Interaction between breathers and fluxon "bundles" with microshorts, Abrikosov vortices, and local currents in distributed Josephson junctions

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(Submitted 28 November 1986)
Zh. Eksp. Teor. Fiz. 93, 1293–1298 (October 1987)

The interaction of fluxons and breathers with point inhomogeneities in distributed Josephson junctions is studied by a Hamiltonian formalism. The conditions for the pinning of a breather at a microshort or a microresistance are determined. The conditions for the decay of a breather at an Abrikosov vortex are also determined. The critical currents for the detachment of a pair of solitons from an Abrikosov vortex are calculated.

1. INTRODUCTION

The quanta of magnetic flux—fluxons—moving in distributed Josephson junctions are known to be described by soliton solutions of the sine-Gordon equation, which can be written in the form of a Hamiltonian system for the variables φ and φ_t. Modified sine-Gordon equations describe junctions with inhomogeneities which give rise to corresponding increments in the Hamiltonian. In particular, a distributed Josephson junction with a microshort—a local region of a high superconducting current, with dimensions much smaller than the Josephson penetration depth of the magnetic field—are described by the equation[1]

$$\varphi_{tt}=\varphi_{xx}-\sin\varphi-\mu\delta(x)\sin\varphi,\quad \mu>0 \tag{1}$$

or by the Hamiltonian density

$$H=H^{sG}+H^{p},$$

where

$$H^{sG}=0.5\varphi_t^2+0.5\varphi_x^2+1-\cos\varphi, \tag{2}$$

$$H^{p}=\mu\delta(x)(1-\cos\varphi). \tag{3}$$

Here and below, the coordinate x, along which the solitons are propagating, is expressed in units of the Josephson penetration depth of the magnetic field, the time t is expressed in units of the reciprocal of the Josephson plasma frequency, and the indices x and t mean differentiation with respect to the corresponding variables.

A microresistance—a local thickening of a contact, which is a region of a lowered superconducting current—is described by an analogous expression with $\mu<0$ (Ref. 2).

Another interesting example is an Abrikosov vortex which is pinned in a superconductor parallel to the plane of the junction and perpendicular to the x axis. For this example, the increment in the Hamiltonian density is[3]

$$H^{p}=\eta\delta_x(x)\varphi(x,t) \tag{4}$$

and is the same, in the corresponding units, as the energy of an Abrikosov vortex in the mangetic field of a soliton in the London approximation. The equation describing the junction takes the form

$$\varphi_{tt}=\varphi_{xx}-\sin\varphi-\eta\delta_x(x).$$

Various perturbation-theory approaches have been developed[2,4,5] for studying the interaction of solitons with microinhomogeneities because of the outlook for possible technical applications of distributed Josephson junctions. It has been shown that φ^+ and φ^- solitons are repelled from a microshort and attracted to a microresistance. An Abrikosov vortex has a selective effect: If the polarities of the vortex and soliton are the same, they attract each other, but if the polarities are different they repel each other. In a study of the perturbations of breather and multisoliton solutions, however, these methods become excessively cumbersome and in some cases totally inapplicable.

In this paper we propose an approach for studying modified sine-Gordon equations based on the use of the canonical variables introduced in Ref. 6. We assume that the phase distribution $\varphi(x,0)$ in the distributed Josephson junction initially corresponds to some multisoliton solution. Transforming from the velocities v_i and phases X_i of the solitons and also from the velocities v_i, the frequencies $\cos\nu_i$, the oscillation phases θ_i, and the displacement phases X_i of the breathers to the canonical variables p_m, q_m in this solution, we can describe the evolution of the system by a simple set of Hamilton's equations. We ignore the effect of the continuum (Swihart waves), as we are justified in doing in the case of small perturbations or low velocities.[1,4,5]

The interaction of current vortices with inhomogeneities of the contact gives rise to some new steady states. Techniques involving the sweeping of a weak laser beam over a distributed Josephson junction have recently been developed[7] to observe these new states experimentally. In the present paper, two such states are predicted: a breather pinned by a microresistance and a pair of like fluxons pinned by an Abrikosov vortex.

The dynamics of breathers is also of much interest. Breathers can appear in a junction from Swihart waves under the influence of the displacement current. Their decay gives rise to fluxons of different polarities, which become unipolar upon reflection from the edge of the junction. In the process, a magnetic field may build up in the junction, in the absence of an external field. This effect can be observed by recording voltage-current characteristics[8]; it would be seen as a jump from one zero-field step to another. In Sec. 3 it is shown that the decay of a breather may occur not only under the influence of a displacement current[5] but also as a result of an interaction with an Abrikosov vortex at a constant displacement current or in the absence of such a current.

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Akoh *et al.*[9] recently proposed a method for observing picosecond processes in distributed Josephson junctions in "real time" by means of a stroboscopic converter using Josephson elements. This method can be used to see how well the theoretical equations describe the actual motion of fluxons and breathers.

2. INTERACTION OF A BREATHER WITH A MICROSHORT (OR MICRORESISTANCE)

The energy of the interaction of a breather with a microshort (or microresistance) is given in canonical variables by

$$H^p(p,q)=8\mu\left[\frac{\mathrm{tg}(p_1/16)\sin q_1\,\mathrm{ch}\,q_2}{\mathrm{tg}^2(p_1/16)\sin^2 q_1+\mathrm{ch}^2 q_2}\right]^2. \qquad (5)$$

In the limit $q_2\to\infty$ this expression contributes nothing, telling us that there is no interaction at large range.

Writing Hamilton's equations for the total energy of a breather, and then transforming to the original variables, we find the system of equations

$$\nu_t={}^1/_2\mu W\sin(2\theta)\,\mathrm{tg}^2\nu\,\mathrm{ch}^2 z,$$
$$\theta_t=(1-v^2)^{-1/2}\cos\nu+\mu W\sin^2\theta\cos^{-2}\nu\,\mathrm{tg}\,\nu\,\mathrm{ch}^2 z,$$
$$v_t={}^1/_2\mu W(1-v^2)\sin^2\theta\,\mathrm{tg}^2\nu\,\mathrm{sh}(2z),\quad z_t=v(1-v^2)^{-1/2}\sin\nu \qquad (6)$$

where

$$W=(\mathrm{ch}^2 z-\mathrm{tg}^2\nu\sin^2\theta)(\mathrm{ch}^2 z+\mathrm{tg}^2\nu\sin^2\theta)^{-3},$$

$$z=X(1-v^2)^{-1/2}\sin\nu,$$

which describes the dynamics of a breather in the presence of a microshort (or microresistance). It follows from the first pair of equations that θ always increases monotonically over time, so there are no singular points in any (τ,ν) plane of the phase space of the system. A breather will not decay into solitons (a state with $\nu=\pi/2$) in collisions with a microshort (or microresistance). The reason is that the interaction, although depending on the distance between the microshort (or microresistance) and the soliton, is unaffected by the polarity of the latter.

Steady breather states correspond to singular points in (v,z) planes. Since the derivatives v_t and ν_t are $\sim\mu$, since the derivative z_t is $\sim v$, and since the latter derivative is also small in comparison with $\tau_t\sim\cos\nu$ near an equilibrium point, we find the following simple system of equations, taking z and ν to be constant in a first approximation, and averaging Eqs. (6) over the rapidly oscillating parameter θ:

$$v_t=\mu(1-v^2)\,\mathrm{tg}^2\nu\,\mathrm{sh}(2z)(2\,\mathrm{ch}^2 z-\mathrm{tg}^2\nu)(2\,\mathrm{ch}^2 z+\mathrm{tg}^2\nu)^{-3},$$
$$z_t=v(1-v^2)^{-1/2}\sin\nu. \qquad (7)$$

This system of equations determines an equilibrium point. To analyze the stability of these states it is useful to consider a diagram of the average value of θ over the energy (Fig. 1).

If there is a microshort at the coordinate origin, the origin is a point of marginally stable equilibrium for breathers with frequency

$$\omega\approx\cos\nu<3^{-1/2},$$

although the solitons bound in a breather would separately be repelled from it. This state may be thought of as lying symmetrically with respect to a pair of solitons which attract each other and which repel the center. The minimum energy of the finite motion and the height of the potential barrier are, respectively,

$$16\mu\omega^2(1-\omega^2)(1+\omega^2)^{-2},\quad 4\mu\omega^2(1-\omega^2)^{-1}.$$

They depend on the breather frequency, which determines the strength of the coupling of the solitons. As the frequency decreases, the distance between solitons becomes so small that they can no longer lie on different sides of the microshort; the depth of the potential well approaches zero; and there are no steady states for a breather with $\omega>3^{-1/2}$.

Under the condition $\mu<0$ (Fig. 1) there are evidently always stable equilibrium positions, since solitons are attracted to a microresistance. For a strongly bound pair in a breather, this point is the origin of coordinates; as the binding energy decreases ($\omega<3^{-1/2}$) it would consist of the points

$$x_0=\pm\omega\,\mathrm{arch}\{[(1-\omega^2)/2\omega^2]^{1/2}\}.$$

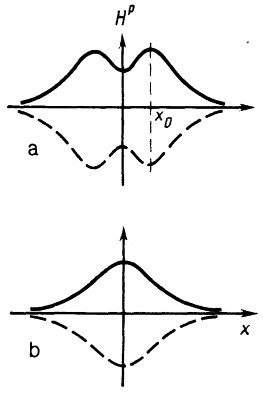

FIG. 1. Effective interaction potentials of a breather with a microshort (solid line) and with a microresistance (dashed line). a—$\omega<3^{-1/2}$; b—$\omega>3^{-1/2}$.

3. DECAY OF A BREATHER INTO SOLITONS IN AN INTERACTION WITH AN ABRIKOSOV VORTEX

By analogy with the arguments above, we express the energy of the interaction of a breather with an Abrikosov vortex in canonical variables:

$$H^p=4\eta\,\mathrm{tg}\left(\frac{p_1}{16}\right)\frac{\mathrm{sh}(p_2/16)\cos(p_1/16)\cos q_1\,\mathrm{ch}\,q_2-\mathrm{ch}(p_2/16)\sin(p_1/16)\sin q_1\,\mathrm{sh}\,q_2}{\mathrm{tg}^2(p_1/16)\sin^2 q_1+\mathrm{ch}^2 q_2}. \qquad (8)$$

Using (8), we can easily write Hamilton's equations, but we will not reproduce those lengthy equations here. We offer only a qualitative analysis.

At small values of η, which correspond to a real distributed Josephson junction, as was shown in Ref. 3, we have $v_t\sim 0$, and $z_t\sim 1$ to lowest order. A variation in z shifts the singular points in the (θ,ν) plane, but it does not result in their disappearance or the creation of new ones. There are no states in which the breather does not move.

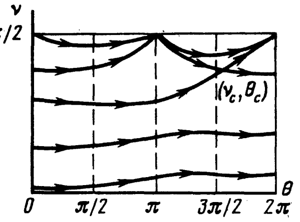

FIG. 2. Phase trajectories of the parameters ν and θ of a breather interacting with an Abrikosov vortex.

Let us examine the topological structure of the (θ,ν) plane in the phase space of the system (Fig. 2). At values $\cos\nu \gg \eta$ we find, in the zeroth approximation in η,

$$\nu_t=0, \quad \theta_t={}^1/_{16}E \operatorname{ctg}\nu.$$

Going to the first order, we find small oscillations of the trajectories, so there are no singular points in this part of the phase space. A breather does not decay into fluxons. For $\cos\nu \ll \eta$ the system reduces to the equations

$$(\cos\nu)_t=\frac{\eta\cos\nu\cos\theta\,\mathrm{sh}\,z}{4(1-v^2)^{1/2}\sin^2\theta}, \quad \theta_t=\frac{\eta\,\mathrm{sh}\,z}{4(1-v^2)^{1/2}\sin\theta},$$
$$\sin\theta/\cos\nu=\mathrm{const}.$$

All the trajectories of the solutions of these equations terminate at the points $\nu=\pi/2$, $\theta=\pi(\mathrm{mod}\ 2\pi)$, which determine the time of the breather decay.

The two ranges of values of the parameter ν and, correspondingly, the two types of trajectories are separated by a separatrix, on which the conditions $\cos\nu\sim\eta^{1/2}$ holds, as is obvious from simple considerations. The energy of the interaction of a breather with an Abrikosov vortex is on the order of η, while the binding energy of the solitons in a breather is $16(1-\sin\nu)$. The separatrix has a minimum at $\theta\approx\pi$. All the trajectories lying above it correspond to breathers which decay into solitons in an interaction with an Abrikosov vortex. Retaining in the equation for θ terms of up to second order in η inclusively, we find the coordinates of the saddle point:

$$\theta_c\approx 3\pi/2, \quad \cos\nu_c=4\eta E(1-v^2)^{-1/2}\,\mathrm{sh}\,z.$$

From the separatrix equation we easily see that at a fixed frequency the probability for the decay of a breather with a relatively high velocity is itself relatively high, and we can determine the frequency region in which splitting is impossible:

$$\omega>(\eta v/2)^{1/2}.$$

4. INTERACTION OF A PAIR OF SOLITONS WITH MICROINHOMOGENEITIES

The two-soliton solution is given explicitly in terms of canonical variables in Ref. 6, among other places. Going through some lengthy but straightforward calculations similar to those presented above, we find the energy of the interaction of two solitons with an Abrikosov vortex from this explicit expression:

$$H^p=-2\eta\Pi\frac{\mathrm{ch}(p_2/8)\,\mathrm{ch}\,q_1+\mathrm{ch}(p_1/8)\,\mathrm{ch}\,q_2}{\Pi^2\,\mathrm{ch}^2[(q_1-q_2)/2]+\mathrm{sh}^2[(q_1+q_2)/2]},$$

where

$$\Pi=\frac{\exp[(p_1-p_2)/8]+1}{\exp[(p_1-p_2)/8]-1}.$$

A stable equilibrium corresponds to a symmetric arrangement of fluxons with respect to the Abrikosov vortex, which has polarity opposite that of the fluxons. Setting $q_1=q_2$, we thus find from Hamilton's equations that the distance between immobile fluxons is $2\ln(\eta/4)$, and the critical value of the uniformly distributed displacement current, which detaches a single fluxon, in $\eta^2/8\pi$. For a fluxon pair $\varphi^+-\varphi^-$ a state with a single soliton at infinity and with another at the Abrikosov vortex is the most preferred state from the energy standpoint.

In an interaction of a pair of fluxons with a microresistance, no pinning will occur, since the corresponding term in the energy,

$$H^p=8\mu\left\{\Pi\frac{\mathrm{ch}[(q_1-q_2)/2]\,\mathrm{sh}[(q_1+q_2)/2]}{\Pi^2\,\mathrm{ch}^2[(q_1-q_2)/2]+\mathrm{sh}^2[(q_1+q_2)/2]}\right\}^2,$$

vanishes in the case of a symmetric arrangement of fluxons. This result agrees with that found in Ref. 2 by a bifurcation perturbation theory, but we see that it is unrelated to the small value of the parameter μ.

5. CONCLUSION

This study has yielded several results:

1) An Abrikosov vortex may lead to splitting of a breather. At a given frequency, the decay probability increases with increasing velocity. Breathers with $\omega>(\eta v/2)^{1/2}$ do not split.

2) The presence of a microresistance does not lead to the decay of a breather, but it is capable of pinning the breather to itself at $\omega>3^{-1/2}$ or at a distance $\omega\,\mathrm{arch}\{[(1-\omega^2)/2\omega^2]^{1/2}\}$ from itself at other frequencies.

3. A microshort does not split a breather, but it does set the stage for a metastable state of equilibrium for frequencies $\omega>3^{-1/2}$. A breather can be extracted from such a state by applying a uniformly distributed replacement current. For other frequencies, there are no stationary points.

4) An Abrikosov vortex, in contrast with a microresistance, is capable of pinning two solitons to itself; the solitons would be positioned symmetrically with respect to the vortex, at a distance of $2\ln(\eta/4)$ from each other. This stable, steady-state configuration exists at displacement current densities from 0 to $\eta^2/8\pi$; in the interval from $\eta^2/8\pi$ to $\eta 2\pi$, states with only a single fluxon are stable; at higher current densities, pinning to an Abrikosov vortex is totally impossible.[3]

All of these effects can be observed experimentally by the techniques mentioned in Sec. 1. The locally distributed replacement current which was studied experimentally in Ref. 9 is also capable of splitting breathers.

We would like to mention the constant interest in this study and the major support of L. G. Aslamazov. We also thank A. T. Filippov and Yu. S. Gal'pern for a useful discussion of these results.

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Translated by Dave Parsons