Propagation of sound in one-dimensional disordered metals at a finite temperature

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A calculation is made of the attenuation coefficient and of the dispersion of the velocity of long-wavelength sound in weakly disordered metals with one-dimensional conductance at low temperatures. It is shown that delocalization of electrons by thermal phonons at temperatures $T < \omega_D$ results mainly in oscillations of the attenuation of sound as a function of the wave number. The amplitude and period of these oscillations are governed by the length of electron jumps between quasistationary localized states. In the case of sound polarized along the conducting chains an increase in temperature enhances greatly the role of the inertial mechanism of the electron-phonon interaction because of a quadratic increase of the conductivity $\sigma(T)$ with temperature.

1. INTRODUCTION

The electron system in one-dimensional ($1D$) conductors is known[1] to be unstable in the presence of any (no matter how weak) disorder of the crystal structure. This is manifested by an exponential localization of all the conduction electrons at absolute zero. Consequently, disordered $1D$ metals are good conductors on the scale of distances less than the localization length and they behave like zero-gap insulators if the dimensions of the samples exceed this length significantly. The frequency dispersion of the conductivity of such systems has been analyzed in detail in Refs. 1 and 2.

Localization of electron states in one-dimensional conductors not only affects the conductivity but also influences significantly the dynamic elastic moduli and, consequently, the propagation of acoustic waves in such conductors. This problem was investigated in detail earlier[3] and it was found that the one-dimensional nature of electron motion is the cause of one additional important effect. A strong Coulomb screening of charges in the course of propagation of long-wavelength acoustic vibrations in a $1D$ metal ($qa \ll 1$, where q is the wave number of sound and a is the lattice constant) suppresses the usual deformation mechanism of the interaction between electrons and sound. Instead we have to allow for an inertial (Stewart-Tolman) mechanism and for a cross-deformation interaction which appears as a result of modulation of the random potential of static defects by the field of the acoustic wave. This is true in the case of weak scattering of electrons by impurities (defects) when $\varepsilon_F \tau_i \gg 1$ (ε_F is the Fermi energy and τ_i^{-1} is the frequency of the electron-impurity backscattering).

These two mechanisms of the electron-phonon interaction (inertial and cross-deformation) are much weaker than the deformation mechanism. Therefore, the dispersion of the velocity and the attenuation of acoustic waves in $1D$ conductors are considerably less than in the usual three-dimensional materials. The relative role of these mechanisms is governed primarily by the relationship between the parameters m/M and $(\varepsilon_F \tau_i)^{-1}$ (m is the effective mass of an electron and M is the mass of an ion).

The problem of propagation of elastic waves in $1D$ metals was solved earlier[3] at the absolute zero. Our aim will be to study the influence of a finite temperature on the spectrum and attenuation of sound in such conductors. The screening of the deformation potential described above is independent of temperature. On the other hand, the electron states are no longer strictly localized at $T \neq 0$ and this undoubtedly should be reflected in the electron elasticity of a crystal.

Up to now the temperature effects in $1D$ systems have been investigated mainly in the case of the electron mobility. Electron transport at temperatures $T \neq 0$ depends strongly on the relationship between the thermal phonon energy and the frequency τ_i^{-1}. In the range $T\tau_i \ll 1$ the motion of electrons is in the form of jumps over distances which are large compared with the localization length. However, the quasistatic nature of the phonon field considered in this limit can enhance significantly the localization of electrons. A rigorous solution of this problem is lacking at present. The main available results were derived sometime ago by Mott and Davis[4] {including the familiar Mott law for the static conductivity $\sigma \propto \exp[-\overline{(T\tau_i)}^{-1/2}]$, deduced using qualitative considerations} and they were also derived in Ref. 5, where a study was made of the frequency dependence $\sigma(\omega)$ in the pair approximation.

The other limiting case $T\tau_i \gg 1$ had been investigated in greater detail. It was shown in Ref. 2 that in this limit the role of phonons is mainly to produce delocalization and dynamic broadening of the energy levels of localized electrons. This makes possible their tunneling between closely spaced localized states and gives rise to a finite static electrical conductivity.

In the present study the absorption and dispersion of the velocity of sound in $1D$ metals will be considered in the range of temperatures defined by the inequalities

$$\tau_i^{-1} \ll T \ll \omega_D, \tag{1}$$

where ω_D is the Debye frequency of the phonons. The first of these inequalities allows us to ignore the localization of electrons because of the scattering by thermal phonons, whereas the second is essential to identify the mechanism of the interaction of these phonons with electrons. In this limiting case the thermal phonons have long wavelengths and the complete screening of the deformation interaction applies to them as well.

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2. GENERAL RELATIONSHIPS

The following equations describe elastic vibrations of a $1D$ metal in the case of arbitrary directions of propagation and polarization of sound[3]:

$$Mn\omega^2 u_i = \left[K_{iklm} + \frac{1}{a^2}\langle T_{ik}T_{lm}\rangle_{q,\omega}\right] \times q_k q_l u_m - i\omega^3\left(\frac{m_0}{e}\right)^2 \sigma(q,\omega)\, u_x \delta_{ix} - \frac{m_0\omega}{a^2 e}[\delta_{ix}\langle jT_{lm}\rangle_{q,\omega} q_l u_m + \langle T_{ik} j\rangle_{q,\omega} q_k u_x]. \quad (2)$$

Here, n is the concentration of ions of charge $Z|e|$; m_0 and e are the mass of a free electron and its charge; ω, $\mathbf{q}$, and $\mathbf{u}$ are the frequency, wave vector, and amplitude of sound; K_{iklm} are the adiabatic elastic moduli of the investigated crystal; $\sigma(q,\omega)$ is the electrical conductivity. The x axis is directed along the conducting chains and δ_{ix} is the Kronecker delta. The angular brackets with the indices q and ω denote the Fourier components of the retarded Green functions of the relevant Heisenberg operators, which are the current density $\hat{j}$ and the stress operator:

$$\hat{T}_{ik}(x) = \psi^+(x) t_{ik}(x)\psi(x),$$
$$t_{ik}(x) = \frac{1}{\tau_i}[\zeta(x)\hat{a}^+\Delta_{ik} + \zeta^+(x)\hat{a}\Delta_{ik}^*],$$
$$\hat{a} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad \hat{a}^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}. \quad (3)$$

The following notation is used in Eq. (3): $\psi(x)$ and $\psi^+(x)$ are the electron field operators in the spinor (in respect of the sign of the electron momentum) representation, $\zeta(x)$ is a complex Gaussian random field equivalent to a random field of impurities with the correlation properties

$$\langle\zeta(x)\zeta^+(x')\rangle = l_i\delta(x-x'),\ \langle\zeta(x)\zeta(x')\rangle = 0,\ l_i = v\tau_i,$$

v is the Fermi velocity of electrons, and Δ_{ik} are the components of a dimensionless complex cross-deformation potential tensor with values $|\Delta_{ik}| \sim 1$.

In the calculation of the correlation functions in Eq. (2) we have to allow not only for the averaging over a realization of the random potential of impurities, but also for the interaction of electrons with thermal phonons. This procedure should be carried out rigorously. If we confine ourselves to the range of temperatures defined by Eq. (1), we find—as shown in Ref. 6—that one other inequality is automatically satisfied:

$$\tau_i/\tau_{ph} \sim T^2/\varepsilon_F\omega_D \ll 1 \quad (4)$$

(τ_{ph} is the relaxation time of electrons interacting with phonons), which means that the scattering on the thermal vibrations of the lattice has little effect on localized electron states formed as a result of the elastic impurity scattering. It follows from the results of Ref. 2 that this influence reduces to a dynamic broadening of the energy levels of localized electrons and can be allowed for by replacing the frequency ω with the complex quantity $\tilde{\omega} = \omega + i/\tau_{ph}$.

A direct calculation of the correlation functions in the last term of Eq. (2) shows that in a statistically homogeneous conductor we have

$$\langle jT_{ik}\rangle_{q,\tilde{\omega}} = -\langle T_{ik}j\rangle_{-q,\tilde{\omega}} = -\langle T_{ik}j\rangle_{q,\tilde{\omega}}.$$

This makes the third term in Eq. (2) zero if sound propagates along or exactly across the conducting chains. In other words, in the case of symmetric directions of sound propagation the induction and cross-deformation mechanisms do not interfere with one another. We shall consider only such cases because this simplifies somewhat the calculations but has no significant influence on the final result.

A calculation of the correlation function $\langle T_{ik}T_{lm}\rangle_{q,\tilde{\omega}}$ requires an accurate allowance for the interference effect in the case of multiple scattering of electrons by impurities. In the approximation of a white-noise impurity potential this can be done exactly by using an original technique developed in Ref. 3 for the calculation of the correlation functions. Very cumbersome intermediate stages, which give rise to finite-difference Berezinskiĭ equations for the conductivity, allow us to represent the correlation function $\langle T_{ik}T_{lm}\rangle_{q,\tilde{\omega}}$ in the following form:

$$\langle T_{ik}T_{lm}\rangle_{q,\tilde{\omega}} = -\frac{\mathrm{Re}(\Delta_{ik}\Delta_{lm}^*)}{\pi\tau_i}\left[\frac{1}{L_c} + i\frac{\tilde{\omega}}{v}F(\tilde{\omega})\right] + \frac{i\tilde{\omega}}{\pi l_i}\,\mathrm{Im}\Delta_{ik}\,\mathrm{Im}\,\Delta_{lm}S_+(q,\tilde{\omega}). \quad (5)$$

Here, $L_c \sim a$ is the correlation length of the random potential of the impurities,

$$F(\tilde{\omega}) = 1 + 2\sum_{n=0}^{\infty} R_{n+1}^2, \quad (6)$$

$$S_+(q,\tilde{\omega}) = S(q,\tilde{\omega}) + S(-q,\tilde{\omega}). \quad (7)$$

The quantities R_{n+1} in Eq. (6) satisfy the following system of the Berezinskiĭ finite-difference equations[1]

$$n(R_{n+1} - 2R_n + R_{n-1}) - \beta R_n = 0,\ R_0 = 1,\ \beta = -2i\tilde{\omega}\tau_i, \quad (8)$$

the solution of which gives

$$R_n = \beta\int_0^\infty dt\, e^{-\beta t}\left(\frac{t}{1+t}\right)^n, \quad n = 0, 1, 2\ldots. \quad (9)$$

The function $S(q,\tilde{\omega})$ in Eq. (7) can be represented by an integral

$$S(q,\tilde{\omega}) = \frac{1}{\beta}e^{\beta}\int_\beta^\infty d\xi\, e^{-\xi}\xi(\xi-\beta)y(\xi), \quad (10)$$

in which $y(\xi)$ is the solution of the equation

$$[\hat{\nu} + i(\varkappa - \tilde{\omega}\tau_i)]y(\xi) = \beta f(\xi) + 2(\xi-\beta)\frac{d}{d\xi}[\xi f(\xi)], \quad (11)$$

where

$$\hat{\nu} = -\frac{d}{d\xi}\xi^2\frac{d}{d\xi} + \xi\frac{d}{d\xi}\xi + \beta\frac{d}{d\xi}\xi\left(\frac{d}{d\xi} - 1\right),$$
$$f(\xi) = -e^{\xi}\,\mathrm{Ei}(-\xi),\ \varkappa = ql_i \quad (12)$$

with the boundary conditions requiring that $y(\xi)$ is finite at $\xi = \beta$ and that $y(\xi) \to 0$ when $\xi \to \infty$.

3. ANALYSIS OF FUNCTIONS $S_+(q,\tilde{\omega})$ AND $F(\tilde{\omega})$

It is shown in Ref. 3 that at high frequencies, when $\omega\tau_i \gg 1$, it follows from Eq. (11) that

$$S(q,\tilde{\omega}) = [1 + i(\varkappa - \tilde{\omega}\tau_i)]^{-1}. \quad (13)$$

At low frequencies, when $\omega\tau_i \ll 1$, we can seek the solution of Eq. (11) by formal expansion in powers of β:

$$y(\xi)=y_0(\xi)+\beta y_1(\xi)+\dots, \tag{14}$$

which, however, is not exactly a Taylor series expansion. We shall show below that the function $y_1(\xi)$ includes a logarithmic dependence on β, i.e., the series of Eq. (14) is functional.

The term $y_0(\xi)$ can be found using the Green function of the equation which is derived from Eqs. (11) and (12) by going to the limit $\beta=0$:

$$G_\varkappa(\xi,\xi')=(\xi\xi')^{-1/2}\exp[(\xi-\xi')/2]K_{\Delta/2}(\xi_{max}/2)I_{\Delta/2}(\xi_{min}/2),$$
$$\xi_{\substack{max\\min}}=\max_{\min}(\xi,\xi'),\quad \Delta=(1+4i\varkappa)^{1/2},\quad \operatorname{Re}\Delta>0, \tag{15}$$

where $K_\nu(x)$ and $I_\nu(x)$ are the Macdonald and Bessel functions with an imaginary argument. The result obtained for the term $S_0(q,\tilde\omega)$, which is the leading term in β, is identical with that obtained in Ref. 3 and it can be written in the form

$$S_0(q,\tilde\omega)=\frac{1}{\beta}\int_0^\infty d\mu\, w(\mu)\frac{\nu^3(\mu)[1+\nu(\mu)/2]}{\nu(\mu)+i\varkappa}, \tag{16}$$

where

$$w(\mu)=\frac{\pi^2}{2}\frac{\mu\,\mathrm{sh}(\pi\mu/2)}{\mathrm{ch}^3(\pi\mu/2)},\qquad \nu(\mu)=\frac{1+\mu^2}{4}.$$

Integration in Eq. (16) represents averaging [with a probability density $w(\mu)$] over quantum fluctuations of the dimensionless relaxation frequency $\nu(\mu)$, which is an eigenvalue of the operator $\hat\nu_0\equiv\hat\nu$ ($\beta=0$) (for details see Ref. 7).

The term of the next order in powers of β in $S(q,\tilde\omega)$ is of the form

$$S_1(q,\tilde\omega)=\int_0^\infty d\xi\, e^{-\xi}\xi[(\xi-1)y_0(\xi)+\xi y_1(\xi)], \tag{17}$$

whereas $y_1(\xi)$ is found from the equation

$$(\hat\nu_0+i\varkappa)y_1(\xi)=g_1(\xi)+g_2(\xi), \tag{18}$$

where

$$g_1(\xi)=f(\xi)-y_0(\xi)/2,\quad g_2(\xi)=(d/d\xi-i\varkappa/\xi)y_0(\xi).$$

Equation (18) has a solution satisfying the necessary boundary conditions, viz.,

$$y_1(\xi)=\int_\beta^\infty d\xi'\, G_\varkappa(\xi,\xi')[g_1(\xi')+g_2(\xi')]. \tag{19}$$

The lower limit in the integral with respect to ξ' in Eq. (19) cannot be assumed to be zero [in contrast to Eq. (17)] because of the irregular behavior of the term $g_2(\xi')$ in the integrand in the limit $\xi'\to 0$.

Since $g_1(\xi')$ has no singularity at low values of ξ', the contribution of the first term in Eq. (19) to $S_1(q,\tilde\omega)$ is readily calculated and in the sum with the first term of Eq. (17) it becomes

$$S_1^{(1)}(q,\tilde\omega)=\frac{1}{2}\int_0^\infty d\mu\frac{w(\mu)\nu(\mu)}{\nu(\mu)+i\varkappa}\Big\{\nu^3(\mu)+\nu(\mu)+2-\frac{\nu^2(\mu)[1+\nu(\mu)/2]}{\nu(\mu)+i\varkappa}\Big\}. \tag{20}$$

The contribution $S_1^{(2)}(q,\tilde\omega)$ due to the second term in Eq. (19) can be represented conveniently by the difference

$$S_1^{(2)}(q,\tilde\omega)=S_1^{(2)}(q,0)-\Delta S_1(q,\tilde\omega),$$

where

$$\Delta S_1(q,\tilde\omega)=\int_0^\infty d\xi\, e^{-\xi}\xi^2\int_0^\beta d\xi'\, G_\varkappa(\xi,\xi')g_2(\xi'). \tag{21}$$

The expression for $S_1^{(2)}(q,0)$ in the case of arbitrary values of $\varkappa$ is very cumbersome, so that we shall consider it initially in the limit $\varkappa\ll 1$. In this range we find that $S_1^{(2)}(q,0)$ is a regular function of $\varkappa$ and can be expanded as a Taylor series. The principal term can be calculated using the limiting expression for the Green function of Eq. (15) when $\varkappa=0$:

$$G_0(\xi,\xi')=(\xi\xi')^{-1}\exp[(\xi-\xi')/2]\times\{\exp[-|\xi-\xi'|/2]-\exp[-(\xi+\xi')/2]\}.$$

The result of such a calculation is $S_1^{(2)}(0,0)=-7/12$. The term linear in $\varkappa$ is then annihilated and the quadratic term is $A\varkappa^2$, where $A\sim 1$ is a numerical coefficient whose exact value is difficult to determine.

We shall calculate $\Delta S_1(q,\tilde\omega)$ by substituting in Eq. (21) the asymptotic expressions $G_\varkappa(\xi,\xi')$ and $g_2(\xi')$ in the case when $|\xi'|\ll 1$. This gives

$$\Delta S_1(q,\tilde\omega)=\frac{\pi}{2^{10}}\frac{(9-\Delta^2)(1+\Delta)^3}{\Gamma^2(1+\Delta/2)}(1-\Delta)\left(\frac{\beta}{4}\right)^{\Delta-1}. \tag{22}$$

A certain refinement must be made here. In Eq. (21) the Green function (15) or the operator (12), obtained in the zeroth order with respect to β, and also the function $g_2(\xi)$ containing the solution $y_0(\xi)$ of Eq. (11) for $\beta=0$, are integrated in the range defined by $\xi'\lesssim\beta$. For these values of ξ' strictly speaking the values of $G_\varkappa$ and $y_0(\xi')$ are invalid, because terms linear in β and dropped from Eqs. (11) and (12) are of the same order of magnitude as the other terms. This leads to an incorrect determination of the numerical coefficient in Eq. (22).

We can find the correct expression for $\Delta S_1(q,\tilde\omega)$ using a procedure found fruitful in the calculation of the dissipative conductivity.[7] This can be done by substituting a variable $\xi=\beta(1+t)$ in Eqs. (10)–(12). Then, the equation for $y(t)\equiv\beta y[\xi(t)]$ becomes

$$-\frac{d}{dt}\left[t(1+t)\frac{dy}{dt}\right]+\beta t\frac{d}{dt}[(1+t)y]+i(\varkappa-\tilde\omega\tau_i)y=p_0(t)+2t\frac{d}{dt}[(1+t)p_0(t)],\qquad p_0(t)\equiv f[\xi(t)]. \tag{23}$$

The solution of this equation in the range $0\leqslant t\ll 1/\beta$ (which corresponds exactly to $\xi'\sim\beta$) in the zeroth order with respect to β can be obtained by expanding in terms of conical functions $P_{-(1+i\mu)/2}(1+2t)$, which are eigenfunctions of the differential operator on the left-hand side of Eq. (23) if $\beta=0$. We shall not consider this procedure in detail but simply mention that the result $\Delta S_1(q,\tilde\omega)$ obtained in this way is half that given by Eq. (22).

Finally, the required function $S_+(q,\tilde\omega)$ obtained in the principal approximation in β is given by Eq. (16), and the term of the next higher order in the range $\varkappa\ll 1$ is

$$S_{1+}(q,\tilde{\omega})=1-2\varkappa\exp(2\varkappa^2\ln(\beta/4))\times\sin(2\varkappa\ln(\beta/4))+O(\varkappa^2). \quad (24)$$

In the limit $\varkappa\gg 1$, this term can be calculated conveniently by direct iteration of Eq. (18), which gives

$$S_{1+}(q,\tilde{\omega})=2/\varkappa^2. \quad (25)$$

We shall now consider the function $F(\tilde{\omega})$ in Eq. (6). It describes the local interaction of an electron with a vibrating impurity at the moment of collision and for this reason it is independent of the spatial dispersion parameter $\varkappa$. Substituting R_n from Eq. (9) into Eq. (6), we obtain the following expression for $F(\tilde{\omega})$:

$$F(\tilde{\omega})=1+\frac{2}{\beta}\left[1+e^{\beta}\int_{\beta}^{\infty}d\xi\,\xi(\xi-\beta)\,\mathrm{Ei}(-\xi)\right], \quad (26)$$

the asymptotes of which are of the following form for different values of the parameter β:

$$F(\tilde{\omega})\approx{}^2/_3(1/\beta+1)+\beta/3, \quad |\beta|\ll 1,$$
$$F(\tilde{\omega})\approx 1+2/\beta^2, \quad |\beta|\gg 1. \quad (27)$$

These limiting expressions demonstrate that the localization of electrons in a $1D$ metal is important also in the description of essentially local effects when the size of the spatial localization of electron states is unimportant. Only at high frequencies corresponding to $|\beta|\gg 1$, when repeated returns of electrons because of the scattering by impurities are unimportant, does the formula for $F(\tilde{\omega})$ become identical with that obtained in the Born approximation in respect of the cross-deformation interaction.

4. ABSORPTION AND DISPERSION OF THE VELOCITY OF SOUND

In this section we shall give the results of a calculation of the dispersion of the velocity and of the absorption of acoustic waves of different polarizations traveling along a conducting chain. The absorption and the dispersion of sound with a wave vector perpendicular to the direction of high conductivity will then be obtained from the derived formulas by a simple substitution of the relevant values of the velocity of acoustic waves and of the components of the tensor of the cross-deformation potential. In particular, if transversely propagating sound is also polarized across the conducting chains or filaments (in which case the vibrations may be longitudinal or transverse), we have to use the formulas for the transverse sound traveling along the chains and relabel the constants. However, if the vibrations are polarized along the chains (when the sound is only transverse), we have to use the results given below for longitudinal sound.

The difference between the cases of the transverse and longitudinal polarizations of sound is that in the case of propagation of transverse waves along the direction of high conductivity the inertial mechanism of the electron-phonon interaction is inactive and only the cross-deformation mechanism remains, so that the contribution of the latter can be investigated independently. In the case of longitudinally polarized sound both mechanisms are important and they may compete with one another.

a) Transverse sound, q‖x

On the right-hand side of Eq. (2) we are now left with just the first term. We shall introduce the adiabatic velocity of the transverse sound using the expression $K_{xyxy}=Mns_t^2$. Then, the change in the velocity of sound $\Delta s_t=s_t(\omega)-s_t$ and the relative attenuation Γ_t/ω are described by the following formulas:

$$\frac{\Delta s_t}{s_t}=-\frac{\tilde{Z}_t}{(p_0 l_i)^2}\left\{|\Delta|^2\frac{l_i}{L_c}-|\Delta|^2\,\mathrm{Im}[\tilde{\omega}\tau_i F(\tilde{\omega})]+\Delta_2^2\,\mathrm{Im}[\tilde{\omega}\tau_i S_+(q,\tilde{\omega})]\right\}, \quad (28)$$

$$\frac{\Gamma_t}{\omega}=\frac{\tilde{Z}_t}{(p_0 l_i)^2}\{|\Delta|^2\,\mathrm{Re}[\tilde{\omega}\tau_i F(\tilde{\omega})]-\Delta_2^2\,\mathrm{Re}[\tilde{\omega}\tau_i S_+(q,\tilde{\omega})]\}. \quad (29)$$

Here, $\tilde{Z}_t=Z(mv^2/4Ms_t^2)\sim 1$, p_0 is the Fermi momentum of the electrons, and $\Delta\equiv\Delta_{xy}=\Delta_1+i\Delta_2$. On the right-hand sides of Eqs. (28) and (29) we have to replace the wave number q with ω/s_t. We shall analyze the dependence of the velocity and attenuation of sound on its frequency and on temperature in various frequency intervals. We shall ignore the first term in the braces of Eq. (28), since it is a relatively small (independent of the frequency and temperature) correction to the adiabatic elastic moduli and appears because of the electron-impurity collisions.

Low frequencies, $\omega\tau_i\ll 1$. We have to consider here several regions, depending on the spatial dispersion parameter $\varkappa=\omega\tau_i v/s_t$. We turn first to the case when $\varkappa\ll 1$. In contrast to conductors with a higher dimensionality, the spatial dispersion cannot be ignored in this range of frequencies for $1D$ metals because of the oscillatory dependence of $S_{1+}(q,\tilde{\omega})$ [Eq. (24)]. Moreover, even when $\omega\tau_i\ll 1$, the time dispersion is important, because at a finite temperature we have a new parameter $\omega\tau_{ph}\gg\omega\tau_i$, which occurs in the expression $|\beta|/4=(\tau_i/2\tau_{ph})[1+(\omega\tau_{ph})^2]^{1/2}$.

Substituting in Eqs. (28) and (29) the asymptotic expressions for $F(\tilde{\omega})$ and $S_+(q,\tilde{\omega})$, we then obtain the following formulas in the $\varkappa\ll 1$ case:

$$\frac{\Delta s_t}{s_t}=\frac{\tilde{Z}_t}{(p_0 l_i)^2}\left\{\frac{|\Delta|^2}{3}-\frac{\Delta_2^2}{2}\left(1-\frac{5}{2}\varkappa^2\right)+\frac{\tau_i}{\tau_{ph}}\left[\frac{2}{3}|\Delta|^2-\Delta_2^2\left(1-2\varkappa\exp\left(2\varkappa^2\ln\frac{|\beta|}{4}\right)\times\sin\left(2\varkappa\ln\frac{|\beta|}{4}\right)\right)\right]\right\},$$

$$\frac{\Gamma_t}{\omega}=\omega\tau_i\frac{\tilde{Z}_t}{(p_0 l_i)^2}\left\{\frac{2}{3}|\Delta|^2\left(1+\frac{\tau_i}{\tau_{ph}}\right)-\Delta_2^2\left[1-2\varkappa\exp\left(2\varkappa^2\ln\frac{|\beta|}{4}\right)\sin\left(2\varkappa\ln\frac{|\beta|}{4}\right)\right]\right\}, \quad (30)$$

from which we can see that in addition to a small absolute dispersion of the velocity and attenuation of transverse sound in $1D$ metals, these quantities depend weakly on temperature (via the parameter τ_i/τ_{ph}). This dependence is manifested firstly by a monotonic quadratic rise of Δs_t and Γ_t with temperature and, secondly, it is contained in the amplitude and period of the relatively weak oscillations of

these quantities. These oscillations should be observed in the range of frequencies defined by the inequalities

$$\varkappa^2|\ln(\beta/4)|\ll 1\ll\varkappa|\ln(\beta/4)|. \qquad (31)$$

Their origin is associated with the existence of the following characteristic length of the electron system:

$$L_\beta=2l_i|\ln(\beta/4)|,$$

which represents the length of a jump of an electron between distant localized states. If $\omega\tau_{ph}\gg 1$, then these are the states whose energy levels differ by an amount equal to a quantum ω of the external field. However, if $\omega\tau_{ph}\ll 1$, the jumps occur between quasistationary states whose levels overlap within a width $1/\tau_{ph}$. Exponential decay of the amplitude of the oscillations in the range of frequencies where the left-hand inequality of Eq. (31) is disobeyed can be attributed to the statistical scatter of the electron jump lengths. This scatter means that only one or two periods of the oscillations can be observed in reality. Oscillations of the type described by Eq. (30) have been predicted earlier for the conductivity of $1D$ metals.[6,7]

In the region of strong spatial dispersion, $\varkappa\gg 1$, the velocity and absorption of transverse sound are described by

$$\frac{\Delta s_t}{s_t}=\frac{\tilde{Z}_t}{(p_0l_i)^2}\left[\frac{|\Delta|^2}{3}\left(1+2\frac{\tau_i}{\tau_{ph}}\right)-\frac{6}{5}\frac{\Delta_2^2}{\varkappa^2}\right],$$
$$\frac{\Gamma_t}{\omega}=\omega\tau_i\frac{\tilde{Z}_t}{(p_0l_i)^2}\left[\frac{2}{3}|\Delta|^2\left(1+\frac{\tau_i}{\tau_{ph}}\right)-2\frac{\Delta_2^2}{\varkappa^2}\right]. \qquad (32)$$

At high frequencies, $\omega\tau_i\gg 1$, the velocity of transverse sound is independent of its frequency and the nature of its temperature dependence is generally the same as that given by Eq. (32). However, in the high-frequency limit the absorption practically ceases to depend on temperature:

$$\frac{\Delta s_t}{s_t}=\frac{\tilde{Z}_t}{(p_0l_i)^2}\left[|\Delta|^2\frac{\tau_i}{\tau_{ph}}+2\Delta_2^2\left(\frac{s_t}{v}\right)^2\right],$$
$$\frac{\Gamma_t}{\omega}=\omega\tau_i\frac{\tilde{Z}_t}{(p_0l_i)^2}(|\Delta|^2-2\Delta_2^2/\varkappa^2). \qquad (33)$$

b) Longitudinal sound, $q\|x$

The equation for longitudinal sound differs from the equation for transverse vibrations by the presence in Eq. (2) of a term with the conductivity $\sigma(q,\tilde{\omega})$, which describes the influence of the inertial interaction of electrons with the lattice. Consequently, the formulas for the velocity and attenuation of longitudinal waves differ from Eqs. (28) and (29) by the replacement of all the transverse elastic moduli with the longitudinal moduli $\Delta=\Delta_{xx}$, and also by the presence on the right-hand sides of these equations of additional terms

$$\left.\begin{matrix}\delta s_l/s_l\\ \delta\Gamma_l/\omega\end{matrix}\right\}=2\omega\tau_i\tilde{Z}_l\left(\frac{m_0s_l}{mv}\right)^2\left(\begin{matrix}\mathrm{Im}\\ \mathrm{Re}\end{matrix}\right)\frac{\sigma(q,\tilde{\omega})}{\sigma_0},\qquad \sigma_0=n_ee^2\tau_i/m. \qquad (34)$$

At low frequencies, $\omega\tau_i\ll 1$, in the region of weak spatial dispersion $\varkappa=\omega l_i/s_l\ll 1$, the correction to the velocity of longitudinal sound because of the inertial interaction is negative [in contrast to Eq. (30)], depends quadratically on the frequency ω, and is independent of temperature:

$$\delta s_l/s_l=-8\zeta(3)\tilde{Z}_l(m_0s_l/mv)^2(\omega\tau_i)^2, \qquad (35)$$

where $\zeta(3)$ is the Riemann zeta function. The inertial correction to the absorption coefficient found in this frequency range can be represented by a sum of a monotonic part

$$(\delta\Gamma_l/\omega)_{mon}=8\zeta(3)\omega\tau_i\tilde{Z}_l(m_0s_l/mv)^2\tau_i/\tau_{ph} \qquad (36)$$

and an oscillatory term with a relatively small amplitude, which appears because of the oscillatory nature of the one-dimensional conductivity. The temperature dependence of the inhomogeneous conductivity of $1D$ metals was analyzed in detail in Ref. 6. It was found that oscillations of $\sigma(q,\tilde{\omega})$ can generally be described by a fairly cumbersome formula which we shall not give here. However, in the range where these oscillations are strongest [see the inequalities in Eq. (31)], the formula for $\sigma(q,\tilde{\omega})$ simplifies and the corresponding correction to the attenuation of sound becomes

$$\left(\frac{\delta\Gamma_l}{\omega}\right)_{osc}=-4\tilde{Z}_l\frac{1-(\omega\tau_{ph})^2}{\omega\tau_i}\left(\frac{m_0s_l^2}{mv^2}\right)^2\left(\frac{\tau_i}{\tau_{ph}}\right)^2$$
$$\times\left[1-\exp\left(2\varkappa^2\ln\frac{|\beta|}{4}\right)\cos\left(2\varkappa\ln\frac{|\beta|}{4}\right)\right]. \qquad (37)$$

Going now to the region of strong spatial dispersion, $\varkappa\gg 1$, we find that Eq. (34) yields the following asymptotic expressions:

$$\delta s_l/s_l=-2\tilde{Z}_l(m_0s_l^2/mv^2)^2,$$
$$\delta\Gamma_l/\omega=2\omega\tau_i\tilde{Z}_l\varkappa^{-2}(m_0s_l/mv)^2[2(s_l/v)^2+\tau_i/\tau_{ph}]. \qquad (38)$$

This result is not affected by a further increase in the velocity of sound and by transition to the high-frequency range $\omega\tau_i\gg 1$.

It is clear from Eq. (38) that when ω is increased, the contribution of the inertial interaction to the velocity of sound in the region $\varkappa\gg 1$ becomes saturated with respect to the frequency and the Stewart-Tolman attenuation should approach rapidly zero. For this reason, beginning from a certain frequency, which is found by comparing the absorption of Eq. (38) with a formula of the (32) type for Γ_l, the attenuation of longitudinal sound is governed entirely by the cross-deformation mechanism of the electron-phonon interaction.

We shall conclude by noting that in our opinion the results obtained in the present study, particularly geometric resonance oscillations of the absorption of sound, may be used in experimental determination of the electron-phonon interaction constants of $1D$ metals and at temperatures $T<\omega_D$ these constants are components of the tensor Δ_{ik}. Moreover, it should be noted that because of the screening (in the long-wavelength limit) of the deformation potential, the dispersion and attenuation of acoustic waves, and also their temperature dependences are very weak. This can be used to construct various thermally stable acoustoelectronic devices from $1D$ conductors, for example, dispersion-free delay lines characterized by very low losses may be feasible.

We shall make one additional comment. We have analyzed the dispersion and attenuation of sound in the temperature range defined by Eq. (1), which is bounded from above by the Debye frequency. On increase in temperature above this frequency ω_D a relatively weak cross-deformation interaction of electrons with thermal phonons changes to the usual deformation interaction, which alters greatly the elec-

tron-phonon scattering frequency: in the range $T > \omega_D$, we know that $1/\tau_{ph} \propto T$. The inequality (4), which plays an important role in our calculations, is reversed and this means that localized electron states do not form in the available time and the kinetics of conduction electrons is determined entirely by their scattering on phonons. In this situation the absorption and the dispersion of the velocity of sound can be calculated employing the usual transport equation for $1D$ electrons.

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