Excitation of longitudinal ultrasound by a variable electric field in degenerate conductors

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We propose a deformation mechanism which is linear in the amplitude of the external variable electric field for generating longitudinal ultrasound normal to the surface of a degenerate conductor. The effect is due to the anomalous penetration of this field into the conductor, whose carriers have a long mean free path l: perturbations of the electron density are due both to intrinsic elastic lattice vibrations (with wave vector $k \gtrsim \omega/S$), and to induced vibrations (with wave vector $\varkappa \approx (\omega + i\nu)/v_0$, where ω and S are the frequency and speed of the ultrasound, and ν and v_0 are the collision rate and Fermi velocity of the carriers), and the imbalance between these two factors at each point in the conductor is responsible for the generation of ultrasound. We show that the amplitude of the elastic vibrations excited increases with increasing l and ω. Measurements made on monocrystalline tin at liquid-helium temperatures bear out the observability of the effect, which can provide information about the diagonal components of the deformation potential tensor of a degenerate conductor.

In the present paper, we propose a new mechanism for remote excitation of longitudinal ultrasound in degenerate conductors. The prediction of various mechanisms for directly transforming electromagnetic energy into acoustic energy, as well as the search for such mechanisms and their implementation, is of interest in its own right, and has already found its way into practice.[1,2] Moreover, the wave transformation phenomenon can be used to obtain information about important characteristics of conductors, in particular, about the components of the electron deformation potential tensor. Research along these lines has thus far been limited to consideration of the conversion of a transverse electromagnetic wave into ultrasound.[3] When a constant magnetic field H_0 is present, an induction mechanism is responsible for the transformation, and when $H_0 = 0$, it is deformation, which is effective when the anomalous skin effect comes into play. Under those circumstances, an electromagnetic wave generates transverse ultrasound which, as has been demonstrated by a multitude of experimental and theoretical studies,[4–13] makes it possible to determine the nondiagonal components of the deformation potential tensor. The proposed conversion mechanism seems to hold promise for determining the diagonal components of this tensor (see also Ref. 14). When a transverse electromagnetic wave is converted into acoustic form, diffuse reflection of electrons by the surface is responsible for the appearance of concentrated forces right at the boundary, which can be treated as an additional source for the generation of elastic vibrations.[15] As we shall see, this connection of force at the surface also occurs, in the case we are considering, with specular reflection of electrons as well.

1. The penetration of a variable (and, naturally, nonuniform) electric field into a conductor destroys the equilibrium between conductor electrons and the ion lattice of the crystal, which causes the latter to vibrate. The acoustic and macroscopic electromagnetic fields in conducting crystals are related by a well known[16] self-consistent set of equations, which includes the dynamical equation of the theory of elasticity, the kinetic equation for the electron distribution function, and Maxwell's equations. The addition of appropriate boundary conditions to this system enables one, in particular, to determine acoustic displacements based on specified electric and magnetic field strengths, i.e., to solve the problem of electromagnetically excited ultrasound.

It is well known that in a quasistatic longitudinal electric field ($E = E_x$), the surface of a conductor occupying the region of space $x \geqslant 0$ is subject to a pressure

$$P = -\int_0^\infty dx E(x,t)\rho_e(x,t) = E^2(0,t)/8\pi, \quad (1)$$

where ρ_e is the unneutralized charge density induced by the field. A varying pressure naturally generates an acoustic wave in the conductor. This conversion mechanism, which is usually known as electrostatic acoustic transformation (ESAT), has been observed experimentally many times (see Ref. 17, for example). It is clear that the force corresponding to this mechanism is quadratic in the applied variable field strength.

In degenerate conductors, however, even to a linear approximation in $E(0)$, there is another possible mechanism for generating longitudinal sound which is associated the anomalous occurrence of a variable electric field in a conductor whose carriers have a long mean free path.[18] It is due to the deformational interaction of the nonequilibrium electron subsystem with acoustic vibrations of the lattice (see Ref. 16):

$$\rho(\omega^2 u + S^2 u'') = \langle \Lambda_{xx} \chi'(x) \rangle, \quad (2)$$

where Λ_{xx} is the component of the deformation potential after subtraction of its mean value at the Fermi surface (to order of magnitude, Λ_{xx} is the same as the Fermi energy ε_F). In the dynamical equation (2), ρ is the crystal density, S is the speed of sound,[1)] and angular brackets signify an average over the Fermi surface; the temporal factor $\exp(-i\omega t)$ has been omitted both from displacements ($u = u(x)$) and from the nonequilibrium increment $(\partial f_0/\partial\varepsilon)\chi$ to the electron distribution function f_0; primes denote

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differentiation with respect to x.

The force on the right-hand side of (2) acts on the ion lattice of the crystal. At the same time, the conductor as a whole is electrically neutral, and to a linear approximation in the external electric field, it must remain in mechanical equilibrium. This means (cf. Ref. 15) that its surface is subject to additional pressure from the nonequilibrium electron gas, due to the integral of the force (2) over the entrie thickness of the sample, and taken with the opposite sign. Thus, if the surface of the sample is not subject to any external mechanical forces, the boundary condition there can be written in the form

$$\rho S^2 u'(0) = -\int_0^\infty dx \langle \Lambda_{xx} \chi'(x) \rangle = \langle \Lambda_{xx} \chi(0) \rangle. \tag{3}$$

2. In order to solve this problem quantitatively, it is necessary to assume a specific form for the dispersion relation of the carriers and the boundary condition imposed on their distribution function. We will illustrate the effect of the deformation mechanism for transformation with the simple case in which the electron Hamiltonian is quadratic in the quasimomentum, and the xx-component of the deformation potential tensor can be represented in the form

$$\Lambda_{xx} = \Lambda (3v_x^2/v_0^2 - 1),$$

where $v_0 = (v_x)_{\max}$ is the electron Fermi velocity. In that case, the force on the right-hand side of (2) can easily be represented, with the aid of the kinetic equation and Maxwell's equations, in terms of the electric field $E(x)$ within the metal [we assume that its value $E(0)$ outside the metal is given]:

$$u'' + \frac{\omega^2}{S^2} u = \frac{\Lambda}{4\pi e \rho S^2} \left[E'' - \alpha^2 \left(E + \frac{E - E(0)}{\varepsilon - 1} \right) \right]. \tag{4}$$

Here ε is the permittivity of a conductor with carrier density n,

$$1 - \varepsilon = \frac{4\pi n e^2}{m\omega(\omega + i\nu)} = \frac{\omega_0^2}{\omega(\omega + i\nu)}, \tag{5}$$

and ω_0 is the plasma frequency, which under actual experimental conditions is much greater, for metals and semimetals, than both the freuqency of the electric field and ν, the reciprocal of the electron relaxation time. In the quasistatic case, the parameter $\alpha \equiv 3^{1/2} \omega_0 / v_0$ is practically identical with the decrement of the exponentially decaying part of the electric field. Hence, it is already clear that the latter does not play an important role in the excitation of sound it will be evident in what follows that the effect in question is due to a "pulling field"[18] which arises in the conductor.

The equation to determine the field is obtained by differentiating the Poisson equation with respect to x, giving

$$E'' = -4\pi e \langle \chi' \rangle.$$

Taking acoustic displacements into account, this may be written out explicitly as

$$E(x) - \alpha^{-2} E''(x) = \int_0^\infty dy\, u''(y) [P(x-y) - P(x+y)] + \int_0^\infty dy \left[E(y) - \frac{\alpha^{-2} E''(y)}{1 + \omega/i\nu} \right] [Q(x-y) - Q(x+y)], \tag{6}$$

$$P(x) \equiv \frac{i\omega}{\langle 1 \rangle e E(0)} \left\langle \frac{\Lambda_{xx}}{v_x} \exp\left(\frac{i\omega - \nu}{v_x} |x| \right) \right\rangle_+,$$
$$Q(x) \equiv \frac{\nu - i\omega}{\langle 1 \rangle} \left\langle v_x^{-1} \exp\left(\frac{i\omega - \nu}{v_x} |x| \right) \right\rangle_+, \tag{7}$$

the subscript " + " signifies integration over that part of the Fermi surface where $v_x > 0$.

Equation (6) assumes specular reflection of electrons by a metal surface.[2)] Note that when sound is excited by a transverse electromagnetic wave, the surface force is zero when electron reflection is specular, since at the surface, electrons do not lose the momentum acquired from the electric field. In the present case, the acquired momentum is directed along the normal, and the nature of the reflection does not play so important a role. The occurrence of the surface force (3) is a consequence of a loss of momentum by electrons at the boundary of the metal. Rigorously specular reflection simplifies the treatment; as shown previously in Refs. 18 and 19, taking surface scattering of the carriers into account does not qualitatively change the distribution of the longitudinal electric and acoustic fields in the conducing half-space.

The set of equations (4) and (6) and the boundary condition (3) completely describe the self-consistent distribution of the acoustic and electric fields in the conducting half-space $x \geqslant 0$. Continuing the unknown functions $u(x)$ and $E_1(x) = E(x) - E(0)/\varepsilon$ to negative x as odd functions, this system may be solved using Fourier transform techniques.[3)] Eliminating E_1, for the Fourier transform of the acoustic field

$$u_p \equiv \varkappa \int_{-\infty}^{\infty} dx\, u(x) \exp(i\varkappa p x), \qquad \varkappa \equiv (\omega + i\nu)/v_0 \tag{8}$$

we obtain

$$u_p = \frac{2i}{p} \left\{ u(+0) \left(1 + \frac{a^2}{D_p} \right) + \frac{\lambda/\gamma}{1 - p^2(1 + i\nu/\omega)/3\varepsilon} \left(1 + \frac{a^2 - p^2}{D_p} \right) \right\}, \tag{9}$$

where we have used the notation

$$\lambda = \frac{\Lambda E(0)}{4\pi e \rho S^2 (1 + i\nu/\omega)}, \qquad \gamma = \frac{-3\varepsilon}{4\pi\rho} \frac{\omega\Lambda}{e S v_0}, \qquad a = \frac{\omega}{\varkappa S}, \tag{10}$$

and $u(+0)$ is the boundary displacement, which must be determined from (3). The dispersion relation

$$D_p = p^2 - a^2 + \gamma \left(1 - p^2 \frac{1 + i\nu/\omega}{3\varepsilon} \right) \times \frac{(p^2 - 3) Q_p + 3}{p^2/3(1-\varepsilon) + (1 - Q_p)(1 + i\nu p^2/3\omega(1-\varepsilon))} \tag{11}$$

may be expressed in terms of the Fourier transform of the kernel $Q(x)$ [see (7)]:

$$Q_p = \frac{1}{2p} \ln \frac{1+p}{1-p}.$$

It is possible to prove that in the lower half of the p-plane, the solution u_p has only two singular points: close to $-a$, there is a zero of the dispersion relation (11) at $-p_0$, and there is a branch point at $p = -1$. The region near the zero of the permittivity of the metal [the latter is equal to the denominator in (11) multiplied by $3(1-\varepsilon)p^{-2}$] contains no singular points of u_p, i.e., the exponentially decaying part of the electric field drops out of consideration, as indicated above. This circumstance is a consequence of the special form of the deformation force (2) ($\langle \Lambda_{xx} \rangle = 0$), which reflects the quasi-neutrality of the metal. With all this in mind, the solution satisfying the boundary condition (3) can be represented in the form

$$u(x) = \frac{\lambda}{\gamma} \int_C \frac{dp e^{-i\varkappa px}}{i\pi p D_p} \left\{ \frac{p^2-a^2}{1-p^2(1+i\nu/\omega)/3\varepsilon} - \int_{C_2} \frac{dp}{p^2 D_p} \frac{p^2-a^2}{1-p^2(1+i\nu/\omega)/3\varepsilon} \left(\int_C \frac{dp}{p^2 D_p} \right)^{-1} \right\}, \tag{12}$$

where the integration contour $C = C_1 + C_2$, consists of two "loops": the first (C_1) encompasses the pole at $-p_0$, and the second surrounds the cut from the branch point at $p = -1$ to infinity along the real p-axis. The acoustic wave $u_{ac}(x)$ is the residue of the integrand in (12) at $p = -p_0$:

$$u_{зв}(x) = 2\lambda e^{i\varkappa p_0 x} \int_{C_2} \frac{dp}{p^2 D_p} \left(\frac{p_0^2-a^2}{1-p_0^2(1+i\nu/\omega)/3\varepsilon} - \frac{p^2-a^2}{1-p^2(1+i\nu/\omega)/3\varepsilon} \right) \times \left[\gamma p_0 D'_{-p_0} \int_C \frac{dp}{p^2 D_p} \right]^{-1}. \tag{13}$$

The prime here denotes a derivative with respect to p. Note that there is an integral over C_2 in the numerator. It is well known (see Refs. 18 and 19) that there is a branch point because the electron gas is degenerate, and the electrons have a maximum (Fermi) velocity v_0. Electrons with $v_x = v_0$ produce an anomalous component of the longitudinal electric field—the "pulling field"—which penetrates a distance on the order of the mean free path $l = v_0/\nu$ into the metal. It is apparent from eq. (13) that it is precisely this field which is responsible for the linear acoustic excitation mechanism.

3. Equations (12) and (13) provide an exact solution of the problem for arbitrary values of the input constants (which permit a quasiclassical description). As applied to the usual conditions, however, these expressions contain the very small parameter

$$a^2 \frac{1+i\nu/\omega}{3(1-\varepsilon)} = \left(\frac{\omega}{\alpha S} \right)^2 = \frac{1}{3} \left(\frac{v_0}{S} \frac{\omega}{\omega_0} \right)^2 \ll 1, \tag{14}$$

i.e., the square of the ratio of the Debye length to the acoustic wavelength. In a typical metal, at a frequency $\omega = 10^9 \sec^{-1}$, we have $(\omega/\alpha S)^2 \approx 10^{-7}$, and this grows very slowly (proportional to $n^{-1/3}$) with decreasing electron density, so we will not consider cases in which the inequality of (14) does not hold, which are only possible at very high frequencies in degenerate semiconductors. Making the basic approximation in $(\omega/\alpha S)^2$, the dispersion relation (11) may be written in the form

$$D_p = p^2 - a^2 + \gamma[3 + p^2 Q_p/(1-Q_p)],$$

and its zero is at

$$-p_0 \approx -a - \begin{cases} i\pi\gamma/4, & |a| \gg 1 \\ -4\gamma/5a, & |\gamma|^{1/2} \ll |a| \ll 1 \end{cases}, \tag{15}$$

$$i\varkappa p_0 \approx \frac{i\omega}{S} - \left(\frac{\Lambda}{mSv_0} \right)^2 \frac{nm\omega}{\rho v_0} \begin{cases} \dfrac{3\pi}{4}, & l \gg \dfrac{S}{\omega} \\ \dfrac{6}{5} \dfrac{\omega v_0}{\nu S}, & \dfrac{S}{v_0} \dfrac{S}{\omega} \ll l \ll \dfrac{S}{\omega} \end{cases}. \tag{15'}$$

It is necessary to have $|\gamma|^{1/2} \ll |a|$ in order for p_0 not to differ too much from ϑ. Note that when $|a| \gg 1$, $|\gamma| \sim 1$, while when $|a| \ll 1$, $|\gamma| \sim \omega/\nu$, whereupon we get the constraint $l \gg (S/v_0)(S/\omega)$ in the second line of Eq. (15').

Thus, the acoustic wave excited by the electric field takes the form

$$u_{ac}(x, t) = A \exp(i\varkappa p_0 x - i\omega t). \tag{16}$$

For typical metals, the collisionless damping coefficient (with $v_0/\nu \gg S/\omega$), which is well known from Ref. 20, is of order ω/v_0, and the amplitude, according to (13) and (14), is

$$A = \frac{2\lambda}{\gamma p_0 D'_{-p_0}} \left(\int_{C_2} dp \frac{p_0^2 - p^2}{p^2 D_p} \Big/ \int_C \frac{dp}{p^2 D_p} \right). \tag{17}$$

A straightforward but somewhat tedious calculation of the integral in the numerator (performed numerically for $|a| \to 0$) gives

$$\int_{C_2} \frac{dp}{D_p} \left(\frac{p_0^2}{p^2} - 1 \right) \approx \begin{cases} a^{-2} \ln |a|, & |a| \gg 1 \\ -0.164\ldots, & |a| \ll 1 \end{cases}.$$

The normalizing integral in (17) is basically determined by the residue at $-p_0$. As a result,

$$A \approx \frac{-\Lambda E(0)}{8\pi e \rho S v_0} \begin{cases} \ln \dfrac{(\omega l/S)^2}{1+\omega^2/\nu^2}, & l \gg \dfrac{S}{\omega} \\ \left(\dfrac{\omega l}{S} \right)^2 0{,}328\ldots, & \dfrac{S}{v_0} \dfrac{S}{\omega} \ll l \ll \dfrac{S}{\omega} \end{cases}. \tag{18}$$

The amplitude of the ultrasound which is excited first increases rapidly with $\omega l/S$; as might be expected, the "collisionless" regime is the most favorable, where the electron mean free path l is much greater than the acoustic wavelength S/ω. For $\omega \gg \nu$, the frequency dependence and the dependence on the mean free path approaches saturation

$$A \approx -(\Lambda E(0)/4\pi e \rho S v_0) \ln(v_0/S).$$

We can compare this result with the amplitude of an acoustic wave generated by the quadratic transformation mechanism:

$$|u_{ESAT}| = E^2(0)/16\pi\rho S\omega$$

(see Ref. 17). Since the latter falls off with increasing frequency, we find that at frequencies greater than

$$\omega_1=\frac{ev_0E(0)}{4\Lambda\ln(v_0/S)}\approx\frac{eE(0)}{mv_0\ln(v_0/S)},$$

the linear deformation mechanism for generating ultrasound is dominant. This estimate is made under the assumption that ω_1 corresponds to perfect collisionless conditions, i.e., that $\omega_1\gg\nu$, but in fact, there is little change if the much weaker constraint $\omega_1\gg(S/v_0)\nu$ is employed. Assuming for the numerical estimate that $E(0)=1$ CGSE unit, we obtain $\omega_1\approx10^8$–10^9 sec^{-1}.

In pure conductors at low temperatures, the experimental conditions for the two effects are the same. The presence of an acoustic signal at the frequency of the applied field (and not at double the frequency, as in the case of ESAT and an increaing amplitude as a function of $\omega l/S$ would be indicative of a linear conversion mechanism.

4. The wave (16) does not exhaust the displacement field excited in the conductor: there is another significant contribution at high frequencies $\omega>\nu$ from the branch point u_p, usually referred to as a quasiwave (cf. Ref. 19). This part of the displacement field $u_{an}(x)$ is described by the integral over C_2 in (12), and can be put in the form

$$u_{an}(x)=-\lambda\int_1^\infty\frac{dp\,e^{i\varkappa px}(W^2+\pi^2/4p^2)^{-1}}{p^2[1-2\gamma+2W\gamma/(W^2+\pi^2/4p^2)]-a^2+6\gamma},$$

where

$$W(p)=1+\frac{1}{2p}\ln\frac{p-1}{p+1}. \tag{19}$$

When $x\ll l$, the damping in this expression with increasing x is due only to the increase in oscillation frequency of the integrand, and is not exponential, i.e., a quasiwave is typically produced in the conductor at a distance of one mean free path length l of the carriers. The values of p which are important in determining the magnitude of the integral (19) lie in an interval $v_0/\omega x$ above the lower limit, so for $x\gg S/\omega$, the integrand can be expanded in powers of $(p/a)^2$.

$$u_{an}(x)\approx\frac{\lambda}{a^2}\int_1^\infty\frac{dp\exp(i\varkappa px)}{\{1+(1/2p)\ln[(p-1)/(p+1)]\}^2+\pi^2/4p^2},$$

$$x\gg S/\omega.$$

We have also dropped terms of order ν/a^2 compared with unity. It is then not hard to see that u_{an} is much less than A [see (17)], so that right down to a depth of the order of the distance traversed by an electron in one period of the field (v_0/ω), the solution $u(x,t)$ is actually an acoustic wave (16), moving at speed S. At large depths, however, collisionless damping takes place, and for $\omega\gg\nu$, the ultimate asymptotic acoustic wave in the conductor turns out to be a quasiwave propagating at the Fermi velocity v_0,

$$u_{an}(x,t)\approx\frac{i\Lambda E(0)}{\pi e\rho v_0^2}\frac{v_0}{\omega x}\ln^{-2}\left(\frac{\omega x}{v_0}\right)\exp\left(\frac{i\omega-\nu}{v_0}x-i\omega t\right),$$

$$x\gtrsim\frac{v_0}{\omega}\ln\frac{v_0}{S}. \tag{20}$$

5. The physical nature of the linear conversion mechanism is the following. A varying electric field which penetrates a metal (to a very small depth of the order of the Debye length $v_0/\omega\sqrt{3}$, to a first approximation) causes oscillations of the electron density, which in a degenerate Fermi gas are carried by electrons into the metal at the Fermi velocity v_0 a distance equal to their mean free path length.[18] Being associated by virtue of the deformation potential with lattice displacements, these oscillations of the electron density excite both intrinsic vibrations (with wave vector $k\approx\omega/S$) and induced vibrations $(\varkappa\approx(\omega+i\nu/v_0)$ of the lattice. These waves do not cancel one another anywhere in the metal, thus giving rise to the observed effect.

To conclude this point, we make the following comment. Just as in other boundary-value problems, it appears here at first glance that there is some difficulty associated with the fact that "bulk" effects such as the deformation potential and even the quasiparticle dispersion relation lose their meaning, strictly speaking, at microscopic distances from the crystal boundary. Our calculations indicate, however, that in the present case the behavior of the deformation force [see (2), (4)] at macroscopic depths comparable to the wavelength of the excited acoustic wave is important [in the integral (17) for the amplitude, only wave vectors $\varkappa_p$ less than or of order ω/S are significant]. The extrapolation we have made therefore does not detract from the applicability of the results obtained.

6. Preliminary measurements made at liquid-helium temperatures on monocrystalline tin bear out the observability of the effect. The measurements were made by the pulse-echo method at a frequency $\omega\approx6.3\cdot10^7$ sec^{-1}, with the ultrasound propagating in the direction of the [001] fourth-order symmetry axis; at $T=4.2$ K, the parameter $kl\approx12$. The linear-generation amplitude was approximately 1% of the amplitude of ultrasound excited by the ESAT mechanism at twice the frequency, which is qualitatively consistent with our calculations.

As can be seen from Eqs. (16) and (17), quantitative measurements of the amplitude of longitudinal ultrasound excited by a longitudinal electric field in degenerate conductors can serve as a source of information about the diagonal components of the deformation potential tensor, to which we plan to devote a separate communication.

[1)]We assume that the normal to the surface of the sample corresponds to a "good" crystallographic direction, along which a purely longitudinal wave can propagate.

[2)]Equation (4), which derives only from the Boltzmann equation (with the collision integral in the form $\nu(\chi-\langle\chi\rangle)/\langle1\rangle$) and Poisson's equation, does not depend on the nature of the reflection.

[3)]As follows from the continuity condition for electric displacement, the field in the interior of the metal tends to the limit $E(0)/\varepsilon$, so that $E_1(+\infty)=0$. It is easily seen that the constant $E(0)/\varepsilon$ also drops out of the deformation force (4).

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