

$$f(X)=f_1(E-l\Delta),\quad -A_1+lA\leqslant X\leqslant lA,$$
$$f(X)=f_2(E-l\Delta),\quad lA\leqslant X\leqslant(l+1)A-A_1,\tag{9}$$

where, as can be seen from Fig. 1, the function $f_1(E)$, which is prescribed on the left slope of the fundamental ($l=0$) period, is defined in the energy interval $[0,E_0]$, while the function $f_2(E)$, which is prescribed on the right slope of this period, is defined in the energy interval $[0,E_0+\Delta]$.

The electron-impurity collision integral (8) relates the value of the function $f_1(E)$ at the point X_b (see Fig. 1) lying in the fundamental period with the values of $f_2(E+\Delta)$ and $f_2(E)$ at the points X_a and X_b (nearest neighbors) respectively, as well as with the values of $f_1(E-\Delta)$ and $f(E+\Delta)$ in the neighboring wells, etc. Taking account of the fact that the probability for electron scattering by the impurities decreases exponentially with increasing jump distance, we shall, in the case (of interest to us here) of slowly varying superlattice potentials, i.e., for $\lambda<A$, limit ourselves in (8) to only the contribution of the nearest neighbors. The densities of states on the slopes of the fundamental period are in fact functions of the energy: $m(X_b)=m_1(E)$, $m(X_c)=m_2(E)$. On account of the periodicity of dE/dX, the density of states at the point X_a will be $m(X_a)=m_2(E+\Delta)$. Let us again set $w(X_b-X_a)=w_1(E)$, $w(X_c-X_b)=w_2(E)$ and

$$g_1(E)=\frac{w_1(E)}{m_1(E)m_2(E+\Delta)},\qquad g_2(E)=\frac{w_2(E)}{m_1(E)m_2(E)}.\tag{10}$$

Then from (4), (5), and (8) it follows that the distribution function $f_1(E)$ satisfies the equation

$$-(\tau_{im}/\tau_{ph})V_0\frac{d}{dE}\left\{K_1(E)\left[T\frac{df_1}{dE}+f_1(1-f_1)\right]\right\}$$
$$=g_1(E)[f_2(E+\Delta)-f_1(E)]+g_2(E)[f_2(E)-f_1(E)],\tag{11a}$$

where we have taken account of the fact that, in our notation, $u(X_b)\hbar s/\lambda\ V_0m_1(E)$, and have set $K[u(X_b)]=K_1(E)$. The function $f_2(E)$ in the energy interval $[\Delta,E_0]$ satisfies a similar equation:

$$-(\tau_{im}/\tau_{ph})V_0\frac{d}{dE}\left\{K_2(E)\left[T\frac{df_2}{dE}+f_2(1-f_2)\right]\right\}$$
$$=g_1(E-\Delta)[f_1(E-\Delta)-f_2(E)]+g_2(E)[f_1(E)-f_2(E)], \quad (11b)$$

where we have set

$$K[u(X_c)]=K_2(E), \quad u(X_c)\hbar s/\lambda=V_0m_2(E)$$

and have taken account of the fact that $m(X_d)=m_1(E-\Delta)$ and $w(X_d-X_c)=w_1(E-\Delta)$, since $X_d(E)-X_c(E)=X_b(E-\Delta)-X_a(E-\Delta)$. In the energy interval $[0, \Delta]$ we should discard the first term on the right-hand side of Eq. (11b), and in the interval $[E_0, E_0+\Delta]$, the second term on the same side of the equation, since in these energy intervals the electron at X_c does not have neighbors to the right and left, respectively, of it.

Equations (11) for the determination of the functions $f_i(E)$ constitute a complicated nonlinear system of second-order differential-difference equations. We shall formulate the boundary conditions to this system as the need arises.

Before proceeding to solve the system (11), let us note that the collision integrals (5) and (8) have completely different effects on the distribution function. The electron-phonon collision integral tends to establish local thermodynamic equilibrium on the slopes of the individual wells, whereas the electron-impurity collisions lead to the transfer of the charge carriers from well to well, and hence to the spreading of the distribution function, because of its periodicity, over the well. If $u_0=\lambda^2V_0/\hbar sA$ is the characteristic local drift velocity and $K_0=u_0^{-2}$ is the characteristic value of the coefficient $K(E)$, then the electron-phonon collision integral (5) can be estimated as

$$\mathrm{St}_{ph}[f]\sim\frac{1}{\tau_{ph}^*}\frac{\hbar s}{\lambda}\frac{d}{dE}\left[T\frac{df}{dE}+f(1-f)\right],$$

where $\tau_{ph}^*=\tau_{ph}u_0$ is the effective electron-phonon collision time, which has the meaning of energy relaxation time, and $\hbar s/\lambda$ is the characteristic energy transfer. Let us, for the purpose of estimating the collision integral (8), introduce the characteristic probability for the occurrence of a jump during the scattering of an electron by the impurities, $w_0=\exp\{-(\Delta X)^2/2\lambda^2\}$ (ΔX is the characteristic jump distance), and the mean density of states $m_0=u_0\hbar s/\lambda V_0$. Then $\mathrm{St}_{im}[f]\sim[f-f]/\tau_{im}^*$, where $\tau_{im}^*=\tau_{im}m_0/w_0$ is the effective electron-impurity collision time.

Let us estimate the ratio

$$\tau_{im}^*/\tau_{ph}^*=(\tau_{im}/\tau_{ph})(\hbar s/\lambda V_0)w_0^{-1}.$$

For the case of the inversion layer on the surface of a silicon sample located in the typical magnetic field $B=1.5\times10^5$ Oe, the time $\tau_{ph}\approx3\times10^{-11}$ sec, and it is convenient to express the quantity τ_{im} in terms of the time $\tau^{(0)}$ characterizing the scattering of an electron by the short-range impurity centers in the absence of a magnetic field: $\tau_{im}=(2\pi)^{1/2}\tau^{(0)}V_0/\hbar\omega_c^*$, which can be estimated from the low-temperature mobility, $\tau^{(0)}\approx2\times10^{-12}$ sec.[11] Further, setting $\hbar s/\lambda=4$ K and $\hbar\omega_c^*\approx100$ K, we find $\tau_{im}^*/\tau_{ph}^*\approx10^{-2}w_0^{-1}$. Thus, the ratio of the effective scattering times is a rapidly varying function of the energy. In the vicinity of the energy E^* at which the above probabilities for jumps to the nearest neighbors are equal, i.e., at which $g_1(E^*)=g_2(E^*)$, the characteristic jump distance satisfies $\Delta X\approx A/2$. In the region $E<E^*$, the length of the longer jump should be taken as the characteristic jump distance, since it is precisely the longer jump that corresponds to the transfer of the electron from the well in question to the neighboring well. In the region $E>E^*$ an analogous argument applies to the holes.

Of greatest interest is the case in which the intensity of the electron-impurity scattering is high, i.e., in which $\tau_{im}^*\ll\tau_{ph}^*$, in the entire important energy region. The inequality $\tau_{im}^*\ll\tau_{ph}^*$ holds for not-too-slowly varying superlattice potentials, i.e., for the case in which $A/\lambda\lesssim3$ or 4. Then the current in the system is generated largely as a result of the elastic hops of the electrons on the impurities, and the role of the phonons reduces to that of determining the principal (symmetric) part of the distribution function. In this case the time τ_{im}^* is similar to the usual electron scattering time in momentum space, since the ability of an electron to acquire energy in an electric field can be expressed in terms of this quantity. Indeed, a jump of an electron from a given well to a neighboring one is, on account of the periodicity of $f(X)$, equivalent to a shift in the electron energy in the well in question by an amount $\Delta=eFA$. Consequently, the rate of energy acquisition by an electron in the field is equal to $(\dot{E})_{inel}=\Delta/\tau_{im}^*$. On the other hand, the energy loss by the electron as a result of the emission of phonons with characteristic energy $\hbar s/\lambda$ is equal to $(\dot{E})_{inel}=\hbar s/\lambda\tau_{ph}^*$. Therefore, the nature of the solution to the steady-state kinetic equation depends essentially on the magnitude of the parameter $\gamma=(\dot{E})_{el}/(\dot{E})_{inel}=\Delta\lambda\tau_{ph}^*/\hbar s\tau_{im}^*$. In the case when $\tau_{im}^*\ll\tau_{ph}^*$, the transition from the $\gamma\ll1$ to the $\gamma\gg1$ regime occurs in relatively weak electric fields. Increasing F, we obtain different characteristic cases, which are analyzed below in Subsecs. a), b), and c).

In the opposite limiting case of very slowly varying superlattice potentials, i.e., for $A/\lambda\gtrsim6$ or 7, on account of the smallness of w_0 we have $\tau_{im}^*\gg\tau_{ph}^*$. The electron-impurity collision integral will be nonzero only in the vicinity of the extrema of the magnetic band (6). Therefore, the distribution function is determined almost everywhere by the electron-phonon collision integral, the current in the system is generated as a result of the interaction with the phonons, and the role of the impurity scattering reduces to the closure of the current in those regions around the extrema where the collision integral (5) is equal to zero. This case is analyzed below in Subsec. d). Finally, in the region of extremely strong electric fields, i.e., in the region $\Delta>V_0$, where the derivative $E'(x)$ is of constant sign, the collision integral (8) is identically equal to zero, and (5) leads to a solution similar to the one for the homogeneous case, which was analyzed earlier by Erukhimov.[23]

a) The linear case

Let us begin the solution of the system (11) with the simplest linear case, i.e., the case of weak electric fields, in which $\gamma\ll1$. Furthermore, we assume $\Delta\ll\varepsilon_0$, where $\varepsilon_0=V_0(\lambda/A)^2$ is the characteristic scale of the variation of the functions $g_i(E)$—the coefficients in the equations (11)—whose dependence on energy is the most critical. (The characteristic energy ε_0 can be found from the estimate for the index of the exponential function in the formula (8).) Besides being energy dependent, the coefficients $K_i(E, F)$, g_i

(E, F) also depend on the electric field intensity F as a parameter. Let us, using the condition $\Delta \ll \varepsilon_0$, expand these quantities in power series in F, limiting ourselves to the lowest nonvanishing terms. Furthermore, in the linear approximation it is natural to seek the distribution function in the form $f_i(E) = f_0(E) + f_i^{(1)}(E)$, where the small corrections $f_i^{(1)}(E)$ are linear in the electric field and $f_0(E)$ is the Fermi function with lattice temperature T. As the boundary conditions to the system (11) in the present case, let us take the condition for the vanishing of the corrections $f_i^{(1)}$ at points far from the Fermi level μ_0, which in turn can be determined from the normalization condition, namely, the condition for the maintenance of the number of electrons in the well at the equilibrium value.

Linearizing the system (11) in the usual manner, we obtain for the corrections $f_i^{(1)}$ the expressions

$$f_i^{(1)} = \frac{1}{2} g_1^{(0)} (g_1^{(0)} + g_2^{(0)})^{-1} \Delta \frac{df_0}{dE}, \quad f_2^{(1)} = -f_1^{(1)}, \tag{12}$$

hwere we have set $g_i^{(0)} = g_i(\mu, 0)$. As can be seen from the solutions (12), the corrections $f_i^{(1)}$ indeed decrease rapidly in the region $|E - \mu_0| > T$. As usual, the dependence of the solution on the parameter γ manifests itself only in the corrections that are nonlinear in the electric field.

b) The warming up of the electron gas

As the electric field intensity increases, the parameter γ increases, and in the limit $\gamma \gg$ the electron acquires energy from the electric field faster than it dissipates it through the emission of phonons. This leads to the spreading of the distribution function along the slopes of the well, so that the characteristic energy scale T_e of the variation of the functions $f_i(E)$ is greater than the lattice temperature. Therefore, the quantities $f_i(E \pm \Delta)$ entering into (11) can be expanded in power series in the small parameter $\Delta/T_e \ll 1$. Assuming also that $\Delta/\varepsilon_0 \ll 1$, and normalizing the coefficients g_i and K_i to their characteristic values K_0 and g_0 in the region of energies far from the magnetic-band edges, i.e., setting $K_i/K_0 = \bar{K}_i$ and $g_i/g_0 = \bar{g}_i$, we can write the system (11) in the form

$$-\frac{\Delta}{\gamma}\frac{d}{dE}\{\bar{K}_1[T(df_1/dE) + f_1(1-f_1)]\} = (\bar{g}_1+\bar{g}_2)[f_2 - f_1] + \bar{g}\left[\Delta(df_2/dE) + \frac{\Delta^2}{2}(d^2f_2/dE^2)\right], \tag{13a}$$

$$-\frac{\Delta}{\gamma}\frac{d}{dE}\{\bar{K}_2[T(df_2/dE) + f_2(1-f_2)]\} = (\bar{g}_1+\bar{g}_2)[f_1 - f_2] + \left(-\Delta\frac{d\bar{g}_1}{dE} + \frac{\Delta^2}{2}\frac{d^2\bar{g}_1}{dE^2}\right)[f_1 - f_2] - \bar{g}_1\left(\Delta\frac{df_1}{dE} - \frac{\Delta^2}{2}\frac{d^2f_1}{dE^2}\right) + \Delta^2\frac{d\bar{g}_1}{dE}\frac{df_1}{dE}. \tag{13b}$$

As can be seen from (13), $f_2 - f_1 \sim \Delta/T_e \ll 1$. Therefore, let us seek the solution to this system in the form $f_i = \Phi + f_i^{(1)}$, where the corrections $f_i^{(1)}$ are proportional to the first power of the parameter Δ/T_e. Neglecting the left members in (13), since $\gamma \gg 1$ in the present case, we find

$$f_2^{(1)} - f_1^{(1)} = -\bar{g}_1(\bar{g}_1+\bar{g}_2)^{-1}\Delta(d\Phi/dE). \tag{14}$$

Adding the equations (13), and using (14), we obtain an equation for the determination of the unknown function $\Phi(E)$:

$$\frac{1}{\gamma}\frac{d}{dE}\left\{(\bar{K}_1+\bar{K}_2)\left[T\frac{d\Phi}{dE} + \Phi(1-\Phi)\right]\right\} + \Delta\frac{d}{dE}\left\{\bar{G}(E)\frac{d\Phi}{dE}\right\} = 0, \tag{15}$$

where $\bar{G} = \bar{g}_1\bar{g}_2/(\bar{g}_1+\bar{g}_2)$. Assuming that the function $\Phi(E)$ decreases to zero in the vicinity of the upper magnetic-band edge (i.e., as $E \to E_0$), and setting $T_e(E) = T + \gamma\Delta\bar{G}/(\bar{K}_1+\bar{K}_2)$, we find from (15) that

$$\Phi(E) = \left[\exp\left\{\int_{\mu}^{E} dE'/T_e(E')\right\} + 1\right]^{-1}, \tag{16}$$

where the integration constant μ is determined from the condition for the constancy of the number of particles in the well.

Let us note that the electron temperature T_e which we have formally introduced is a rapidly varying function of the energy (on account of the critical dependence of the function $\bar{G}(E)$ on the energy), so that, generally speaking, the function $\Phi(E)$ differs greatly from the Fermi function. We can only speak of a local electron temperature in the corresponding energy (coordinate) region. Let the chemical potential μ_0 of the system in equilibrium lie below the energy E^* at which $g_1(E^*) = g_2(E^*)$. Then in not too strong electric fields the quantities $g_i(\mu)$ differ greatly from each other, and as the characteristic value we should take $g_0 = \min\{g_1(\mu), g_2(\mu)\}$, since it is precisely the smaller of the two quantities $g_i(\mu)$ that enters into the definition of τ_{im}^*. For $\mu < E^*$, $g_0 = g_1(\mu) \ll g_1(E^*) \ll g_2(\mu)$. Hence $\bar{G}(\mu) \approx 1$, and since $\bar{K}_i \approx 1$ always, the quantity $T_e(\mu) \approx T + \gamma\Delta/2$. Depending on the magnitude of the ratio $\tau_{\text{im}}^*/\tau_{\text{ph}}^*$, we can have a case of weak ($\gamma\Delta < T$) or strong ($\gamma\Delta > T$) detachment of the electron temperature at $E = \mu$. But irrespective of the magnitude of $\gamma\Delta$, the electron temperature is higher, i.e., $T_e(E) \gg T_e(\mu)$, in the energy region $E > \mu + \Delta$, since the quantity $\bar{G}(E)$ increases exponentially in the region $E > \mu$ with a characteristic scale of variation ε_0, attaining the maximum value at $E = E^*$: $\bar{G}(E^*) = g_1(E^*)/2g_1(\mu) \gg 1$. Therefore, the function $\Phi(E)$ has in the energy region $E > \mu + \varepsilon_0$ a long plateau extending right up to energies of the order of $2E^* - \mu$ (for the approximately symmetric function $\bar{G}(E) = G(2E^* - E)$), where we again have $\bar{G}(E) \approx 1$. The amplitude of this plateau depends on the relation between $T_e(\mu)$ and ε_0. If $T_e(\mu) < \varepsilon_0$, then in the region of energies of order ε_0 in the vicinity of $E = \mu$, the function $\Phi(E)$ decreases like the Fermi function with temperature $T_e(\mu)$, and is able to decrease to the value $\exp[-\varepsilon_0/T_e(\mu)]$ before the growth of T_e with increasing E begins to have an effect. Since in this case the amplitude of the plateau is small, the normalization constant satisfies $\mu \approx \mu_0$.

c) The case of strong electric fields

As can be seen from the preceding subsection, in medium (warming) electric fields the symmetric part of the distribution function $\Phi(E)$ still retains some features of the equilibrium case: $\Phi(E) \approx 1$ at the bottom of the magnetic band, and, subsequently, after the plateau has come to an

end, rapidly decreases to a value practically equal to zero. But in strong electric fields the parameter γ can be so large that the distribution function, under the action of the electron-impurity collisions, greatly flattens out, and becomes roughly constant over the entire extent of the magnetic band. In this case the analysis carried out in Subsec. b) is inapplicable: we must take account of the fact that, in the vicinity of the magnetic-band edges, in an interval Δ (which constitutes a substantial fraction of the amplitude V_0 of the potential in strong electric fields), the right member of Eq. (11b) does not contain either the first or second term. Let us first consider the commensurable case in which the interval Δ is contained in the left slope of the fundamental period (see Fig. 1) an integral number of times, so that $E_0/\Delta = n$ is a whole number. Then the interval Δ will be contained in the right slope of this period exactly $n+1$ times. Let us set in the s-th interval $E = E_s + \varepsilon, E_s = (s-1)\Delta$, $0 \leqslant \varepsilon \leqslant \Delta$. Let us also set $f_i(E_s+\varepsilon) = f_{is}(\varepsilon)$, $g_i(E_s+\varepsilon) = g_{is}(\varepsilon)$, $i = 1,2$. Then from (11) we obtain a system of equations relating the distribution functions from neighboring intervals:

$$\hat{D}_{1s}(f_{1,s}(\varepsilon)) = g_{1,s}[f_{2,s+1}(\varepsilon) - f_{1,s}(\varepsilon)] + g_{2,s}[f_{2,s}(\varepsilon) - f_{1,s}(\varepsilon)], \tag{17a}$$

$$\hat{D}_{2s}(f_{2,s}(\varepsilon)) = g_{1,s-1}[f_{1,s-1}(\varepsilon) - f_{2,s}(\varepsilon)](1-\delta_{1,s}) + g_{2,s}[f_{1,s}(\varepsilon) - f_{2,s}(\varepsilon)],$$

where $s = 1, 2, \ldots, n$, δ_{ik} is the Kronecker symbol, and for $s = n+1$ we have the single equation

$$\hat{D}_{2,n+1}(f_{2,n+1}(\varepsilon)) = g_{1,n}[f_{1,n}(\varepsilon) - f_{2,n+1}(\varepsilon)]. \tag{17b}$$

The differential operators standing on the left-hand sides of the equations (11) and proportional to the small parameter $\gamma^{-1} \ll 1$ have been denoted by the $\hat{D}_{is}(f_{is})$ in (16). Let us seek the solution to the system (17) in the form of a series in powers of the parameter γ^{-1}: $f_{is} = f_{is}^{(0)} + f_{is}^{(1)} + \ldots$. In the zeroth approximation in this parameter, we discard the left members of the equations (17), and find that the $f_{is}^{(0)}$ are determined by the single function $f^{(0)}$, defined in the interval Δ, so that $f_{is}^{(0)} = f^{(0)}$. In the next approximation we substitute into the left members of the system (17) the function $f^{(0)}$; into the right members, the corrections $f_{is}^{(0)}$. Considering the function $f^{(0)}$ and, with it, the left members, $\hat{D}_{is}(f^{(0)}) = \hat{D}_{is}^{(0)}$, of the system (17) to be prescribed, we can successively express the $f_{is}^{(1)}(\varepsilon)$ in terms of the function $f_{2,1}^{(1)} = f^{(1)}(\varepsilon)$, which is also defined in the interval Δ:

$$f_{1,k}^{(1)}(\varepsilon) = f^{(1)}(\varepsilon) + \sum_{l=1}^{k-1} \frac{T_{1,l} + T_{2,l}}{g_{1,l}} + \sum_{l=1}^{k} \frac{T_{1,l-1} + T_{2,l}}{g_{2,l}}, \tag{18a}$$

$$f_{2,k+1}^{(1)}(\varepsilon) = f^{(1)}(\varepsilon) + \sum_{l=1}^{k} \left\{ \frac{T_{1,l} + T_{2,l}}{g_{1,l}} + \frac{T_{1,l-1} + T_{2,l}}{g_{2,l}} \right\}, \tag{18b}$$

where we have set

$$T_{i,l} = \sum_{s=1}^{l} \hat{D}_{i,s}^{(0)}, \qquad T_{i0} \equiv 0.$$

The indicated procedure for obtaining the corrections $f_{i,k}^{(n)}$ can be continued indefinitely, and, thus, the functions $f_{is}(\varepsilon)$ can be expressed in terms of the function $f_{2,1}(\varepsilon) = f^{(0)} + f^{(1)} + \ldots$, which is defined in the interval Δ. The form of the function $f^{(0)}$ can be found on the basis of the following arguments. It is easy to see that the sum over s of the right members of the equations (17) vanishes identically. Therefore, for the corresponding system of equations for the determination of the corrections $f_{is}^{(1)}$ to be solvable, we must require that the sum of the left members of the equations (17) vanish, i.e., that

$$\sum_{s=1}^{n} \hat{D}_{1,s}(f^{(0)}) + \sum_{s=1}^{n+1} \hat{D}_{2,s}(f^{(0)}) = 0. \tag{19}$$

The relation (19) is actually a differential equation for the determination of the function $f^{(0)}$. Integrating it once, we obtain

$$T\, df^{(0)}/dE + f^{(0)}(1-f^{(0)}) = C\Big[\sum_{s=1}^{n} K_{1,s}(\varepsilon) + \sum_{s=1}^{n+1} K_{2,s}(\varepsilon)\Big]^{-1}, \tag{20}$$

where C is a normalization constant, proportional to the density of the electric current flowing along the superlattice axis. Another constant, which is required for the determination of $f^{(0)}$, can be found from the matching condition, $f^{(0)}(0) = f^{(0)}(\Delta)$, necessary for the continuity of $f^{(0)}(E)$ over the entire extent of the magnetic band. It is clear that the symmetric part of the distribution function $f^{(0)}(E)$ is periodic, with period Δ, in the magnetic band.

On the basis of similar arguments, we can obtain an equation for the determination of the function $f^{(1)}(\varepsilon)$. But for the computation of the currents in the system, it is sufficient to know the difference $f_{2,k}^{(1)} - f_{1,k}^{(1)}$, which, with allowance for (20), can be written in the form

$$f_{2,k}^{(1)} - f_{1,k}^{(1)} = C\frac{\tau_{im}}{\tau_{ph}}\frac{V_0}{g_{2,k}}\frac{d}{dE}\Big[\Big(\sum_{s=1}^{k-1} K_{1,s}(\varepsilon) + \sum_{s=1}^{k} K_{2,s}(\varepsilon)\Big) \times \Big(\sum_{s=1}^{n} K_{1,s}(\varepsilon) + \sum_{s=1}^{n+1} K_{2,s}(\varepsilon)\Big)^{-1}\Big] \tag{21}$$

The general (incommensurable) case, in which $E_0 = n\Delta + \Delta'$, $\Delta' < \Delta$, can be analyzed, using practically the same procedure developed above for solving the system (11). It is just sufficient to divide the interval $[0, E_0 + \Delta]$ into successive intervals of length Δ' and $\Delta - \Delta'$, and write down a system of equations similar to (17) in each of the intervals, the total number of which will now be $2n+3$. Of these equations only the $n+1$ equations pertaining to the $\Delta - \Delta'$ intervals and the $n+2$ equations pertaining to the Δ' intervals are coupled. Each of these systems of equations can be solved similarly to the system (17), and the symmetric part of the distribution function $f^{(0)}(E)$ is "sewn" together from the portions defined in the Δ' and $\Delta - \Delta'$ intervals.

The possible variation of the function $f^{(0)}(E)$ in the Δ interval depends on the form of the functions $K_i(E)$ and decreases with increasing number n of intervals, the function $f^{(0)}(\varepsilon)$ tending to a constant as Δ decreases. At the same time the amplitude of the aperiodic monotonic component connected with the corrections $f_{is}^{(1)}$ begin to increase, and the solution constructed in the present subsection joins the solution given in Subsec. b).

d) The case of weak impurity scattering

Let us now consider the case $\tau_{\text{im}}^* \gg \tau_{\text{ph}}^*$, which can be called the case of weak impurity scattering. As we have

shown above, this situation is clearly realized in the limit $A \gg \lambda$, when the electron-impurity collision integral can be neglected. Discarding the right members of the equations (11), and integrating once, we obtain

$$K_i[T(df_i/dE)+f_i(1-f_i)]=C_i, \tag{22}$$

where the C_i are integration constants. But in narrow regions around the extrema of the dispersion curve (6), where the conditions for phonon emission are not fulfilled and the coefficients K_i vanish, the collision integral (8) is relatively large because of the growth of the corresponding overlap integrals in these regions. The scattering of the electrons by the impurities in the vicinity of the extrema ensures the closure of the current in these regions, and, furthermore, brings together the values of the distribution functions f_i on different sides of an extremum. Therefore, the effect of the impurity scattering (an effect which is important only in the indicated narrow regions) on the form of the functions f_i in the principal energy region lying far from the magnetic-band edges can be qualitatively taken into account in the form of the corresponding boundary conditions on Eqs. (22). Specifically, we equate the distribution functions $f_i(E)$ at the bottom of the well (see Fig. 1) and in the vicinity of its edges, which, on account of the periodicity of the distribution function, leads to the conditions

$$f_1(0)=f_2(0), \quad f_1(E_0)=f_2(E_0+\Delta), \tag{23}$$

and also set $-C_1=C_2=J$, which, with allowance for the sign, implies the equality of the electric-current densities at the sides of the well. Then it is not difficult to show that, at lattice temperatures $T \lesssim V_0$ and a magnetic-band occupancy $\nu < \frac{1}{2}$, the current in the system is always exponentially small because of the fact that $f_i \sim \exp\{-E_0/T\}$ near the magnetic-band edges. For the constant J, which is proportional to the electric current density, we have

$$J \sim \exp\{-E_0/T\}(1-\exp\{-\Delta/T\}). \tag{24}$$

Everywhere above we spoke of electrons, assuming that the magnetic band occupancy satisfies $\nu \lesssim \frac{1}{2}$. In the case $\nu \gtrsim \frac{1}{2}$ the obtained results can easily be reformulated for the holes in the magnetic band.

§ 3. COMPUTATION OF THE DISSIPATIVE AND HALL CURRENTS

As is well known,[18,24] for a prescribed density matrix $\hat{f}$ of the system the average densities of the dissipative, j_x, and Hall, j_y, currents of the system can be computed from the formulas

$$j_x=\frac{1}{L_xL_y}\sum_{N,X} e\omega_c{}^2\lambda[2(N+1)]^{1/2}\,\mathrm{Im}\,f_{N,N+1}(X), \tag{25a}$$

$$j_y=\frac{1}{L_xL_y}\sum_{NX}\left\{e\omega_c{}^2\lambda[2(N+1)]^{1/2}\,\mathrm{Re}\,f_{N,N+1}(X)+\frac{c}{B}\frac{dE}{dX}f_N(X)\right\}, \tag{25b}$$

where the $|N,X\rangle$ are the basis wave functions of the unperturbed Hamiltonian $\hat{H}_{0e}$, (1). The off-diagonal density-matrix elements $f_{N,N+1}$ entering into these expressions can be expressed by means of the standard procedure[18] in terms of the diagonal elements $f_N(X)$, which we computed in the preceding section with the aid of the kinetic equation (4). After a number of tedious but simple transformations we can reduce (25) into a form convenient for the computation of the currents in the above-mentioned cases a), b), and c) from §2:

$$j_x=(e/2\pi\lambda\tau_{im}V_0)\int_0^{E_0} g_2(E)(f_2-f_1)\,dE, \tag{26a}$$

$$j_y=(e/2\pi\hbar A)\left[\int_0^{E_0}(f_2-f_1)\,dE+\int_{E_0}^{E_0+\Delta} f_2\,dE\right]. \tag{26b}$$

It is easy to see that (26a) takes account of the total number of electrons crossing in unit time the cross section $X=0$ (the bottom of the well in Fig. 1) as a result of impurity scattering. As we saw in §2, the electric-current component due to the electron-phonon collisions vanishes in the vicinity of the magnetic-band extrema; therefore, the formula (26a) gives the total dissipative current in the system. (Let us recall that we are neglecting the hopping of the carriers from slope to slope during the scattering by the phonons.) For an arbitrary cross section ($X \neq 0$) the formula equivalent to (26a) has a more complicated form. The expression (26b) shows that the Hall current in the system is proportional to the drift velocity averaged over the distribution $f(E)$.

Substituting the solutions (12) into (26), we obtain at sufficiently low lattice temperatures $T < \varepsilon_0$ in the linear case the expressions

$$j_x=\frac{e}{2\pi\lambda}\frac{1}{\tau_{im}}\frac{eFA}{V_0}G(\mu_0), \tag{27a}$$

$$j_y=\frac{e^2}{2\pi\hbar}F\frac{g_1(\mu)}{g_1(\mu)+g_2(\mu)}. \tag{27b}$$

These formulas give the implicit dependence of j_x and j_y on the magnetic band occupancy ν, which is connected with the position of the chemical potential μ of the system relative to the bottom of the band through the normalization condition ($T \ll V_0$):

$$\frac{\lambda}{AV_0}\int_0^{\infty} f_0(E)\left[\frac{1}{m_1(E)}+\frac{1}{m_2(E)}\right]dE=\nu. \tag{28}$$

Since $G(E)$ has a sharp peak of width ε_0 at $E=E^*$, in the present ($\lambda < A$) case the quantity j_x, as a function of the occupancy, has a sharp peak inside the magnetic band, in contrast to the case $\lambda \gtrsim A$, in which j_x has its maximum near band edges.[16] In this same region of energies $\sim\varepsilon_0$ the function $j_y(\nu)$ undergoes a steep jump in the vicinity of E^*, since for $\mu < E^*$ we have $g_1(\mu) \ll g_2(\mu)$, while for $\mu > E^*$, in contrast, we have $g_1(\mu) \gg g_2(\mu)$. Thus, this dependence has a stepwise form, characteristic of the quantum Hall effect (a similar result has been obtained by Aĭzin and Volkov[17] in the linear response formalism), $j_y(\nu)$ being, with exponential accuracy a multiple of $Ne^2/2\pi\hbar$. In the present case the shelves in the plot of $j_y(\nu)$ arise not as a direct consequence of the localization of the electron states, but as a result of the fact that the isoenergetic levels in the vicinity of the bottom of the well are strongly coupled by the electron-impurity interaction, and they have, with exponential accuracy, the same occupancy.

In the opposite ($T > \varepsilon_0$) limit, it is interesting to note that j_x and j_y are rapidly varying functions of the lattice temperature. For example, in the region $\mu < E^*$ the quantity $G(\mu)$ in (27a) is replaced by $G(E^*)(\varepsilon_0/T)\exp[-(E^*-\mu)/T]$, while the ratio $g_1(\mu)/g_2(\mu)$ in (27b) is replaced by $\exp[-(E^*-\mu)/T]$. This is due to the fact that the occupancy of the states with energy of the order of E^*, which make the effective contribution to the dissipative and Hall currents, increases exponentially with increasing temperature. For $3 \lesssim A/\lambda \lesssim 4$ and $30 \lesssim V_0 \lesssim 50$ K, the characteristic energy ε_0 lies in the range from 2 to 5 K, so that the indicated temperature dependences occur in a region convenient for experimental observation.

As F increases in the $\gamma \gg 1$ limit, the electron gas begins to warm up (see Subsec. b in §2), which leads to the appearance of critical dependences of j_x and j_y on the electric field intensity. Assuming the lattice temperature to be low, specifically, that $T \ll \gamma\Delta$, and substituting the expression (14) into (26), we find that in the region $\mu < E^*$

$$j_x = \frac{e}{2\pi\lambda}\frac{1}{\tau_{im}}\frac{eFA}{V_0}G(\mu)\left[1 + C_1\frac{2(E^*-\mu-\varepsilon_0)}{T_e(\mu)}\exp\left\{-\frac{\varepsilon_0}{T_e(\mu)}\right\}\right], \quad (29a)$$

where $C_1 \sim 1$; the second term in the brackets is connected with the appearance in the distribution function $\Phi(E)$ of a long plateau of width $2(E^*-\mu-\varepsilon_0)$ and height $\sim\exp[-\varepsilon_0/T_e(\mu)]$. The nonlinear function $j_x(F)$ arises on account of the fact that $T_e(\mu) \sim F^2$. Similar computations for the Hall current yield

$$j_y = \frac{e^2}{2\pi\hbar}F\left[\frac{g_1(\mu)}{g_2(\mu)} + C_2\frac{\varepsilon_0}{T_e(\mu)}\exp\left\{-\frac{\varepsilon_0}{T_e(\mu)}\right\}\right], \quad (29b)$$

where $C_2 \sim 1$. The second term in the formula (29a) is equal to the first term in the characteristic field

$$\Delta_1^x = [(\hbar s/\lambda)\varepsilon_0\tau_{im}^*/\tau_{ph}^* \ln(V_0/\varepsilon_0)]^{1/2}$$

(see Fig. 2), which can be considered to be the beginning of the nonlinear regime. The critical dependence $j_x(F)$ occurs in a fairly narrow range of fields right up to the field $\Delta_2 = \Delta_1^x ln^{1}(V_0/\varepsilon_0)$ in which the plateau height saturates ($T_e(\mu) \approx \varepsilon_0$). Notice that the nonlinear regime in the Hall current can arise much earlier, specifically, in a field of intensity

$$\Delta_1^y = \Delta_1^x\{\ln(V_0/\varepsilon_0)/\ln[g_2(\mu)/g_1(\mu)]\}^{1/2},$$

and end in the same field Δ_2. For $\Delta > \Delta_2$ the Hall current practically does not differ from the ideal current, whereas the dissipative current saturates, and then there appears a decreasing section in the dependence $j_x(F)$ in a characteristic field Δ_3. The appearance of this section is due to the fact that the distribution function is roughly a constant over the entire extent of the magnetic band in a strong electric field (see Subsec. c in §2). Substituting the expression (21) into (26), we find

$$j_x = \frac{ev(1-\nu)}{2\pi\lambda\tau_{ph}\Delta}\int_0^{E_0}[K_1(E)+K_2(E)]\,dE \approx 2\nu(1-\nu)\frac{e}{2\pi\lambda^2}\frac{A\hbar s/\lambda}{\tau_{ph}^* eFA}. \quad (30)$$

In deriving (30) we assumed that $n = E_0/\Delta \gg 1$, and replaced the corresponding sums in (21) by integrals. It is interesting to note that the expression (30) contains only the characteristics of the electron-phonon scattering, although in the case under consideration this scattering is much weaker than the electron-impurity scattering. This circumstance has a simple physical explanation. Because of the intense scattering by the impurities, the electron "spreads" along the chain of isoenergetic states belonging to different wells, and then slowly, over a time period of the order of τ_{ph}^*, hops from chain to chain, losing in each hopping act energy of the order of $\hbar s/\lambda$. The electron traverses a distance A before losing a total energy of Δ. Consequently, the mean electron velocity along the X axis is $A(\hbar s/\lambda)/\tau_{ph}^*\Delta$.

The characteristic field Δ_3 essentially gives the lower limit of applicability of the expressions obtained in Subsec. c of §2. Consequently, this field can be estimated on the basis of the argument that, near the upper magnetic-band edge, the corrections $f^{(1)}$, (18), are of the order of $f^{(0)}$. Replacing the sums in (18) by integrals, we find that $f_{ik}^{(1)} \sim \varepsilon_0/T_e(V_0)$. Since $T_e(V_0) = T_e(\mu)G(V_0)/G(\mu) < T_e(\mu)$ (see (16)), $\Delta_3 = \Delta_2[G(\mu)/G(V_0)]^{1/2}$.

Finally, in the region of fields $\Delta > \Delta_4 = V_0$, where the electrons can be scattered only by the phonons, the function $j_x(F) \sim F^{-2}$, similarly to the homogeneous case investigated by Erukhimov.[23]

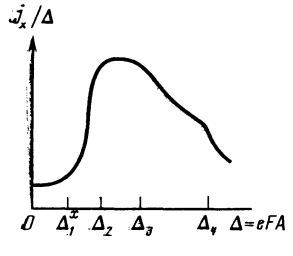

FIG. 2.

§4. CONCLUSION

Above we have studied the dynamics of the electrons of a given partially occupied Landau level, neglecting the possible coupling between the levels, as well as the electron-electron interaction. Let us consider the effect of these factors on the results obtained. Let us, for example, estimate the ratio of the occupancies, f_0 and f_1, of the ground and first excited Landau levels. Since $\hbar\omega_c^* \gg V_0$, in a fairly broad region of electric-field intensities the magnetic subbands of the various Landau levels lying within the limits of a small number of superlattice constants do not intersect, and the interband transitions are over large distances satisfy $\Delta X \gg A$ (see Fig. 3); therefore, these transitions are of low probability, and can give rise to only small corrections to the distribution functions f_0 and f_1 found from Eq. (4) without allowance for the interband transitions. But it is precisely the interband transitions that determine the ratio f_1/f_0 of the occupancies of neighboring levels. This ratio is easily estimated with the aid of a simple iteration procedure based on the exponential smallness of the ratio of the intensities of the interband and intraband electron transitions. It is precisely in the equation

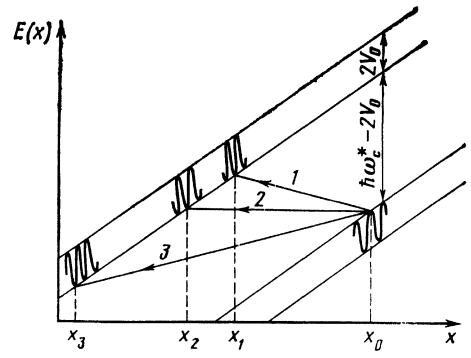

FIG. 3.

for f_0 that we shall take account of the interband transitions to the neighboring $N=1$ Landau level. Subsequently, we shall eliminate from this equation the large $\mathrm{St}(f_0, f_0)$-type collision terms describing the intraband transitions to the $N=0$ level by summing this equation over X. As a result we obtain the following relation between the amplitudes f_0 and f_1:

$$\sum_X \{\mathrm{St}_{im}[f_0(X); f_1(X')] + \mathrm{St}_{ph}[f_0(X); f_1(X')]\} = 0. \quad (31)$$

The estimation from (31) of the desired ratio f_1/f_0 is easiest in the case of large F, when the distribution functions $f_i \approx$ const in the magnetic bands. It is easy to see that in this case the most intense particle exchange occurs between the top of the lower and the bottom of the upper magnetic band. It follows from the laws of conservation of energy and momentum that the scattering by the phonons occurs with maximum effectiveness when the x component of the phonon momentum is equal to zero. The electron-jump distance in this case (the processes 1 and 3 in Fig. 3) is accordingly given by

$$X_0 - X_{1,3} = \lambda(\hbar\omega_c^* - 2V_0)/(eF\lambda \pm \hbar s/\lambda),$$

and the overlap integrals (see Ref. 23)

$$J_i = \exp\{-(X_0 - X_i)^2/2\lambda^2\}.$$

A similar formula relates the overlap integral J_2 in the expression for the probability for electron-impurity scattering (the process 2 in Fig. 3) with the corresponding jump distance[25] $X_0 - X_2 = (\hbar\omega_c^* - 2V_0)/eF$. The minimum energy of the phonon absorbed in the process 1 corresponds to the quite large quantity $(\hbar\omega_c^* - 2V_0)\hbar s/eF\lambda^2$; at low temperatures T the number of such phonons is exponentially small, and the process 1 is of low probability. Therefore, the dominant contribution to the population of the upper level is made by the electron-impurity scattering process 2, and the contribution of the phonon emission processes (the arrow 3 in Fig. 3) is negligible. On the other hand, the upper level is depopulated most rapidly through the emission of phonons in the process that is the inverse of the process 1 in Fig. 3. Comparing the principal exponential factors in the expressions for the rates of population, J_2 and depopulation, J_1, of the upper level, we find that, for

$$(\hbar s/eF\lambda^2)^3[(\hbar\omega_c^* - 2V_0)/(\hbar s/\lambda)]^2 > 1 \quad (32)$$

the occupancy of this level is exponentially small, and, contrary to the assumption made in Ref. 26, is not described by the Boltzmann law with any of the intraband "temperatures."

Owing to the fact that the initial and final points of the most probable jumps cannot coincide exactly with the extrema of the bands, the corrections to the indicated distances $X_0 - X_i$ are small, provided that $X_0 - X_i \gg A$. In this case the probabilities for the indicated transitions practically do not depend on, for example, the shape of the potential $V_0(x)$, whose only effect is to effectively decrease the cyclotron quantum by the width $2V_0$ of the magnetic band. Let us also note that, in the lowest order perturbation theory, the electrons are not allowed by the energy and momentum conservation laws to undergo interlevel transitions in the course of the scattering by each other. Assuming that $\hbar s/\lambda \approx 4$ K and $\hbar\omega_c^* \approx 100$ K, we see that the inequality (32) is violated only in fields $F \sim 5 \times 10^3$ V/cm.

At equilibrium the electron density in a superlattice is inhomogeneous. This leads to the appearance in the system of a periodic self-consistent potential with amplitude of order $V_c \approx e^2 A/2\pi\varkappa\lambda^2$ ($\varkappa$ is the mean permittivity). This estimate has been obtained under the assumption that the fundamental harmonic in the spectrum of the function $n_e(x)$ is the one with period A. If $V_c/V_0 \ll 1$, then this potential can be ignored. In the opposite case it should be included in the definition of the total potential $V_0(x)$. The effect of the self-consistent potential on our results will be greatest in the field-intensity range $[\Delta_2, \Delta_3]$ (see Fig. 2), since for $\Delta < \Delta_2$ the height of the plateau is small, and the variation of the self-consistent potential with the electric-field intensity is insigficant. But if $\Delta > \Delta_3$, then the electron density in the system is approximately uniform, and the self-consistent potential can generally be ignored.

As to the scattering of the electrons by one other, in a number of cases its intensity is substantially suppressed. The laws of conservation of momentum and energy indicate that two electrons in states X_1 and X_2 in the magnetic band can scatter each other only when $E'(X_1) \approx E'(X_2)$, i.e., only when the states are on different sides of the point X^* of inflection of the function $E(X)$, (6). If $\mu < E(X^*)$, and the electron temperature is not high, then the occupancy of one of these states is exponentially small, and the probability for interelectron scattering is $\sim \exp\{-|E(X_1) - E(X_2)|/T_e\}$. Furthermore, in the case of a highly asymmetric shape of the $E(X)$ curve, when $|V_0 - E(X^*)| \ll V_0$, the electron-electron collision integral contains an additional small parameter $\sim |V_0 - E(X^*)|A/\lambda V_0$, due to the narrowness of the range of allowed energy transfers that occur during scattering. The effect of the electron-electron collisions can be appreciable when the distribution function $f(E)$ is of the same order of magnitude in the entire domain of definition (see Subsec. c in §2). The magnitude of the electron-electron collision integral is then proportional to the amplitude of the $f(E)$ oscillations, and decreases as the oscillations abate, since $f(E) = \text{const}$ satisfies this integral identically. If the $f(E)$ oscillation amplitude is high, then it can decrease under the action of the electron-electron collisions. But the order of magnitude of the current j_x, (30), determined by the phonon-emission intensity will not vary in the process.

In conclusion let us note the most interesting cases of the results obtained above.

1. In weak electric fields the components of the conductivity tensor for the system depend exponentially on the extent of occupation of the magnetic band by electrons and on the phonon gas temperature.

2. Critical nonlinear dependences of the Hall and dissipative currents on the electric-field strength arise in relatively weak electric fields, specifically in fields of intensity $F \sim 10$ V/cm.

3. In this case the electron-temperature approximation for the description of the electron gas in magnetic bands is often inapplicable. Even in comparatively weak fields the charge distribution function differs greatly from the Fermi function.

4. Even in fields of appreciable intensity, when the electrons are uniformly distributed at the lowest Landau level, the occupancy of the top level can remain exponentially small.

These results are in fact a consequence of the inhomogeneity of the system in the case when $\lambda < A$, a situation which leads, in particular, to the critical dependence of the electron-impurity transition probability on the electron energy in the magnetic band.

As the magnetic field intensity is increased, the values of the characteristic electric fields $F_i = \Delta_i/eA$ on the current-voltage characteristics of the system vary slightly (see Fig. 2), and, owing to the decrease of λ/A, the indicated critical dependences of the components of the system's conductivity tensor on the electric field, temperature, and carrier concentration will be more pronounced.

We thank G. R. Aĭzin and V. A. Volkov, as well as V. B. Sandomirskiĭ and the participants of his seminar, for useful discussions of the results of our paper.

[1]K. von Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. **45**, 494 (1980).
[2]R. E. Prange, Phys. Rev. B **23**, 4802 (1981).
[3]R. F. Kazarinov and S. Luryi, Phys. Rev. B **25**, 7626 (1982).
[4]S. V. Iordansky, Solid State Commun. **43**, 1 (1982).
[5]S. A. Tragman, Phys. Rev. B **27**, 7539 (1983).
[6]H. Aoki and T. Ando, Solid State Commun. **38**, 1079 (1981).
[7]D. J. Thouless, J. Phys. C **14**, 3457 (1981).
[8]N. A. Usov and F. R. Ulinich, Zh. Eksp. Teor. Fiz. **86**, 644 (1984) [Sov. Phys. JETP **59**, 376 (1984)].
[9]K. Yashihiro *et al.*, Prog. Theor. Phys. Suppl. No. 84, 215 (1985).
[10]É. M. Baskin, L. I. Magarill, and M. V. Éntin, Zh. Eksp. Teor. Fiz. **75**, 723 (1978) [Sov. Phys. JETP **48**, 365 (1978)].
[11]T. Ando, A. Fowler, and F. Stern, Electronic Properties of Two-Dimensional Systems [Translated from English], Mir, Moscow, 1985.
[12]N. A. Usov, F. R. Ulinich, and Yu. B. Grebenschikov, Solid State Commun. **43**, 723 (1982).
[13]B. I. Shklovskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **36**, 43 (1982) [JETP Lett. **43**, 51 (1982)].
[14]V. M. Pudalov and S. G. Semenchinskiĭ, Zh. Eksp. Teor. Fiz. **86**, 1431 (1984) [Sov. Phys. JETP **59**, 838 (1984)].
[15]S. M. Apenko and Yu. E. Lozovik, Zh. Eksp. Teor. Fiz. **89**, 573 (1985) [Sov. Phys. JETP **62**, 328 (1985)].
[16]G. R. Aĭzin and V. A. Volkov, Zh. Eksp. Teor. Fiz. **87**, 1469 (1984) [Sov. Phys. JETP **60**, 844 (1984)].
[17]G. R. Aĭzin and V. A. Volkov, Preprint No. 10, Institute of Radio Engineering and Electronics, Academy of Sciences of the USSR, 1985.
[18]P. S. Zyryanov and M. I. Klinger, Quantum Theory of Electron Transport Phenomena in Crystalline Semiconductors [in Russian], Nauka, Moscow, 1976, Chap. VII.
[19]G. Ebert, K. von Klitzing, K. Ploog, and G. Weimann, J. Phys. C **16**, 5441 (1983)].
[20]M. E. Cage *et al.*, Phys. Rev. Lett. **51**, 1374 (1983).
[21]G. M. Gusev, Z. D. Kvon, and R. A. Cheremnykh, J. Phys. C **18**, L1083 (1985).
[22]F. Stern and W. E. Howard, Phys. Rev. **163**, 816 (1967).
[23]M. Sh. Erukhimov, Fiz. Tekh. Poluprovodn. **3**, 194 (1969) [Sov. Phys. Semicond. **3**, 162 (1969)].
[24]E. Adams and T. Golstein, Problems of the Quantum Theory of Irreversible Processes [Translated from English], IIL, Moscow, 1961, p. 255.
[25]B. A. Tavger and M. Sh. Erukhimov, Zh. Eksp. Teor. Fiz. **51**, 528 (1966) [Sov. Phys. JETP **24**, 354 (1967)].
[26]G. R. Aĭzin and V. A. Volkov, Fiz. Tverd. Tela (Leningrad) **27**, 475 (1985) [Sov. Phys. Solid State **27**, 290 (1985)].

Translated by A. K. Agyei