On the instability of the uniform precession of magnetization in ^{3}He-A

É. B. Sonin

A. F. Ioffe Physicotechnical Institute, USSR Academy of Sciences, Leningrad
(Submitted 25 March 1987)
Zh. Eksp. Teor, Fiz. **93**, 904–906 (September 1987)

The nature of the instability of uniform magnetization precession, recently discovered experimentally and theoretically in the A-phase of ^{3}He, is discussed. The threshold for the onset of this instability is determined.

It is now established both theoretically[1] and experimentally,[2,3] that the uniform precession of the spins in ^{3}He-A is unstable with respect to excitation of spatially inhomogeneous spin oscillations. Some time ago, similar parametric instabilities were studied for the case of ferro- and antiferromagnetic materials.[4,5] The analysis of the instability of the uniform precession in ^{3}He-A carried out in the present communication has the purpose of enhancing the understanding of the nature of this instability and of following the indicated analogy, as well as of determining the threshold for the onset of the instability.

To obtain these goals it is convenient to restrict one's attention to the case of weak nonlinearity (i.e., of a small angle between the precessing spin and the magnetic field). The spin dynamics equations from which we start have the form: ($[\mathbf{VW}] = \mathbf{V}\times\mathbf{W}$ denotes vector product, as usual in Russian literature—Translator's note)

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma\left[\mathbf{M}\frac{\delta F}{\delta \mathbf{M}}\right] - \gamma\left[\mathbf{d}\frac{\delta F}{\delta \mathbf{d}}\right] + D\Delta\mathbf{M},$$
$$\frac{\partial \mathbf{d}}{\partial t} = -\gamma\left[\mathbf{d}\frac{\delta F}{\delta \mathbf{M}}\right] + \varkappa\left[\mathbf{d}\left[\mathbf{d}\frac{\delta F}{\delta \mathbf{d}}\right]\right]. \tag{1}$$

Where $\mathbf{M}$ is the magnetization, γ is the gyromagnetic ratio, D is the spin diffusion coefficient, $\varkappa$ is the Leggett-Takagi relaxation parameter, $\delta F/\delta\mathbf{M}$ and $\delta F/\delta\mathbf{d}$ are the functional derivatives of the free energy density

$$F = M^2/2\chi - \mathbf{MH} - {}^1/_2 V_d(\mathbf{dl})^2 + {}^1/_2 A(\nabla \mathbf{d})^2, \tag{2}$$

which includes the uniform magnetization energy, the Zeeman energy, the dipole energy, and the gradient energy,[1)] χ is the susceptibility, and $\mathbf{H}$ is the constant magnetic field. Comparing the A-phase with an antiferromagnet one must take into account the fact that the orbital vector $\mathbf{l}$, which remains immobile in spin dynamics experiments, corresponds to a field with uniaxial anisotropy, whereas the spin vector $\mathbf{d}$ corresponds to the antiferromagnetic vector normalized to unity.

Assume the field $\mathbf{H}$ to be oriented along the z axis, and the vector $\mathbf{l}$ along the x axis. For the sequel it suffices to write only three equations out of those contained in the vector equations (1):

$$\frac{\partial M_x}{\partial t} = \omega_L M_y + \gamma A(d_y\Delta d_z - d_z\Delta d_x) + D\Delta M_x, \tag{3}$$

$$\frac{\partial M_y}{\partial t} = -\omega_L M_x + \frac{\chi}{\gamma}\Omega^2 d_z d_x + \gamma A(d_z\Delta d_x - d_x\Delta d_z) + D\Delta M_y, \tag{4}$$

$$\frac{\partial d_z}{\partial t} = \frac{\gamma}{\chi}(M_x d_y - M_y d_x) - \frac{\chi}{\gamma^2}\varkappa\Omega^2 d_x^2 d_z, \tag{5}$$

where $\omega_L = \gamma H$ is the Larmor frequency and $\Omega = \gamma(V_d/\chi)^{1/2}$ is the longitudinal NMR frequency. In the case of a small precession angle (linear transverse NMR) M_x, the quantities M_y and d_z are small of first order, whereas d_y is small of second order. Therefore $d_x \approx 1 - d_z^2/2$. The nonlinear correction to d_z in the dipole energy (to which the term $\sim\Omega^2$ in the right-hand side of Eq. (4) is related) is decisive for the appearance of the instability. In antiferromagnetic materials a similar role is played by the nonlinearity in the energy of the uniaxial anisotropy.[5] All other nonlinearities may be neglected, without loss of generality. After this, making use of the strong field condition $\omega_L \gg \Omega$, as well as of the smallness of the relaxation terms, it is easy to eliminate M_y and d_z from the equations (3)–(5) ($d_y = 0$ as a second-order quantity) and thus derive a wave equation for M_x:

$$\frac{\partial^2 M_x}{\partial t^2} + \omega_\perp^2 M_x - \frac{\Omega^2}{2}\frac{M_x^3}{M_p^2} - c^2\Delta M_x - 2D\Delta\frac{\partial M_x}{\partial t} + 2\Gamma\frac{\partial M_x}{\partial t} = 0, \tag{6}$$

where $\omega_\perp = (\omega_L^2 + \Omega^2)^{1/2}$ and $\Gamma = (\chi\varkappa/2\gamma^2)\Omega^4/\omega_L^2$ are respectively the frequency and the reciprocal of the relaxation time of linear homogeneous transverse NMR. $c = \gamma(A/\chi)^{1/2}$ is the speed of the spin waves, $M_p = \chi H$ is the equilibrium magnetization.

The solution of Eq. (6) corresponding to uniform precession should be of the form

$$M_x = M_\perp(t)\sin\varphi_0(t), \tag{7}$$

where $\varphi_0(t)$ is a rapidly varying function of t and $M_\perp(t)$ is a slowly varying function. Substituting (7) into (6) and neglecting the higher harmonics of $\sin 3\varphi_0$ in the nonlinear term, and then setting the coefficients of $\sin\varphi_0$ and $\cos\varphi_0$ equal to zero, we obtain the precession frequency and its decay law

$$\omega_0^2 = \left(\frac{\partial\varphi_0}{\partial t}\right)^2 = \omega_\perp^2 - \frac{3}{8}\Omega^2\frac{M_\perp^2}{M_p^2}, \quad M_\perp(t) = M_0 e^{-\Gamma t}. \tag{8}$$

In Eq. (6) the sign in front of the nonlinear term and is the same as the sign of the term proportional to ΔM_x (Lighthill's condition); this leads to the appearance of a modulation instability.[6] In order to make it manifest it is necessary to introduce in the amplitude $M_\perp$ and in the phase φ_0 of the uniform precession small additions that vary slowly over space, to derive linearized equations for these quanti-

 0038-5646/87/090509-02$04.00

ties, and to determine when these additions increase with time. This was the approach taken by Fomin,[1] who operated with equations written in terms of the Euler angles. However, one can also carry out the analysis in a manner more traditional for parametric instabilities in magnetism. We consider small spatially inhomogeneous oscillations on the background of uniform precession. For this purpose we set

$$M_x = M_\perp(t)\sin\varphi_0(t) + M_k(t)\cos kx. \tag{9}$$

Substituting Eq. (9) into Eq. (6) we obtain a linear equation from M_k:

$$\frac{\partial^2 M_k}{\partial t^2} + \left[\omega_k^2 + \frac{3}{4}\Omega^2\frac{M_\perp^2}{M_p^2}\cos 2\omega_0 t\right]M_k + 2(\Gamma + Dk^2)\frac{\partial M_k}{\partial t} = 0, \tag{10}$$

where

$$\omega_k^2 = \omega_\perp^2 + c^2k^2 - \frac{3}{4}\Omega^2\frac{M_\perp^2}{M_p^2}.$$

We note that in the limit $k\to 0$ the frequency ω_k does not coincide with the uniform precession frequency ω_0, since no matter how small k is, the equation (10) describes a weakly nonuniform mode which differs from the uniform precession mode excited in the nonlinear regime. We obtain by means of the standard method for the growth rate p of the nonuniform mode

$$p^2 = \frac{1}{4}\left[\left(\frac{3}{8}\frac{\Omega^2\beta^2}{\omega_L}\right)^2 - (\omega_k - \omega_0)^2\right] - (\Gamma + Dk^2)^2, \tag{11}$$

where $\beta \approx M_\perp/M_p$ is the small precession angle. After substitution of ω_0 and ω_k in the expression (11) makes the latter different from the one obtained by Fomin (in the limit of small β) only by the presences of the reciprocal of the uniform relaxation time Γ. The consideration of Γ is necessary for the determination of the instability threshold, since the spin diffusion does not guarantee stability for small k and yields small corrections to the magnitude of the threshold if $D\omega_L/c^2 \ll 1$, which is well satisfied in experiments. The threshold value of β_c is determined by the condition $p = 0$ for a k corresponding to the maximum of p^2, when the resonance condition $\omega_0 = \omega_k$ is satisfied:

$$\beta_c = 4(\Gamma\omega_L)^{1/2}/3^{1/2}\Omega. \tag{12}$$

The instability considered here is analogous to the Suhl second-order instability, when two uniform precession magnons are converted into a pair of magnons with wave vectors **k** and −**k**. In antiferromagnets such instabilities have been considered for the case of the absence of an external field (Ref. 5), when for the threshold value of the deflection angle of the sublattice magnetization one obtains in place of Eq. (11) a quantity $\sim(\Gamma/\Omega)^{1/2}$. In an antiferromagnet it is considerably more difficult than in ^{3}He-A to reach the region of strong magnetic fields, since, as a rule, fields of the order of 100 kOe are necessary, field strengths which exceed H_{EA} (see Ref. 7) which is the analog of the dipole field $H_d = \Omega/\gamma \sim 30$ Oe in ^{3}He-A.

Substituting the values of Γ, ω_L and Ω from Ref. 3 into Eq. (12) we obtain $\beta_c = 10°$. Experimentally the instability became noticeable for an angle of 40°. But for the experimental determination of the instability threshold it is necessary that the observation time should be considerably larger than the decay time $1/\Gamma$, which is hard to realize in a pulsed regime, where the experiments of Ref. 2, 3 have been carried out. Therefore, in order to determine the instability threshold it is necessary to do experiments in a steady state regime.[2)]

As the modulated instability develops, energy is pumped from the uniform mode into the nonuniform mode. When a sufficient degree of such pumping is attained one can observe a phenomenon of "reversibility", when pumping back of energy into the uniform mode begins (Ref. 6). This may be the cause of the nonmonotonic decrease of the induction signal for the large β observed in the experiment (see Fig. 5 of Ref. 3).

In conclusion the author expresses his gratitude to Yu. M. Bun'kov, E. I. Golovenchin, V. V. Dmitriev, Yu. M. Mukharskiĭ, V. A. Sanina, and I. A. Fomin for quite useful discussions of the questions touched upon above.

[1)]In general A is a tensor, but for simplicity we restrict our attention to the case when **d** varies only along one direction.

[2)]This circumstance was pointed out to the author by Yu. M. Bun'kov.

[1]I. A. Fomin, Zh. Eksp. Teor. Fiz. 78, 2392 (1984) [Sov. Phys. JETP 51, 1203 (1984)].

[2]A. S. Borovik-Romanov, Yu. M. Bun'kov, V. V. Dmitriev, and Yu. M. Mukharskiĭ, ZhETF Pis. Red. **39**, 390 (1984) [JETP Lett. **39**, 469 (1984)].

[3]Yu. M. Bun'kov, V. V. Dmitriev, and Yu. M. Mukharskiĭ, Zh. Eksp. Teor. Fiz. **88**, 1218 (1985) [Sov. Phys. JETP **61**, 228 (1985)].

[4]H. Suhl, J. Phys. Chem. Sol. **1**, 209 (1957).

[5]A. J. Heeger and P. Pincus, Phys. Rev. Lett. **10**, 53 (1963). A. J. Heeger, Phys. Rev. **131**, 608 (1963).

[6]M. I. Rabinovich and D. I. Trubetskov, Vvedenie v teoriyu kolebanii i voln (Introduction to the theory of oscillations and waves), Nauka, Moscow 1984, Sec. 20.2.

[7]E. A. Turov, Fizicheskie svoĭstva magnitouporyado-chennykh kristallov (Physical properties of magnetically ordered crystals). Izd-vo AN SSSR, 1963, p. 77

Translated by Meinhard E. Mayer