Radiation from channeled positrons in a hypersonic wave field

A. R. Mkrtchyan, R. A. Gasparyan, and R. G. Gabrielyan

Institute of Applied Physics Problems, Academy of Sciences of the Armenian SSR
(Submitted 18 February 1986)
Zh. Eksp. Teor. Fiz. **93**, 432–436 (August 1987)

The radiation from a channeled positron in the field of a longitudinal or transverse hypersonic standing wave is considered. It is shown for the case of planar channeling that the spectral intensity distribution has a resonance as a function of the hypersound frequency.

The radiation emitted by relativistic charged particles channeled in crystals has become an active research topic in recent years in connection with the problem of producing a tunable and powerful source of radiation.[1–4] It has been shown theoretically that channeled electrons and positrons can radiate narrow beams of hard γ-rays, and there has been considerable interest in the experimental verification of the existence of this radiation.[5–7]

The possibility of charged-particle channeling in the presence of external periodic perturbations in a crystal was subsequently examined from the point of view of controlling the intensity of this radiation. The first investigations in this direction were published in Refs. 8 and 9, which reported studies of the effect of external electromagnetic waves, transverse ultrasonic waves, and superlattices on the radiation emitted by channeled charged particles. This work was confined to the consideration of relatively low-energy charged particles (for which the dipole approximation is valid), taking into account resonant transitions between the energy levels in the channel. In particular, it was shown that ultrapowerful acoustic waves were necessary to produce an appreciable increase in radiation intensity, and it seemed unrealistic to expect such waves to be produced experimentally.

In this paper, we consider the possibility of producing sources of hard radiation with controllable intensity, produced by channeling positrons in crystals in which acoustic waves satisfying certain definite conditions are excited. Two cases are considered, namely, the excitation of longitudinal and transverse hypersonic waves.

LONGITUDINAL WAVES

Suppose that a single crystal, in which standing hypersonic waves are produced along the channeling axis, is entered by an ultrarelativistic positron with $\gamma \sim 10^4$ at an angle $\vartheta \ll \vartheta_L$, where γ is the relativistic factor and ϑ_L the critical channeling angle. If, as in Ref. 10, we confine our attention to the first approximation in the expansion of the z-periodic potential into a Fourier series, the potential energy of the positron in the channel can be specified in the following form in the harmonic approximation when a longitudinal hypersonic wave is present:

$$U(x,z)=V_0[1-\mu\cos(2\pi z/\lambda_s)]x^2+V_1\cos(2\pi z/\lambda_s), \quad (1)$$

where $V_0 = 4U_0/d^2$, U_0 is the maximum potential energy of the positron in the channel, d is the channel width, V_1 is a constant, μ is a small parameter that depends on the acoustic power, and λ_s is the hypersound wavelength. In (1), we have neglected the time-dependence of the potential since $\omega_s\tau \ll 1$, where τ is the positron channeling time and ω_s the frequency of the hypersonic wave.

To determine the trajectory of an ultrarelativistic positron in the channel as it travels through the potential field (1) within the framework of classical theory, we consider the following set of equations:

$$\ddot{x}+\omega_0^2x=\mu x\omega_0^2\cos(2\pi z/\lambda_s), \quad \ddot{z}=-c^{-2}\ddot{x}\dot{x}\dot{z}. \quad (2)$$

The solution of (2) by the method of successive approximations yields the following expression for the trajectory:

$$\mathbf{r}(t)=\mathbf{i}x(t)+\mathbf{k}\bar{v}_zt,$$

$$x(t)=x_m\sin\omega_0t-\mu x_m\frac{\omega_0^2}{2}\left[\frac{\sin\omega_+t}{\omega_+^2-\omega_0^2}-\frac{\sin\omega_-t}{\omega_-^2-\omega_0^2}+\frac{2\sin\omega_0t}{\Omega^2-4\omega_0^2}\right], \quad (3)$$

where

$$x_m=v_{0\perp}/\omega_0,\quad \omega_\pm=\Omega\pm\omega_0,\quad \omega_0=(2V_0/m_0\gamma)^{1/2},\quad \Omega=2\pi c/\lambda_s,$$
$$\gamma=(1-\beta^2)^{-1/2},\quad \beta=v_0/c,\quad \bar{v}_z=v_0[1-B(\xi)/2],$$

$$B(\xi)=\frac{1}{2}\left(\frac{x_m\omega_0}{c}\right)^2\left[1+\frac{4\mu(\xi^2-2)\sin2\pi\xi}{\pi\xi(\xi^2-4)^2}-\frac{2\mu}{\xi^2-4}\right],$$
$$\xi=\frac{\Omega}{\omega_0}=\frac{\lambda_0}{\lambda_s},$$

$v_{0\perp}$ is the initial transverse velocity, m_0 is the rest mass of the positron, $\lambda_0 = 2\pi c/\omega_0$ is the wavelength of mechanical oscillations of the positron in the channel, and **i** and **k** are unit vectors. We note that, in deriving (2), we neglected the deceleration of the particle due to energy losses by radiation, since these losses were small.[2] The deceleration will be taken into account elsewhere.

The spectral and angular dependence of the radiation emitted by the channeled particle, whose trajectory is described by (3), can be found by the method developed in Ref. 11 for undulator radiation. When the condition for the validity of the dipole approximation ($x_m\omega_0/c \ll 1/\gamma$) is violated, which occurs for ultrarelativistic positrons with $\gamma \sim 10^4$, the spectral distribution of the k th radiation harmonic assumes the form

$$\frac{dI_k}{d\omega}=\frac{e^2}{2\pi\bar{\beta}_z\omega c}\int_0^{2\pi}|a_k(\omega,\varphi)|^2d\varphi, \quad (4)$$

where

$$|a_k(\omega,\varphi)|^2=(k\omega_0)^2\Big[J_k^2(k\varkappa)\left(\frac{1}{n_x^2}-\frac{\omega^2}{\gamma^2k^2\omega_0^2}\right)$$
$$-\mu k\varkappa J_k(k\varkappa)\,\mathrm{Im}\,F(\xi)\left(\frac{1}{n_x^2}-\frac{\omega^2}{\gamma^2k^2\omega_0^2}\right)$$
$$+\mu\varkappa J_k(k\varkappa)f(\xi)\left(\frac{1}{n_x^2}-\frac{\omega}{k\omega_0}\right)\Big],$$

$$F(\xi)=\frac{1}{2\pi}\int_0^{2\pi}\left\{\frac{\sin[(1+\xi)\psi]}{\xi(\xi+2)}+\frac{\sin[(1-\xi)\psi]}{\xi(\xi-2)}+\frac{2\sin\psi}{\xi^2-4}\right\}$$
$$\cdot\exp[i(k\psi-k\varkappa\sin\psi)]d\psi,$$

$$f(\xi)=\frac{2\sin 2\pi\xi}{\pi\xi(\xi^2-4)},\quad k\varkappa=\frac{\omega n_x x_m}{c},$$
$$\omega=k\omega_0\Big/\left(1-\frac{\bar v_z}{c}\cos\vartheta\right),$$

$n_x = \sin\vartheta\cos\varphi$, ϑ is the angle between the direction of emission and the channel axis, φ is the aximuthal angle of emission, and J_k the Bessel function.

To simplify the analysis of our results, we reproduce the expression for the intensity of the first harmonic of the channeled positron at maximum radiation frequency $\omega_{1m} = 2\omega_0\gamma^2/[1 + B(\xi)\gamma^2]$:

$$\left.\frac{dI_1}{d\omega}\right|_{\omega=\omega_{1m}}=\frac{e^2\omega_0}{c}\frac{B(\xi)\gamma^2}{1+B(\xi)\gamma^2}. \tag{5}$$

It is clear from this expression that the radiation intensity as a function of the sound frequency ω_s has a dispersive character. Actually, when the hypersound wavelength λ_s approaches the collision length $\lambda_0/2$, the radiation intensity increases as shown in Fig. 1. The maximum frequency of the first-harmonic radiation exhibits a similar behavior (Fig. 2). The results reproduced in Figs. 1 and 2 correspond to the channeling of a positron in the (110) direction in diamond for an initial positron energy $E_0 = 16$ GeV, $\mu = 0.2$, $\vartheta_0 = 0.46\times10^{-4}$, $U_0 = 22.8$ eV, and $d = 1.26$ Å.

The effect of the hypersonic wave on the intensities of the individual harmonics, the total intensity of all the harmonics, and the integrated intensity is illustrated in Fig. 3, which gives a plot of (4) for the total intensity of four harmonics $\Sigma_k dI_k/d\omega$ ($k = 1,2,3,4$), in the absence ($\mu = 0$) and presence ($\mu = 0.4$) of the sound waves near resonance ($\xi = 1.7$). It is clear from Fig. 3 that the intensities of all the harmonics increase, and the intensities of all the higher harmonics increase more appreciably. Moreover, the integrated radiation intensity increases by a factor of 1.5 as compared with the $\mu = 0$ case. As μ increases the ratio of the integrated intensity in the presence of the hypersonic wave to the integrated intensity in the absence of these waves increases still further.

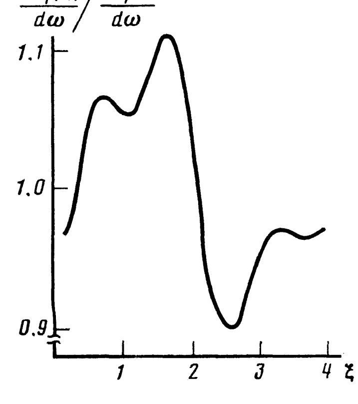

FIG. 1. Relative intensity of the first harmonic at the maximum emission frequency as a function of $\xi = \lambda_0/\lambda_s$ for a longitudinal hypersonic wave.

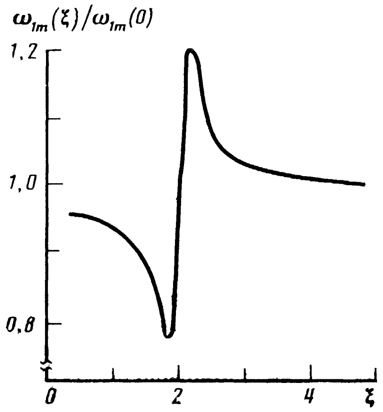

FIG. 2. Relative maximum first-harmonic frequency as a function of ξ.

TRANSVERSE WAVE

We shall suppose that a transverse hypersonic wave propagates along the z-axis in the crystal and displaces the atomic planes in the direction of the x-axis (which is perpendicular to the propagation axis) by the amount $A\cos(2\pi z/\lambda_s)$, where A is the displacement amplitude. The potential in the channel in the presence of this wave can be written in the form

$$U(x,z)=V_0[x-A\cos(2\pi z/\lambda_s)]^2. \tag{6}$$

The trajectory of the positron in this potential is

$$\mathbf{r}(t)=\mathbf{i}x(t)+\mathbf{k}\bar v_z t,$$
$$x(t)=x_m\sin\omega_0 t-A\omega_0^2(\omega_0^2-\Omega^2)^{-1}(\cos\omega_0 t-\cos\Omega t), \tag{7}$$

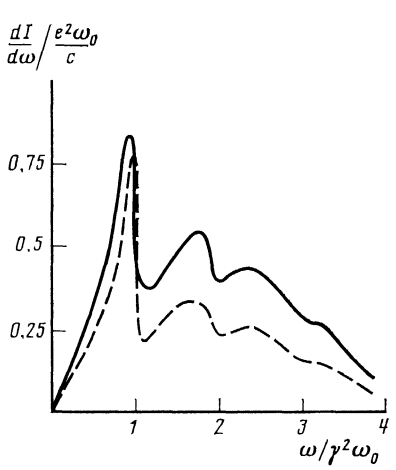

FIG. 3. Spectral distribution of radiation emitted by a channeled positron in the case of a longitudinal hypersonic wave: $\mu = 0.4$ (solid line) and $\mu = 0$ (broken line).

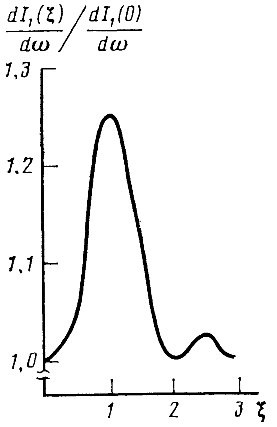

FIG. 4. Relative first-harmonic intensity at the maximum emission frequency as a function of $\xi = \lambda_0/\lambda_s$ in the case of a transverse hypersonic wave.

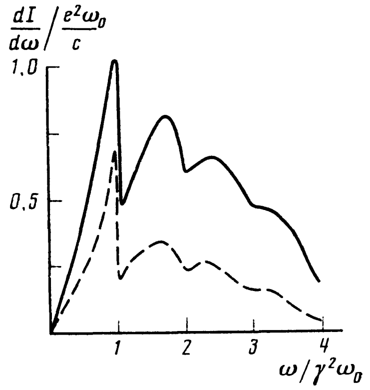

FIG. 5. Spectral distribution of the intensity emitted by a channeled positron in a transverse hypersonic wave for $\mu = 0.2$ (solid line) and $\mu = 0$ (broken line).

where

$$\bar{v}_z = v_0[1 - B(\xi)/2], \quad \mu = A/x_m,$$

$$B(\xi) = \frac{1}{2}\left(\frac{x_m\omega_0}{c}\right)^2\left[1 + \frac{4\mu\xi^2\sin^2\pi\xi}{\pi(\xi^2-1)^2}\right] \qquad (8)$$

and the remaining notation is the same as in (3).

In this case, the spectral distribution of the k th harmonic of the radiation intensity is given by (4) except that the function $B(\xi)$ is very different [see (8)]. It is clear from (8) that, in the case of the transverse wave, the function $B(\xi)$ and, consequently, $dI_1/d\omega$, have resonances at $\lambda_s = \lambda_0$ ($\xi = 1$). When λ_s approaches λ_0, both the intensity of the spectral distribution and the integrated radiation intensity increase very substantially (cf. Figs. 4 and 5). It is clear from Fig. 5 that, even at low acoustic power ($\mu = A/x_m = 0.2$), the integrated radiation intensity increases by a factor of 2.32. As μ increases, this ratio can increase until dechanneling processes begin to play the dominant part.

Note that the radiation emitted by a channeled particle on the field of the transverse hypersonic wave depends on the angle of incidence ϑ_0 on the crystal for fixed A, or on the sign of A for fixed ϑ_0. Actually, when $\mu > 0$, it follows from (4) and (7) that the intensity increases, whereas for $\mu < 0$ it decreases. In other words, the particle can be channeled or dechanneled, depending on the sign of μ.

Thus, the radiation emitted by a channeled positron shows a resonance dependence on the frequency of longitudinal and transverse hypersonic waves excited in the crystal, and this results in a sharp and controllable amplification of the intensities of the individual harmonics and, correspondingly, of the entire emission.

[1]M. A. Kumakhov, Phys. Lett. A **57**, 17 (1976).
[2]M. A. Kumakhov, Zh. Eksp. Teor. Fiz. **72**, 1489 (1977) [Sov. Phys. JETP **45**, 781 (1977)].
[3]M. A. Kumakhov and Kh. G. Trikalinos, Zh. Eksp. Teor. Fiz. **78**, 1623 (1980) [Sov. Phys. JETP **51**, 815 (1980)].
[4]V. A. Bazylev and N. K. Zhevago, Usp. Fiz. Nauk **137**, 605 (1982) [Sov. Phys. Usp. **25**, 565 (1982)].
[5]A. O. Agan'yants, Yu. A. Vartanov, G. A. Vartapetyan, *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **29**, 554 (1979) [JETP Lett. **29**, 505 (1979)].
[6]S. A. Vorob'ev, V. N. Zabaev, B. N. Kalinin, *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **29**, 414 (1979) [JETP Lett. **29**, 376 (1979)].
[7]M. J. Alguard, R. L. Swent, R. H. Pantell, *et al.*, Phys. Rev. Lett. **42**, 1148 (1979).
[8]V. G. Barshevsky, I. Ya. Dubovskaya, and A. O. Grubich, Phys. Lett. A **77**, 61 (1980).
[9]J. Ikezi, Y. R. Lin-Liu, and T. Ohkawa, Phys. Rev. B **30**, 1567 (1984).
[10]M. A. Kumakhov and R. Wedell, Phys. Status Solidi B **76**, 119 (1976).
[11]D. F. Alferov, Yu. A. Bashmakov, and E. G. Bessonov, Tr. Fiz. Inst. Akad. Nauk SSSR **80**, 100 (1975).

Translated by S. Chomet