Weakly damped magneto-impurity waves in metals

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We have calculated the transverse dynamic conductivity of metals with magneto-impurity electronic states, and have investigated the influence of these states on the spectrum and decay of electromagnetic waves. We predict a new class of waves, which we call "magneto-impurity" waves, and show that localization of electrons on impurities in a magnetic field makes possible the propagation of low-frequency helicon waves with left-handed polarization (antihelicon waves), which in pure samples cannot propagate. This gives rise to new branches of the high-frequency electromagnetic excitation spectrum, whose frequencies lie in narrow transmission bands around the frequencies of resonance transitions from localized levels of the electrons to Landau levels. We calculate the surface impedance of a metal with magnetic impurity states, and analyze its dependence on frequency. The field distribution of magneto-impurity waves in metals is discussed.

I. INTRODUCTION

The basic reason for the existence of various types of weakly decaying electromagnetic waves in metals subjected to an external magnetic field **H** (Refs. 1, 2) is collective motion of the conduction electrons. This motion occurs in two forms. We will refer to the first of these as "collective drift" (either Hall or polarization) of the electrons in planes perpendicular to the magnetic field. In uncompensated metals (i.e., with different concentrations of electrons N_e and holes N_h), Hall drift gives rise to helicon waves,[3] while in semimetals (Bi, Sb, etc.), collective polarization drift, which arises because of the temporal dispersion of the electromagnetic field, leads to the appearance of magneto-plasma waves (Alfvén and fast magnetosonic waves).[4] The longitudinal drift motion of the electrons (parallel to **H**) in quantizing magnetic fields can lead to collective oscillations with linear dispersion laws (quantum electromagnetic waves; see, e.g., Ref. 2).

Another type of collective motion which can give rise to weakly damped electromagnetic excitations is related to various resonances in the electron-hole system. Thus, near the cyclotron resonance frequency in metals we find cyclotron waves[5,6]; Doppler-shifted cyclotron resonance leads to various "dopplerons" (see, e.g., Ref. 7); near resonance frequencies corresponding to transitions between magnetic surface levels, we find specific surface waves,[8] etc.

It can be proved in general that near any resonance there must exist a concomitant electromagnetic wave,[2,9] and that the frequency of any such resonance coincides with some limiting frequency in the spectrum of this kind of collective oscillations.

In this paper, we predict a new class of weakly damped electromagnetic waves in metals which arise because of scattering resonances of the electrons, and investigate their properties theoretically. In particular, we will discuss collective oscillations caused by resonant transitions of electrons from localized magneto-impurity states to Landau levels.

Impurities play a dual role in a metal.[10,11] On the one hand, they limit the mean free path of conduction electrons, and determine the collision-induced damping of the electromagnetic waves. On the other hand, impurity atoms can radically change the structure of the electronic energy spectrum of a metal, leading to the appearance of localized and quasilocalized states.[11] Such states, which consist of electrons trapped on attractive impurities, can exist in the absence of a magnetic field only when the strength of the impurity potential exceeds some critical value.[12] In a quantizing magnetic field, due to the quasi-one-dimensional character of the electronic motion, states of an electron bound to an attractive impurity can appear for any value of the impurity potential.[13,14] These states of electrons localized on impurities in a magnetic field (both local[13,14] and quasilocal[15,16]) are called "magneto-impurity" states. Resonance transitions from various occupied magneto-impurity states to Landau levels lying above the Fermi energy can also give rise to a new class of electromagnetic wave, which we will also refer to as "magneto-impurity."[1)] A distinctive feature of these waves is the absence of any spatial-dispersion effects in that part of the high-frequency conductivity which describes the resonance transitions. In addition, magneto-impurity waves can exist only in the absence of collisionless (i.e., Landau) damping.

In the next section we will calculate the transverse high-frequency conductivity (including the resonance transitions) of a metal with a spherical Fermi surface, in the Faraday geometry $\mathbf{q}\|\mathbf{H}\|z$ ($\mathbf{q}$ is the wave vector). In Section 3 we will find the spectrum, damping and polarization of low-frequency magneto-impurity waves which are similar to helicon and magnetoplasma waves; in particular, we show that it is possible to propagate helicon waves with left-hand polarization (antihelicon waves) in metals with magneto-impurity electronic states. It is well known that such waves cannot propagate in a pure electronic conductor.[1,2] In the fourth section, we will discuss high-frequency magneto-impurity waves which appear near the frequencies of resonance transitions between magneto-impurity states and Landau levels. The fifth section is devoted to calculating the surface impedence and field distribution of a magneto-impurity wave in a semi-infinite sample.

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2. CONTRIBUTIONS OF MAGNETO-IMPURITY STATES TO THE TRANSVERSE DYNAMIC CONDUCTIVITY OF METALS

According to the Kubo formula,[18] the dissipative part of the dynamic conductivity tensor $Re\,\sigma_{\alpha\beta}(\omega)$ can be expressed as a configuration average over the two-particle retarded Green's function. For the case of impurities with short effective ranges, it is well known[19] that this average can be replaced by the product of two averaged one-particle Green's functions. Then the expression for the dissipative part of the conductivity can be cast in the following form:

$$\mathrm{Re}\,\sigma_{\alpha\beta}(\omega)=2\pi\frac{e^2}{\omega}\sum_{\varkappa_1\varkappa_2}v^{\alpha}_{\varkappa_1\varkappa_2}v^{\beta}_{\varkappa_2\varkappa_1}\int_{-\infty}^{\infty}d\varepsilon\,\rho_{\varkappa_1}(\varepsilon)\rho_{\varkappa_2}(\varepsilon+\omega)\times[f(\varepsilon)-f(\varepsilon+\omega)].$$

Here, e is the magnitude of the electron charge, ω is the frequency, $\varkappa=(n,k_y,k_z)$ is the set of quantum numbers for electrons in the Landau representation, $\mathbf{v}_{\varkappa_1\varkappa_2}$ is the matrix element of the velocity operator, $\rho_\varkappa(\varepsilon)$ is the spectral density of the one-particle Green's function averaged over the impurity positions,[18] and $f(\varepsilon)$ is the Fermi distribution. Planck's constant $\hbar$ is taken to be unity. For small impurity concentrations n_i and frequencies far from the cyclotron frequency $\Omega=eH/mc$ (i.e., for $|\omega-\Omega|\gg\nu$, where ν is a collision frequency) we can limit ourselves to terms linear in the impurity concentration in calculating the spectral density $\rho_\varkappa(\varepsilon)$.[20] As a result of these calculations we obtain

$$\mathrm{Re}\,\sigma_{xx}(\omega)=\frac{e^2\Omega}{2m\omega}\int_0^{\infty}d\varepsilon\,\tau^{-1}(\varepsilon-\omega)[f(\varepsilon-\omega)-f(\varepsilon)]\sum_{n=0}^{N(\varepsilon)}g_n(\varepsilon)\times\left[\frac{n+1}{(\omega+\Omega)^2}+\frac{n}{(\omega-\Omega)^2}\right]+(\omega\to-\omega),\qquad(1)$$

where $g_n(\varepsilon)$ is the partial density of electronic states at the nth Landau level; the summation over n in (1) is limited to the largest value of $N(\varepsilon)$ for which the radical contained in g_n is still real; the frequency of electron-impurity collisions equals[13]

$$\tau^{-1}(\varepsilon)=2\pi n_iU_0^2g(\varepsilon)\{[1-U_0F(\varepsilon)]^2+[\pi U_0g(\varepsilon)]^2\}^{-1}.$$

In this equation the scattering of electrons by an isolated impurity is taken into account exactly. The quantity $g=\Sigma_ng_n$ is the total density of states, while the function $F(\varepsilon)$ is proportional to the trace of the real part of the Green's function of the impurity-free metal. It appears in I. Lifshitz'[11] equation for the localized and quasilocalized energy levels

$$1-U_0F(\varepsilon)=0.$$

From now on, the scattering potential U_0 ($U_0>0$) will be expressed in terms of the scattering length a ($U_0=-2\pi a/m$).

In calculating the dissipative part of the conductivity (1) it is necessary to take into account the resonance character of the electron-impurity scattering amplitude. In the case of an impurity which attracts electrons, a quasilocalized level is located beneath every Landau level and a localized level lies beneath the lowest Landau level (with $n=0$).[13–16] A conduction electron whose energy is close to the energy of a quasilocalized level ε_j undergoes resonant scattering with a probability which is described by the Breit-Wigner formula. The collision frequency takes the form

$$\tau^{-1}(\varepsilon)=\frac{2n_i}{|F'|}\sum_j\frac{\Gamma_j}{(\varepsilon-\varepsilon_j)^2+\Gamma_j^2}+\nu.\qquad(2)$$

The positions of the quasilocalized (or localized) levels are given by the equation

$$\varepsilon_j=\Omega(j+{}^1/_2)-\Delta,\quad\Delta={}^1/_2\Omega(a/l_H)^2,$$

where Δ is the binding energy of an electron and $l_H=(c/eH)^{1/2}$ is the magnetic length. The half-width of the jth quasilocalized level Γ_j depends weakly on the index j[15,16]; the prime on F in (2) implies a derivative with respect to energy. The sum on j includes contributions from all the resonances, while the quantity ν represents the contribution from potential scattering by impurities in the Born approximation.[1,2]

Let us substitute (2) into (1) anc calculate the contribution to the conductivity from resonance scattering. We will assume that the temperature satisfies $T\gg\Gamma_j$. Then the difference in the Fermi functions in (1) can be considered slowly varying and can be taken outside the integral over energy. The contribution of resonant scattering to the transverse conductivity is found to equal

$$\sigma_r=\sum_j\sigma_j,$$

where

$$\sigma_j(\omega)=\frac{e^2n_i}{2\pi ml_H^4|F'|\omega}\sum_{n=0}^{N(\varepsilon_j)}\left[\frac{n+1}{(\omega+\Omega)^2}+\frac{n}{(\omega-\Omega)^2}\right]\times[f(\varepsilon_j)-f(\varepsilon_j+\omega)]k_{nj}^{-1},$$

$$k_{nj}=(2m)^{1/2}[\varepsilon_j+\omega-\Omega(n+{}^1/_2)+i\Gamma_j]^{1/2}.\qquad(3)$$

The radical in (3) is defined so that its real part is positive; this ensures that the dissipative part of $\sigma_j(\omega)$ is positive. The contribution to the complex conductivity from potential scattering [the term ν in (2)] under conditions of quasiclassical quantization $\Omega\ll\varepsilon_F$ (ε_F is the Fermi energy) can be found from classical scattering theory[2]; the approximation implies neglect of quantum oscillations of the Shubnikov–de Haas type arising from modulation of the electronic density of states. These oscillations have small amplitudes, scaling with the parameter $(\Omega/\varepsilon_F)^{1/2}$ which measures the validity of the quasiclassical approximation. For the localized level $j=0$, the width Γ_0 in (3) must be set equal to zero.

The sum (3) will contain one (for $\omega<\Omega$) or several (for $\omega>\Omega$) special terms due to transitions from the magneto-impurity levels to their corresponding Landau levels. For these special terms the energy difference $\varepsilon_j+\omega-\Omega(n+1/2)$ is as small as possible, implying that there is a group of special terms in expression (3) for the conductivity which possess singularities in frequency of the form $(\omega-\omega_{nj})^{-1/2}$, along with a collection of terms which are regular in ω. We can replace the nonsingular part of the electron-impurity scattering by an integral and include it in ν; the corresponding contribution to the conductivity in the absence of a magnetic field was discussed earlier.[21]

By making use of the fact that the quasilocalized levels practically coincide in energy with the cyclotron frequency Ω, we can express all the singular terms of the conductivity in the form

$$\delta\sigma_s(\omega)=\frac{\omega_p^2}{4\pi\omega_s}\alpha_s\left(\frac{\omega_s}{\omega-\omega_s+i\Gamma}\right)^{1/2}. \tag{4}$$

Here, ω_p is the electron plasma frequency; $\omega_s=\Delta+s\Omega$ ($s=0, 1,...$) are the resonance frequencies for transitions from magneto-impurity levels to Landau levels; Γ is a characteristic value of the half-width of that quasilocalized state which corresponds to the Fermi energy. The dimensionless parameter α_s equals

$$\alpha_s=2\frac{n_i}{N_c}\frac{\Omega}{\omega_s}\left(\frac{\Delta}{\omega_s}\right)^{1/2}\sum_j\left[\frac{s+j+1}{(1+\Omega/\omega_s)^2}+\frac{s+j}{(1-\Omega/\omega_s)^2}\right]$$
$$\times[f(\varepsilon_j)-f(\varepsilon_j+\omega_s)]. \tag{5}$$

The quantity α_s can be interpreted as a kind of effective oscillator strength of the resonance transition with frequency ω_s. It differs from the usual oscillator strength[9] in that the singularity in the conductivity (4) is square-root-like rather than a first-order pole. The effective oscillator strength α_s incorporates the Pauli principle, which is introduced through the difference of Fermi functions in (5); this implies that the quantity α_s will exhibit quantum oscillations with magnetic field as successive quasilocalized levels ε_j pass through the Fermi surface. While this transition is taking place, α_s exhibits a strong dependence on temperature. For the case of the transition from the localized level $j=0$ to its respective Landau level, only one term need be retained in the sum (5).

3. THE EFFECT OF MAGNETO-IMPURITY STATES ON LOW-FREQUENCY WAVES

In this section we will discuss the effect of electronic magneto-impurity states on the spectrum and damping of low-frequency waves. Assume the wave frequency is much lower than the cyclotron frequency, while the wavelength is much longer than the Larmor radius r. Then in the low-frequency region there can be only one resonance transition from a quasilocalized level to its Landau level for those intervals of magnetic field where the Fermi surface is located between these two levels, $\varepsilon_j<\varepsilon_F<\Omega\,(j+1/2)$ (Fig. 1). In this case the resonance frequency $\omega_0=\Delta$ coincides with the electron binding energy. Let us consider separately the helicon and magneto-plasma waves.

a) Magneto-impurity helicon and antihelicon waves ($N_e\neq N_h$)

The dispersion relation which determines the spectrum and damping of circularly polarized waves ($E_\pm=E_x\pm iE_y$) has the form

$$\frac{c^2q^2}{\omega_p^2}=\alpha_0\left(\frac{\omega_0}{\omega_0-\omega-i\Gamma}\right)^{1/2}\mp\frac{\omega}{\Omega}+\frac{i\nu\omega}{\Omega^2}. \tag{6}$$

The first term on the right-hand side of (6) arises from the special term (4); the second is connected with the Hall conductivity (for simplicity we set $N_h=0$) while the last term is caused by the dissipative conductivity

$$\sigma_{xx}=N_e e^2\nu/m\Omega^2.$$

In the absence of magneto-impurity states ($\alpha_0=0$), Eq. (6) determines the spectrum and damping of helicon waves [the

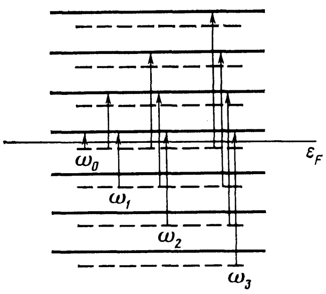

FIG. 1. Diagram of resonance transitions between magneto-impurity levels (the horizontal dashed lines) and Landau levels (the continuous lines).

lower sign in front of the second term in (6)], and damped antihelicon waves of the opposite circular polarization [the upper sign in (6)] in the local regime. According to (5), the quantity α_0 in the case under discussion is

$$\alpha_0=4\frac{n_i}{N_e}\frac{\omega_0}{\Omega}\left(N_F+\frac{1}{2}\right)$$
$$\times\left\{f\left[\Omega\left(N_F+\frac{1}{2}\right)-\omega_0\right]-f\left[\Omega\left(N_F+\frac{1}{2}\right)\right]\right\},$$

where N_F is the integer part of ε_F/Ω.

Let us first analyze the spectrum and damping of antihelicon waves. Due to the appearance of the additional term (4) in the conductivity, a transmission window for antihelicon waves appears in the phase space (q, w) as ν, $\Gamma\to 0$. It follows from (6) that the character of the dispersion curves is determined by the parameter

$$\xi=\alpha_0\Omega/2\omega_0,$$

which depends on the impurity concentration, magnetic field and temperature. In Fig. 2 we illustrate the antihelicon wave dispersion curves for various values of ξ. For small ξ ($\xi<3^{-3/2}\approx 0.19$) there are two antihelicon branches with normal and anomalous dispersion. As ξ increases, the limiting frequencies (on the long-wavelength side) $\omega_\pm=\omega_\pm(q=0)$ approach each other and reduce to $\omega_k=2\omega_0/3$ as $\xi\to 3^{-3/2}$. For larger values of ξ ($>3^{-3/2}$), both branches coalesce into one dispersion curve. The coordinate minimum of the curve $q(\omega)$ equals

$$q_m=\frac{\omega_p}{c}\left[\frac{\omega_0}{\Omega}(3\xi^{2/3}-1)\right]^{1/2},\qquad \omega_m=\omega_0(1-\xi^{2/3}).$$

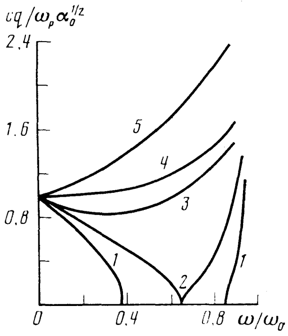

FIG. 2. Dispersion curves for antihelicons ($1-\xi=0.15$, $2-\xi=\xi_k=3^{-3/2}$, $3-\xi=0.5$, $4-\xi=2$ and helicons ($5-\xi=0.2$).

It is obvious that the minimum exists only for $3^{-3/2} < \xi < 1$. For $\xi > 1$, the antihelicon dispersion curve becomes monotonic. The limiting frequency of the antihelicon spectrum as $q \to \infty$ equals ω_0. We present here analytic expressions for the dispersion functions $\omega_\pm(q)$ for $\xi < 3^{-3/2}$:

$$\omega_-(q) = {}^1/_3\omega_0[1-2\eta-2(1+\eta)\cos{}^1/_3(\varphi+\pi)], \quad (7)$$

$$\omega_+(q) = {}^1/_3\omega_0[1-2\eta+2(1+\eta)\cos{}^1/_3\varphi], \quad (8)$$

where

$$\cos\varphi = 1-54\xi^2/(1+\eta)^3, \quad \eta = c^2q^2\Omega/\omega_p^2\omega_0.$$

In particular, for $\xi \ll 1$ the limiting frequencies equal

$$\omega_- = 2\omega_0\xi, \quad \omega_+ = \omega_0(1-4\xi^2). \quad (9)$$

If $\xi > 3^{-3/2}$, the dispersion curve of the magneto-impurity antihelicon wave is given by Eq. (7).

The decay of antihelicon waves is determined by the same dispersion relation (6). Assuming that the damping rate $\gamma(\omega)$ is much smaller than ω, we obtain

$$\gamma(\omega) = \frac{|h'|}{2^{1/2}h''}\left\{\left[1+4\frac{h''^2}{h'^4}\left(\xi\frac{\Gamma}{\Omega}\left(\frac{\omega_0}{\omega_0-\omega}\right)^{3/2}+\frac{\nu\omega}{\Omega^2}\right)^2\right]^{1/2}-1\right\}^{1/2}, \quad (10)$$

where

$$h(\omega) = \alpha_0\left(\frac{\omega_0}{\omega_0-\omega}\right)^{1/2} - \frac{\omega}{\Omega}.$$

The primes on h denote derivatives with respect to frequencies. In deriving (10) we expanded the right-hand side of (6) in the small imaginary part of ω through quadratic terms. It is necessary to take the quadratic terms into account, because for $3^{-3/2} < \xi < 1$ the coefficient of the term which is linear in γ vanishes at the point ω_m.

Let us now turn to an analysis of the magneto-impurity helicon waves, corresponding to the + sign in the relation (6). The spectrum begins at a threshold value of

$$q_0 = \frac{\omega_p}{c}\left(2\xi\frac{\omega_0}{\Omega}\right)^{1/2} \quad (11)$$

and increases monotonically, approaching the limiting frequency ω_0 asymptotically (Fig. 2, curve 5). The decay of this wave for small ν and Γ is given by the equation

$$\gamma(\omega) = \frac{\xi\Gamma[\omega_0/(\omega_0-\omega)]^{3/2}+\nu\omega/\Omega}{\xi[\omega_0/(\omega_0-\omega)]^{3/2}+1}. \quad (12)$$

When $\xi \to 0$, the threshold $q_0 \to 0$ and the dispersion and damping of helicon waves (12) takes its usual value.[1-3]

What distinguishes magneto-impurity waves from analogous collective excitations in the absence of bound electronic impurity states is the appearance of a new limiting frequency ω_0 above which the propagation of these waves is found to be impossible because of their strong resonance damping. An additional peculiarity is the appearance of the unusual antihelicon propagation.

b) Magneto-plasma waves ($N_e = N_h$)

We will calculate the spectrum and damping of these waves within an isotropic two-component model. The dispersion equation for both orthogonal linear polarized oscillations with $\mathbf{q}\|\mathbf{H}$ are found to be the same, and have the form

$$q^2 = \alpha_0\frac{\omega_p^2}{c^2}\left(\frac{\omega_0}{\omega_0-\omega-i\Gamma}\right)^{1/2} + \frac{\omega^2}{v_a^2} + \frac{i\nu\omega}{v_a^2}, \quad (13)$$

where v_a is the Alfvén velocity and ν is the average relaxation frequency.[1,2] The second and third terms on the right side of (13) contain the joint contribution of electrons and holes.[2] The singular term in the presence of impurities which attract electrons is due only to these latter carriers, since there are no magneto-impurity hole states in the field of donors. The region of applicability of Eq. (13) is determined by the inequality

$$\nu, qv_F^{(e,h)} \ll \omega \ll \Omega,$$

where v_F is the Fermi velocity of the carriers. The condition $qv_F \ll \omega$ is necessary in order that collisionless Landau damping be absent; this damping appears when we take into account the anisotropic dispersion relations for electrons and holes. The requirement on the average collision frequency ensures that the corresponding contribution to the damping rate is small.

As ν, $\Gamma \to 0$ the dispersion curve $\omega(q)$ begins at the threshold value (11), and then increases monotonically, approaching the limiting frequency ω_0. Consequently, the dispersion of this wave is normal.

The damping rate to first order in ν and Γ equals

$$\gamma(\omega) = \frac{\nu}{2} + \frac{\alpha_0\Omega_e^2 m_e\Gamma}{4(m_e+m_h)\omega_0\omega}\left(\frac{\omega_0}{\omega_0-\omega}\right)^{3/2}. \quad (14)$$

This equation can be used for $\gamma \ll \omega$. The damping (14) is a nonmonotonic function of frequency. It has a minimum at $\omega = 0.4\omega_0$, at which

$$\gamma_{min} = \frac{\nu}{2} + \frac{5}{8}\left(\frac{5}{3}\right)^{3/2}\frac{\alpha_0 m_e\Omega_e^2\Gamma}{(m_e+m_h)\omega_0^2}.$$

Note that we can ignore the magneto-impurity resonant transitions in analyzing the propagation of weakly damped waves if $\xi \ll 1$, since the special terms in the dispersion relations (6) and (13) can be neglected.

4. HIGH-FREQUENCY MAGNETO-IMPURITY WAVES

We will now discuss the high-frequency region $\omega > \Omega$. Let us first set $\mathbf{q}\|\mathbf{H}$, $qr \ll 1$, $\omega < \omega_p$; this allows us to neglect spatial dispersion in calculating the conductivity, and to neglect the displacement current. The resonance frequencies in this region equal $\omega_s = \Delta + s\Omega$, where $s = 1, 2,...$; the corresponding resonances originate from two sources: transitions from quasilocalized levels to Landau levels and transitions from localized levels to Landau levels. If $\xi_j < \varepsilon_F < \Omega(j+1/2)$, the lowest resonance frequency equals ω_0. Subsequent resonances are located at the frequencies ω_1, ω_2,..., which lie in the high-frequency region (Fig. 1). For $\Omega(j-1/2) < \varepsilon_F < \varepsilon_j$, however, transitions at the frequency ω_0 are forbidden by the Pauli principle. The first allowed resonance transition then takes place at frequency ω_1. In the case of transitions between the local level and its Landau level, the first absorption maximum is located at the frequency ω_1 only in the ultra-quantum limit. If, however, $\varepsilon_F \gg \Omega$, the first absorption maximum is located at the frequency ω_{N_F}, where N_F is the number of occupied Landau levels. Transitions at lower resonance frequencies are forbidden by the Pauli principle.

In addition to the special term (4), in the dispersion relation we will leave the classical expression[1,2]

$$\sigma=\frac{\omega_p^2}{4\pi\omega^2}(\nu+i\omega),$$

where $\omega \gg \Omega$, ν. Because

$$\operatorname{Im}\sigma/|\sigma_{xy}|=\Omega/\omega\ll 1,$$

the Hall conductivity can be discarded. As a result, the dispersion relation for linearly-polarized waves has the form

$$\frac{c^2q^2}{\omega_p^2}=\alpha_s\left(\frac{\omega_s}{\omega_s-\omega-i\Gamma}\right)^{1/2}-1+\frac{i\nu}{\omega_s}. \qquad (15)$$

From this expression, it is clear that as $\alpha_s \to 0$ the wave vector is imaginary, i.e., the wave does not propagate. Inclusion of the special term due to resonance scattering of electrons by impurity atoms in a magnetic field gives rise to solutions of (15) in the form of a series of linearly-polarized electromagnetic waves with frequencies in the vicinity of the resonance frequencies ω_s.

The solutions to Eq. (15) have the form $\omega_s(q) - i\gamma_s(q)$, where

$$\omega_s(q)=\omega_s\left[1-\frac{\alpha_s^2}{(1+c^2q^2/\omega_p^2)^2}\right] \qquad (16)$$

is the dispersion law and

$$\gamma_s(q)=\frac{2\nu\alpha_s^2}{(1+c^2q^2/\omega_p^2)^3}+\Gamma \qquad (17)$$

is the damping rate. In Fig. 3 we show the functions (a) $\omega_1(q)/\omega_1$ and (b) $10^4\gamma_1(q)/\omega_1$ versus cq/ω_p, calculated for the spectral parameters of bismuth[22] with donor impurities [for example Te or Se (Ref. 23)]. The magnetic field is parallel to the twofold axis; we took $a=10^{-7}$ cm, $n_i/N_e=10^{-2}$, $H=10^4$ Oe. We note that in this case the condition $qr \ll 1$ is violated only when $cq/\omega_p > 30$.

From (17) it is clear that for small q the decay of magneto-impurity waves is determined both by potential scattering of electrons from impurity atoms and by the width of the quasilocalized level. As q grows, the first term in (17) decreases, and γ_s approaches Γ. The maximum value of the damping rate $\gamma_s(0)=2\nu\alpha_s^2+\Gamma$ is small compared to the wave frequency; hence, we can speak of transmission bands in the metal for waves with the dispersion law (16). The width of the sth band equals $\delta\omega_s=\omega_s\alpha_s^2$; it must exceed the total width of the Landau level and the quasilocalized level which define the transition. For the first band ($s=1$),

$$\frac{\delta\omega_1}{\nu}=\frac{\Omega^2}{\nu\omega_0}\left(\frac{2n_i}{N_e}\right)^2.$$

For the parameter values quoted above, i.e., for Bi–Te, this ratio is larger than one if $n_i > 10^{12}$ cm^{-3}. On the other hand, the sample cannot be too highly doped, lest we violate condition $n_i a l_H^2 < 1$, which ensures the existence of the magneto-impurity state.[14] In the case under discussion here, this is equivalent to $n_i < 10^{17}$ cm^{-3}. If $s \gg 1$ we obtain

$$\delta\omega_s=\frac{4\Omega}{s^2}\left(\frac{2n_i}{N_e}\right)^2\left(\frac{\omega_0}{\Omega}\right)^3.$$

As s increases, the width of the band decreases and eventually becomes comparable to $\nu+\Gamma$. In this case, it is no longer meaningful to treat the system as if it has transmission bands.

5. SURFACE IMPEDANCE AND FIELD DISTRIBUTION OF MAGNETO-IMPURITY WAVES

Let us discuss the surface impedance and field distribution of a metallic half-space when a normally incident external electromagnetic wave excites waves with dispersion laws (6), (13), and (15). In the absence of spatial dispersion, the diagonal component of the impedance $Z_{xx}=R_{xx}-iX_{xx}$ equals

$$Z=4\pi\omega/c^2q(\omega),$$

where $q(\omega)$ is a solution to the dispersion equation. In this case, we identify the correct square-root branch of $q(\omega)$ from the condition $\operatorname{Im}q(\omega)>0$, which ensures that the wave decays into the bulk metal. In discussing helicon waves it is convenient to calculate the circular components of the impedance $Z_\pm$.[1,2]

The electric field distribution of the left-handed circular component of an antihelicon wave in the metal takes the form

$$E_+(z)=-iq_+^{-1}(\omega)E_+'(0)\exp[iq_+(\omega)z], \qquad (18)$$

where $q_+(\omega)$ corresponds to the upper sign in (6). The z-axis is directed into the bulk of the metal. The prime denotes

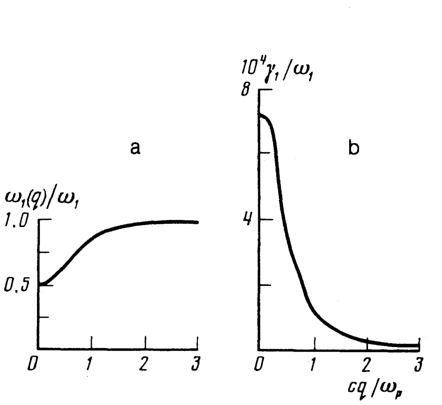

FIG. 3. Dispersion law (a) and damping rate (b) of the $s=1$ magneto-impurity wave.

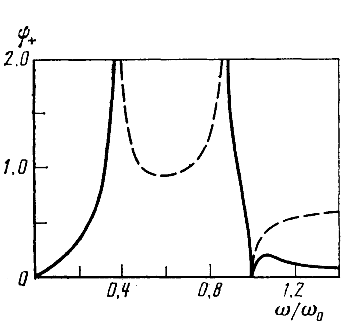

FIG. 4. Dependence of the real (continuous curve) and imaginary (dashed curve) parts of the surface impedance for antihelicon waves versus frequency for $\xi=0.15$.

a derivative with respect to z. It is clear that the field (18) in the bulk of the sample is exponentially damped.

If ν, $\Gamma \to 0$, we obtain for the real and imaginary parts of the quantity

$$\psi_+ = \frac{cZ_+\omega_p\alpha_0^{1/2}}{4\pi\omega_0}\left(Z_+ = \frac{4\pi\omega}{c^2 q_+(\omega)}\right)$$

the following expressions

$$\mathrm{Re}\,\psi_+(x) = \begin{cases} (2\xi)^{1/2} x (C-x)^{-1/2}, & 0 \leqslant x \leqslant x_-,\ x_+ < x \leqslant 1,\\ 0, & x_- < x < x_+,\\ \xi^{1/2} x (x^2 + C^2)^{-1/2}[(x^2+C^2)^{1/2} - x]^{1/2}, & x \geqslant 1, \end{cases} \quad (19)$$

$$-\mathrm{Im}\,\psi_+(x) = \begin{cases} 0, & 0 \leqslant x < x_-,\ x_+ < x \leqslant 1,\\ (2\xi)^{1/2} x (x-C)^{-1/2}, & x_- < x < x_+,\\ \xi^{1/2} x (x^2 + C^2)^{-1/2}[(x^2+C^2)^{1/2}) + x]^{1/2}, & x \geqslant 1, \end{cases} \quad (20)$$

where $x = \omega/\omega_0$, $C(x) = 2\xi\,|1-x|^{-1/2}$. A plot of the functions (19) and (20) for $\xi = 0.15$ is given in Fig. 4. The impedance has square-root singularities at the points $x_\pm = \omega_\pm/\omega_0$, which are the limiting frequencies for antihelicon waves. Near ω_0, the functions (19) and (20) are proportional to $|1-x|^{1/4}$. When ξ increases until it is close to $\xi_k = 3^{-3/2}$, the limiting frequencies approach $x_k = 2/3$. At this point, $\mathrm{Re}\,\psi+$ has a $|x - x_k|^{-1}$ singularity.

For finite values of ν and Γ these singularities are washed out. In this case, we obtain from (6)

$$\begin{pmatrix} \mathrm{Re}\,\psi_+ \\ -\mathrm{Im}\,\psi_+ \end{pmatrix} = 2^{-1/2} x \left[\left(A - \frac{x}{2\xi}\right)^2 + \left(B + \frac{\nu}{\Omega}\frac{x}{2\xi}\right)^2\right]^{-1/2} \left\{\left[\left(A - \frac{x}{2\xi}\right)^2 + \left(B + \frac{\nu}{\Omega}\frac{x}{2\xi}\right)^2\right]^{1/2} \pm \left(A - \frac{x}{2\xi}\right)\right\}^{1/2}, \quad (21)$$

where

$$\begin{pmatrix} A(x) \\ B(x) \end{pmatrix} = 2^{-1/2}\left[(1-x)^2 + \left(\frac{\Gamma}{\Delta}\right)^2\right]^{-1/2} \times \left\{\left[(1-x)^2 + \left(\frac{\Gamma}{\Delta}\right)^2\right]^{1/2} \pm (1-x)\right\}^{1/2}.$$

The impedance (21) has asymmetric resonance maxima at the points $x_\pm$, where

$$\psi_+(x_\pm) = x_\pm\left[B(x_\pm) + \frac{\nu}{\Omega}\frac{x_\pm}{2\xi}\right]^{-1/2} e^{-\pi i/4}.$$

The width of the resonance curve is

$$\delta x_\pm = |\varphi_\pm'|^{-1}\left[B(x_\pm) + \frac{\nu}{\Omega}\frac{x_\pm}{2\xi}\right],$$

where

$$\varphi(x) = A(x) - x/2\xi.$$

Since the expansion of $\varphi(x)$ near the point x_k starts with a quadratic term, the impedance (21) has a symmetric maximum at this point. The line width of the resonance is now determined by the relation

$$\delta x_k = 2|\varphi_k''|^{-1}\left[B(x_k) + \frac{\nu}{\Omega}\frac{x_k}{2\xi}\right],$$

where as before the primes denote derivatives with respect to x.

The surface impedance for magneto-impurity waves can be obtained from (13). As ω increases, the real part R_{xx} of the impedance first grows as ω, then reaches a maximum which is followed by a minimum located near ω_0. As $\omega \to \infty$ the quantity R_{xx} saturates at $4\pi v_a/c^2$. Initially the imaginary part X_{xx} is $\sim\omega$, then it passes through a maximum; as $\omega \to \infty$ it decreases with a dependence like $X_{xx} = 2\pi v_a \nu/c^2\omega$.

The field distribution of the sth wave solution to the dispersion law (16) is determined by the expression

$$E_x^{(s)}(z) = -iq_s^{-1}(\omega)E_x'(0)\exp[iq_s(\omega)z],$$

where $q_s(\omega)$ is obtained from (15). The impedance for these waves equals

$$Z_{xx}^{(s)}(\omega) = \frac{4\pi}{c^2}\frac{\omega}{q_s(\omega)}.$$

It has asymmetric resonance maxima at the points $\omega_{0s} = \omega_s(0) = \omega_s(1-\alpha_s^2)$. Near resonance,

$$R_s(\omega) = X_s(\omega) = R_s(\omega_{0s})\frac{\gamma_s}{[(\omega-\omega_{0s})^2 + \gamma_s^2]^{1/2}},$$

where

$$R_s(\omega_{0s}) = \frac{4\pi}{c}\frac{\omega_{0s}}{\omega_p}\left(\frac{\delta\omega_s}{\gamma_s}\right)^{1/2},$$

while γ_s equals (17) for $q = 0$. Between the impedance maxima there is a minimum in the vicinity of the resonance frequency ω_s. If ν, $\Gamma \to 0$, near ω_s we have

$$Z_s(\omega) = \frac{4\pi\omega_s}{c\omega_p}\alpha_s^{-1/2} e^{-\pi i/4}\left(\frac{\omega-\omega_s}{\omega_s}\right)^{1/4}, \quad \omega \geqslant \omega_s;$$

$$R_s(\omega) = \frac{4\pi\omega_s}{c\omega_p}\alpha_s^{-1/2}\left(\frac{\omega_s-\omega}{\omega_s}\right)^{1/4}, \quad X_s = 0, \quad \omega \leqslant \omega_s;$$

when ν and Γ are included, these singularities are smoothed out.

[1)]A brief communication about these waves was published in Ref. 17.

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