Dynamics of electron motion and of emission of radiation during axial quasichanneling

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An investigation is made of the spectral characteristics of the radiation emitted by relativistic electrons during axial quasichanneling in thick single crystals. It is shown that in the region of a maximum the spectral intensity of the radiation emitted by channeled electrons is several times higher than the intensity of the radiation generated as a result of quasichanneling. This is due to rapid diffusion of the transverse energies of quasichanneled electrons. Above-barrier electrons makes the main contribution to the hard part of the spectrum.

Motion of relativistic charged particles at low angles to crystallographic axes and planes is accompanied by the emission of strong electromagnetic radiation in the x-ray and gamma wavelength ranges (for a review see Ref. 1). The characteristics of such radiation depend strongly on the nature of motion of particles in the electrostatic field of atomic chains and planes. The greatest amount of work has been done on the spectral and integral characteristics of the radiation emitted as a result of planar channeling and quasichanneling of electrons and positrons in the soft part of the spectrum when the frequency of the emitted photons is $\hbar\omega < 0.1E$, where $E = \gamma mc^2$ is the kinetic energy of a particle and γ is the Lorentz factor. A solution of the transport equation was used in Refs. 2 and 3 to demonstrate the important influence of multiple scattering on the spectral characteristic of the radiation emitted as a result of motion of particles in the field of atomic planes in thick single crystals. A similar problem was tackled by the computer modeling method in Ref. 4. These investigations have demonstrated that a correct interpretation of the available experimental results requires allowance for the contribution made to the emitted radiation by above-barrier (quasichanneled) particles with infinite transverse trajectories. In the case of axial channeling of negative particles the problem is complicated by the need to solve the two-dimensional transport equation of Beloshitskiĭ and Kumakhov[5] together with the one-dimensional transport equation for quasichanneled particles. This problem was considered in Ref. 6. Use was made of a model potential of an atomic chain $U(\rho) \propto \alpha/\rho$ (ρ is the distance from a chain). This potential has a singularity in the limit $\rho \to 0$. The emission spectra of channeled electrons were calculated in the approximation of closed transverse trajectories on the basis of a theory developed in Refs. 7 and 8. The contribution of quasichanneled electrons to the radiation was ignored in Ref. 6.

We report a study of the spectral and spectral-angular characteristics of the radiation which appears as a result of axial quasichanneling of electrons in thick single crystals in the range of relatively soft frequencies $\hbar\omega \ll E$. The importance of this problem is due to the circumstance that in the case of thick single crystals the above-barrier electrons represent a much larger part of the initial beam.

The characteristic features of the radiation spectrum obtained in a quasichanneling case were considered qualitatively in Ref. 9 using the approximation of rectilinear transverse trajectories. The problem was considered in greater detail in a review paper.[10] The quantum aspects of the radiation obtained in the quasichanneling case were discussed in Ref. 11.

In recent papers[12,13] it was suggested that the transport equation method developed in Ref. 5 and used in the present study is unsuitable for the investigation of axial channeling of electrons. For example, the procedure of averaging of the diffusion coefficients in the kinetic equations was linked in Ref. 13 to the condition that the change in the parameters of a trajectory be small during the period of transverse motion when such averaging is a consequence of a statistical equilibrium in the transverse phase space. This problem was considered by many authors (for a review see Ref. 14). It was shown that in the case of planar channeling a statistical equilibrium in respect of the coordinates is established after several oscillations of a particle in a channel. In the case of axial channeling of electrons such an equilibrium distribution appears much more rapidly because of the considerable anharmonicity of the potential and because of the stronger multiple scattering. Therefore, the correctness of the procedure of averaging of the transport equation over an equilibrium distribution is in no way related to conditions of the $\Delta\overline{E}_\perp/E_\perp < 0.1$ type, where $\Delta\overline{E}_\perp$ is the change in the transverse energy of an electron during one revolution around a chain. A condition of the $\Delta\overline{E}_\perp/E_\perp < 0.1$ type used in Ref. 13 as the criterion of validity of the method of averaging of the transport equations is in conflict with the generally accepted ideas (see, for example, the theory of the Krylov-Bogolyubov method[15]). Investigations of the motion of electrons in the course of axial channeling, carried out by Vorob'ev and Taratin using a computer modeling method,[16] gave results in agreement with the numerical solution of the transport equation.

The accelerator in Serpukhov[17] was used to study the angular distribution of electrons of 10 GeV energy transmitted along ⟨111⟩ channels in a silicon crystal. It was found that the total number of particles emerging from a crystal 0.8 mm thick at angles to the axis less than the critical channeling angle was greater than in the initial beam. This was due to the bulk capture of single electrons into the axial channeling regime, investigated theoretically in Refs. 16 and 18. In a thickness of 0.8 mm the channeled electrons in silicon made on the average 50–60 revolutions about the axis when the beam energy was 10 GeV. However, in spite of multiple scat-

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tering, the number of particles in the channel increased, whereas according to the conclusions reached in Ref. 13 there should be no channeled particles in crystals of this thickness (see Fig. 1 in Ref. 13). Therefore, the view expressed in Refs. 12 and 13 that the channeling of electrons should not occur at gigaelectron-volt energy is in conflict with the experimental results and those obtained by numerical modeling.

1. DYNAMICS OF MOTION OF QUASICHANNELED ELECTRONS

If a beam of electrons is incident on a crystal at an angle ψ to the crystallographic axis, the fraction of particles which become channeled can be calculated from

$$N_c=3\left(\frac{a_F}{\rho_0}\right)^2\left\{\exp\left[2\frac{\psi^2}{\psi_L^2}+\frac{d}{Ze^2}U_L(\rho_0)\right]-1\right\}^{-1}, \quad (1)$$

where ρ_0 is the transverse radius of the channel; a_F is the Thomas-Fermi screening parameter; Z is the atomic number of atoms in the crystal; d is the distance between atoms in a chain; $\psi_L = (4Ze^2/dE)^{1/2}$ is the critical Lindhard angle. Equation (1) is derived for the Lindhard potential $U_L(\rho)$. If $\psi \sim 0.15\psi_L$, then about 50–70% of the initial beam becomes channeled.

Multiple scattering by electrons in a crystal and by thermal vibrations of the lattice nuclei results in a redistribution of the beam electrons in the space of transverse energies causing dechanneling and then the channeled electrons have two integrals of motion: the transverse energy $E_\perp$ and the momentum M of motion relative to a chain. Consequently the evolution of channeled electrons with depth of penetration into a crystal is described by a two-dimensional distribution function $f(E_\perp, M, z)$ satisfying the Fokker-Planck transport equation obtained for negative particles by Beloshitskiĭ and Kumakhov[5]:

$$\frac{\partial f}{\partial z}=\frac{\partial}{\partial E_\perp}\left[D_{\varepsilon\varepsilon}T\frac{\partial}{\partial E_\perp}\left(\frac{f}{T}\right)\right]+\frac{\partial}{\partial E_\perp}\left[D_{\varepsilon\mu}T\frac{\partial}{\partial M}\left(\frac{f}{T}\right)\right]+\frac{\partial}{\partial M}\left[D_{\mu\varepsilon}T\frac{\partial}{\partial E_\perp}\left(\frac{f}{T}\right)\right]+\frac{\partial}{\partial M}\left[D_{\mu\mu}T\frac{\partial}{\partial M}\left(\frac{f}{T}\right)\right], \quad E_\perp<0, \quad (2)$$

where T is the period of transverse motion and the diffusion coefficients D_{ij} are expressed in terms of the rms angle of multiple scattering which includes the contribution made to the scattering by electrons and thermal vibrations of the crystal nuclei (for details see Ref. 5).

Quasichanneled electrons have a positive transverse energy and move in a field of many atomic chains. The spatial distribution of quasichanneled electrons is homogeneous in a transverse plane, which corresponds to a uniform distribution of above-barrier particles in respect of the impact parameters relative to a chain. The only integral of motion is then the transverse energy $E_\perp$. In that sense the axial channeling of above-barrier electrons is analogous to the motion of positrons in the field of atomic chains and the prediction made in Ref. 13 that in the region of above-barrier motion the diffusion coefficient of the momenta should tend to infinity is meaningless, because in the present model a uniform distribution of the angular momenta of electrons is ensured automatically in the range $E_\perp > 0$. The distribution function of quasichanneled electrons $f(E_\perp, z)$ satisfies the one-dimensional transport equation[5]:

$$\frac{\partial f}{\partial z}=\frac{\partial}{\partial E_\perp}\left(\left\langle\frac{\overline{\Delta E_\perp^2}}{2\Delta z}\right\rangle\frac{\partial f}{\partial E_\perp}\right), \quad E_\perp>0, \quad (3)$$

where the angular brackets represent averaging over all the impact parameters of an electron relative to the axis. The diffusion coefficient in Eq. (3) rises linearly on increase in the transverse energy. If the potential is $U(\rho) \propto \alpha/\rho$, we then have[18]

$$\left\langle\frac{\overline{\Delta E_\perp^2}}{2\Delta z}\right\rangle=(E_\perp+U_m)\,\delta, \quad \delta=\frac{E}{2L_{rad}}\left(\frac{21}{E[\mathrm{MeV}]}\right)^2, \quad (4)$$

where $L_{\rm rad}$ is the radiation length and U_m is the height of the potential barrier. An allowance is made in Eq. (4) for the fact that we can ignore the scattering by the crystal electrons in the case of the above-barrier particles.

Equation (4) allows us to estimate the region in the space of transverse energies where the majority of the quasichanneled electrons is located at a depth z:

$$0<E_\perp<E_\perp^{max}=z\delta\,[1+(1+2U_m/z\delta)^{1/2}], \quad (5)$$

where δ is defined in Eq. (4). For example, in the case of the $\langle 111\rangle$ channeling in a silicon crystal of thickness 250 μ at beam energies of 600 MeV, 1.2 GeV, and 5 GeV for an initial angle of incidence $\psi < \psi_L$ on a crystal almost all the quasichanneled electrons are within the transverse energy range $0 < E_\perp < E_\perp^{\max}$, where $E_\perp^{\max} = 19U_m$, $12U_m$, and $3U_m$, respectively. This estimate is in good agreement with the results of a numerical solution of the transport equation.

A numerical method for the simultaneous solution of Eqs. (2) and (3) was described in detail in Ref. 18. In calculating the spectral characteristics of the radiation it is of interest to consider the distribution function $\bar f(E_\perp, z)$ averaged over the thickness of a crystal:

$$\bar f(E_\perp, z)=\frac{1}{z}\int_0^z f(E_\perp, z)\,dz. \quad (6)$$

Figure 1 shows the results of a calculation of the depth-averaged distribution function (6) representing quasichanneled electrons in a $\langle 111\rangle$ silicon crystal of thickness $z = 250$ μ when the beam energy is 0.6, 1.2, and 5 GeV. The initial angle of entry relative to the atomic chain is $\psi = 0.15\,\psi_L$. The initial distribution of quasichanneled electrons $f(E_\perp, z = 0)$ is such that for the angle of entry ψ these elec-

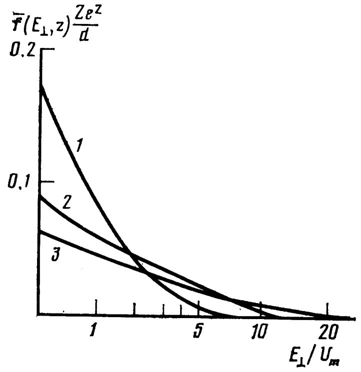

FIG. 1. Depth-averaged distribution function of quasichanneled electrons in a $\langle 111\rangle$ silicon crystal of thickness 240 μ. The initial angle of entry into a channel is $\psi = 0.15\,\psi_L$ and the beam divergence is 10^{-4} rad. Curves 1, 2, and 3 correspond to the energies 5, 1.2, and 0.6 GeV. The values of Z, e, and d are the same as in Eq. (1).

trons occupy states with transverse energies $0 < E_\perp < E\psi^2/2$. When the chain potential is $U(\rho) \propto \alpha/\rho$, the initial distribution function of the transverse energies of the above-barrier electrons is

$$f(E_\perp, z=0) = \begin{cases} 2(\alpha/\rho_0)^2(\alpha/\rho_0 - E_\perp + E\psi^2/2)^{-3}, & 0<E_\perp<E\psi^2/2 \\ 0, & E_\perp > E\psi^2/2 \end{cases}. \tag{7}$$

This distribution has a sharp maximum at $E_\perp = E\psi^2/2$. However, as electrons penetrate into a crystal, the value of the distribution function at the maximum falls rapidly and at depths of several microns (for $E\sim 1$ GeV) the maximum disappears.

A crystal is regarded as thin if the thickness is such that the range of transverse energies where the bulk of the quasichanneled electrons is concentrated does not exceed the potential-barrier height:

$$z \leqslant z_1 = U_m/4\delta. \tag{8a}$$

If as a result of diffusion the range of transverse energies of the quasichanneled particles is approximately $\Delta E_\perp \sim 10 U_m$, a crystal can be regarded as thick:

$$z \geqslant z_2 = 9U_m/2\delta. \tag{8b}$$

The results of a numerical solution of the transport equations (2) and (3) show that the faster the diffusion of above-barrier particles the faster the dechanneling of electrons out of the bound state. At depths $z\sim z_1$ the fraction of the initial beam which becomes channeled is ~25–35%, and at depths $z\sim z_2$ only 2–5% of the particles remain in the bound state (if the initial angle of entry of electrons into a channel is $\psi<\psi_L$). At energies of $E\sim 1$ GeV in the case of ⟨111⟩ of silicon we have $z_1 = 10\,\mu$, $z_2 = 215\,\mu$, whereas for ⟨111⟩ of germanium we have $z_1 = 4.6\,\mu$ and $z_2 = 85\,\mu$.

2. PROBLEM OF THE CALCULATION OF SPECTRAL CHARACTERISTICS OF THE RADIATION EMITTED BY ELECTRONS IN AN AXIALLY SYMMETRIC POTENTIAL

The results of calculations of the spectral characteristics of the radiation emitted by electrons crossing thick single crystals are reported in Ref. 6. The potential of an atomic chain is assumed to be $U(\rho)\propto\alpha/\rho$. The transverse trajectories of channeled electrons for this potential are ellipses with a focus coinciding with the axis. The difficulties encountered due to the divergence of the Coulomb potential at low values of ρ were avoided in Ref. 6 by ignoring electrons with small angular momenta. Moreover, the potential $U(\rho)\propto\alpha/\rho$ ignores the precession of transverse trajectories of channeled electrons, although such precession occurs in the case of axial channeling of negative particles.[19] A theory developed in Ref. 20 is free of these shortcomings and it gives general expressions for the calculation of the spectral characteristics of the radiation generated as a result of axial channeling in an arbitrary axially symmetric field of an atomic chain. In the dipole approximation the spectral density of the radiation intensity per unit length is[20]

$$\frac{d^2I}{d\omega\,dz} = \frac{e^2\omega}{2c^3}\sum_{n=0}^{\infty}\Big[\omega_n^{(+)2}|\rho_n^+|^2 f\Big(\frac{\omega}{2\omega_n^{(+)}\gamma^2}\Big) + \omega_n^{(-)2}|\rho_n^-|^2 f\Big(\frac{\omega}{2\omega_n^{(-)}\gamma^2}\Big)\Big], \tag{9}$$

$$\rho_n^\pm = \frac{1}{T}\int_0^T \rho(t)\exp[i\varphi(t) + i(n\omega_0\pm\Omega)t]\,dt, \tag{10}$$

where

$$\omega_n^{(\pm)} = n\omega_0\pm\Omega,\qquad f(x) = (1-2x+2x^2)\eta(1-x),$$

$\eta(x)$ is the Heaviside step function; $\omega_0 = 2\pi/T$; T is the period of transverse radial oscillations; $\Omega = \Delta\varphi/T$; $\Delta\varphi$ is the angle of precession; $\rho(t)$ and $\varphi(t)$ are the transverse polar coordinates of an electron; c is the velocity of light.

A characteristic feature of Eq. (9) is that for a given nth harmonic the spectrum acquires a doublet structure. Moreover, radiation appears at the zeroth harmonic ($n=0$) which corresponds to the precession frequency shifted because of the Doppler effect.

The Fourier components of the transverse coordinates of an electron given by Eq. (10) admit an analytic solution for the potential of an atomic chain in the form[19]

$$U(\rho) = \begin{cases} -\alpha/\rho + \alpha/\rho_0, & \rho_1<\rho<\rho_0 \\ -U_m + \beta\rho^2, & \rho<\rho_1 \end{cases}, \tag{11}$$

where the parameters ρ_1 and β are deduced from the condition of continuity of the potential (11) and its derivative when $\rho = \rho_1$.

3. SPECTRAL AND SPECTRAL-ANGULAR CHARACTERISTICS OF RADIATION OF AXIALLY SYMMETRIC QUASICHANNELED ELECTRONS

The spectral-angular distribution of the radiation energy into a solid-angle element $d\Omega$ is[21]

$$\frac{d^2I}{d\omega\,d\Omega} = \frac{e^2\omega^2}{4\pi^2c^3}|[\mathbf{n}\mathbf{j}_\omega]|^2,\qquad \mathbf{j}_\omega = \int_{-\infty}^{\infty}\mathbf{v}(t)\exp[i\omega t - i\mathbf{k}\mathbf{r}(t)]\,dt, \tag{12}$$

where $\mathbf{r}(t)$ and $\mathbf{v}(t)$ are the coordinate and the velocity of a particle; $\mathbf{k} = \omega\mathbf{n}/c$ is the wave vector of the emitted photon; $\mathbf{n}$ is a unit vector in the direction of the radiation.

If a charged particle moves at a low angle to a crystallographic axis, Eq. (12) can be simplified. Bearing in mind the smallness of the polar angle of the radiation $\theta\ll 1$, we find that to within terms of the order of $\sim(\theta v_\perp)^2$ (see Ref. 22),

$$\frac{d^2I}{d\omega\,d\Omega} = \frac{e^2\omega^2}{4\pi^2c^3}(|[\mathbf{n}_\rho\mathbf{j}_\rho]|^2 + |j_z\theta - \mathbf{n}_\rho\mathbf{j}_\rho|^2), \tag{13}$$

$$\mathbf{j}_\rho = \int_{-\infty}^{\infty}\dot{\boldsymbol{\rho}}\exp(i\omega t - i\mathbf{k}\mathbf{r})\,dt,\qquad j_z = \int_{-\infty}^{\infty}\exp(i\omega t - i\mathbf{k}\mathbf{r})\,dt, \tag{14}$$

where $\mathbf{n}_\rho(\cos\varphi, \sin\varphi)$ is a unit vector in the direction of the projection of the momentum of a photon on a transverse plane and φ is the azimuthal angle of the radiation.

If the maximum angle of deflection of the axis by the field and the angle at which an electron moves relative to an atomic chain are small compared with the effective angle of emission of the radiation, we can use the dipole approximation. In this case Eqs. (13) and (14) can be simplified further. After integration with respect to the azimuthal angle, we obtain

$$\frac{d^2I}{d\omega\theta\,d\theta} = \frac{e^2\omega^4}{8\pi c^3}|\rho_\omega|^2(\theta^4 + \gamma^{-4}), \tag{15}$$

$$\rho_\omega = \int_{-\infty}^{\infty} \rho(t) \exp\left[\frac{i\omega t}{2}(\theta^2+\gamma^{-2})\right] dt.$$

The approximate formula (15) is valid if

$$2EE_\perp/(mc^2)^2 \ll 1 \tag{16}$$

(m is the rest mass on an electron).

The Fourier components of the current (14) can generally be integrated analytically for the chain potential $U(\rho) \propto \alpha/\rho$. The expressions obtained in this case can be found in Ref. 11. In the dipole approximation Eq. (15) for the potential $U(\rho) \propto \alpha/\rho$ gives[11]

$$\frac{d^2 I}{d\omega\theta\, d\theta} = \frac{2}{\pi c}\left(\frac{e\omega a}{c}\right)^2 \frac{\theta^4+\gamma^{-4}}{(\theta^2+\gamma^{-2})^2} \exp(\pi\nu)\, Y(\nu\varepsilon),$$

$$Y(\nu\varepsilon) = K_{i\nu}'^2(\nu\varepsilon) + (1-\varepsilon^{-2}) K_{i\nu}^2(\nu\varepsilon), \quad \nu = \omega(\theta^2+\gamma^{-2})/2\omega_0,$$
$$\omega_0^2 = 2E_\perp (c/a)^2/E, \quad a = \alpha/2E_\perp, \quad \varepsilon = (1+b^2/a^2)^{1/2}, \tag{17}$$

where b is the impact parameter of an electron relative to an atomic chain, and $K_{i\nu}$ and $K'_{i\nu}$ are the Macdonald function and its derivative with respect to the argument. In rough calculations we can use the approximate formula for

$$K_{i\nu}(x) \approx (\pi/2x)^{1/2} \exp(-x-\nu^2/2x), \quad x > 1.$$

The spectral-angular characteristics of the radiation generated as a result of the scattering of a particle in the field of an isolated chain can be calculated using Eqs. (14), (15), and (17).

We shall now average these expressions over all the impact parameters. Since the spatial distribution of quasichanneled electrons is uniform in a transverse plane, the average number of collisions per unit length of the trajectory of electrons with impact parameters from b to $b+db$ with a chain is $dn = (v_\perp/\pi\rho_0^2 c)db$. In the case of the Coulomb potential we obtain

$$\frac{d^3 I}{d\omega\, dz\, \theta\, d\theta} = \frac{a}{\pi\rho_0^2}\left(\frac{2E_\perp}{E}\right)^{1/2} \int_{\varepsilon_{min}}^{\varepsilon_{max}} \frac{d^2 I}{d\omega\theta\, d\theta} \frac{\varepsilon\, d\varepsilon}{(\varepsilon^2-1)^{1/2}},$$
$$\varepsilon_{min} = (1+u_\perp^2/a^2)^{1/2}, \quad \varepsilon_{max} = (1+\rho_0^2/a^2)^{1/2}, \tag{18}$$

where u is the amplitude of thermal vibrations of the crystal lattice nuclei.

It follows from the above formulas that the characteristic frequencies of the radiation are $\omega \propto 2\omega_0\gamma^2$ and that the spectral intensity per unit length of the trajectory can be calculated to within an order of magnitude using the formulas

$$\hbar\omega_1 = \frac{8\hbar c\gamma^2}{3a_F}\left(\frac{2Ze^2}{dE}\right)^{1/2}, \quad I_0 = \frac{3a_F}{\hbar c}\left(\frac{e\gamma}{\pi\rho_0}\right)^2 \left(\frac{2Ze^2}{dE}\right)^{3/2}. \tag{19}$$

As in the case of the channeled electrons, the characteristic frequencies of the radiation at relatively low energies increase proportionally to $E^{3/2}$ and the spectral intensity is $d^2I/d\omega dz \propto E^{1/2}$.

Figure 2 shows the spectral-angular distributions of the radiation emitted by 10 GeV electrons channeled in $\langle 111 \rangle$ silicon, calculated allowing for the nondipole nature of the radiation and assuming various transverse energies. The calculations were carried out using Eqs. (14) and (18) at a fixed frequency amounting to $\omega = 0.5\omega_1$, where ω_1 is defined by Eq. (19). The Fourier components of the current of Eq.

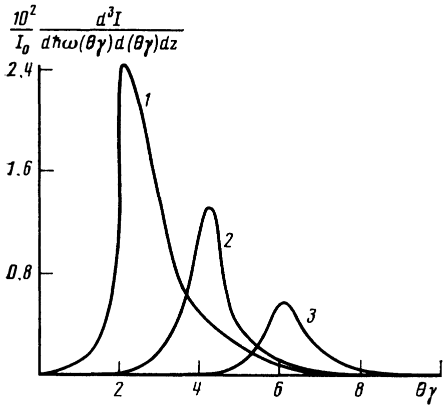

FIG. 2. Spectral-angular density of the intensity of the radiation emitted by quasichanneled electrons with different transverse energies traveling along the $\langle 111 \rangle$ axis in silicon when the beam energy is 10 GeV and the frequency is $\omega/\omega_1 = 0.5$. Curves 1, 2, and 3 correspond to transverse energies $0.5U_m$, $2U_m$, and $4.6U_m$. The values of I_0 and ω_1 are given by Eq. (19).

(14) were found using the results of Ref. 11. Curves 1, 2, and 3 in Fig. 2 correspond to the transverse energies $0.5U_m$, $2U_m$, and $4.6U_m$. The spectral-angular distribution of the radiation of channeled electrons in a crystal of infinite length is such that a fixed angle θ there is a line spectrum of frequencies corresponding to different harmonics.[7,8,22] In the case of above-barrier electrons this situation is different. It is clear from Fig. 2 that the energy emitted at a given frequency by an above-barrier electron with a transverse energy $E_\perp = U_m$ is concentrated mainly within an angular interval $\Delta\theta \sim 2\lambda^{-1}$ near the value

$$\theta \sim [2(E_\perp + U_m)/E]^{1/2} - \gamma^{-1}. \tag{19a}$$

This formula reflects the fact that the angular distribution of the emitted photons repeats approximately (to within $\Delta\theta$) the angular distribution of quasichanneled electrons.

An increase in the transverse energy reduces the intensity of the radiation emitted by quasichanneled electrons proportionally to $E_\perp^{-1/2}$.

Figure 3 gives the results of a calculation of the averaged (over all the impact parameters) spectral intensity of the radiation per unit length of the trajectory of quasichanneled electrons with a transverse energy $E_\perp = U_m$ in silicon. The results are given for various beam energies: curves 1, 2, and 3 correspond to 600 MeV, 5 GeV, and 15 GeV. The spectrum of the radiation of above-barrier electrons, like the

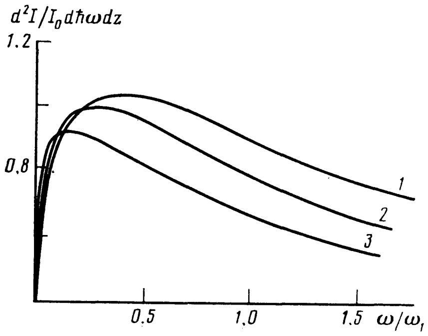

FIG. 3. Spectral density of the intensity of the radiation of above-barrier electrons of transverse energy $E_\perp = U_m$ traveling along the $\langle 111 \rangle$ axis in silicon. Curves 1, 2, and 3 correspond to the beam energies of 0.6, 5, and 15 GeV.

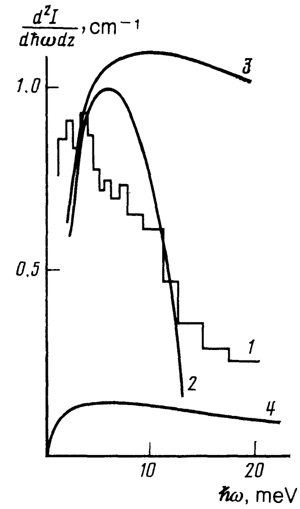

FIG. 4. Radiation spectrum of 600-MeV electrons traveling in a silicon crystal of thickness 250 μ. Curve 1 represents the experimental results reported in Ref. 23, curve 2 is the radiation spectrum of channeled electrons calculated allowing for the effect of precession of transverse orbits, curve 3 is the radiation spectrum of channeled electrons calculated in the approximation of closed transverse orbits (see Ref. 6), and curve 4 is the radiation spectrum of quasichanneled electrons.

spectrum of channeled electrons, has a characteristic maximum in the frequency range $\omega \sim (0.1\text{–}0.8)\omega_1$. An increase in the beam energy results in a departure from the condition for the dipole radiation [Eq. (16)]. The radiation spectrum then shifts toward relatively low ("soft") frequencies, i.e., the characteristic frequencies of the radiation increase on increase in the energy more slowly than $\omega \propto E^{3/2}$. This improves the degree of monochromaticity of the spectrum of the radiation emitted by quasichanneled electrons when the beam energy is increased (see Fig. 3). In the dipole approximation the intensity of the radiation rises proportionally to $E^{1/2}$. On increase in the electron energy this dependence becomes weaker and if the condition $E \gg (\gamma\theta mc^2)^2/U_m$ is obeyed, the spectral intensity of the radiation observed as a result of quasichanneling falls on increase in the energy proportionally to $E^{-1/2}$. In the case of $\langle 111 \rangle$ silicon and the angles of emission $\theta \sim \gamma^{-1}$ this energy is ~ 20 GeV. Similar properties are exhibited also by the spectral characteristics of the radiation emitted by channeled electrons.[22]

4. CALCULATION OF THE SPECTRAL CHARACTERISTICS OF RADIATION IN THICK SINGLE CRYSTALS

Figure 4 gives the results of a theoretical interpretation of the experimental results obtained for a 600-MeV electron beam along the $\langle 111 \rangle$ direction in a silicon crystal. Curve 1 represents the experimental results taken from Ref. 23 and curve 2 gives the results of calculations of the radiation emitted by electrons, carried out in the dipole approximation using Eqs. (9) and (10) and averaging over the distribution function $\bar{f}(E_\perp, M, z)$ of electrons in a channel. Curve 3 gives the results of a calculation of the radiation emitted by channeled electrons when the potential is $U(\rho) \propto \alpha/\rho$, carried out using the approximation of closed transverse orbits, similar to that used in Ref. 6. We can see that allowance for the precession of the transverse orbits improves greatly the agreement between the theory and experiment. In the range of high ("hard") frequencies a calculation carried out using the potential $\sim \alpha/\rho$ overestimates the results greatly because of the high degree of anharmonicity of the Coulomb potential compared with the real potential.

A calculation of the radiation spectrum of above-barrier electrons (curve 4 in Fig. 4) was carried out in the dipole approximation, which is valid at the energies under consideration. In this case the averaging of Eq. (18) over all the impact parameters gives the formula

$$\frac{d^2I}{d\hbar\omega\, dz} = I_0\left(\frac{dE_\perp}{Ze^2}\right)^{1/2} \Omega \int_\Omega^\infty \left[1 - \frac{2\Omega}{\nu}\left(1 - \frac{\Omega}{\nu}\right)\right]$$
$$\times e^{\pi\nu} \int_{\varepsilon_{min}}^{\varepsilon_{max}} Y(\nu\varepsilon) \frac{\varepsilon\, d\varepsilon\, d\nu}{(\varepsilon^2 - 1)^{1/2}}, \tag{20}$$

where $\Omega = \omega/\omega_1$; $\varepsilon_{\min}$ and $\varepsilon_{\max}$ have the same meaning as in Eq. (18); $Y(\nu\varepsilon)$ is defined by Eq. (17).

The expression (20) has to be averaged also with a distribution function (6):

$$\frac{\overline{d^2I}}{d\hbar\omega\, dz} = \int_0^\infty \frac{d^2I}{d\hbar\omega\, dz} \bar{f}(E_\perp, z)\, dE_\perp. \tag{21}$$

It follows from Fig. 4 that the spectral density of the intensity of the radiation emitted by channeled electrons in the region of a maximum in the case of a thick crystal exceeds the intensity of the radiation of quasichanneled electrons by a factor ~ 8. However, at higher frequencies the reverse is true: the radiation is dominated by quasichanneled electrons. The maximum in the radiation spectrum of above-barrier particles coincides approximately with the position of the maximum in the radiation spectrum of electrons, but the degree of monochromaticity of the radiation in the case of channeling in crystals of large thickness is considerably higher. This is due to the fact that in the case of thick single crystals characterized by $z \gtrsim z_2$ the distribution function of quasichanneled electrons is strongly broadened in respect of the transverse energies (Fig. 1). More than half of the contribution to the spectral intensity of the radiation of above-barrier electrons in the region of the maximum of the spectrum comes from electrons with transverse energies in the range $0 < E_\perp < 2U_m$. It is possible to ignore the contribution of electrons of energies $E_\perp \gtrsim 6U_m$ to the spectral intensity in the region of the maximum.

Calculations of the spectral characteristics of the radiation per unit length of the trajectory traveled by above-barrier 1.2-GeV electrons in silicon shows that the intensity of the radiation in the region of the maximum increases by a factor of 1.7 compared with $E = 0.6$ GeV, i.e., the spectral density of the radiation of above-barrier electrons per unit distance increases on increase in the energy of the beam faster than in accordance with the $E^{1/2}$ law in a crystal of the same thickness. This is due to the fact that at high energies a relatively large fraction of quasichanneled electrons has low transverse energies and, consequently, emits more intensely. The rise of the radiation intensity in a thick single crystal faster than the $E^{1/2}$ law applies also to channeled electrons,[6] which is due to a linear rise of the number of channeled particles on increase in the beam energy E.

5. TOTAL ENERGY LOSSES OF ABOVE-BARRIER ELECTRONS DUE TO EMISSION OF RADIATION

The energy radiated by a charged particle throughout the time of interaction with an isolated chain is given by the expression[11]

$$\Delta W=\frac{2e^2\gamma^4c}{3E^2}\int_{-\infty}^{\infty}|\nabla U(\rho(t))|^2\,dt. \qquad (22)$$

If the potential is given by Eq. (11) and also if $\beta=0$ and $\rho_1=u_\perp$, then averaging of Eq. (22) over all the impact parameters gives, in accordance with Eq. (18),

$$\frac{dE}{dz}=E_0\left(\frac{dE_\perp}{Ze^2}\right)^2\int_{\varepsilon_{min}}^{\varepsilon_{max}}[F(\varphi_1)-F(\varphi_2)]\frac{\varepsilon\,d\varepsilon}{(\varepsilon^2-1)^3},$$
$$F(\varphi)=(1+\varepsilon^2/2)\varphi+(\varepsilon^2\cos\varphi/2+2\varepsilon)\sin\varphi, \qquad (23)$$
$$\cos\varphi_1=-\frac{1}{\varepsilon},\qquad \cos\varphi_2=\min\left\{1,\varepsilon^{-1}\left[\frac{a(\varepsilon^2-1)}{u_\perp}-1\right]\right\},$$
$$E_0=\frac{e^2}{3\pi}\left(\frac{8u_\perp EU_m}{3a_F\rho_0m^2c^4}\right)^2,$$

where ε and a are defined in Eq. (17). A similar formula given in Ref. 11 differs from Eq. (23) by the method of averaging over all the impact parameters.

In the case of high transverse energies the trajectories of electrons can be regarded as rectilinear. In this case the dependence $\rho(t)$ in Eq. (22) has the simple form

$$\rho(t)=(b^2+v_\perp^2t^2)^{1/2},$$

where b is the impact parameter of an electron with a transverse velocity $v_\perp$. In this approximation if the potential is $U\propto a/\rho$, we obtain

$$\frac{dE}{dz}=\frac{1}{6}\left(\frac{e\gamma^2U_m}{\rho_0E}\right)^2. \qquad (24)$$

The same result is obtained from Eq. (23) in the case of high transverse energies.

Using the standard Lindhard potential $U_L(\rho)$ we find that the asymptotic value of the energy losses due to radiation obtained in the approximation of rectilinear trajectories is

$$\frac{dE}{dz}=\frac{8e^2}{3}\left(\frac{Ze^2\gamma^2}{\rho_0\,dE}\right)^2\left(\ln\frac{1+c_1}{1+c_2}+\frac{1}{2c_1}-\frac{1}{2c_2}\right),$$
$$c_1=(1+3a_F^2/u_\perp^2)^{1/2},\qquad c_2=(1+3a_F^2/\rho_0^2)^{1/2}. \qquad (25)$$

Figure 5 shows the dependence of the energy lost by above-barrier electrons due to the emission of radiation on the transverse energy in $\langle 111\rangle$ silicon. Curve 1 represents calculations carried out using Eq. (23), curve 2 gives the results of calculations carried out assuming the potential $U\propto a/\rho$, and the dashed line is the asymptotic value of the energy losses calculated from Eq. (24). We can see that if the transverse electron energy is $E_\perp>2U_m$, we can use the simple formula (24). A calculation for the Lindhard potential made using Eq. (25) gives a result for a $\langle 111\rangle$ silicon crystal which differs by just 1% from Eq. (24). The Coulomb potential (curve 2) can be used only in the case of high transverse energies $E_\perp>10U_m$.

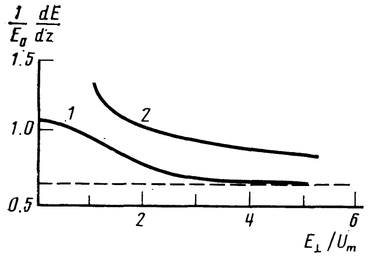

FIG. 5. Dependences of the total energy losses due to emission of radiation on the transverse energy of quasichanneled electrons traveling along the $\langle 111\rangle$ axis in silicon. Curve 1 represents calculations carried out using Eq. (23), curve 2 shows the calculations for the potential $U\propto a/\rho$, and the dashed line gives the calculations using the asymptotic formula (24).

TABLE I.

E, GeV	Asymptotic formula (24), ΔE, MeV	Losses during quasichanneling ΔE, MeV	Losses during channeling ΔE, MeV (Ref. 6)
0.6	0.26	0.26	0.22
1.2	1.10	1.22	4.8
5.0	19.2	16.1	200

The results of a calculation of the total energy losses ΔE in the case of quasichanneling and channeling in a $\langle 111\rangle$ silicon crystal of thickness 240 μ are presented in Table I for different beam energies.

The radiation losses of channeled electrons increase on increase in the beam energy much faster than the losses of above-barrier electrons in a crystal of a given thickness. If $E=0.6$ GeV, a silicon crystal of this thickness is thick and, therefore, a calculation carried out using the exact distribution function gives the same result as the simple formula (24). If $E=5$ GeV, a crystal of thickness 240 μ is not thick (see Fig. 1), i.e., the number of electrons in the channeling regime is large and, therefore, the asymptotic formula overestimates the results compared with the exact calculations. In the case of thick crystals characterized by $z\gtrsim z_2$ the energy losses are comparable for the channeling and quasichanneling regimes. At high beam energies the energy losses due to the emission of radiation by channeling electrons in moderately thick crystals $z<z_2$ may be an order of magnitude higher than the energy losses of above-barrier electrons.

6. CONCLUSIONS

When an electron beam is incident at a small angle $\psi<\psi_L$ relative to a crystallographic axis, the fraction of the particles trapped into states with finite transverse motion is given by Eq. (1). The other electrons of energies $E_\perp>0$ are distributed in the space of the transverse energies in accordance with the distribution function (7) which has a sharp maximum at $E_\perp=E\psi^2/2$. As the beam penetrates into a crystal, this maximum disappears rapidly at depths $z\sim z_1$ given by the system of equations (8) and almost all above-barrier electrons have transverse energies within the range $0<E_\perp<E_\perp^{\max}$ given by Eq. (5). Then single crystals are thick if $z\gtrsim z_2$ [see Eq. (8)].

The spectral-angular characteristics of the radiation of above-barrier electrons are governed by the dynamics of their motion in the field of many atomic chains. The strongest intensity of radiation is obtained for electrons with low transverse energies. An increase of the transverse energy reduces the spectral intensity of the radiation in accordance with the $E_\perp^{-1/2}$ law. In contrast to the channeling case, the spectral-angular density of the radiation intensity of above-barrier electrons at frequencies near the maximum is such that the emitted energy is concentrated in an angular interval $\Delta\theta\sim 2\gamma^{-1}$ near the value θ defined by Eq. (19). The emission of harder photons is concentrated in a narrower angular interval $\Delta\theta$ located near the angle θ [Eq. (19)].

The spectral density of the intensity of the radiation emitted by above-barrier electrons has a characteristic maximum located approximately at the position of the maximum of the spectrum of channeled particles, but because of the rapid diffusion in the space of transverse energies, quasichanneled electrons rapidly leave the region with low transverse energies. Consequently, in thick crystals characterized by $z \sim z_2$ the main contribution to the spectral intensity in the region of the maximum of the spectrum comes from channeled electrons and not from above-barrier electrons. For example, in the case of $\langle 111 \rangle$ silicon of thickness 240μ when $E = 0.6$ GeV, the intensity of the radiation of channeled electrons in the region of the maximum of the spectrum exceeds the intensity of the radiation emitted by quasichanneled electrons by a factor of 8, although channeled electrons represent a much smaller fraction of the beam. At higher frequencies the reverse is true: above-barrier electrons emit more intensely. An analysis shows that over half the contribution to the spectral intensity of the radiation in the case of quasichanneling in thick single crystals is due to electrons with transverse energies $0 < E_\perp < 2U_m$.

At relatively low beam energies E when the condition (16) is satisfied we can use the dipole approximation. Then, in the case of thin single crystals characterized by $z \lesssim z_1$ the intensity of the radiation of quasichanneled electrons increases on increase in the energy in accordance with the $E^{1/2}$ law, and the characteristic frequencies of the radiation vary proportionally to $E^{3/2}$. The nondipole effects begin to play a role as the beam energy is increased. The degree of monochromaticity of the spectrum increases, the dependence of the spectral intensity on the energy E becomes weaker than the $E^{1/2}$ law, and at sufficiently high beam energies the intensity falls as $E^{-1/2}$. Similar properties are exhibited also by the spectral characteristics of the radiation emitted by electrons in the channeling regime.

In the case of sufficiently thick single crystals characterized by $z \gg z_2$ the energy losses due to the emission of electromagnetic radiation in all angles are mainly due to the radiation emitted by quasichanneled electrons. If $z \lesssim z_2$, the reverse is true: the energy losses due to the radiation of channeled electrons are several times higher than the energy losses in the quasichanneling case and in the case of thin single crystals characterized by $z \sim z_1$ this ratio may be more than order of magnitude.

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