The hump in the ultrahigh-energy cosmic-ray spectrum

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Interaction of the protons of ultrahigh-energy cosmic rays with 2.7 K relict radiation leads to appearance of features in the differential energy spectrum in the form of a hump due to the production of e^+e^- pairs, a dip due to e^+e^- pairs, a photopion hump, and a "black-body cutoff," which occur in sequence with increase of the energy. In this work we have given a physical interpretation of these features and a method of calculation in the approximation of continuous energy loss. We have calculated the spectra of one source and (diffuse) spectra from many sources. We take into account cosmological effects, including the increase of the energy loss with increase of the red shift. All four features appear distinctly in the spectrum of a single source for large propagation times. The diffuse spectra may have only a sharply expressed photopion hump and a black-body cutoff. In Local-Supercluster Models of galaxies the existence of a hump is predicted for a broad range of propagation time.

1. INTRODUCTION

The spectrum of ultrahigh-energy cosmic rays from remote metagalactic sources has a characteristic feature in the form of a "black-body cutoff" at $E \sim 3 \cdot 10^{19}$ eV. This feature, which was predicted by Greisen[1] and by Zatsepin and Kuz'min,[2] arises as a consequence of the interactions of protons or cosmic-ray nuclei with the 2.7 K relict radiation. Since that time many calculations have been made of the spectra of metagalactic ultrahigh-energy cosmic rays,[3-12] the principal results of which can be summarized as follows.

1) Protons of ultrahigh energy lose their energy as a consequence of the red shift, production of e^+e^- pairs ($p + \gamma \to p + e^+ + e^-$), and photoproduction of pions ($p + \gamma \to N + \pi$). The energy loss of nuclei is due to the red shift, pair production ($A + \gamma \to A + e^+ + e^-$), photodisintegration ($A + \gamma \to (A-1) + N$), and photoproduction of pions. The energy loss is determined mainly by the interaction of cosmic rays with relict photons, and the role of optical and infrared radiation is small for most models which have been discussed.[12]

2) The energy at which the spectrum steepens (the black-body cutoff) depends on the propagation time of the cosmic rays and consequently on the model of the distribution of cosmic-ray sources in the Universe. It is convenient to describe the cutoff energy by the quantity $E_{1/2}$ (Ref. 10) at which the integral spectrum with inclusion of the black-body steepening becomes a factor of two smaller than the power-law extrapolation from low energies. For a Universe uniformly filled with nonevolving proton sources, $E_{1/2} = 5.8 \cdot 10^{19}$ eV for a broad range of values of the exponent of the integral production spectrum $1.1 \leqslant \gamma_g \leqslant 1.7$. The cutoff energy $E_{1/2}$ for nuclei does not differ greatly from $E_{1/2}$ for protons.

3) In models taking into account the evolution of the sources (namely, with inclusion of an increase in the past of the spatial density of sources and/or of their luminosity) the spectrum has a more complicated form. In particular, in the integral spectrum before the cutoff there is a flat step. This feature was first noted by Hillas[3] in the spectrum of evolving cosmic-ray sources, and calculations for quasars and Seifert galaxies with inclusion of their observed evolution were carried out by us in Ref. 9.

4) If cosmic-ray sources form a small group around our Galaxy, then the black-body cutoff of the spectrum can be shifted to the region of higher energies or can be absent completely as a consequence of the smallness of the propagation time of the cosmic rays. A realistic candidate for such a group is a Local Supercluster of galaxies. The Local-Supercluster Model has been discussed in Refs. 4, 10, and 11.

All of the conclusions listed above regarding the shape of the spectrum of metagalactic high-energy cosmic rays were obtained for integral spectra, since ten years ago just such experimental spectra were obtained, as the result of the poor statistics. Recently Hill and Schramm[13] took a new step, and analyzed the differential spectrum. Here they actually interpreted the feature of the integral spectrum (the flat step in front of the black-body cutoff) in terms of the differential spectrum. It turned out that it consists of a small dip and a hump—precursors of the black-body cutoff. Hill and Schramm consider that the cause of the appearance of these features in their calculations is due to taking into account recoil protons and to use of collision terms in the kinetic equation instead of the approximation of continuous energy loss. We shall show here that the approximation of continuous energy loss satisfactorily describes the spectrum features discussed, and the cause of their absence in earlier studies is due merely to the calculation of the integral spectra, in which the dip and hump appeared in integrated form as a flat step.

Most authors have included in the kinetic equation a term which describes the collision of protons with relict photons in the approximation of continuous energy loss in the form $\partial(b(E)\, j(E))/\partial E$ ($b(E) = dE/dt$ is the proton energy loss due to the red shift and to production of e^+e^- pairs and pions), since it automatically takes into account slowed down protons. The production of e^+e^- pairs must be discussed as a process of continuous energy loss, since the proton in a $p\gamma$ collision loses a small fraction of its energy $\delta = 2m_e/m_p = 10^{-3}$. In regard to the photopion process, for the spectral features of interest here the important energy

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region is that in which the pion is produced near threshold. The fraction of the energy lost by a proton in this energy region is $\delta \simeq \mu/m_p \approx 0.15$, and consequently the continuous-energy-loss approximation is rather good. At very high energies, which in this case are of purely academic interest, the proton loses about half of its energy in a collision. What is the accuracy of the continuous-energy-loss approximation in this case? For very high energies, at which only the photopion process is important, two equations for propagation of cosmic rays were discussed in Ref. 8: an equation with collision terms which take into account fluctuations in $p\gamma$ collisions, and an equation in the continuous-energy-loss approximation. Both equations were solved analytically. The ratio of the solutions is $(\gamma_g + 1)/2$. Consequently for flat production spectra ($\gamma_g \leqslant 1.6$), which are usually discussed, the accuracy of the continuous-energy-loss approximation is better than 30% even at the highest energies. This problem has been discussed in greater detail in Ref. 12.

In the present work we investigate the fine structure of the cosmic-ray differential spectrum in the continuous-energy-loss approximation and devote particular attention to the physical interpretation of the hump and dip—the precursors of the cutoff (Section 3). These features have been studied quantitatively in a single-source model (with a fixed propagation time, Section 4) and in a multiple-source model (Section 5). In Section 6 we have given a comparison with the results of Hill and Schramm,[13] and in Section 7 we have formulated the principal conclusions.

2. ENERGY LOSS OF ULTRAHIGH-ENERGY COSMIC-RAY PROTONS AND THE RELATION BETWEEN THE ENERGIES AT THE TIME OF PRODUCTION AND AT THE TIME OF OBSERVATION

The energy loss of ultrahigh-energy protons in the epoch with red shift $z=0$ is due to the red shift $(1/E)(dE/dt) = H_0$, where $H_0 = 75$ km/sec·Mpc is the Hubble constant, to e^+e^- pair production ($p+\gamma \to p + e^+ + e^-$), and to pion production ($p+\gamma \to \pi$'s $+ N$) in interaction with the 2.7 K relict radiation.

The proton energy loss due to interaction with relict radiation of temperature T can be written in the form

$$\beta_0(E) \equiv -\frac{1}{E}\frac{dE}{dt}$$

$$= \frac{ckT}{2\pi^2\Gamma^2(c\hbar)^3}\int\limits_{\varepsilon_0}^{\infty} d\varepsilon_r \sigma(\varepsilon_r) f(\varepsilon_r)\varepsilon_r[-\ln(1-\exp(-\varepsilon_r/2\Gamma kT))], \quad (1)$$

where Γ is the proton Lorentz factor, k is the Boltzmann constant, $\hbar$ is the Planck constant, c is the velocity of light, ε_r is the energy of the relict photon in the proton rest system, $\sigma(\varepsilon_r)$ is the interaction cross section, $f(\varepsilon_r)$ is the fraction of the energy lost by the proton in one collision, and ε_0 is the threshold of the considered reaction in the proton rest system.

For the pair-production process new calculations of $\sigma(\varepsilon_r)$ and $f(\varepsilon_r)$ have been carried out and special attention has been given to low energy values, which are not present in Ref. 6; at all other energies good agreement is obtained with Ref. 6. For $E \ll m_e m_p/kT = 2.1\cdot 10^{18}$ eV (i.e., $\varepsilon_r/m_e - 2 \ll 1$), when the reaction occurs with photons from the high-energy tail of the Planck distribution and $f(\varepsilon_r)$ and $\sigma(\varepsilon_r)$ are determined by their threshold values

$$f_{th} = \frac{2m_e}{m_p}, \quad \sigma(\varepsilon_r) \approx \frac{\pi}{12}\alpha r_0^2\left(\frac{\varepsilon_r}{m_e} - 2\right)^3,$$

we obtain from Eq. (1)

$$\beta_0(E) \equiv -\frac{1}{E}\frac{dE}{dt}$$

$$= \frac{16c}{\pi}\frac{m_e}{m_p}\alpha r_0^2\left(\frac{kT}{\hbar c}\right)^3\left(\frac{\Gamma kT}{m_e}\right)^2\exp\left(-\frac{m_e}{\Gamma kT}\right), \quad (2)$$

where $\alpha = 1/137$ and r_0 is the electron radius. The approximation (2) can be used up to $E \lesssim 1\cdot 10^{18}$ eV.

At higher energies the energy loss is determined by the photopion reactions ($p+\gamma \to p + \pi^0$, $p + \gamma \to n + \pi^+$). The cross sections for these reactions are well known, and the kinematics is simple. For the case of production of several pions the results of Ref. 14 were used. The energy loss calculated by us for ultrahigh-energy protons is shown in Fig. 1. For energies $E < \varepsilon_0 m_p/2kT = 3\cdot 10^{20}$ eV at which the cross section and the fraction of energy lost by the proton in one collision are determined by their threshold values,

$$\sigma(\varepsilon_r) \approx \sigma'(\varepsilon_r - \varepsilon_0), \quad f(\varepsilon_r) = \frac{\varepsilon_r}{m_p}\frac{1+\mu^2/2\varepsilon_r m_p}{1+2\varepsilon_r/m_p},$$

the energy loss can be written in the form

$$-\frac{1}{E}\frac{dE}{dt} = \frac{2(kT)^3\sigma'\varepsilon_0^2}{\pi^2 c^4\hbar^3 m_p}\exp\left(-\frac{\varepsilon_0}{2\Gamma kT}\right), \quad (3)$$

where $\varepsilon_0 = \mu(1+\mu/m_p)$ is the threshold of the $p+\gamma \to \pi + N$ reaction in the rest system of the proton and $\sigma' = 6.8\cdot 10^{-36}$ cm^2/eV. Equation (3) is accurate to 20% up to an energy $E = 3\cdot 10^{20}$ eV. At very high energies $E \gg \varepsilon_0 m_p/2kT = 3\cdot 10^{20}$ eV the cross section is $\sigma(\varepsilon_r) = 1\cdot 10^{-28}$ cm^2, $f(\varepsilon_r) \to f_0 \approx 0.5$, and $-(1/E)dE/dt \approx c f_0 n\sigma = 1.8\cdot 10^8$ y^{-1}.

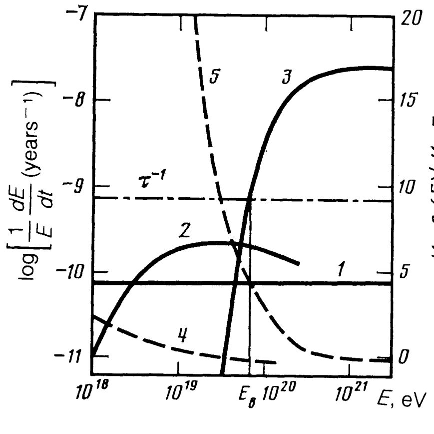

FIG. 1. Relative energy loss of protons in relict radiation $\beta_0(E) = (1/E)(dE/dt)$ in the contemporary epoch ($z=0$). Curve 1 is the energy loss due to the red shift. Curves 2 and 3 are respectively the loss to production of e^+e^- pairs and pions in relict photons. The dashed lines show the derivatives $d\ln\beta_0(E)/d\ln E$ (right-hand scale) for pair production (curve 4) and pion production (curve 5). For purposes of illustration we have drawn in the values of the quantities τ^{-1} and E_b, where τ is the propagation time and E_b is an energy approximately equal to the cutoff energy.

Knowing the energy loss of the proton $\beta_0(E)$ at the present epoch $z=0$, we can calculate the energy $E_g(z)$ which the proton should have in the production epoch (red shift z) if for $z=0$ its energy is E. In the past the density of relict photons and their energy were greater respectively by $(1+z)^3$ and $(1+z)$ times, and therefore for the energy loss in epoch z we have

$$\beta(E, z)=(1+z)^3\beta_0((1+z)E), \quad b(E, z)=(1+z)^2 b_0((1+z)E). \tag{4}$$

The loss to red shift increases slowly with z:

$$\beta_{r.sh}(E, z)=H_0(1+z)^{3/2}. \tag{5}$$

Here and below we shall always use the Einstein–de Sitter cosmological model ($\Omega=1$), in which the age of the Universe t and the red shift z are related as follows:

$$t={}^2/_3 H_0^{-1}(1+z)^{-3/2}. \tag{6}$$

Then $E_g(z)$ can be found from the equation

$$(1/E)\,dE/dt=-\beta(E, z), \tag{7}$$

where $\beta(E,z)$ is the sum of three terms which describe the energy loss to the red shift $\beta_{r,\mathrm{sh}}(E,z)$, to e^+e^- pair production $\beta_{\mathrm{pair}}(E,z)$, and to pion production $\beta_{\mathrm{pion}}(E,z)$ calculated from the data of Fig. 1 and with use of Eqs. (4)–(6).

Below we shall everywhere use the relation

$$\lambda(E, z)=E_g(z)/E, \tag{8}$$

which is given by Eq. (8), in the form of a table of values for various E and z (or the propagation time τ).

3. THE HUMP: ITS PHYSICAL INTERPRETATION

The existence of a hump in the differential spectrum of ultrahigh-energy protons is a direct consequence of the black-body cutoff and conservation of the total number of protons in the spectrum. We shall consider a source of ultrahigh-energy protons with a power spectrum $F(E)\,dE=KE^{-(\gamma+1)}dE$ at a distance r from the Earth. Assume that the protons are moving in a straight line with a propagation time $\tau=r/c$. All protons with energy $E>E_b$, which can be estimated roughly from the condition $\beta(E_b)\sim\tau^{-1}$ (see Fig. 1), are slowed down during their time of flight to an energy $E<E_b$. However, the energy of the slowed down protons cannot decrease substantially below E_b, since the energy loss falls off exponentially with decrease of the energy, and as a result the protons are accumulated (pile up) at $E\sim E_b$ in the form of a hump. Here the integral spectrum remains simply a monotonically falling function of the energy. We shall actually consider two differential spectra: with the energy loss turned off ($\sim E^{-(\gamma+1)}$ at all energies) and with inclusion of the energy loss (a spectrum with a hump, which begins at an energy denoted as E_0). It is understandable that the number of particles with energy above E_0, $j(\geqslant E_0)$, is the same for the two spectra. If we now shift slightly to an energy E which is inside the hump, then the number of particles in the spectrum with loss will be equal to the number of particles in the spectrum without loss, $j_0(\geqslant E)$, minus the number of particles which as a consequence of slowing down have transferred to the part of the hump between energies E_0 and E. With increase of E this number of particles increases and consequently the integral spectrum with energy loss, $j(>E)$, monotonically becomes steeper than the spectrum without inclusion of energy loss, $j_0(>E)\sim E^{-\gamma}$. Let us turn now to a quantitative discussion. We shall consider the case of a nearby source with a power-law production spectrum $F(E_g)dE_g=KE_g^{-(\gamma+1)}dE_g$, where $F(E_g)$ is the total number of particles emitted by the source per second with energy E_g. We shall assume that the propagation time $\tau=r/c$ is rather small, for example, $\tau\sim3\cdot10^8$ years. In this case it is possible to neglect the change in the temperature of the relict radiation T, and this means that the proton energy loss $b(E)=-dE/dt$ does not depend on time.

The total number of protons at a distance $r'=ct$ from the source $F(E,t)=4\pi(r')^2 j(E,t)$ satisfies the equation

$$\partial F(E, t)/\partial t-\partial(b(E)F(E, t))/\partial E=0, \tag{9}$$

which for the initial condition $F(E_g, 0)=KE_g^{-(\gamma+1)}$ has the solution

$$F(E, \tau)=KE_g^{-(\gamma+1)}\,b(E_g)/b(E), \tag{10}$$

where $E_g=\lambda(E,\tau)E$ (see Section 2).

First we shall show that the integral spectrum

$$F(>E, \tau)=\int_E^\infty dEKE_g^{-(\gamma+1)}\frac{b(E_g)}{b(E)} \tag{11}$$

is monotonically falling and has no peaks. For this purpose we shall prove that in Eq. (11)

$$b(E_g)/b(E)=dE_g/dE. \tag{12}$$

The solution of the energy-loss equation $dE/dt=-b(E)$ can be represented in the form of a trajectory $\varphi(E',t')$ fixed by the value of E_g at a production time t_g (the age of the Universe):

$$\varphi(E', t', t_g)=E_g. \tag{13}$$

Since E_g is an integral of motion along the trajectory, we have

$$\frac{dE_g}{dt'}=\frac{\partial\varphi(E',t',t_g)}{\partial t'}+\frac{\partial\varphi(E',t',t_g)}{\partial E'}\frac{\partial E'}{\partial t'}=0 \tag{14}$$

and

$$\frac{\partial\varphi(E,t,t_g)}{\partial t}=\frac{\partial\varphi(E,t,t_g)}{\partial E}b(E), \tag{15}$$

where E is the energy at the time of observation t. The energy intervals at the time of production dE_g and at the time of observation dE are related as follows:

$$dE_g=\frac{\partial\varphi(E,t,t_g)}{\partial E}dE. \tag{16}$$

The energy loss at the time of production is

$$\frac{\partial E_g}{\partial t_g}\equiv\frac{\partial\varphi(E,t,t_g)}{\partial t_g}=-b(E_g). \tag{17}$$

Substituting $a\varphi/\partial E$ from (15) into (16) and using $\partial\varphi(E,t,t_g)/\partial t=-\partial\varphi(E,t,t_g)/\partial t_g$, since in the stationary problem discussed here $\varphi(E,t,t_g)=\varphi(E,t-t_g)$, we obtain (12). Finally, from (11) and (12), using the relation $E_g=\lambda(E,z)E$, we arrive at

$$F(>E,\tau)=\lambda^{-\gamma}(E,\tau)\frac{K}{\gamma}E^{-\gamma}. \tag{18}$$

Since $\lambda(E,\tau)$ is a smooth function of the energy, it follows from (8) that the integral spectrum has no other feature except a monotonic steepening which begins at some energy E_b.

Let us return to the differential spectrum (10) and analyze the ratio of the spectrum with energy loss and the spectrum without loss. We shall call this ratio the modification factor

$$\eta(E,\tau)\equiv F(E,\tau)/KE^{-(\gamma+1)}=\lambda^{-\gamma}(E,\tau)\beta(E_g)/\beta(E). \tag{19}$$

We shall consider the case of a very close source, in which $\tau^{-1}>\beta(E)$ for all energies, for example, $\tau=1\cdot10^7$ years (see Fig. 1). For this case analytic solutions can be obtained with all features of the spectrum in explicit form (although of small magnitude). Using the following approximate formulas: $\lambda(E,\tau)\approx1+\beta(E)\tau$ and $\beta(E_g)\approx\beta(E)[1+(d\beta/dE)\tau E]$, we obtain

$$\eta(E,\tau)\approx1+\beta(E)\tau(d\ln\beta(E)/d\ln E-\gamma), \tag{20}$$

which is valid both for the case of energy loss in pair production and for energy loss to pion photoproduction. First let us consider the energy region in which the photopion loss is dominant. It can be seen from Eq. (20) that for all energies $d\ln\beta(E)/d\ln E>\gamma$ and $\eta(E,\tau)$, according to Eq. (3), increases with increase of E exponentially, forming a hump. At some energy the difference $d\ln\beta(E)/d\ln E-\gamma$ changes sign and the hump goes over to a cutoff. At very high energies $E>5\cdot10^{20}$ eV $d\ln\beta(E)/d\ln E\to0$ and $\eta\to1-\gamma\tau/t_0$, where $1/t_0$ is the limiting value of $\beta_{\rm pion}(E)$ at very high energies (see Fig. 1). Taking the photopion loss at $E<3\cdot10^{20}$ eV in the form $\beta(E)=(1/\tau_\pi)\exp(\varepsilon_\pi/E)$, where $\varepsilon_\pi\approx3\cdot10^{20}$ eV and $\tau_\pi=1.7\cdot10^7$ years, as in (3), we can obtain from (20)

$$\eta(E,\tau)\approx1+\frac{\tau}{\tau_\pi}\exp\left(-\frac{\varepsilon_\pi}{E}\right)\left(\frac{\varepsilon_\pi}{E}-\gamma\right), \tag{21}$$

which quantitatively demonstrates the exponential rise of the hump and its transition to a steepening of the spectrum—a cutoff.

Let us turn now to the energy region in which energy loss to pair production is dominant. At low energies, according to (2), $\beta(E)=AE^2\exp(-\varepsilon_p/E)$, where $\varepsilon_p=m_em_p/kT=2.15\cdot10^{18}$ eV. Substituting this expression into (20), we obtain

$$\eta(E,\tau)\approx1+AE^2\exp\left(-\frac{\varepsilon_p}{E}\right)\left[\frac{\varepsilon_p}{E}-(\gamma-2)\right]. \tag{22}$$

In the real problem the exponential rise predicted by (22) at $E<\varepsilon_p$, is not visible, since at such low energies the loss to the red shift, which we have not taken into account, is dominant over the loss to pair production. When the situation changes (see Fig. 1) the energy dependence of $\beta_{\rm pair}$ is weak and the hump produced is not so sharp as in the case of photopion loss. At the point where $\beta_{\rm pair}$ reaches a maximum, $d\ln\beta(E)/d\ln E=0$ and, according to (20), $\eta(E,\tau)\to1$. At high energies $\eta(E,\tau)$ drops until the photopion hump no longer compensates for this drop. Therefore a dip is formed. Since particles slowed down from the energy region where there is a cutoff accumulate near it and during their time of flight do not reach the energy region under discussion, the numbers of particles in the dip and hump from pair production are approximately equal.

For the very short propagation times considered here, changes in the spectrum as a consequence of pair production and pion production are very small. With increase of τ, first the photopion hump and the cutoff appear appreciably, and then at still greater distances—the hump and dip from pair production appear. This statement is illustrated by Fig. 3, which will be discussed in the next section.

The magnitude of the hump is greater in the case of flat production spectra, since a greater number of particles slow down as the result of energy loss and are accumulated in the form of a hump.

4. THE SINGLE-SOURCE SPECTRUM

Let us consider the real case of the spectrum of a single source located at an arbitrary distance. We shall assume also that all particles have the same propagation time as in the case of straight-line propagation. The total number of particles produced by the source per second is related to the energy release (luminosity) as

$$F(E_g)\,dE_g=\gamma(\gamma-1)L_p(E_g+1)^{-(\gamma+1)}\,dE_g$$
$$\approx\gamma(\gamma-1)L_pE_g^{-(\gamma+1)}\,dE_g, \tag{23}$$

where E_g and L_p are measured respectively in GeV and GeV/c. The age of the Universe at the time of production we shall call t_g (red shift z_g), and at the contemporary epoch ($z=0$) we shall call it t_0. For a flat Einstein–de Sitter Universe $t_0=2/(3H_0)$ and $1+z_g=(t_0/t_g)^{2/3}$. The flux at the point of observation is

$$j(E)\,dE=F(E_g)\,dE_g/(1+z)4\pi R^2(t_0)f^2(r), \tag{24}$$

where r is the cosmological space coordinate, $f(r)=r$ for a flat Universe, and r is related to the radius $R(t)$ of the Universe as follows:

$$r=c\int_{t_g}^{t_0}dt/R(t). \tag{25}$$

Using $R(t)=R(t_0)(t/t_0)^{2/3}$, we obtain

$$rR(t_0)=\frac{2c}{H_0}\left[1-\frac{1}{(1+z_g)^{1/2}}\right], \tag{26}$$

$$j(E)\,dE=\frac{F(E_g)\,dE_g}{(16\pi c^2/H_0^2)[(1+z_g)^{1/2}-1]^2}. \tag{27}$$

The relation between E_g and E is given as before by (8), but the relation between dE_g and dE is more complicated than (12), since now the trajectory $E_g=\varphi(E,t,t_g)$ does not satisfy the condition $\varphi(E,t,t_g)=\varphi(E,t-t_g)$ as in the stationary case.

As will be shown below, for the case of a nonstationary Universe the relation between dE_g and dE can be written in the form

$$\frac{dE_g}{dE}=(1+z_g)$$
$$\times\exp\left[\int_0^{z_g}\frac{dz}{H_0}(1+z)^{1/2}\left(\frac{db_0^{b-b}(E')}{dE'}\right)_{E'=\lambda(E,z)(1+z)E}\right], \tag{28}$$

where $b_0^{b-b}(E') = \partial E'/\partial t'$ is the proton energy loss due to interaction of the proton with relict radiation in the contemporary epoch ($z=0$). The derivative $db_0(E)/dE$ is shown in Fig. 2 as a function of energy. For proof of (28) we shall consider a trajectory $E_g = f(t)$ passing through a point (E, t_0), $f(t_0) = E$, and another very close trajectory $E'_g = f(t) + y(t)$ such that $y(t) \ll f(t)$ for all t from t_0 to t_g. The infinitely small quantity $y(t_0) = \varepsilon$ is dE in (28). Using the definition of the energy loss $\partial E'_g/\partial t = -b(E'_g, t)$, we obtain

$$\frac{\partial f}{\partial t}+\frac{\partial y}{\partial t}=-b(f+y,t)\approx-\left[b(f,t)+\left(\frac{\partial b(E,t)}{\partial E}\right)_{E=f(t)}y(t)\right],$$

from which

$$\frac{\partial y}{\partial t}=-\left(\frac{\partial b(E,t)}{\partial t}\right)_{E=f(t)}y(t). \qquad (29)$$

In Eq. (29) from the expression for the energy loss we shall separate the term corresponding to the loss to the red shift and the terms which describe the interaction with the relict radiation:

$$\left(\frac{\partial b(E,z)}{\partial E}\right)_{f(z)}=\left(\frac{\partial b(E,z)}{\partial E}\right)^{r.sh}_{f(z)}+\left(\frac{\partial b(E,z)}{\partial E}\right)^{b-b}_{f(z)}, \qquad (30)$$

where

$$\left(\frac{\partial b(E,z)}{\partial E}\right)^{r.sh}_{f(z)}=H_0(1+z)^{3/2}, \qquad (31)$$

$$\left(\frac{\partial b(E,z)}{\partial E}\right)^{b-b}_{f(z)}=(1+z)^2\left(\frac{\partial b(E,0)}{\partial E}\right)_{E=(1+z)f(z)}. \qquad (32)$$

Substituting (31) and (32) into (29) and replacing the variable t by z, we obtain

$$\frac{dy}{dz}=\frac{y}{H_0(1+z)^{5/2}}\left[H_0(1+z)^{3/2}+(1+z)^2\left(\frac{\partial b(E,z)}{\partial E}\right)_{E=(1+z)f(z)}\right]. \qquad (33)$$

Integrating (33) from $z=0$ to $z=z_g$ and using the notation $y(z_g) = dE_g$, we arrive at (28). It is easy to see that for $z \ll 1$ Eq. (28) goes over into Eq. (12).

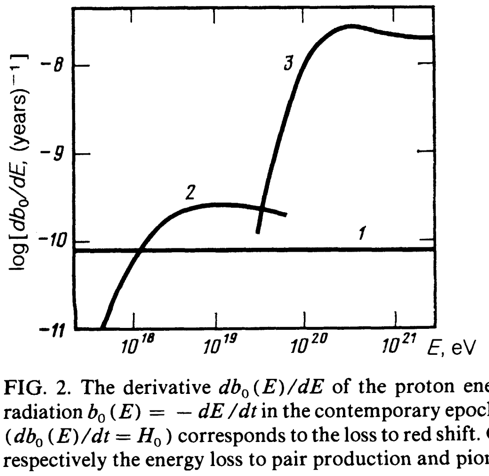

FIG. 2. The derivative $db_0(E)/dE$ of the proton energy loss in relict radiation $b_0(E) = -dE/dt$ in the contemporary epoch ($z=0$). Curve 1 ($db_0(E)/dt = H_0$) corresponds to the loss to red shift. Curves 2 and 3 are respectively the energy loss to pair production and pion production.

From (27) and (23) we obtain the magnitude of the flux from a single source:

$$j(E)=\frac{H_0^2}{16\pi c^2}\gamma(\gamma-1)L_pE^{-(\gamma+1)}\frac{[\lambda(E,z_g)]^{-(\gamma+1)}}{[(1+z_g)^{1/2}-1]^2}\frac{dE_g}{dE}, \qquad (34)$$

where dE_g/dE is given by Eq. (28).

The modification factor

$$\eta(E,z)=\frac{[\lambda(E,z)]^{-(\gamma+1)}}{[(1+z)^{1/2}-1]^2}\frac{dE_g}{dE} \qquad (35)$$

is shown as a function of the energy E in Fig. 3 for two values of γ ($\gamma = 1.1$ and 1.6). In this figure we have shown for each pair of curves the values of the propagation time $\tau = t_0 - t_g$, where t_0 and t_g have been found from (6) respectively for $z=0$ and $z = z_g$.

The modification factor $\eta(E, \tau)$ behaves as was discussed in Section 3. For small values of τ, for which the analysis in Section 3 is quantitatively valid, we can see a small hump which comes before the steepening (cutoff) of the spectrum. With increase of τ (the straight line τ^{-1} is shifted downward in Fig. 1) the cutoff energy decreases, and consequently a larger number of particles slow down and are collected in the form of a hump. The hump increases and becomes sharply expressed, as was mentioned in Section 3. In discussing the magnitude of the hump (Fig. 4), it is necessary to recall that the modification factor $\eta(E,\tau)$ is proportional to $E^{\gamma+1}j(E)$ and consequently with increase of τ, when the cutoff energy decreases, the number of particles in the hump is multiplied by the progressively decreasing factor $E^{\gamma+1}$. This explains the decrease in the size of the hump in Fig. 4 with increase of τ. For very large τ the dip and hump from pair production become appreciable, as was suggested

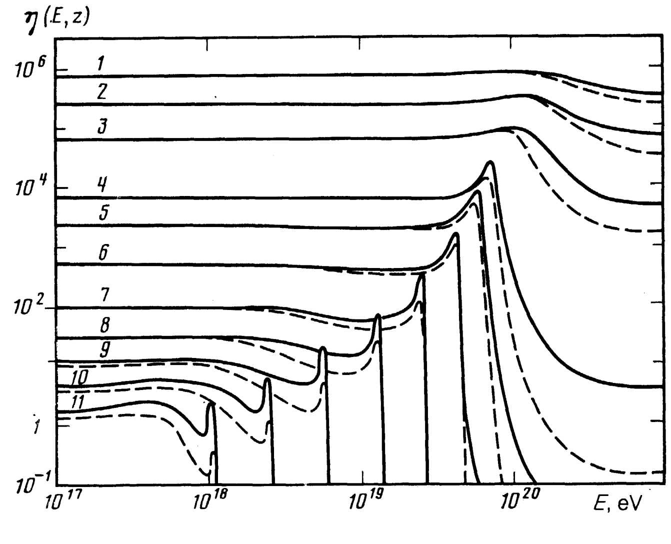

FIG. 3. Modification factor for a single-source spectrum $\eta(E,\tau)$ given by Eq. (35) with $\gamma = 1.1$ (solid lines) and 1.6 (dashed lines). Each pair of curves corresponds to various values of τ (or z): 1—$\tau = 3\cdot10^7$ years ($z = 2.3\cdot10^{-3}$); 2—$\tau = 5\cdot10^7$ years ($z = 3.8\cdot10^{-3}$); 3—$\tau = 1\cdot10^8$ years ($z = 7.7\cdot10^{-3}$); 4—$\tau = 3\cdot10^8$ years ($z = 2.4\cdot10^{-2}$); 5—$\tau = 5\cdot10^8$ years ($z = 4.0\cdot10^{-2}$); 6—$\tau = 1\cdot10^9$ years ($z = 8.5\cdot10^{-2}$); 7—$\tau = 2\cdot10^9$ years ($z = 0.19$); 8—$\tau = 3\cdot10^9$ years ($z = 0.33$); 9—$\tau = 4\cdot10^9$ years ($z = 0.51$); 10—$\tau = 5\cdot10^9$ years ($z = 0.76$); 11—$\tau = 6\cdot10^9$ years ($z = 1.17$).

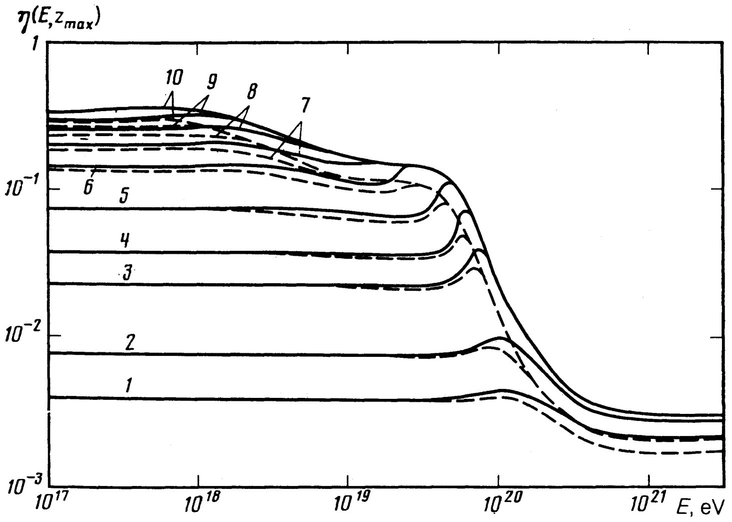

FIG. 4. Modification factor for diffuse spectra $\eta(E, \tau_{max})$, given by Eq. (38) with $\gamma = 1.1$ (solid lines) and 1.6 (dashed lines). The curves correspond to various values of τ_{max} (or z_{max}) for the outer boundary of the surface enclosing the sources: 1—$\tau_{max} = 5\cdot 10^7$ years ($z_{max} = 3.8\cdot 10^{-3}$); 2—$\tau_{max} = 1\cdot 10^8$ years ($z_{max} = 7.7\cdot 10^{-3}$); 3—$\tau_{max} = 3\cdot 10^8$ years ($z_{max} = 2.4\cdot 10^{-2}$); 4—$\tau_{max} = 5\cdot 10^8$ years ($z_{max} = 4.0\cdot 10^{-2}$); 5—$\tau_{max} = 1\cdot 10^9$ years ($z_{max} = 8.5\cdot 10^{-2}$); 6—$\tau_{max} = 2\cdot 10^9$ years ($z_{max} = 0.19$); 7—$\tau_{max} = 3\cdot 10^9$ years ($z_{max} = 0.33$); 8—$\tau_{max} = 4\cdot 10^9$ years ($z_{max} = 0.51$); 9—$\tau_{max} = 5\cdot 10^9$ years ($z_{max} = 0.76$); 10—$\tau_{max} = 6\cdot 10^9$ years ($z_{max} = 1.17$).

in Section 3. For flat production spectra the absolute number of particles subjected to slowing down and which form the hump is larger than for steep production spectra. This can be seen in Fig. 4 from comparison of the humps for $\gamma = 1.1$ and 1.6.

5. DIFFUSE SPECTRA

Let us consider a sphere of large radius filled with sources with density n and with an absorber at the center. Let particles be propagated in straight lines. We shall consider the case of cosmological evolution of the sources, i.e., we shall take into account the increase of the density of sources in the accompanying volume $n'(z)$ and of the luminosity $L_p(z)$ with increase of the red shift z: $n'(z)L_p(z) = (1+z)^m n_0' L_p(0)$.

The flux from an elementary volume of this sphere is

$$dj=n(z)\,dVF(E_g,\ z)\,dE_g/(1+z)4\pi R_0^2 f^2(r). \qquad (36)$$

Using the relations

$$R(z)dr=cdt,\ \ dt=-(^3/_2)t_0(1+z)^{-5/2}dz,\ \ R_0/R(z)=(1+z),$$
$$L_p(z)n(z)=(1+z)^{m+3}L_p(0)n_0$$

and integrating (36) out to the boundary of the sphere considered, which corresponds to a red shift z_{max}, we obtain

$$j(E)=\frac{3}{8\pi}R_0 n_0 F(E)\int_0^{z_{max}}\frac{dz}{(1+z)^{5/2}}(1+z)^m\lambda^{-(\gamma+1)}(E,z)\frac{dE_g}{dE}. \qquad (37)$$

The modification factors $\eta(E, z_{max})$ for the case of nonevolving sources ($m = 0$) are given in Fig. 4.

$$\eta(E,z_{max})=\int_0^{z_{max}}\frac{dz}{(1+z)^{5/2}}(1+z)^m\lambda^{-(\gamma+1)}(E,z)\frac{dE_g}{dE}. \qquad (38)$$

The curves in Fig. 4 are the result of summation of curves for individual sources shown in Fig. 3. The humps become less sharply expressed for two reasons: 1) humps from sources at different distances occur at different energies and, consequently, do not enhance each other in the summation; 2) the low-energy, flat portions of the spectra of the individual sources add up and the hump becomes less appreciable against this background. It can be seen from Fig. 4 that an appreciable hump in the diffuse spectrum is formed for $0.008 \leqslant z_{max} \leqslant 0.2$ (or 10^8 years $\leqslant \tau_{max} \leqslant 2\cdot 10^9$ years). For a sphere of smaller radius the hump is essentially not perceptible as the result of the too short propagation time, and for larger radii the humps from remote sources appear at low energies where the flat background from the relatively close sources is large. A dip appears only for large radii (or $\tau_{max} \sim 1 - 3\cdot 10^9$ years) for the reasons set forth in Section 4. Evolution enhances the contribution of the remote sources, and consequently for such models there is neither a hump nor a dip in the spectrum.

6. DISCUSSION

To make a special comparison of the results of our calculations with the results of Hill and Schramm,[13] we calculated the spectrum of a single source ($\gamma = 1.5$) for the same propagation-time values as in Ref. 13. For all propagation times significant discrepancies are observed; they become particularly noticeable for very large propagation times. In contrast to the results of Hill and Schramm, our calculations give a substantially earlier cutoff of the spectra (earlier by an order of magnitude for $\tau > 5\cdot 10^9$ years). We see the cause of this in the allowance for cosmological evolution of the relict radiation.

Let us trace the motion of a proton in the reverse direction: from the time of observation t to the time t_g of production in the source. The higher the particle energy at time t_g turns out to be, the smaller is the flux observed at time t. With motion of the proton backward in time, its energy increases and for sufficiently large initial energies it can fall into the region of dominance of photopion energy loss (see Fig. 1), where the rise of the energy is significantly enhanced as a result of the steep rise of the energy loss. The energy E at the time of observation for which a proton in the production epoch t_g falls in the region of photopion loss is just the energy of the black-body cutoff.

When the cosmological evolution of the relict radiation is taken into account, the energy of the cutoff decreases since, first, the energy at which curve 2 in Fig. 1 (loss to production of e^+e^- pairs) intersects curve 3 (photopion loss) decreases by a factor $(1+z)$ in the epoch with red shift z and, second, the energy loss increases by a factor $(1+z)^3$.

Let us describe this effect quantitatively. Consider a particle with $E \sim 1\cdot10^{19}$ eV at $z=0$. The increase of its energy with increase of z is described by the equation

$$\frac{dE}{dz}=\frac{E}{H_0}\frac{1}{(1+z)^{5/2}}\beta(E,z), \tag{39}$$

where $\beta(E,z)=(1+z)^3\beta_0((1+z)E)$ is the relative energy loss in epoch z. At an energy $5\cdot10^{18}\ \text{eV}\leqslant E\leqslant 5\cdot10^{19}$ eV the function $\beta_0(z)$ can approximately be considered to be a constant quantity β_0 (see Fig. 1). Then for this energy interval we obtain

$$E(z)=E_0\exp\left\{\frac{2}{3}\frac{\beta_0}{H_0}[(1+z)^{3/2}-1]\right\}. \tag{40}$$

As was explained above, if in (40) $E(z)=E_c/(1+z)$, where E_c is the point of intersection of curves 2 and 3 for $z=0$ (Fig. 1), then E_0 is the energy of the black-body cutoff E_{b-b} and we have

$$E_{b-b}=\frac{E_c}{(1+z)}\exp\left\{-\frac{2}{3}\frac{\beta_0}{H_0}[(1+z)^{3/2}-1]\right\}. \tag{41}$$

For $z\ll1$ the expansion of (41) in z gives $E_{b-b}\approx E_c\exp(-\beta_0\tau)$, where $\tau=z/H_0$ is the propagation time, while for $z\gtrsim1$ the cutoff energy E_{b-b} given by (41) decreases significantly.

In Ref. 13 the spectrum of a single source was calculated, and its most characteristic feature turned out to be a dip in the spectrum. In our work in addition to the single-source spectra we calculated also spectra of multiple sources. In the single-source spectra there are both a dip and a hump, whereas in the multiple-source spectra (diffuse spectra) the hump turns out to be a more noticeable feature. A dip in the diffuse spectra can appear as an artificial incidental effect not related to the production of e^+e^- pairs. For example, in models in which for $E\lesssim1\cdot10^{19}$ eV the observed particles have a Galactic origin and for $E\gtrsim1\cdot10^{19}$ eV they have a metagalactic origin, a dip associated with the joining of these two spectra appears.

7. CONCLUSIONS

The interaction of ultrahigh-energy protons with the 2.7 K relict radiation leaves in the shape of the differential energy spectrum its autograph in the form of the following features, which follow each other with increase of the energy: a hump from generation of e^+e^- pairs, a dip from the same process, a photopion hump, and the black-body cutoff of the spectrum. The two humps are produced by protons of higher energies as the result of their slowing down. As a consequence of the similarity of the energy-loss curves for protons and nuclei, the differential spectra of the latter have the same features.

These features appear in complete form in the spectrum of a single source located at a large distance from the observer (Fig. 3). For a nearby single source (propagation time $\tau\lesssim3\cdot10^8$ years) only the photopion hump with a subsequent black-body cutoff can be observed. The hump appears more distinctly for flat production spectra, since in this case a larger number of particles are slowed down and pile up in the form of a hump.

In the multiple-source spectrum (diffuse spectra) the hump and dip from e^+e^- pairs turn out to be practically unobservable; the photopion hump also is more weakly expressed than for the spectrum of a single remote source. The reason is that the humps from remote sources at different distances occur at different energies and consequently are added incoherently, whereas the low-energy parts of the spectra, in the background of which the hump must be distinguished, enhance each other substantially in the addition. The photopion hump is an observable feature of the diffuse spectrum if the maximum propagation time (from the outer boundary of the group of sources) $\tau_{\max}$ lies in the range from 10^8 to 10^9 years (see the curves in Fig. 4 for times $(1\text{–}5)\cdot10^8$ years). For a universe uniformly filled with sources, the spectral features discussed are very small and disappear completely in the case of cosmological evolution of the sources.

If the "flat" component of the spectrum, which is observed[15] for $E\geqslant10^{19}$ eV, is generated by sources in a local supercluster of galaxies, then our calculation predicts the existence of an appreciable hump in the spectrum (see the curves in Fig. 4 for times $(1\text{–}5)\cdot10^8$ years). Here in the spectrum there is also a dip, which has, however, a trivial origin: it is a natural hollow between the hump on the high-energy side and the steep component of the spectrum on the low-energy side, which most likely has a galactic origin.

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