

Bloch oscillations of electrons and instability of space-charge waves in superconductor superlattices

A. A. Ignatov and V. I. Shashkin

Institute of Applied Physics, USSR Academy of Sciences

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A kinetic approach is used to investigate the instability of space-charge waves in semiconductor superlattices in a strong electric field that induces Bloch oscillations in the minibands. Solution of the Boltzmann transport equation with a Bhatnagar-Gross-Krook collision integral yielded for the dielectric constant an expression that makes it possible to take into account, within a unified approach, the influence exerted on the high-frequency response of superlattices by resonance effects and by effects due to carrier drift and diffusion. It is shown that when the strong spatial dispersion is taken into account the spectrum of the space-charge waves are characterized in a wide range of superlattice parameters by a set of stable resonant Bloch superlattices and by an unstable drift-relaxation mode.

1. INTRODUCTION

When an electric field is applied to an ideal crystal having a sufficiently narrow resolved energy band, the carrier motion is localized and the particles oscillate at the Bloch frequency $\Omega = eE_0d/\hbar$, where E_0 is the electric field strength and d is the crystal period. The carrier dynamics in a band of finite width was investigated in sufficient detail in connection with the theory of interband optical transitions.¹ Various approaches to the description of the particle motion under these conditions, starting with Bloch's trail-blazing work, were critically analyzed in recent papers,^{2,3} in which it was shown that if the lower-lying band gap is wide enough the probability of electron interband tunneling is negligibly small for a sufficiently large number of Bloch-oscillation periods. The presence of inelastic collisions is known to result in carrier drift, in dissipation of the applied-field energy and, by virtue of the limited electron energy in the band, in negative differential conductivity (NDC) at $E_0 > \hbar\nu/ed$,^{1,4-7} where ν is the characteristic relaxation frequency of the distribution function.

In a recently realized new class of multilayer semiconductor structure [superlattices⁸ (SL)] the artificially produced periodicity makes it possible to vary in a wide range the parameters of the energy spectrum and to realize conditions for NDC on account of bounded motion of the electrons in an allowed subband (miniband) of the SL.⁹ It must be noted here, however, that notwithstanding the large number of investigations of the hf properties of SL,¹⁰ there is still no clear idea of the spatial and temporal scales over which the space-charge waves (SCW) in SL become subject to the instability that governs the electric properties of the diode superlattice structures usually investigated in experiment.^{11,12} One of the approaches to the solution of this question is connected with consideration of the instability of the low-frequency drift-relaxation wave of the SCW spectrum,¹³⁻¹⁵ an instability that can be described in terms of quasihydrodynamic equations that take into account the NDC and the drift and diffusion of carriers localized in a narrow LS miniband. A consequence of the developed instability of the W connected with this branch of the spectrum is generation of particles that move with drift velocity, or of

strong electric-field domains localized near inhomogeneities of the structure (such as its boundaries)—the Gunn effect.¹³⁻¹⁵

At the same time, the oscillatory character of the carrier motion in the SL narrow miniband makes possible instability development at much higher frequencies. As noted in Refs. 8 and 16, by interacting with the high-frequency field, carriers oscillating in a miniband can produce negative absorption at a frequency equal to or a multiple of the Bloch frequency. It is important to note here in this situation there is produced in the system a characteristic high-frequency-response spatial-dispersion scale connected with the swing $x_0 \approx \Delta/eE_0$ of the Bloch oscillations of the electrons in the electric field (Δ is the miniband width). The problem calls therefore for a kinetic treatment.

An approach to the calculation of the high-frequency conductivity (the dielectric constant) of an SL in a strong electric field with allowance for spatial-dispersion effects, based on an analogy with the calculation of the conductivity of a magnetoactive plasma, was formulated in Ref. 17. However, the model used there for the collision integral $St = -\nu(f - f_0)$, as will be shown below, makes it impossible, by virtue of the nonconservation of the number of scattered particles ($\int St \, dp \neq 0$), to take into account the influence of the carrier drift and diffusion effects on the SCW spectrum. This, naturally, leaves out of consideration the low-frequency drift-relaxation branch of the spectrum. At the same time, the quasihydrodynamic approach used in Refs. 14 and 15 to describe the instability of this branch does not take into account the system's resonant properties due to the Bloch oscillations of the electrons.

We have investigated in the present study the spectrum of the space-charge waves in superconductor superlattices in a strong electric field, by solving the Boltzmann transport equation with a Bhatnagar-Gross-Krook (BGK) collision integral.¹⁸ This approach permits an adequate account of the influence exerted on the SCW spectrum both by resonance effects due to Bloch oscillations of the electrons and by effects connected with carrier drift and diffusion under conditions of strong spatial dispersion. It becomes possible as a result to determine the conditions under which either low-frequency (Gunn) instability or oscillations near the Bloch

frequencies are produced in the system. This is the main purpose of the present paper.

2. INITIAL EQUATIONS

We describe the energy spectrum of the electrons in the lower miniband, which is filled in accordance with the Boltzmann statistics, in the tight-binding approximation⁸:

$$\varepsilon = \frac{\Delta}{2} \left(1 - \cos \frac{p_x d}{\hbar} \right) + \frac{p_{\perp}^2}{2m}. \quad (1)$$

Here Δ is the miniband width, p_x the quasimomentum component along the SL axis, and p_{\perp} and m the carrier transverse quasimomentum and mass.

To describe the response of the electrons to external fields, we use the Boltzmann transport equation with a BGK collision integral, which permits adequate allowance for the particle-number conservation law $\int \text{St } \mathbf{dp} = 0$ for scattering in an inhomogeneous field.¹⁸ If the external fields are oriented along the SL axis (vertical transport in the diode configuration), integration of the Boltzmann equation with a BGK collision integral with respect to p_{\perp} makes the problem one-dimensional:

$$\frac{\partial f}{\partial t} + v(p) \frac{\partial f}{\partial x} + eE(x) \frac{\partial f}{\partial p} = -v \left(f - \frac{n(x)}{n_0} f_0 \right), \quad (2)$$

where

$$v(p) = \partial \varepsilon / \partial p = v_0 \sin(pd/\hbar), \quad v_0 = \Delta d / 2\hbar,$$

v_0 is the characteristic particle velocity in the band (we omit hereafter the subscript z , since the problem is now one dimensional),

$$f_0 = \frac{n_0 d}{2\pi \hbar I_0} \exp \left(\frac{\Delta}{2\kappa T} \cos \frac{pd}{\hbar} \right)$$

is the equilibrium distribution function normalized to n_0 , $n(x)$ is the electron density in the x plane, $I_n = I_n(\Delta/2\kappa T)$ is a modified Bessel function of argument $\Delta/2\kappa T$, T is the lattice temperature, ν is the characteristic relaxation frequency of the distribution function, and n_0 is the equilibrium carrier density.

We supplement the transport equation (2) with the Poisson equation and with the condition that the total current be continuous. This formulates completely the electro-dynamics of the problem in the one-dimensional situation:

$$\frac{\partial E}{\partial x} = \frac{4\pi e}{\varepsilon} (n(x) - n_0), \quad (3)$$

$$j^{tot} = j(x) + \frac{\varepsilon}{4\pi} \frac{\partial E}{\partial t}, \quad (4)$$

where

$$n(x) = \int f \mathbf{dp}, \quad j = e \int v(p) f(p) dp,$$

ε is the lattice dielectric constant and the integration is within the limits of the 3D Brillouin zone.

Equation (2) is valid within the framework of the quasiclassical description of the particle dynamics in the miniband^{2,3}:

$$eEd, \quad \hbar\omega \ll \Delta_0, \quad (5)$$

$$eEd, \quad \hbar\nu \ll \Delta. \quad (6)$$

Here Δ_0 is the width of the forbidden miniband. By virtue of (5), we neglect the Zener tunneling and the optical transitions in the upper minibands. At the same time, if (6) is satisfied, the swing $x_0 = \Delta/eE_0$ of the Bloch oscillations in the electric field exceeds the period d of the structure; this is equivalent in above-threshold fields $E_0 \gtrsim \hbar\nu/ed$ to a sufficiently large electron mean free path $d \lesssim 2v_0/\nu$.

The choice of the collision integral in (2) in the BGK form is a rather crude approximation of carrier-scattering processes. Nonetheless, in the case of homogeneous high-frequency fields the nonlinear SL response, calculated on the basis of (2), is well enough confirmed by results of numerical simulation by the Monte Carlo method when account is taken of scattering by optical phonons and impurities.⁹

3. DIELECTRIC CONSTANT

Following the usual approach to SCW investigations,¹³ we set apart in the structure the volume $0 < x < L$, in which a small inhomogeneous perturbation

$$E(x, t) = E_0 + E_1(x) e^{-i\omega t} \quad (7)$$

is superimposed on the homogeneous constant field E_0 . The high-frequency field $E_1(x)$, as well as the perturbations of all the remaining quantities that enter in the problem, permits in the segment $0 < x < L$ a representation in the form of Fourier series:

$$\Phi(x) = \sum_k \Phi(k) e^{ikx}, \quad (8)$$

where $k = 2\pi m/L$ and m are integers. Linearization of the Boltzmann transport equation leads to the following relation for the Fourier component of the perturbations of the distribution function, of the field, and of the particle density:

$$\frac{\partial f_1}{\partial \varphi} + i(\alpha + \beta \sin \varphi) f_1 = -\frac{\Omega_1}{\Omega} \frac{\partial F_0}{\partial \varphi} + \frac{\nu n_1}{\Omega n_0} f_0, \quad (9)$$

where $\Omega_1 = eE_1 d / \hbar$ is the amplitude of the high-frequency field, $\varphi = pd/\hbar$ is the dimensionless quasimomentum, $\alpha = -(\omega + i\nu)/\Omega$, $\beta = kv_0/\Omega$ is the normalized wave number of the perturbations, and F_0 is the electron distribution in the constant homogeneous electric field E_0 :

$$F_0 = \frac{n_0 d}{2\pi \hbar I_0} \sum_{m=-\infty}^{\infty} I_m \frac{\nu}{\nu + im\Omega} e^{im\varphi}. \quad (10)$$

We obtain the connection, in the Fourier representation, between the perturbations of the electric field, of the current and of the particle density from Eqs. (3) and (4):

$$ik\Omega_1 = \frac{4\pi e^2 d}{\varepsilon \hbar} n_1, \quad (11)$$

$$\delta_{k,0} j_1^{tot} = j_1(k) - \frac{i\omega \varepsilon}{4\pi} E_1(k), \quad (12)$$

where $\delta_{k,0}$ is the Kronecker delta.

From the formal viewpoint, the method of finding the solution of (9) is perfectly similar to the calculation of the dielectric constant of a magnetoactive plasma,^{17,20} with the periodicity of the coefficients in (9) determined by the periodic dependence of the particle energy on the quasimomentum (1). Solving (9) by the method given in Ref. 20, we obtain for the high-frequency conductivity that relates the Fourier components of the current and the field

$$j_1(k) = \sigma(\omega, k) E_1(k), \quad (13)$$

$$\sigma(\omega, k) = -\frac{\varepsilon v}{4\pi} \sum_{m,s=-\infty}^{\infty} \frac{i^{-m} I_m}{I_0} \left[1 - \frac{\omega_p^2 m}{\beta \Omega (\nu + im\Omega)} \right] \times \frac{s J_s(\beta) J_{s+m}(\beta)}{(\alpha - s)}, \quad (14)$$

where $\omega_p = (4\pi e^2 n_0 v_0 d / \varepsilon \hbar)^{1/2}$, and the dielectric constant of the SL is determined, according to (12), by the usual relation

$$\varepsilon(\omega, k) = \varepsilon + i \frac{4\pi \sigma(\omega, k)}{\omega}. \quad (15)$$

It is easy to verify that the expression obtained for the dielectric constant has a symmetry property that follows from the causality principle (the Kramers-Kronig relation)²⁰:

$$\varepsilon^*(-\omega, -k, E) = \varepsilon(\omega, k, E). \quad (16)$$

In addition, since the preferred direction in the problem is determined by the electric field, we have

$$\varepsilon(\omega, k, E) = \varepsilon(\omega, -k, -E). \quad (17)$$

Expression (15) describes the SL response to an external inhomogeneous hf field with account taken of the strong spatial and temporal dispersions. The characteristic scale of the spatial dispersion $\beta \approx 1$, $k \approx x_0^{-1}$ is the swing $x_0 \approx \Delta / eE_0$, and the scale of the temporal dispersion is the Bloch frequency. In the limit of homogeneous fields $\beta \rightarrow 0$ expression (14) goes over into the result of Ref. 21. In this case the resonance takes place only at the fundamental frequency $\omega \approx \Omega$. Allowance for spatial dispersion leads, according to (14), to a resonant response at the harmonics of the Bloch frequency $\omega = n\Omega$.

We note here that the model of the collision integral $St = -\nu(f - f_0)$ used in Ref. 17 to calculate the dielectric constant of an SL in a strong electric field makes it possible to take into account the influence of the spatial dispersion only on the conductivity component that is proportional to the carrier density [second term in the square brackets of (14)], which has, naturally, been defined in the limit of weakly inhomogeneous high-frequency fields at $\omega_p^2 / \Omega^2 \gg \beta$, $\Omega > \nu$. At the same time, the response of the system to an external field contains collision-induced terms that lead to drift and diffusion at arbitrary powers of β . They stem from the second term in the right-hand side of (9), which takes into account the conservation of the scattered particles. To avoid misunderstandings, we note that the contribution to the convective current of the electrons $j = e \int v f d p$, associated with this term, is sometimes set apart as a component of the "diffusion vector" (Ref. 22). In our case, however, it is convenient to use the Poisson equation (11) and to include this contribution in the high-frequency conductivity (dielectric constant) of the system. We make use thereby of the terminology of Ref. 18. Thus, relation (14) accounts for the influence of both plasma and drift-diffusion effects on the linear response of SL within the framework of a single approach.

4. DRIFT-RELAXATION AND RESONANCE (BLOCH) MODES

It is known that the temporal evolution of the initial perturbation in an active medium is determined by the solu-

tion of the dispersion equation $\varepsilon(\omega, k) = 0$ for real values of the wave numbers k . Examination of expressions (14) and (15) shows readily that if the complex frequency $\omega(k)$ is a solution of the dispersion equation, then $-\omega^*(-k)$ is also its solution, i.e., we can confine ourselves in the analysis of the SCW spectrum to the region $\text{Re} \omega(k) > 0$, $-\infty < k < \infty$, and construct the solution in the region $\text{Re} \omega(k) < 0$ by using the indicated symmetry property. We note right away that since the system has a preferred direction connected with the electric field, the roots of the dispersion equation must satisfy the symmetry relations for equilibrium isotropic media. It is obvious only that $\omega(k, E) = \omega(-k, -E)$.

Neglecting spatial dispersion effects, the determination of the system natural-oscillation spectrum satisfying the relation $\varepsilon + i \cdot 4\pi \sigma(\omega, k \rightarrow 0) / \omega = 0$ reduces to solving a third-order equation, with real coefficients, for the quantity $\delta = -i\omega/\nu$, where ω is the complex frequency:

$$\delta^3 + 2\delta^2 + (Z^2 + \gamma + 1)\delta - \gamma(Z^2 - 1) = 0. \quad (18)$$

Here

$$Z = edE_0 / \hbar \nu, \quad \gamma = \omega_0^2 / (\Omega^2 + \nu^2), \quad \omega_0^2 = \omega_p^2 I_1 / I_0,$$

where ω_0^2 is the square of the carrier plasma frequency. The solution of (18) takes in an approximation with low plasma frequencies $\gamma \ll \max\{|1 - Z^2|^{-1/2}, Z\}$ the form

$$\omega_1 = i \frac{\omega_0^2 (Z^2 - 1)}{\nu [(Z^2 + 1)^2 + \omega_0^2 / \nu^2]}, \quad (19)$$

$$\omega_{2,3} = \pm (\Omega^2 + \gamma \nu^2)^{1/2} - i\nu. \quad (20)$$

Relation (19) describes the relaxational (aperiodic) build-up of the perturbation in the above-threshold field $Z > 1$. It will be made clear below that at $k \neq 0$ this solution corresponds to the drift-relaxation branch of the spectrum with a growth rate and a phase velocity that depend on the wave vector. Equation (20) yields damped oscillations of the electric field at the hybrid plasma-Bloch frequency. In the limit $\gamma \rightarrow 0$ these oscillations correspond to the Bloch resonant branch of the spectrum. It is important to note here that according to the Routh-Hurwitz criterion, there are no growing solutions with $\text{Im} \omega > 0$ at arbitrary values of the plasma frequencies (of the parameter γ) in below-threshold fields ($Z < 1$), and that at $Z > 1$ there is a single root with $\text{Im} \omega > 0$. This proves at the same time that at $k = 0$ only the relaxation mode of the spectrum is unstable, and the oscillations near the Bloch frequency are damped.

Allowance for the spatial dispersion leads to the onset of propagating waves near the Bloch frequency and its harmonics, in analogy with the case of cyclotron waves in a magnetized plasma.²⁰ In the limit $\Omega \gg \nu$, $\Omega \gg \omega_p$, when the plasma resonance has high Q , the double series in (14) can be summed with the aid of the Bessel-function addition theorem,²³ while the SCW spectrum near a resonance of multiplicity l can be represented in the form

$$\omega = l\Omega - i\nu + i\nu J_l(\beta) \left[\frac{J_l(\beta + iy)}{I_0(y)} + i \frac{\omega_p^2}{\Omega^2 \beta} \left(\frac{J_l(\beta + iy)}{I_0(y)} - J_l(\beta) \right) \right], \quad (21)$$

where $y = \Delta / 2\kappa T$, $l = 0, 1, 2, 3, \dots$. The solution with $\text{Re} \omega < 0$ ($l < 0$) is obtained in accordance with the symmetry property mentioned above. At $l = 0$ Eq. (21) describes the drift-relaxation branch of the spectrum. Indeed, in the

limit $\beta \ll 1$ we obtain, on the basis of (21), an ordinary dispersion relation in the form^{13,14}

$$\omega = kv_a - i(\omega_m + k^2 D), \quad (22)$$

where the coefficients

$$\omega_m = -\omega_p^2 I_1 / \nu I_0 Z^2, \quad (23)$$

$$v_a = v_0 I_1 / I_0 Z, \quad (24)$$

$$D = \frac{v_0^2}{4\nu} \left(2 + \frac{I_2}{I_0} \right) \frac{1}{Z^2} \quad (25)$$

determine respectively the differential Maxwellian frequency, the drift velocity, and the diffusion coefficient in a strong electric field $Z \gg 1$ and at relatively low carrier density $\omega_p \ll \Omega$.

To analyze the SCW spectrum in a wide range of values of the parameter ω_p/ν and wave numbers β we use a numerical solution of the dispersion equation by the iteration method. The calculation results for a Bloch resonance with sufficiently high Q , $Z = \Omega/\nu = 4$, $\Delta/2\kappa T = 1$, at different values of ω_p/ν , are shown in Fig. 1. The frequency region $\text{Re } \omega$ contains the drift-relaxation mode and two resonant ($l = 1, 2$) modes of the spectrum. As seen from the figure, the conditions

$$\text{Re } \omega(k) = -\text{Re } \omega(-k), \quad \text{Im } \omega(k) = \text{Im } \omega(-k),$$

are met for the drift-relaxation branch, obviously as a consequence of the general symmetry property of the solutions. With increase of the plasma frequencies, in accordance with (19), the growth rate of the instability of the drift-relaxation mode increases, whereas the resonant mode ($l = 1$) is stable all the way to values $\omega_p/\nu \approx 20$ corresponding to quite high Q of the plasma resonance. At $k = 0$ the oscillation frequency of the resonant branch follows the analytic estimate (20). At high values of ω_p/ν the instability of the resonant branch sets in at $\beta \approx 2.5$ and is of geometric origin. In this situation,

the SCW length is comparable with the swing of the electron Bloch oscillations. Numerical calculations show that the instability region appears near Ω when the rather stringent conditions $\omega_p/\Omega \gtrsim 5$, $Z = \Omega/\nu \gtrsim 3$, and $\Delta/2\kappa T \gtrsim 1$ are simultaneously met. It is important also to note that even under these conditions the growth rate of the resonant branch is quite small compared with that of the drift-relaxation branch. With further increase of the parameter ω_p/ν the hybrid frequency of the resonant branch can exceed the second harmonic of the Bloch frequency, leading to a restructuring of the SCW spectrum. We ignore this case, for owing to the high collision frequencies and the large effective mass of the carriers in the narrow SL minibands it is difficult to obtain a plasma resonance with high- Q $\omega_p/\nu \gtrsim 20$.

For typical values of the SL structure parameters $\omega_p/\nu \approx 1.76$, $\Delta/2\kappa T \approx 1.74$, $Z = 1.5$, corresponding approximately to the GaAs-Al_{0.25}Ga_{0.75}As samples with period $d = 90 \text{ \AA}$ investigated in Ref. 12 at $T \approx 100 \text{ K}$, $\Delta \approx 30 \text{ meV}$, $\epsilon = 13$, $n_0 \approx 2.2 \cdot 10^{16} \text{ cm}^{-3}$, $v_0 \approx 2 \cdot 10^7 \text{ cm/s}$, and $\nu \approx 5 \cdot 10^{12} \text{ s}^{-1}$; the numerically calculated SCW spectrum is shown in Fig. 2. It can be seen from this figure that the resonant branch is strongly damped, whereas the drift-relaxation branch is unstable in a wide range of wave numbers. In the entire wave-number region corresponding to the SCW instability, the spectrum of the drift-relaxation branch is well described by a second-order polynomial of form (2), so that the type of instability can be determined by using the criterion $|\omega_m| \geq v_d^2/4D$, formulated for this case and describing the transition from convective to absolute instability.^{13,24} In the normalized variables shown in Figs. 1 and 2, the absolute-instability criterion takes the form

$$|\tilde{\omega}_m| \geq \frac{\beta_0}{2} \frac{d}{d\beta} (\text{Re } \tilde{\omega}), \quad (26)$$

where $\tilde{\omega} = \omega/\nu$, $\tilde{\omega}_m$ is the growth rate at $\beta = 0$, $d(\text{Re } \tilde{\omega})/d\beta$ is the effective drift velocity, and β_0 is the limiting wave-

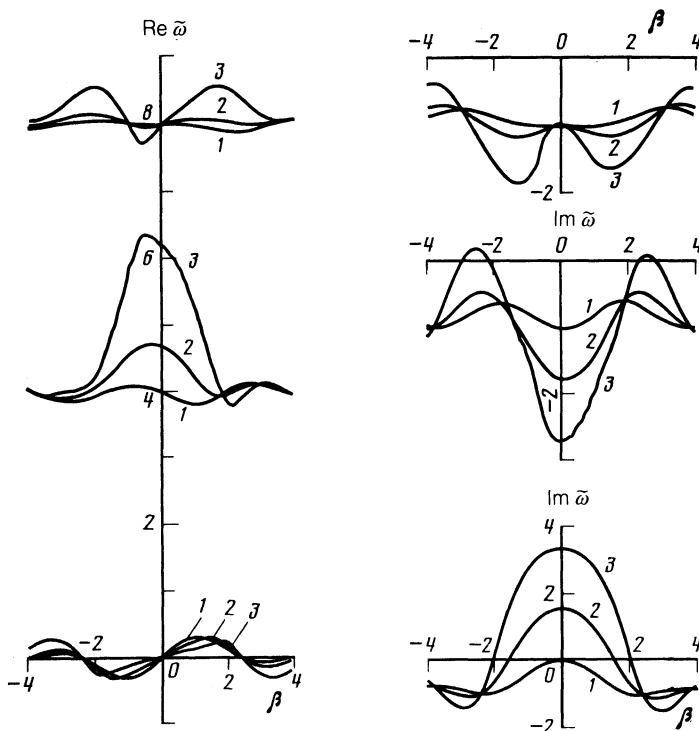


FIG. 1. Spectrum of space-charge waves in superlattices in strong electric field at various carrier densities. The region $\text{Re } \tilde{\omega}$ ($\tilde{\omega} = \omega/\nu$) includes a drift-relaxation and two resonant branches at $\Delta/2\kappa T = 1$, $Z = 4$; Curves 1— $\omega_p/\nu = 1$, 2— $\omega_p/\nu = 10$, 3— $\omega_p/\nu = 20$.

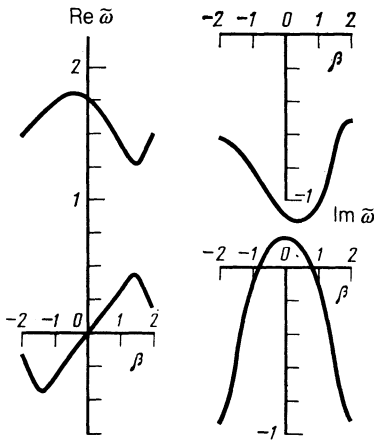


FIG. 2. Space-charge wave spectrum in low- Q plasma resonance; $\Delta/2\kappa T \approx 1.74$, $Z = 1.5$, $\omega_p/v \approx 1.77$.

vector value at which the growth rate vanishes. As seen from Fig. 2, according to (26) the instability is absolute in this case.

As shown in Ref. 24, in systems with NDC, drift, and carrier diffusion, the criterion (26) determines significantly the consequence of the evolution of the SCW instability. Under conditions of convective instability there is realized a regime with traveling electric-field domains, accompanied by pulsations of the current in the external circuit. At sufficiently high carrier density, $|\omega_m| > v_d^2/4D$, a regime is realized with a static domain localized near an inhomogeneity of the structure (for example, near its boundaries); this leads to switchover effects on the current-voltage characteristic of the structure. A convenient estimate of the density at which the criterion (26) is satisfied is obtained on the basis of (23)–(25) at $E_0 \gg E_i$:

$$n_0 > \begin{cases} \epsilon E_0^2/16\pi\kappa T, & \kappa T \gg \Delta \\ \epsilon E_0^2/6\pi\Delta, & \kappa T \ll \Delta \end{cases} \quad (27)$$

We note finally that the approximation with a quasiclassical description of the W spectrum is valid at $\lambda \gg d$ (λ is the length of the SCW); this is equivalent to

$$\pi\Delta/\beta edE_0 \gg 1. \quad (28)$$

In the wave-number region shown in Figs. 1 and 2 we have $\pi/\beta \approx 1$. Equation (28) reduces then to the initial condition (6) for the validity of the quasiclassical description.

5. CONCLUSION

The foregoing kinetic analysis of the space-charge-wave spectrum in semiconductor SL shows that at typical values of the SL parameters the character of the instability development is determined by the low-frequency drift-relaxation branch. At the same time, the high-frequency resonant (Bloch) branches of the spectrum ($\text{Re } \omega \approx n\Omega$) are stable in a wide range of SL parameters, except in the cases of very high Q of the Bloch and plasma resonances. Thus, the build-up of high-frequency oscillations of an electric field and of space charge in superlattices at frequencies comparable with the Bloch frequency is hindered by the low electron mobility due to the small width of the miniband and to the high collision frequency.

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