The effect of thermal vibrations of atoms on the scattering and radiation of ultrarelativistic particles in crystals

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We examine the effect of thermal vibrations of atoms on the scattering and radiation of ultrarelativistic electrons and positrons in crystals. Equations are derived which describe the incoherent scattering and radiation of fast particles, and which hold for arbitrary orientations of the crystal axes relative to the incident beam. The analysis of the radiation process includes the effects of radiative reaction. We show that for motion along the crystallographic axis, the cross sections for incoherent scattering and radiation from superbarrier electrons is independent of the transverse energy of the particles, and differs only by the Debye–Waller factor from the scattering and radiation cross sections in an amorphous medium. For positrons, the incoherent scattering and radiation cross sections in a crystal oriented to the beam are found to be much smaller than the corresponding cross sections in an amorphous medium. Our results generalize the corresponding results of Ter-Mikaelyan [Zh. Eksp. Teor. Fiz. **25**, 296 (1953); Vliyanie sredy na elektromagnitnye protsessy pri vysokikh energiyakh (Effect of the Medium on Electromagnetic Processes at High Energies), Armenian Academy of Sciences, Erevan (1969)] to the situation in which the Born theory for the coherent interaction of particles with atoms in a lattice (in the Born approximation for electrons and positrons, the radiation and scattering cross sections are the same for both types of particles) is no longer valid.

1. INTRODUCTION

When an ultrarelativistic electron or positron moves in a crystal at a small angle with respect to one of the crystallographic axes or planes, coherent and interference effects occur during radiation which give rise to sharp maxima in the spectral density of the radiation, and which produce high intensity and polarization of the radiation at the maxima.[1–4] These effects result from the periodic arrangement of the atoms in the crystal lattice.

Because of thermodynamic fluctuations, however, the positions of the atoms in the crystal always have a certain spread about their equilibrium values, so it is important to know how this factor affects the interference radiation of a particle in a crystal. Ter-Mikaelyan[1] was the first to carry out such an investigation, working within the scope of the Born approximation in quantum electrodynamics. He showed that taking thermal vibrations of atoms into consideration leads to a reduction of radiation intensity at the interference maxima, and to the appearance of a term in the expression for the radiation spectrum which describes the incoherent effects taking place during emission. The latter term makes a critical contribution to the radiation at high frequencies.

The results obtained in Refs. 1–4 are valid if the field in the crystal weakly perturbs the particle motion, i.e., if particle motion in the crystal can be assumed to be almost rectilinear, and furthermore, if the scattering angle per unit coherence length is small compared to the characteristic radiation angle of a relativistic particle.[5–7] When these conditions fail to hold, new effects occur in the radiation.

The way the radiation is affected by particle-trajectory distortion in the discontinuous potential of either an array of atoms in a crystal or the crystallographic planes has recently been the subject of a great deal of work (see the reviews in Refs. 7–9 and references therein), in which it has been shown that for motion along the crystallographic axes and planes, particle radiation spectra differ significantly from the corresponding results obtained with the Born theory of coherent radiation.

In the present paper, we study the effects of thermal vibrations of atoms on the radiation and scattering of ultrarelativistic particles in crystals when the Born theory for coherent interaction of particles with the atoms in the lattice is no longer valid. It should be pointed out that some aspects of this problem touching on the radiation process of the particles have also been studied in Refs. 10 and 11 using classical and quantum electrodynamics. The main results in those papers, however, pertain to low frequencies, where the equations for the spectral density and angular-spectral density of the radiation simplify considerably. This constraint is lifted in the present paper, which makes it possible to investigate the influence of thermal vibrations of atoms in the lattice on the high-frequency radiation emitted by fast particles (the frequency ω is of the order of the particle energy ε), as well as on the total radiative energy losses by particles in the crystals.

In Sec. 2, we present general equations for the spectral and angular-spectral radiation densities, enabling us to take radiative reaction into account when considering radiation from fast particles in a crystal.

In Secs. 3 and 4, we average these equations over the thermal vibrations of atoms and the various particle trajectories in the crystal. Our main focus is on the features of radiation by superbarrier electrons and positrons in a crystal, which make the major contribution to emission over a wide range of particle angles of incidence ψ relative to the

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crystallographic axes, and it is shown that at small enough angles ψ, the incoherent parts of the electron and positron radiation cross sections are significantly different. In Sec. 5, we study the effect of thermal vibrations of atoms on the scattering of fast particles in crystals.

All of the results in the present paper are based on the assumption that particle motion in the crystal can be treated by classical mechanics. This requires that the effective interaction constant of the particle with the atoms in the lattice be large compared with unity, and that the number of levels of transverse motion also be large. These conditions hold at sufficiently high particle energies ($\varepsilon \gtrsim 100$ MeV).[6,8]

2. SPECTRAL DENSITY OF RADIATION

The radiation of an ultrarelativistic electron (or positron) develops over a large spatial region aligned parallel to its momentum vector, whose extent grows with energy. At high energies many collisions can take place between the electron and the atoms of the medium within this region. If the motion is then at a small angle to one of the crystallographic axes, and the coherence length[1)] $l = 2\varepsilon\varepsilon'/m^2\omega$ (ε and ε' are the initial and final electron energies, m is the electron mass, and ω is the frequency of the emitted photon) is much larger than the lattice constant a, then the effective interaction constant of the particle with the atoms in the lattice will be large. The motion and radiation of the particle in the crystal can then be described by a quasi-classical approximation.[6,12] The angular-spectral radiation density, with radiative reaction taken into account, can be expressed in terms of the particle's trajectory in the medium.[12] To terms of order m/ε, this quantity (see Eq. (20.31) of Ref. 12) can be written as

$$\frac{d\mathscr{E}}{d\omega\, do} = \frac{e^2}{8\pi^2}\frac{\varepsilon^2+\varepsilon'^2}{\varepsilon'^2}\omega^2\left[\,|[\mathbf{nI}]|^2 + \frac{\omega^2 m^2}{\varepsilon^2(\varepsilon^2+\varepsilon'^2)}|I|^2\right], \quad (2.1)$$

where $\varepsilon' = \varepsilon - \omega$, $\mathbf{n}$ is a unit vector in the direction of the radiation, and

$$\mathbf{I} = \int_{-\infty}^{\infty} dt\mathbf{v}(t)\exp\left[\,i\frac{\varepsilon}{\varepsilon'}\omega(t-\mathbf{nr}(t))\,\right],$$

$$I = \int_{-\infty}^{\infty} dt\exp\left[\,i\frac{\varepsilon}{\varepsilon'}\omega(t-\mathbf{nr}(t))\,\right].$$

Here $\mathbf{r}(t)$ is the particle trajectory in the medium ignoring radiation, and is governed by the equation

$$\frac{d}{dt}\frac{m\mathbf{v}(t)}{(1-\mathbf{v}^2(t))^{1/2}} = -\nabla\sum_k u(\mathbf{r}-\mathbf{r}_k), \quad (2.2)$$

where $u(\mathbf{r}-\mathbf{r}_k)$ is the potential energy due to interaction of the particle with an atom located at $\mathbf{r}_k$.

If within a coherence length the scattering angle ϑ_e is small compared with the characteristic radiation angle $\sim m/\varepsilon$ for a relativistic particle, we can expand (2.1) in the parameter $\varepsilon\vartheta_e/m$. To first order in this expansion (corresponding to the dipole approximation), the spectral density of the radiation takes the form

$$\frac{d\mathscr{E}}{d\omega} = \frac{e^2}{4\pi}\frac{\varepsilon^2+\varepsilon'^2}{\varepsilon\varepsilon'}\omega\int_{\delta}^{\infty}\frac{dv}{v^2}\left[\,1-2\frac{\delta}{v}\left(1-\frac{\delta}{v}\right)\right.$$

$$\left.\times\left(1-\frac{\omega^2}{\varepsilon^2+\varepsilon'^2}\right)\right]|\mathbf{W}(v)|^2, \quad (2.3)$$

$$\mathbf{W}(v) = \int_{-\infty}^{\infty} dt\,\mathbf{v}_\perp(t)\exp(ivt),$$

where $\delta = \omega m^2/2\varepsilon\varepsilon'$, $\mathbf{v}_\perp(t)$ is the particle velocity at time t in the plane orthogonal to the initial velocity $\mathbf{v}$ ($v_\perp \ll v$), and the variable v is related to the emission angle ϑ_γ by $v = \delta(1+\varepsilon^2\vartheta_\gamma^2/m^2)$.

We consider the case in which the radiation is produced over a length l which is much greater than the lattice constant a. In calculating the quantity $\mathbf{W}(v)$ in (2.3), we can then assume that the particle velocity changes discontinuously as a result of collisions with individual atoms. We then have

$$|\mathbf{W}(v)|^2 \approx \sum_{n,k}\vartheta_n\vartheta_k\exp[iv(t_n-t_k)], \quad (2.4)$$

where ϑ_n is the scattering angle for a collision with the nth atom:

$$\vartheta_n \approx -\frac{1}{\varepsilon}\frac{\partial}{\partial\boldsymbol{\rho}}\int_{-\infty}^{\infty} dtu(\mathbf{r}(t)-\mathbf{r}_n),$$

t_n is the time at which the collision takes place, and $\boldsymbol{\rho}$ is the impact parameter.

Equations (2.1) and (2.3) generalize the corresponding equations of the classical radiation theory of a charged particle in an external field (see Ref. 13, for example) to the case in which radiative reaction is important.

3. EFFECT OF THERMAL OSCILLATIONS OF ATOMS ON THE RADIATION FROM A PARTICLE MOVING IN A CRYSTAL

Equation (2.1) must be averaged over the thermal spread in atomic positions in the lattice, and over particle trajectories in the crystal. In the dipole approximation, averaging of the spectral density of the radiation is simplified considerably, extending only to the quantity $|\mathbf{W}(v)|^2$.

Because of thermodynamic fluctuations, the positions of atoms in the lattice will always have some spread $\mathbf{u}_n$ relative to their equilibrium positions $\mathbf{r}_n^0$:

$$\mathbf{r}_n = \mathbf{r}_n^0 + \mathbf{u}_n. \quad (3.1)$$

The quantity $|\mathbf{W}(v)|^2$ appearing in (2.3) must be averaged over $\mathbf{u}_n$.

From here on, we will be interested in the radiation from a particle moving at a small angle ψ to one of the crystallographic axes (the z-axis), where the most important orientational effects of particle radiation and scattering are manifest. In that case, it is well known[14] that the particle trajectory is basically determined by the average continuous potential of the array of atoms in the crystal and by the lattice potential averaged over the z-coordinate:

$$U(\boldsymbol{\rho}) = \frac{1}{L}\int dz\sum_n u(\mathbf{r}-\mathbf{r}_n), \quad (3.2)$$

where L is the crystal thickness and $\boldsymbol{\rho} = (x,y)$ is a point in a plane orthogonal to the z-axis. The deviation of the lattice potential from the mean (3.2), $\delta U = U_c(\mathbf{r}) - U(\boldsymbol{\rho})$, will then be determined by fluctuations of the particle trajectory relative to its trajectory in the field $U(\boldsymbol{\rho})$. If these fluctuations are small, then we can expand (2.4) in δU.[2)] To first order in this expansion, $|\mathbf{W}(v)|^2$ can be written in the form

$$|W(\nu)|^2=\frac{1}{(2\pi)^4\varepsilon^2}\sum_{n,n'}\int d^2g\,d^2g'\,\mathbf{g}\mathbf{g}'\,B_n(\mathbf{g})B_{n'}^*(\mathbf{g}')$$
$$\cdot\exp[i(\mathbf{g}\mathbf{u}_n-\mathbf{g}'\mathbf{u}_{n'})+i\nu(t_n-t_{n'})], \qquad (3.3)$$

where

$$B_n(\mathbf{g})=\exp\{-i\mathbf{g}(\boldsymbol{\rho}(t_n)-\boldsymbol{\rho}_n^0)\}\int dz U(\mathbf{g},z),$$

$$U(\mathbf{g},z)=\int d^2\rho\, e^{i\mathbf{g}\boldsymbol{\rho}}u(\boldsymbol{\rho},z),$$

$\boldsymbol{\rho}(t)$ is the particle trajectory in the continuous field due to the array of atoms in the crystal, and $\boldsymbol{\rho}_n^0$ is the position of a line of atoms in the x-y plane.

Assuming for simplicity that the $\mathbf{u}_n$-distribution of atoms in the lattice is Gaussian, with each atom having a mean squared displacement $\overline{u^2}$, we find after averaging (3.3) over $\mathbf{u}_n$ that

$$\overline{|W(\nu)|^2}=\sum_n A(\boldsymbol{\rho}(t_n))+\left|\sum_n e^{i\nu t_n}\frac{d}{\varepsilon}\frac{\partial}{\partial\boldsymbol{\rho}}U_r(\boldsymbol{\rho}(t_n))\right|^2, \qquad (3.4)$$

where

$$A(\boldsymbol{\rho})=\frac{1}{(2\pi)^4\varepsilon^2}\int d^2g\,d^2g'\,\mathbf{g}\mathbf{g}'B_n(\mathbf{g})B_n(\mathbf{g}')$$
$$\cdot\left[\exp\left\{-\frac{1}{2}\overline{u^2}(\mathbf{g}-\mathbf{g}')^2\right\}-\exp\left\{-\frac{1}{2}\overline{u^2}(\mathbf{g}^2+\mathbf{g}'^2)\right\}\right], \qquad (3.5)$$

d is the distance between atoms along the z-axis, and $U_r(\boldsymbol{\rho})$ is the continuous potential of an isolated array of atoms:

$$U_r(\boldsymbol{\rho})=\frac{1}{4\pi^2 d}\int d^2g B_n(\mathbf{g})\exp\{-\overline{u^2}g^2/2\}.$$

In the case we are considering, where particle motion takes place at a small angle ψ to one of the crystallographic axes, the time interval $\Delta t=t_{n+1}-t_n$ between successive particle collisions with atoms of the lattice is[3] d/v. To order of magnitude, the change in the function $\rho(t_n)$ during this time interval is

$$\Delta\rho=\rho(t_{n+1})-\rho(t_n)\sim d\max(\psi,\vartheta_1),$$

where $\vartheta_1\sim 2Ze^2/\varepsilon R$ is a typical angle for a fast particle to be scattered by an individual atom, $Z|e|$ is the charge on an atomic nucleus, and R is the screening radius of an atom. Thus, for small ψ and large ε, the change in $\rho(t_n)$ between successive collisions of a particle with the atoms of the lattice is small; the functions being summed in (3.4) then vary smoothly with increasing n, and these sums can consequently be replaced by the corresponding integrals over time. As a result of this replacement, Eq. (3.4) takes the form[4]

$$\overline{|W(\nu)|^2}=\frac{1}{d}\int dtA(\boldsymbol{\rho}(t))+\left|\int dt e^{i\nu t}\ddot{\boldsymbol{\rho}}(t)\right|^2, \qquad (3.6)$$

where $\boldsymbol{\rho}(t)$ is determined from the equation

$$\ddot{\boldsymbol{\rho}}=-\frac{1}{\varepsilon}\frac{\partial}{\partial\boldsymbol{\rho}}U(\boldsymbol{\rho}) \qquad (3.7)$$

and $U(\boldsymbol{\rho})$ is the continuous potential (3.2) of the atoms in the crystal.

Thus, taking the effect of thermal fluctuations of atoms on the radiation from a particle moving in a crystal leads to a change in the continuous potential of chains of atoms in the crystal, and to the appearance of a term in the radiation spectrum [first term in (3.6)] which governs incoherent effects during radiation.

4. MEAN SPECTRAL DENSITY OF THE RADIATION FROM A PARTICLE BEAM TRAVERSING A CRYSTAL

To find the intensity of the radiation due to a particle beam traversing a crystal, Eq. (3.6) must be averaged over various particle trajectories in the crystal. In general, such averaging is difficult. We therefore consider some limiting cases.

If the mean field $U(\boldsymbol{\rho})$ of the crystal lattice only weakly perturbs the particle motion, a solution of (3.6) can be found in the form of an expansion in the potential.[6] To lowest order in this expansion, the particle trajectory in the x-y plane, orthogonal to the z-axis, is rectilinear:

$$\boldsymbol{\rho}(t)\approx\boldsymbol{\rho}_0+\mathbf{n}_\perp\psi t, \qquad (4.1)$$

where $\boldsymbol{\rho}_0$ is the entry point and $\mathbf{n}_\perp$ is a unit vector in the x-y plane in the direction of particle motion. In that event, Eq. (2.3) must be averaged over the entry points:

$$\left\langle\frac{d\mathscr{E}}{d\omega}\right\rangle=\frac{1}{L_xL_y}\int d^2\rho_0\frac{d\mathscr{E}}{d\omega}, \qquad (4.2)$$

where L_x and L_y are the linear dimensions of the crystal along the x and y axes. The mean spectral density of the radiation (4.2) is then entirely consistent with the corresponding result given by the Born theory for coherent radiation by fast particles in crystals[2,7]:

$$\left\langle\frac{d\mathscr{E}}{d\omega}\right\rangle=\left\langle\frac{d\mathscr{E}}{d\omega}\right\rangle_B=Ln\omega\left(\frac{d\sigma_n^{(B)}}{d\omega}+\frac{d\sigma_c^{(B)}}{d\omega}\right), \qquad (4.3)$$

where n is the atomic density in the crystal and $d\sigma_c^{(B)}/d\omega$ and $d\sigma_n^{(B)}/d\omega$ are the coherent and incoherent parts of the radiation cross section (see the equations of §8 in the review of Ref. 2).

The quantity $d\sigma_n^{(B)}/d\omega$ differs solely by the Debye–Waller factor $[1-\exp(-\mathbf{g}^2\overline{u^2})]$ from the corresponding cross section for a fast particle and an isolated atom:

$$\frac{d\sigma_n^{(B)}}{d\omega}=\frac{e^2}{12\pi^3m^2\omega}\left(1+\frac{\varepsilon'^2}{\varepsilon^2}+\frac{\omega^2}{2\varepsilon^2}\right)$$
$$\cdot\int d^2g\mathbf{g}^2|U(\mathbf{g})|^2(1-e^{-\mathbf{g}^2\overline{u^2}}). \qquad (4.4)$$

As a result, the incoherent part of the radiation cross section for a particle in a crystal is found to be somewhat smaller (5%–25%, depending on the crystal temperature) than the radiation cross section from an isolated atom in an amorphous medium.

The expansion (4.1) is valid if $\psi\gg\psi_c$, where $\psi_c=(2U_0/\varepsilon)^{1/2}$, with $U_0=2Ze^2/d$ [U_0 defines the order of magnitude of the potential energy $U(\boldsymbol{\rho})$]. When this condition fails to hold, the effect of trajectory deviations from a straight line must be taken into account. The influence of this factor on the coherent part of the radiation cross section has been investigated in a great many papers in recent years (for example, see Ref. 15 and the reviews in Refs. 7–9). We will now show that when $\psi\lesssim\psi_c$, the deviation of a particle from a straight-line trajectory can have a significant effect on the incoherent part of the radiation cross section as well.

First, we note that there is a wide frequency range over which the incoherent part of the radiation cross section

makes a critical contribution to the radiation from a particle beam traversing a crystal. Thus, the Born theory of coherent radiation tells us that in the frequency range for which $2R/\psi \gg \delta^{-1} \gg d$, the cross section $d\sigma_n^{(B)}/d\omega$ is the major contributor to the radiation (in this frequency range, $d\sigma_c^{(B)}/d\omega$ is small). Clearly, the analogous situation must also hold for $\psi \lesssim \psi_c$, where one must take account of the effect of nonrectilinear particle motion on the radiation. When $\psi \lesssim \psi_c$, particle motion in the crystal can either be finite (channelization) or infinite (superbarrier motion) with respect to the z-axis.

In moving through a crystal, a superbarrier particle encounters different chains of atoms lying parallel to the z-axis, and if the motion is then far from the close packed atoms of the crystal planes, these collisions can be treated as random (in which case there is a sizable spread in impact parameters for successive collisions). If the distance l over which the radiation is produced is then small by comparison with the mean free path of the particle between successive collisions with chains of atoms, $l_{fr} \sim \bar{b}/\psi$, where $\bar{b}$ is the mean distance between atomic chains, we can ignore interference of the radiation from different chains. The mean value of the quantity $\overline{|\mathbf{W}(\nu)|^2}$ in (2.3) is then determined solely by the peculiarities of particle interaction with the field $U_r(\boldsymbol{\rho})$ due to an individual chain of atoms:

$$\langle \overline{|\mathbf{W}(\nu)|^2} \rangle = N_r \int_{-\infty}^{\infty} \frac{db}{\bar{b}} \left[\frac{1}{d} \int_{-\infty}^{\infty} dt A(\boldsymbol{\rho}_r(t)) + \left| \int_{-\infty}^{\infty} dt e^{i\nu t} \ddot{\boldsymbol{\rho}}_r(t) \right|^2 \right], \tag{4.5}$$

where N_r is the number of collisions with chains of atoms, b is the impact parameter of a chain, and $\boldsymbol{\rho}_r(t)$ is the particle trajectory in the field of an individual chain of atoms.

The major contribution to the first term in (4.5) comes from small values of $\rho (\rho \lesssim R)$. With ρ in this range, the potential of a chain can be taken to be cylindrically symmetric. To exploit this symmetry of the potential, it is convenient to transform (4.5), using the relation $dt = d\rho/\dot{\rho}$, where

$$\dot{\rho} = \psi (1 - U_r(\rho)/\varepsilon_\perp - b^2/\rho^2)^{1/2}, \qquad \varepsilon_\perp = \varepsilon \psi^2/2,$$

from an integration over time to an integration over the variable ρ (see Ref. 15, for example). In the present case, the quantity $A(\boldsymbol{\rho}_r)$ depends only on the distance $|\boldsymbol{\rho}_r|$ between the particle and the axis of a chain.

Noting further that $N_r = Ln\psi\bar{b}d$, we obtain after integrating over b

$$N_r \int \frac{db}{\bar{b}} \frac{1}{d} \int dt A(\rho(t)) = 2\pi Ln \int_{\rho^*}^{\infty} \rho \, d\rho A(\rho), \tag{4.6}$$

where $\rho^* = 0$ for electrons [and for positrons with $\varepsilon_\perp > U_r(0)$]; when $\varepsilon_\perp < U_r(0)$ for positrons, ρ^* is determined from the equation $\varepsilon_\perp = U_r(\rho^*)$.

Thus, according to (2.3), (4.5), and (4.6), the mean spectral density of the radiation from a superbarrier particle moving through a crystal can be written as a sum of two terms, giving the incoherent and coherent radiation effects:

$$\left\langle \frac{d\mathscr{E}}{d\omega} \right\rangle = \left\langle \frac{d\mathscr{E}_n}{d\omega} \right\rangle + \left\langle \frac{d\mathscr{E}_c}{d\omega} \right\rangle, \tag{4.7}$$

where

$$\left\langle \frac{d\mathscr{E}_n}{d\omega} \right\rangle = \frac{2Lne^2}{3m^2} \left(1 + \frac{\varepsilon'^2}{\varepsilon^2} + \frac{\omega^2}{2\varepsilon^2} \right) \varepsilon^2 \int_{\rho^*}^{\infty} \rho \, d\rho \, A(\rho), \tag{4.8}$$

$$\left\langle \frac{d\mathscr{E}_c}{d\omega} \right\rangle = \frac{Lne^2 \psi d}{4\pi} \frac{\varepsilon^2 + \varepsilon'^2}{\varepsilon\varepsilon'} \cdot \omega \int_0^{\infty} \frac{d\nu}{\nu^2} \left[1 - 2\frac{\delta}{\nu} \left(1 - \frac{\delta}{\nu} \right) \left(1 - \frac{\omega^2}{\varepsilon^2 + \varepsilon'^2} \right) \right] \cdot \int_{-\infty}^{\infty} db \left| \int_{-\infty}^{\infty} dt e^{i\nu t} \ddot{\boldsymbol{\rho}}_r(t) \right|^2. \tag{4.9}$$

In the frequency range where $2R/\psi \gg \delta^{-1} \gg d$, the second term in (4.7) is vanishingly small. In that range, the first term in (4.7) gives the most important contribution to the radiation, and is responsible for incoherent effects during emission.

For electrons [and for positrons, when $\varepsilon_\perp > U_r(0)$], we can carry out the integration over ρ in (4.8). It is then easy to prove that the incoherent part of the spectral density is the same as the result from the Born theory:

$$\left\langle \frac{d\mathscr{E}_n}{d\omega} \right\rangle = Ln\omega \frac{d\sigma_n^{(B)}}{d\omega}. \tag{4.10}$$

For positrons with $\varepsilon_\perp < U_r(0)$, according to (4.8), the mean spectral density rapidly decreases with decreasing ψ, and when $\psi \ll \psi_c$, it is found to be significantly less than the corresponding results from the Born theory. This is related to the fact when $\psi < \psi_c$ positrons cannot approach too closely to the axis of a chain, where the gradient of the potential is a maximum.

Integrating (4.7) over frequency, we obtain the total radiative energy losses $\Delta\mathscr{E}$ of superbarrier particles in a crystal. If then $\varepsilon \ll m^2 d$, these energy losses will be governed mainly by the first term in (4.7), which is responsible for incoherent radiative effects. The ψ-dependence of $\Delta\mathscr{E}$ will then have the same factor as the ψ-dependence of $\langle d\mathscr{E}_n/d\omega \rangle$.

These equations hold if fluctuations of the integrals of the motion which determine the particle trajectory are small throughout the thickness of the crystal, compared with the integrals of the motion themselves.[5)] If the fluctuation growth process develops adiabatically (as must essentially happen over a coherence length), and the particle distribution function is known in terms of the quantities which determine the particle trajectory in the crystal, then the formulas derived can also be used to study radiation in thicker crystals. To do so, we need only average the radiation spectrum over the given particle distribution. For superbarrier particles, we then arrive at the following expression for the spectral density of the radiation:

$$\left\langle \frac{d\mathscr{E}}{d\omega} \right\rangle_L = \int_0^L dz \int d\varepsilon_\perp f(\varepsilon_\perp, z) L^{-1} \left\langle \frac{d\mathscr{E}}{d\omega} \right\rangle, \tag{4.11}$$

where $L^{-1}\langle d\mathscr{E}/d\omega \rangle$ is the spectral density (4.7) per unit length of the radiation of a superbarrier particle with a fixed value of $\varepsilon_\perp$, and $f(\varepsilon_\perp, z)$ is the particle distribution function over $\varepsilon_\perp$ at a depth z.

As noted above, the high-frequency radiation spectral density for superbarrier electrons is determined by the incoherent part of the radiation cross section, which is independent of $\varepsilon_\perp$. Therefore, even when there is a significant redistribution of such particles in transverse energy, their radiation spectrum is given by

$$\left\langle \frac{d\mathscr{E}}{d\omega} \right\rangle_L = Ln\omega \frac{d\sigma_n^{(B)}}{d\omega}. \tag{4.12}$$

We emphasize that this result holds if a particle is above the energy barrier over its whole path through the crystal, and furthermore, if over the whole path of an electron in the crystal one can neglect the radiative contribution of the term responsible for coherent radiative effects.

For positrons moving along one of the crystallographic axes in a crystal, significant redistribution of particles over $\varepsilon_\perp$ takes place in a distance which, to order of magnitude, is equal to the dechannelization length. This distance is much greater for positrons than for electrons; thus, for 1 GeV particles in a silicon crystal, the dechannelization length for positrons is several millimeters, while for electrons, it is of the order of several tens of micrometers.[17,18] Therefore, even for fairly thick crystals (several hundred micrometers thick), the spectral density of radiation from positrons will be determined by the initial transverse energy distribution of the particles. Thus, if a collimated beam of positrons enters a crystal along a crystallographic axis (z-axis), the $\varepsilon_\perp$-distribution of particles will be given by

$$f(\varepsilon_\perp) = \frac{2\pi\rho_0(\varepsilon_\perp)}{S} \frac{d\rho_0}{d\varepsilon_\perp}, \tag{4.13}$$

where $\rho_0(\varepsilon_\perp)$ is the positron entry point on the crystal, which is related to $\varepsilon_\perp$ in the present case by $\varepsilon_\perp = U_r(\rho_0)$, and S is the area of a basis cell in a plane perpendicular to the z-axis. With this distribution function, the positron high-frequency radiation spectral density will take the form

$$\left\langle \frac{d\mathscr{E}^+}{d\omega} \right\rangle_L = \frac{2Lne^2}{3m^2}\left(1 + \frac{\varepsilon'^2}{\varepsilon^2} + \frac{\omega^2}{2\varepsilon^2}\right)\frac{\pi\varepsilon^2}{S}\int_0^\infty \rho_0^3\, d\rho_0 A(\rho_0). \tag{4.14}$$

The major contribution to this integral comes from values of ρ_0 of the order of the screening radius of the atomic potential, $\rho_0 \sim R$. It is then easy to demonstrate, making use of this estimate, that to order of magnitude, the positron high-frequency radiation spectral density (4.14) is $\pi R^2/S$ times less than the radiation spectral density (4.10) for superbarrier electrons. In particular, for particles passing through a silicon crystal along the $\langle 100\rangle$ axis, $\pi R^2/S \approx 1/30$.

Note that for electrons a moderate, and for positrons a considerable, reduction in high-frequency radiation spectral density for a crystal oriented along the beam, compared with the Bethe–Heitler result, has been observed experimentally.[19–25] The results obtained above account for these experiments: the observed behavior is due to the effect of thermal vibrations of atoms on the incoherent part of the radiation cross section of a fast particle in a crystal, and to the redistribution of particle flux density in the crystal as a function of impact parameter.

For electrons with $\psi < \psi_c$, finite motion as well as infinite motion is possible in the field of one (or a few) chains of atoms in the crystal. A channelized electron more frequently comes close to the axis of a chain, and when it passes through the region occupied by the atomic nuclei of the lattice, spends more time there than a superbarrier electron (to order of magnitude, the time spent by electrons passing through that region is $\Delta t_u \sim 2(\overline{u^2})^{1/2}/v_\perp$, where $v_\perp \approx v\psi_c \times (1 + \varepsilon_\perp/U_0)^{1/2}$ is the transverse velocity of the electron in the region in question; since $v_\perp$ increases with increasing $\varepsilon_\perp$, the time interval Δt_u is greater for a channelized electron ($\varepsilon_\perp < 0$) than for a superbarrier electron ($\varepsilon_\perp > 0$). This is the reason why for electron channelization, the yield of inelastic processes associated with small impact parameters, such as electron reactions with nuclei, secondary electron emission, and others, should be greater than for superbarrier particle motion. (Existing experimental data on the strong dependence of the yield of electron–nuclear reactions[18] and secondary electron emission from crystals of various thicknesses[26] tends to confirm the existence of such an effect at small particle penetration depths into a crystal.) The incoherent part of the radiation cross section of a channelized electron should therefore be greater than that for a superbarrier particle.

Note that at high energies, practically all of the particles in a beam will execute motion whereby a channelized electron passes through the region occupied by the lattice atomic nuclei. This is related to the fact that at high energy, so-called **Retherton** motion, in which particles do not approach the axis of a chain, is unstable to asymmetry in the potential of the channel[27] (the potential of a chain of atoms in a crystal is always asymmetric at large distances from the chain), and to particle scattering from the electron subsystem of the crystal lattice.[16]

An extended stay of a channelized electron in the region of the lattice atomic nuclei, however, leads to significant fluctuations in the particle trajectory relative to its behavior in the field $U(\boldsymbol{\rho})$. Specifically, one finds that even when an electron passes through the indicated region only once, the change in the energy of transverse motion $\delta\varepsilon_\perp$ is comparable to the magnitude of $\varepsilon_\perp$.[16] A description of particle motion under these conditions, where over certain parts of the trajectory, the particle is subjected to significant field fluctuations, is beyond the scope of the present paper, and we will therefore not dwell in detail on an analysis of the incoherent part of the radiation cross section for channelized electrons. We only note that due to multiple scattering from thermal vibrations of atoms in the lattice, there is an extremely rapid dechannelization of the particles, wherein they switch from subbarrier to superbarrier states. Experimental data presently exist[18–26] which tend to confirm that this process develops at particle energies of order 1 GeV over a length of the order of some tens of micrometers. This means that in crystals much thicker than the dechannelization length, superbarrier particles will make a definite contribution to the incoherent part of the radiation cross section.

5. MEAN SQUARED ANGLE FOR MULTIPLE SCATTERING OF A FAST CHARGED PARTICLE IN A CRYSTAL

In the preceding sections, we considered the effect of thermal vibrations of atoms on radiation by relativistic particles in crystals, and showed that in the dipole approximation, the only influence that this factor has shows up in the quantity $|\mathbf{W}(\nu)|^2$. At $\nu = 0$, this is the square of the scattering angle of a particle in the crystal:

$$\vartheta^2 = |\mathbf{W}(0)|^2 = \sum_{n,k} \vartheta_n \vartheta_k, \tag{5.1}$$

so the results obtained above can also be used to analyze the influence of thermal vibrations of atoms on the mean

squared multiple-scattering angle of a particle moving through the atomic lattice. We now examine this process, assuming as before that particle motion in the crystal takes place at a small angle ψ to one of the crystallographic axes.

In that event, according to Eqs. (5.1) and (3.6), the square of the particle-scattering angle in a crystal, averaged over thermal vibrations of the atoms in the lattice, can be written in the form

$$\bar{\vartheta}^2=\overline{|W(0)|^2}=\frac{1}{d}\int dt\,A(\rho(t))+\sum_{n,n'}\vartheta_n^{(r)}\vartheta_{n'}^{(r)}, \quad (5.2)$$

where $A(\rho)$ is defined by Eq. (3.5), and $\vartheta_n^{(r)}$ is the particle-scattering angle in the field of the continuous potential of the nth chain of atoms.

The first term in (5.2) governs the incoherent scattering of particles from thermal vibrations of the atoms in the lattice, and the second governs scattering in the field of the continuous potential of the chains of atoms.

Equation (5.2) must still be averaged over different particle trajectories. If particle collisions with different chains of atoms can then be considered to be random, the results of averaging the first term in (5.2) will be determined by Eq. (4.5). The actual procedure for averaging the second term in (5.2) requires special consideration. This is because particle scattering in the continuous field of each chain of atoms takes place only along the azimuthal angle φ in a plane orthogonal to the axis of the plane (see Fig. 15 of the review in Ref. 7), so the averaging must be extended over the angle φ. Then the mean value of φ can be of order unity.

The overall scattering angle ϑ_N for a particle scattering from N chains of atoms in a crystal is related to the azimuthal scattering angle φ_N from these chains by

$$|\vartheta_N|=2\psi\sin\frac{\varphi_N}{2}. \quad (5.3)$$

Making use of this equation, we can determine the mean squared multiple-scattering angle $\langle\vartheta_N^2\rangle$ for a fast particle in the continuous field of the atomic chains.

We note in this regard that

$$\varphi_N=\varphi_{N-1}+\varphi_N^*, \quad (5.4)$$

where φ_N^* is the scattering angle from the Nth chain. This angle φ_N^* is determined by the impact parameter of the chain, b_N:

$$\varphi_N^*=\pi-2b_N\int_{\rho_0}^{\infty}\frac{d\rho}{\rho^2}\left(1-\frac{U_r(\rho)}{\varepsilon_\perp}-\frac{b_N^2}{\rho^2}\right)^{-1/2}, \quad \varepsilon_\perp=\frac{\varepsilon\psi^2}{2}. \quad (5.5)$$

Taking this relation into account, the mean value $\langle\vartheta_N^2\rangle$ can be written in the form

$$\langle\vartheta_N^2\rangle=\left(\prod_{n=1}^{N}\int\frac{db_n}{\bar{b}}\right)4\psi^2\sin^2\frac{\varphi_1^*+\varphi_2^*+\ldots+\varphi_N^*}{2}. \quad (5.6)$$

It is then easy to obtain a recursion relation for $\langle\vartheta_N^2\rangle$:

$$\langle\vartheta_N^2\rangle=\langle\vartheta_1^2\rangle+\langle\vartheta_{N-1}^2\rangle(1-\langle\vartheta_1^2\rangle/2\psi^2), \quad (5.7)$$

where

$$\langle\vartheta_1^2\rangle=4\psi^2\int\frac{db}{\bar{b}}\sin^2\frac{\varphi_1^*(b)}{2}.$$

The solution of this recursion relation takes the form

$$\langle\vartheta_N^2\rangle=2\psi^2[1-(1-\langle\vartheta_1^2\rangle/2\psi^2)^N]. \quad (5.8)$$

When $\langle\vartheta_1^2\rangle\ll 2\psi^2$ and $N\gg 1$, this equation is transformed into[6)]

$$\langle\vartheta^2\rangle_c=2\psi^2\left[1-\exp\left(-2ndL\psi\int db\sin^2\frac{\varphi(b)}{2}\right)\right]. \quad (5.9)$$

Thus, the mean squared value of the multiple-scattering angle for a fast particle in a crystal is of the form

$$\langle\vartheta^2\rangle=\langle\vartheta^2\rangle_n+\langle\vartheta^2\rangle_c, \quad (5.10)$$

where $\langle\vartheta^2\rangle_n$ is defined by Eq. (4.6).

For superbarrier electrons [and positrons with $\varepsilon_\perp>U_r(0)$], $\langle\vartheta^2\rangle_n$ can be written in the form

$$\langle\vartheta^2\rangle_n=\frac{Ln}{4\pi^2\varepsilon^2}\int d^2g\,g^2|U(g)|^2(1-e^{-g^2\overline{u^2}}). \quad (5.11)$$

When $\overline{u^2}\to\infty$, this equation goes into the corresponding result $\langle\vartheta^2\rangle_a$ from the theory of multiple scattering of fast particles in an amorphous medium. Thus, $\langle\vartheta^2\rangle_n$ differs from the corresponding expression for an amorphous medium only by the Debye–Waller factor[7] $[1-\exp(-g^2\overline{u^2})]$. At room temperature, $\langle\vartheta^2\rangle_n$ is found to be somewhat smaller than $\langle\vartheta^2\rangle_a$ (by 5%–25%, depending on the crystal in which scattering takes place). Note also that $\langle\vartheta^2\rangle_n$ is independent of ψ. According to (4.5), for positrons with $\varepsilon_\perp<U_r(0)$, $\langle\vartheta^2\rangle_n$ decreases rapidly with decreasing ψ.

As indicated by Eq. (5.9), the quantity $\langle\vartheta^2\rangle_c$ is a sensitive function of the angle ψ of particle incidence at the crystal, as it relates to a crystallographic axis, and it exceeds $\langle\vartheta^2\rangle_a$ over a wide range of angles ψ (see Fig. 16 of Ref. 7). This in no way means, however, that the first term in (5.10) can always be neglected in comparison with the second. The term $\langle\vartheta^2\rangle_c$ is related to particle scattering in the continuous field of the chains of atoms in the crystal. But the energy of transverse particle motion $\varepsilon_\perp=\varepsilon\psi^2/2$ (Refs. 7, 14) is conserved in such a field, which in turn results in scattering only being possible along the azimuthal angle φ. The term $\langle\vartheta^2\rangle_n$ controls the variation of transverse energy, and consequently governs the broadening of the annular distribution of scattered particles in the radial direction. Such distributions have been observed, for instance in an experiment described in Ref. 17.

[1)]We use a system of units in this paper in which the speed of light c and Planck's constant $\hbar$ are equal to one.

[2)]We essentially require that particle trajectory fluctuations be small over a coherence length.

[3)]Allowing for fluctuations in the times t_n associated with thermal spread of atoms in the lattice along the z-axis leads to minor corrections in $t_{n+1}-t_n=d/v$ of order u_z/d, which we ignore.

[4)]When $\omega\ll\varepsilon$ and we make use of (3.6), Eq. (2.3) is transformed into the corresponding equations of Refs. 10 and 11, which were derived on the basis of classical and quantum considerations of radiation by a particle in a crystal.

[5)]For superbarrier particles, this condition is satisfied over a wide energy range.[16]

[6)]In that case, Eq. (5.9) can also be derived by considering particle scattering from chains of atoms in the crystal using the kinetic equation method of Refs. 28 and 29.

[7)]A similar result was obtained in Ref. 30 in a study of multiple scattering of fast particles in a crystal, for motion close to the crystallographic planes at angles much greater than the critical angle for planar channelization.

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