Quasi-classical theory of potential scattering of an electron in the field of a laser wave

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(Submitted 22 October 1986; resubmitted 10 February 1987)
Zh. Eksp. Teor. Fiz. **93**, 95–103 (July 1987)

We construct the wave function of an electron in the field of a laser wave in the quasi-classical approximation outside the framework of perturbation theory in terms of the interaction with the field and the scattering potential. Using this we obtain for the potential scattering cross section a new expression which generalizes the well known results of Bunkin, Fedorov, and Berson.

The scattering of an electron by an atom in the presence of an electromagnetic wave is of great interest for the study of the optical breakdown of gases[1] and of the interaction of laser radiation with a plasma.[2] The cross section of the process depends on the strength F_0, the frequency ω_0, and the polarization λ of the wave, and the possible change in energy of the electrons is observed experimentally when one studies the energy distribution of the scattered electrons.[3]

An important feature in the construction of the theory of scattering in an external electromagnetic field is the necessity to take into account the interaction with the field when describing the asymptotic states. However, the theory of the interaction of an atom or a molecule with the field is itself a problem which is difficult to solve.[4–6]

In the field of a laser wave there occurs a shift and a broadening of the energy levels of the initial E_i and the final E_f states of the atom. The conditions for the applicability of perturbation theory when there is a level shift

$$\Delta E_{i,f}^{(2)} \ll |E_{i,f}-E_k| \tag{1}$$

and a level broadening

$$\gamma=(2m_e|E_{i,f}|)^{1/2}\omega_0/eF_0 \gg 1 \tag{2}$$

depend on the strength and the frequency of the wave, and also on the actual spectrum of the atom.[4–6]

Moreover, the condition for a power-law dependence of the scattering cross section on the field strength is determined in the Born approximation

$$e\mathbf{F}_0\mathbf{q}/m_e\omega_0^2 \ll 1$$

by the magnitude of the momentum transfer $\hbar\mathbf{q}$ and, in general, is independent of the conditions (1) and (2).[7,2] We can thus pose the problem of the scattering of an electron in the presence of an electromagnetic wave by considering the interaction of the atomic electron with the field by using perturbation theory, whereas the interaction of the incident electron with the field is taken into account exactly.

A further simplification of the problem of potential scattering by a fixed center is possible if we neglect the polarization of the atomic electrons by the field of the electromagnetic wave or by the incident electron. There is one known exactly soluble problem of the potential scattering of a nonrelativistic electron by a δ-potential in a circularly polarized field.[8] In the general case it is necessary to use approximate methods to solve the Schrödinger equation.

Under conditions when perturbation theory with respect to the interaction with the electromagnetic field is applicable while the atomic potential is taken into account exactly, results are known for single-photon transitions in a Coulomb potential,[9] and for two-photon transitions in a δ-potential[10] and in a Coulomb potential.[11–13] In the quasi-classical approximation two-photon transitions were considered for scattering by a Coulomb potential in Ref. 14, and single-photon transitions for scattering by a power-law potential in Ref. 15.

The Born series of the theory of scattering by exact free electron wave functions in a wave is another variant of perturbation theory. A characteristic feature of the first Born approximation, first considered by Bunkin and Fedorov,[7] is the factorization of the cross section into a cross section for scattering without a field and a factor depending on the strength, frequency, and polarization of the wave. The second Born approximation and the conditions for factorization of the cross section in that case were considered in Ref. 16.

Kroll and Watson's paper[17] set the start of a wide use of a low-frequency approximation in the framework of which the cross section for the scattering in the field was factorized into the exact cross section for scattering without a field and a Born factor. We note that Refs. 18–20 are devoted to attempts at a more rigorous foundation of the approximation as well as to considerations of various processes in this approximation. The problem of the factorization of the scattering cross section in the framework of the theory of sudden perturbations was considered in Refs. 21.

In the eikonal approximation, which was developed in a number of papers,[22–24] the final expression for the scattering amplitude is obtained in the form of a three-dimensional integral which does not permit effective numerical calculations. In this connection one must note the relatively simple expression for the cross section for scattering in a field in the quasi-classical approximation based upon the classical description of the motion of the incident particle.[25]

The aim of the present paper is to obtain a general formula for the cross section for scattering in the field of a laser wave, generalizing the results of the Born[7] and the quasi-classical[25] approximations. In the first section we construct the wave function of an electron in the field of a laser wave in the quasi-classical approximation, and we use it in the second section to find the scattering cross section in the eikonal approximation. We consider in the third section limiting and particular cases of the eikonal approximation.

 0038-5646/87/070053-05$04.00

§1. WAVE FUNCTION OF AN ELECTRON IN THE FIELD OF A WAVE

Let the quasi-classical approximation,

$$\frac{1}{2\pi}\frac{d\lambda}{dx}\ll 1,$$

where $\lambda(x) = 2\pi\hbar p(x)$ is the de Broglie wavelength, be valid for the incident electron. We look for a solution of the nonstationary Schrödinger equation

$$(i\hbar\partial/\partial t-H)\Psi(\mathbf{r},t)=0 \tag{3}$$

with Hamiltonian

$$H=-\hbar^2\nabla^2/2m_e+U(\mathbf{r})+e\mathbf{F}(t)\mathbf{r}$$

in the form

$$\Psi(\mathbf{r},t)=\exp[i\sigma(\mathbf{r},t)/\hbar]. \tag{4}$$

Substituting (4) in (3) and neglecting terms proportional to $\hbar$ we get the Hamilton-Jacobi equation

$$\partial\sigma/\partial t+(\nabla\sigma)^2/2m_e+U(\mathbf{r})+e\mathbf{F}(t)\mathbf{r}=0, \tag{5}$$

which has in the limiting case $\mathbf{F}(t) = 0$, for high energies ε_i, $U/\varepsilon_i \ll 1$, a solution of the form

$$\sigma_0(\mathbf{r},t)=\mathbf{p}_i\mathbf{r}-\varepsilon_i t-v_i^{-1}\int_{-\infty}^{z}U(\rho,z')\,dz', \tag{6}$$

where $\varepsilon_i = p_i^2/2m_e$ and $\mathbf{p}_i = m_e\mathbf{v}_i = \hbar\mathbf{q}_i$ are the electron energy and momentum.

For solving Eq. (5) we can use perturbation theory, choosing as the zeroth approximation the action $\sigma_0(\mathbf{r},t)$:

$$\sigma(\mathbf{r},t)=\sigma_0(\mathbf{r},t)+\tilde{\sigma}_i(\mathbf{r},t),$$

where the term $\tilde{\sigma}_i(\mathbf{r},t)$ is assumed to be a small and slowly varying function of the coordinates as compared to $\sigma_0(\mathbf{r},t)$. We then get from (5), neglecting the second derivatives with respect to the spatial coordinates $\nabla^2\tilde{\sigma}_i$, the equation

$$\partial\tilde{\sigma}_i/\partial t+\nabla\tilde{\sigma}_i\mathbf{v}(t)+(\nabla\tilde{\sigma}_i)^2/2m_e=-e\mathbf{F}(t)\mathbf{r},$$

where $\mathbf{v}(t) = \nabla\sigma_0/m_e$ is the velocity of the classical motion, with the solution

$$\tilde{\sigma}_i(\mathbf{r},t)=-e\mathbf{A}(t)\mathbf{r}-\frac{e^2}{2m_e}\int_{-\infty}^{t}A^2(t')\,dt'+e\int_{-\infty}^{t}\mathbf{A}(t')\mathbf{v}(t')\,dt', \tag{7}$$

$$\mathbf{A}(t)=\int_{-\infty}^{t}\mathbf{F}(t')\,dt'.$$

Finally we get for the wave function of an electron in the potential $U(\mathbf{r})$ and in the field of a laser wave

$$\Psi(\mathbf{r},t)=\tilde{U}(\mathbf{r},t)\tilde{\Psi}(\mathbf{r},t), \tag{8}$$

$$\tilde{\Psi}(\mathbf{r},t)=\exp\left[i\mathbf{q}_i\mathbf{r}-\frac{i}{\hbar}\varepsilon_i t-\frac{i}{\hbar v_i}\int_{-\infty}^{z}U(\rho,z')\,dz' +\frac{ie}{\hbar}\int_{-\infty}^{t}\mathbf{A}(t')\mathbf{v}(t')\,dt'\right],$$

$$\tilde{U}(\mathbf{r},t)=\exp\left[-\frac{ie}{\hbar}\mathbf{A}(t)\mathbf{r}-\frac{ie^2}{2m_e\hbar}\int_{-\infty}^{t}A^2(t')\,dt'\right]. \tag{9}$$

The limits of applicability of the results obtained follow from the smallness condition $\sigma_i \ll \sigma_0$:

$$eF_0/m_e\omega_0=v_F\ll v_i,\quad eF_0a/\hbar\omega_0\ll U_0a/\hbar v_i=\nu. \tag{10}$$

We note that Eq. (8) does not follow from the results of Refs. 22–24 and 26, since the condition $eF_0a(v_i/\omega_0 a) \gg \nu\hbar\omega_0$ for their applicability is not the same as conditions (10).

We apply these results to solve the problem of scattering in the presence of an electromagnetic wave in the eikonal approximation, when it is necessary to satisfy the inequality $q_i a \gg 1$, where a is the range of the scattering potential. In this approximation one is able to study both the limit of the Born approximation, when one of the inequalities

$$U_0\ll\hbar^2/2m_ea^2=\varepsilon_i/(q_ia)^2,$$
$$\nu=U_0a/\hbar v_i=U_0q_ia/2\varepsilon_i\ll 1,$$

is satisfied, and the limit of classical scattering $\nu \gg 1$.

For motion along the z axis $r^2 = \rho^2 + (v_it)^2$ and the velocity $\mathbf{v}(t)$ in (7) is given by the equation

$$\mathbf{v}(t)=\mathbf{v}_i-\frac{\boldsymbol{\rho}}{m_ev_i^2}\int_{-\infty}^{t}\frac{dU}{dt'}\frac{dt'}{t'}=\mathbf{v}_i+\delta\mathbf{v}(t), \tag{11}$$

where the term $\delta\mathbf{v}(t)$ is caused by the action of the scattering potential.

§2. POTENTIAL-SCATTERING AMPLITUDE AND CROSS SECTION

In a monochromatic field $\mathbf{F}(t) = \mathbf{F}_0\cos(\omega_0 t + \Phi_0)$ the potential scattering cross section has the form of a sum of partial cross sections

$$\frac{d\sigma}{d\Omega_f}=\sum_s\left(\frac{d\sigma}{d\Omega_f}\right)_s=\sum_s\left(\frac{m_e}{2\pi\hbar^2}\right)^2\frac{v_s}{v_i}|g_s|^2,$$

where $v_s^2 = 2\hbar\omega_s/m_e$. The quantities g_s and ω_s are given by the equation

$$\sum_s g_s\delta(\omega-\omega_s)=\int_{-\infty}^{\infty}\frac{dt}{2\pi}\exp[i\omega t+i\boldsymbol{\alpha}_0\mathbf{q}_f\cos(\omega_0t+\Phi_0)] \times\int d\mathbf{r}'\exp(-i\mathbf{q}_f\mathbf{r}')U(\mathbf{r}')\tilde{\Psi}(\mathbf{r}',t), \tag{12}$$

where $\alpha_0 = eF_0/m_e\omega_0^2$ is the amplitude of the oscillations in the field, and the function

$$U(\mathbf{r},t)\tilde{\Psi}(\mathbf{r},t) = \Psi(\mathbf{r},t)$$

is the exact wave function of the scattering problem.

In the first Born approximation when

$$\tilde{\Psi}(\mathbf{r}',t)\approx\exp\left(i\mathbf{q}\mathbf{r}'-\frac{i}{\hbar}\varepsilon_qt+\frac{ie\mathbf{q}}{m_e}\int_{-\infty}^{t}\mathbf{A}(t')\,dt'\right) \tag{13}$$

is the wave function of a free electron in the wave we get

$$g_s=(ie^{i\Phi_0})^sJ_s(\boldsymbol{\alpha}_0\mathbf{q}_s)U(\mathbf{q}_s), \tag{14}$$

$$\hbar\omega_s=\varepsilon_i-s\hbar\omega_0=\hbar^2q_{f,s}^2/2m_e, \tag{15}$$

where $J_n(x)$ is a Bessel function, and $\hbar\mathbf{q}_s = \hbar\mathbf{q}_{f,s} - \hbar\mathbf{q}_i$ is the transferred momentum,

$$U(\mathbf{q})=\int d\mathbf{r}\,e^{-i\mathbf{q}\mathbf{r}}U(\mathbf{r}).$$

The cross section for scattering with absorption $s<0$

(or emission $s>0$) of quanta of the external field has the form[7,2]

$$\left(\frac{d\sigma}{d\Omega}\right)_s=\frac{v_s}{v_i}|J_s(\alpha_0\mathbf{q}_s)|^2\left(\frac{d\sigma}{d\Omega}\right)_0^{\mathrm{B}},\qquad(16)$$

where

$$\left(\frac{d\sigma}{d\Omega}\right)_0^{\mathrm{B}}=\left(\frac{m_e}{2\pi\hbar^2}\right)^2|U(\mathbf{q}_s)|^2$$

is the Born cross section for scattering without a field.

In the quasi-classical approximation with the explicit form of the function $\tilde{\Psi}(\mathbf{r},t)$ taken into account the quantities g_s and ω_s are equal to

$$g_s=i\hbar v_i(ie^{i\Phi_0})^s\int d^2\rho\, e^{-i\mathbf{q}\rho}J_s[\alpha_0\mathbf{q}+X_2(\rho)][e^{i\chi(\rho)}-1],$$
$$\hbar\omega_s=\varepsilon_i-s\hbar\omega_0,\qquad(17)$$

where

$$\chi(\rho)=-\frac{1}{\hbar v_i}\int_{-\infty}^{\infty}U(\rho,z')dz'$$

is the eikonal when there is no field, and

$$X_2(\rho)=-\frac{eF_0}{\hbar\omega_0}\left|\lambda\int_{-\infty}^{\infty}e^{i\omega_0 t}\delta\mathbf{v}(t)\,dt\right|\mathrm{sign}(\lambda\rho)\qquad(18)$$

is equal to the ratio of the work done by the electron in the field of the wave to the photon energy. We note that the parameter $X_2(\rho)$ depends on the characteristics of the field and of the scattering potential, while the quantity $\alpha_0\mathbf{q}=X_{fi}$ is determined by the scattering conditions and is independent of the potential.

The scattering amplitude

$$f_s(\vartheta,\gamma)=-m_e g_s/2\pi\hbar^2\qquad(19)$$

depends on the polar ϑ and azimuthal γ scattering angles. The modulus of the scattering amplitude is invariant under the substitution $s\to-s$.

§3. LIMITING CASES OF THE EIKONAL APPROXIMATION

To first order in the scattering potential $U(\mathbf{r})$ the result of the Born approximation (14)–(16) follows from (17).

In the limit as $X_2\to0$ the scattering amplitude in (19) factorizes:

$$f_s(\vartheta,\gamma)=(ie^{i\Phi_0})^s J_s(\alpha_0\mathbf{q})f_0^{\mathrm{eik}}(\vartheta),$$
$$\alpha_0\mathbf{q}=\alpha_0(\mathbf{q}_f-\mathbf{q}_i)=X_{fi},$$

where

$$f_0^{\mathrm{eik}}(\vartheta)=\frac{q_i}{2\pi i}\int d^2\rho\, e^{-i\mathbf{q}\rho}[e^{i\chi(\rho)}-1]$$

is the eikonal scattering amplitude when there is no field. The analog of Bunkin and Fedorov's formula takes in the eikonal approximation the form

$$\left(\frac{d\sigma}{d\Omega}\right)_s=\frac{v_s}{v_i}|J_s(\alpha_0\mathbf{q})|^2\left(\frac{d\sigma_0}{d\Omega}\right)_{\mathrm{eik}},$$

where

$$\left(\frac{d\sigma_0}{d\Omega}\right)_{\mathrm{eik}}=|f_0^{\mathrm{eik}}(\vartheta)|^2$$

is the scattering cross section when there is no field.

The quantity $X_2(\rho)$ can be estimated, using Eq. (18):

$$|X_2|\approx\frac{eF_0\delta va}{\hbar\omega_0 v_i}=\frac{v_F}{v_i}\frac{U_0 a}{\hbar v_i}=\frac{v_F\nu}{v_i}.$$

It follows from (18) that the condition $v_F\ll v_i$, which is the basis of (10), is insufficient when X_2 is small. When $\nu<1$ the condition $v_F\ll v_i$, which in this case is the same as the conditions obtained in Ref. 16, is sufficient for the factorization of the cross section. Under the quasi-classical conditions $\nu\gg1$ it is necessary to require a more stringent condition:

$$(v_F\nu/v_i)\ll1.\qquad(20)$$

Using (17) one can make the result (14) more rigorous, taking $X_2(\rho)$ into account to first order:

$$f_s(\vartheta,\gamma)=(ie^{i\Phi_0})_s[f_0^{\mathrm{eik}}(\vartheta)J_s(X_{fi})+f_1(\vartheta,\gamma)(J_{s-1}(X_{fi})-J_{s+1}(X_{fi}))],\qquad(21)$$

where the amplitude

$$f_1(\vartheta,\gamma)=\frac{q_i}{4\pi i}\int d^2\rho\, e^{-i\mathbf{q}\rho}X_2(\rho)[e^{i\chi(\rho)}-1]\qquad(22)$$

has a simple physical meaning. When electrons are scattered in the absence of a field they can radiate spontaneously. When there is an electromagnetic wave present induced emission takes place which is caused by the interaction of the electron with both the field and the scattering potential. The amplitude of the scattering process which accompanies the induced emission of a photon is determined by the quantity $ie^{i\Phi_0}f_1(\vartheta,\gamma)$.

In the limiting case of classical scattering

$$\tfrac{1}{2}\chi(\rho)\gg1,\quad q\rho\gg1$$

we have for the scattering cross-section

$$d\sigma=\frac{v_s}{v_i}d\sigma_{\mathrm{cl}}\frac{d\gamma}{2\pi}|J_s[X_{fi}+X_2(\tilde{\rho})]|^2,\qquad(23)$$

where $d\sigma_{\mathrm{cl}}=2\pi\rho d\rho$ is the classical scattering cross section and $\tilde{\rho}$ a vector with a modulus which is equal to the solution of the equation

$$\frac{\pi}{2}+\frac{\varepsilon\theta}{2}=\int_{r_0}^{\infty}\frac{\rho\,dr}{r^2}\left[1-\frac{U}{E}-\left(\frac{\rho}{r}\right)^2\right]^{-1/2},$$

$\varepsilon=+1\ (-1)$ for an attractive (repulsive) potential and the direction is determined by the angles $\gamma-\pi$ or γ for attractive and repulsive potentials. If we neglect the interaction with the field in the initial and final states ($X_{fi}=0$) we get the analog of Berson's formula:[25]

$$d\sigma=\frac{v_s}{v_i}\frac{d\sigma_{\text{кл}}\,d\gamma}{2\pi}|J_s[X_2(\tilde{\rho})]|^2.\qquad(24)$$

The results of Ref. 25 refer to the case of scattering in a linearly polarized field with the polarization vector parallel to the direction of the incident electron and the problem is then axially symmetric and the result is independent of the azimuthal scattering angle γ. In the general case there is no such symmetry, a fact reflected in Eqs. (23), (24).

We now consider the case of single-photon emission un-

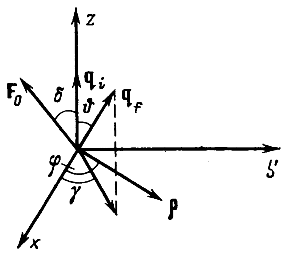

FIG. 1. $\mathbf{q}_i \to \mathbf{q}_f$ scattering by a linearly polarized field of strength $\mathbf{F}_0 = \{F_0 \sin\delta, 0, F_0 \cos\delta\}$.

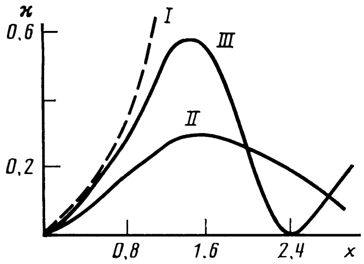

FIG. 2. Cross-section ratio $\varkappa$ as function of the dimensionless parameter x: I—perturbation theory; II—Born and low-frequency approximation; III—eikonal approximation (27).

der conditions when perturbation theory is applicable with respect to the interaction with the external field when there is scattering by a Coulomb potential:

$$U(r)=-Ze^2/r, \quad r<R,$$
$$U(r)=0, \quad r>R.$$

It is necessary to cut off the potential to remove the divergence when we evaluate the eikonal $\chi(\rho)$ which has the form

$$\chi(\rho)=-2\nu \ln(\rho/2R), \quad \rho \ll R.$$

The change in the electron velocity caused by the action of the potential is equal to (11)

$$\delta \mathbf{v}(t)=-\frac{\boldsymbol{\rho}}{m_e v_i^2}\int_{-\infty}^{t}\frac{dU}{dt'}\frac{dt'}{t'}=-\frac{\boldsymbol{\rho} Ze^2}{\rho^2 m_e}\{t(\rho^2+v_i^2t^2)^{-1/2}+1\}. \tag{25}$$

Substituting (25) into (18) we find the quantity $X_2(\boldsymbol{\rho})$ (case of linear polarization, see Fig. 1):.

$$X_2(\boldsymbol{\rho})=-e\mathbf{F}_0\boldsymbol{\rho} Ze^2 K_1(\rho\omega_0/v_i)(\hbar\omega_0\rho\varepsilon_i)^{-1}, \tag{26}$$

where $K_1(x)$ is a Macdonald function with asymptotics

$$K_1(x)=1/x, \quad x\ll 1,$$
$$K_1(x)=(2/\pi x)^{1/2}e^{-x}, \quad x\gg 1.$$

The amplitude of scattering with photon emission when there is no interaction with the field in the initial and final states is equal to

$$f_1(\mathbf{q}_i, \mathbf{q})=ie^{i\Phi_0}f_1(\vartheta, \gamma),$$

where, using (22), (25), (26), we have for the quantity $f_1(\vartheta,\gamma)$

$$f_1(\vartheta,\gamma)=\frac{q_iZe^2v_F}{2\varepsilon_i}\frac{\cos\gamma}{(2R)^{-2i\nu}}\int_0^\infty \rho^{1-2i\nu}J_1(\rho q)K_1\left(\frac{\rho\omega_0}{v_i}\right)d\rho$$
$$=\frac{\nu eF_0 q\cos\gamma}{(2R)^{-2i\nu}\hbar\omega_0}\left(\frac{v_i}{\omega_0}\right)^3(1-i\nu)\Gamma^2(1-i\nu)$$
$$\times\left(\frac{2v_i}{\omega_0}\right)^{-2i\nu}(1-z_0)^{2i\nu-1}{}_2F_1(i\nu, i\nu+1; 2; z_0),$$

where, ${}_2F_1(a,b;c;z)$ is a hypergeometric function, $\Gamma(z)$ the gamma function, and $z_0 = -(qv_i)^2/\omega_0^2$. Comparing the amplitude obtained with the exact $f(\mathbf{q}_i,\mathbf{q})$ of Refs.9 and 27, we get in the limit $\nu = \nu'$ the equation

$$f_1^{\text{eik}}(\mathbf{q}, \mathbf{q}_i+\mathbf{q})=(2q_iR)^{2i\nu}e^{i\pi}f(\mathbf{q}_i, \mathbf{q}).$$

The difference consists in an unimportant phase factor that depends on the cutoff parameter. We note that the condition $\hbar\omega_0/\varepsilon_i \ll 1$ under which the relation $\nu = \nu'$ holds is not a strong restriction in the eikonal approximation.

For single-photon processes the nonrelativistic eikonal approximation is the analog of the quasi-classical theory of the emission of ultra-relativistic electrons (Ref.28; Ref.27, § 96). In the general case the expression for the amplitude of scattering by a Coulomb potential with emission of s photons is rather complicated and we shall therefore consider the limiting case of low frequencies and use Eq. (21) for the scattering amplitude. We then get for the scattering amplitude

$$\left(\frac{d\sigma}{d\Omega}\right)_s=\left(\frac{d\sigma}{d\Omega}\right)_0 f(X_{fi}, s), \tag{27}$$

where

$$\left(\frac{d\sigma}{d\Omega}\right)_0=\frac{4q_i^2\nu^2}{q^4}=\left(\frac{Ze^2}{2m_e v_i^2}\right)^2\sin^{-4}\frac{\vartheta}{2}$$

is the Rutherford cross section.

$$f(x,s)=\left|J_s(x)+x\frac{d}{dx}J_s(x)\right|^2.$$

The cross-section ratio $\varkappa = d\sigma_s/d\sigma_0$ depends only on a single dimensionless parameter X_{fi}. The function $f(x,s)$ is proportional in the perturbation- theory framework to $|x|^{2s}$ and equals $|J_s(x)|^2$ in the Born and the low-frequency approximations. These functions are shown in Fig. 2 for a single-photon process.

§4. CONCLUSION

We have developed in this paper a quasi-classical theory for potential scattering of an electron in the field of a laser wave, generalizing well known results of the Born and quasi-classical approximations which agree with the well known results of Sommerfeld for the single-photon process in scattering by a Coulomb potential, if one uses perturbation theory to consider the interaction with the field. The results allow a generalization also to inelastic processes in an electromagnetic field outside the framework of the Born approximation, but the main difficulty in this case lies in describing the interaction with the field of the target (atom or molecule) outside the perturbation-theory framework.

[1]Yu. P. Raĭzer, *Laser-Induced Discharge Phenomena*, Plenum, New York, 1977.
[2]F. V. Bunkin, A. A. Kazakov, and M. V. Fedorov, Usp. Fiz. Nauk **107**, 559 (1972) [Sov. Phys. Usp. **15**, 416 (1973)].
[3]A. Weingartshofer, J. K. Holmes, J. Sabbagh, and S. L. Chin, J. Phys. **B116**, 1805 (1983).

[4]N. B. Delone and V. P. Kraĭnov, *Atom v sil'nom svetovom pole* (*Atom in a strong light field*) Énergoizdat, Moscow, 1984.
[5]V. A. Kozarskiĭ, N. F. Perel'man, and I. Sh. Averbukh, *Mnogokvantovye perekhody* (*Many-quantum transitions*) Énergoizdat, Moscow, 1985.
[6]L. P. Rapoport, B. A. Zon, and N. L. Manakov, *Teoriya mnogofotonnykh protsessov v atomakh* (*Theory of multi-photon processes in atoms*) Atomizdat, Moscow, 1978.
[7]F. V. Bunkin and M. V. Fedorov, Zh. Eksp. Teor. Fiz. **49**, 1215 (1965) [Sov. Phys. JETP **22**, 844 (1966)].
[8]I. J. Berson, J. Phys. **B8**, 3078 (1975).
[9]A. Sommerfeld, *Atomic Structure and Spectral Lines*, Methuen, London, 1934, Vol. 2, Ch. 7.
[10]S. I. Marmo and A. G. Faĭnshteĭn, Izv. Vyssh. Uchebn. Zaved. Fiz. No. 4, 92 (1984).
[11]N. L. Manakov, S. I. Marmo, and A. G. Faĭnshteĭn, Abstracts, IX All Union Conf. on the Physics of Electron and Atom Collisions, 1984, Part, 2, p. 118.
[12]M. Mohan, J. Phys. **21**, 89 (1983).
[13]M. Gavrila, Phys. Rev. **A32**, 2537 (1985).
[14]V. P. Kraĭnov and S. P. Roshchupkin, Zh. Eksp. Teor. Fiz. **84**, 1302 (1983) [Sov. Phys. JETP **57**, 754 (1983)].
[15]V. I. Kogan and A. B. Kukushkin, Zh. Eksp. Teor. Fiz. **87**, 1164 (1984) [Sov. Phys. JETP **60**, 665 (1984)].
[16]A. F. Klinskikh and L. P. Rapoport, Zh. Eksp. Teor. Fiz. **88**, 1105 (1985) [Sov. Phys. JETP **61**, 649 (1985)].
[17]N. M. Kroll and K. M. Watson, Phys. Rev. **A8**, 804 (1973).
[18]L. Rosenberg, Adv. At. Molec. Phys. **18**, 1 (1982).
[19]M. H. Mittleman, Commun. At. Molec. Phys. **11**, 91 (1981).
[20]F. H. M. Faisal, Commun. At. Molec. Phys. **15**, 119 (1984).
[21]A. M. Dykhne and G. L. Yudin, Usp. Fiz. Nauk **121**, 157 (1977); **125**, 377 (1978) [Sov. Phys. Usp. **20**, 80 (1977); **21**, 549 (1978)].
[22]B. A. Zon, J. Phys. **B8**, L86 (1975).
[23]J. I. Gersten and M. H. Mittleman, Phys. Rev. **A12**, 1840 (1975).
[24]J. Z. Kaminski, Acta Physica **A66**, 517 (1984).
[25]I. Ya. Berson, Zh. Eksp. Teor. Fiz. **80**, 1727 (1981) [Sov. Phys. JETP **53**, 891 (1981)].
[26]M. A. Prasad and K. Unnikrishnan, J. Phys. **B16**, 3443 (1983).
[27]V. B. Berestetskiĭ, E. M. Lifshitz, and L. P. Pitaevskiĭ, *Kvantovaya élektrodinamika* (*Quantum Electrodynamics*) Nauka, Moscow, 1980 [English translation published by Pergamon Press, Oxford].
[28]V. N. Baĭer, V. M. Katkov, and V. S. Fadin, *Izluchenie relyativistskikh élektronov* (*Radiation by relativistic electrons*) Atomizdat, Moscow, 1973.

Translated by D. ter Haar