Electromagnetic structure and scattering of neutrinos in an isotropic medium

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The electromagnetic vertex has been calculated for a neutrino moving in an isotropic dispersive medium. In contrast to the results of our previous work, the complete vertex is represented, including the interaction with both longitudinal and transverse electromagnetic fields in the medium. The physical meaning of a new electromagnetic characteristic of neutrinos—the induced neutrino magnetic moment—is analyzed in detail. Total and differential cross sections for νe scattering have been calculated, and also the neutrino energy loss in matter with inclusion of the neutrino electromagnetic structure which has been found. It is shown that in the low energy region [$p_\nu \sim p_F$, where $E_p^\nu = p_\nu$ is the neutrino energy and $p_F \sim n_0^{1/3}$ is the Fermi momentum of a degenerate plasma (of metals)] the cross section found is roughly 10^3 times greater than the Born cross section calculated in the vacuum standard model of electroweak interactions and averaged over the same state of the electrons of the medium.

1. INTRODUCTION

Recently two new directions have been under development in the theory of weak interactions: a) the interaction of neutrinos with matter is investigated, various properties of which (inhomogeneity, spaciotemporal dispersion, and so forth) lead to a change of the nature of the interaction of neutrinos with a medium; b) the consequences of taking into account the vacuum electromagnetic characteristics of neutrinos are studied.

Problems of the first type have been studied in Refs. 1–3, in which the electromagnetic structure of the neutrino in a medium was investigated and the authors obtained, in particular, the electric form factor[1] and a nonzero electric charge of the neutrino in a medium[3)] (Refs. 2 and 3):

$$e_\nu^{\text{ind}} = -|e|G^{(V)}/4\pi\alpha\sqrt{2}\,r_D^2. \qquad (1)$$

Here $G^{(V)} = 2G_F(g_L^{(\nu e)} + g_R^{(\nu e)}) = G_F(1+4\xi)$ is a vector constant which is invariant to replacement of the neutrino by an antineutrino, where $g_L^{(\nu e)} = 1/2 + \xi$, $g_R^{\nu e} = \xi$; e is the charge of the electron ($e^2 = 4\pi\alpha$, $\alpha = 137^{-1}$), $\xi = \sin^2\theta_W$, where θ_W is the Weinberg angle; r_D is the Debye radius of the plasma. We use the system of units with $\hbar = c = 1$, the Feynman metric $q^2 = q_\mu q^\mu = \omega^2 - \mathbf{k}^2$; $\mu = 0,1,2,3$ (the Latin indices $i,j,k = 1,2,3$), and the standard representation of the Dirac γ matrix, in which $\gamma_5 = \gamma_5^+ = i\gamma_0\gamma_1\gamma_2\gamma_3$.

The existence of the charge (1) leads to an effective long-range property of weak forces in a dispersive medium,[4] changes the excitation spectrum of the medium,[5] and leads to a polarization loss which significantly exceeds the collision loss in the region of relatively low energies. For strong magnetic fields the Čerenkov loss to radiation by a moving neutrino of extraordinary waves also exceeds the collision loss.[6] There is also a change in the multiparticle description of a neutrino gas immersed in a dense medium, in which the contribution of the self-consistent long-range fields to the kinetic equation for neutrinos begins to exceed the contribution of the integral of direct νe collisions (the Vlasov approximation for the νe system).[7]

We emphasize that as a result of the long-range nature of weak forces observed by us previously in a medium,[4,7] the collective interaction, as a result of the thermodynamic nonequilibrium of the neutrino distribution function, can significantly exceed the νe interaction discussed in the present work. Below we investigate the interaction of a single neutrino with the electrons of a dispersive medium, which itself modifies the nature of the νe interaction. This is reflected in the appearance in the neutrino in a medium of an induced electric charge (1) and an induced magnetic moment (13) (see below).

Another interesting direction in investigation of the interaction of neutrinos with matter began with the work of Wolfenstein,[8] who obtained a modification of the oscillation length of neutrinos in a homogeneous medium, and in particular pointed out the possibility of oscillations of massless neutrinos. Interest in these problems was enhanced after publication of Ref. 9, which demonstrated the possibility of resonance enhancement of the oscillations of massive neutrinos associated with taking into account the adiabatic change of the density of the medium when neutrinos of two or more flavors progate in it.

Problems of the second type include those which are concerned with the possible manifestation of the vacuum electromagnetic properties of the neutrino such as anomalous magnetic moment[10] and dipole electric moment.[11] First among these are neutrino spin rotation in an external magnetic field[12] and the possible explanation of the Davis experiments by a change of the helicity of solar neutrinos in the magnetic field of the Sun.[13]

The same vacuum interaction $\mu_\nu^{vac}\sigma_{\rho\sigma}F_{ext}^{\rho\sigma}$ has been the subject also of studies relating to magnetic bremsstrahlung loss, bremsstrahlung in an external field (and in vacuum),[14] and finally electromagnetic vacuum νe scattering.[10]

The present work is devoted to the same process of νe scattering, but with inclusion of the properties of the target medium. Here we consider as targets isotropic dispersive media, including plasma (without an external magnetic field) and also a degenerate plasma of metals and semiconductors, an ultrarelativistic degenerate electron gas, and so forth.

In Section 2 we use the method of thermodynamic Green functions,[15] proceeding from the exact matrix elements corresponding to the Feynman diagrams for the neu-

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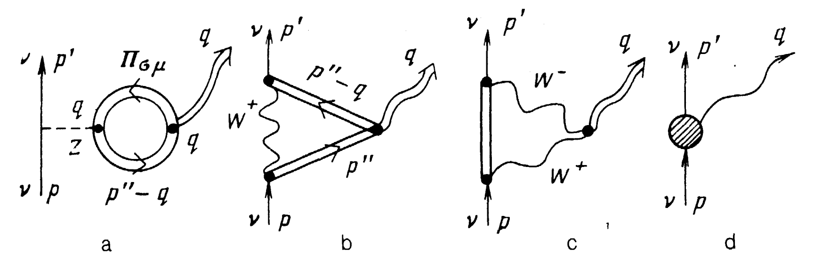

FIG. 1. The electromagnetic vertex Γ_μ of a neutrino in a medium. The double lines denote electron-positron propagators in the medium; A double wavy line is a photon in the medium (plasmon) with momentum q_μ and polarization e_μ. Diagram (d) corresponds to the vacuum counterterm in the unitary gauge used for the vector fields.

trino electromagnetic vertex, to obtain the complete electromagnetic structure of the neutrino in an isotropic medium (including vacuum). In contrast to the results of Refs. 1 and 2 there is an added contribution corresponding to interaction of the neutrino with transverse electromagnetic fields. We analyze in detail the physical meaning of the additional term associated with the induced magnetic moment of the neutrino[16] and calculated by means of the Lamb vertex part (see Fig. 1b).

In Section 3, using the obtained vertex Γ_μ and the Green function of a plasmon in a medium,[15] we first obtain the matrix element of electromagnetic νe scattering (in a medium) in lowest order in the weak interaction constant G_F. Then we calculate the complete average cross section $\langle\sigma_{\nu e}\rangle$ and average energy loss of neutrinos $\langle dE^{(\nu)}/dl\rangle$. The latter are compared with the Born values $\langle\sigma_B^{\nu e}\rangle$ and $\langle dE_B^{(\nu)}/dl\rangle$ averaged over the same electron distribution functions in the target medium and calculated according to the vacuum model of electroweak interactions.

In Section 4 we discuss the results and indicate possible experiments on observation of the predicted new properties of the neutrino interaction with matter.

2. ELECTROMAGNETIC STRUCTURE OF A NEUTRINO IN AN ISOTROPIC MEDIUM

The electromagnetic form factors of the neutrino are deterimined by the matrix elements of the diagrams shown in Fig. 1.

In the S-matrix element of these diagrams[4)]

$$\langle p'q|S|p\rangle = -(2\pi)^4 i\delta^{(4)}(p'+q-p)(4E_pE_{p'}N_{l,\,tr})^{-1}I_\mu e^\mu$$

the electromagnetic current $I_\mu = \Sigma_{n=1}^4 I_\mu^{(n)}$ is determined by the sum of the following contributions:

$$I_\mu^{(a)} = -\frac{ie^3(1-4\xi)}{2\sin^2 2\theta_W}\frac{g^{\rho\sigma}-q^\rho q^\sigma/M_Z^2}{q^2-M_Z^2}$$

$$\times\int\frac{d^4p''}{(2\pi)^4}\bar{\nu}_L(p')\gamma_\rho \operatorname{Sp}\{\gamma_\sigma G(\mathbf{p}'',p_0'') \tag{2}$$

$$\times\gamma_\mu G(\mathbf{p}''-\mathbf{k},p_0''-\omega)\}\nu_L(p),$$

$$I_\mu^{(b)} = -\frac{ie^3}{2\xi}\int\frac{d^4p''}{(2\pi)^4}[\bar{\nu}_L(p')\gamma_\rho G(\mathbf{p}''-\mathbf{k},p_0''-\omega)$$

$$\times\gamma_\mu G(\mathbf{p}'',p_0'')\gamma_\sigma\nu_L(p)]$$

$$\times\frac{g^{\sigma\rho}-(p-p'')^\sigma(p-p'')^\rho/M_W^2}{(p-p'')^2-M_W^2+i\varepsilon}. \tag{3}$$

Here the electron-positron Green function in an isotropic medium is[15]

$$G(\mathbf{p},p_0) = \frac{\hat{Q}^{(-)}(\mathbf{p})+m_e}{2E_p}$$

$$\times\left[\frac{1-(2\pi)^3f^{(-)}(E_p)/2}{E_p-\mu-p_0-i\delta}+\frac{(2\pi)^3f^{(-)}(E_p)/2}{E_p-\mu-p_0+i\delta}\right]$$

$$+\frac{\hat{Q}^{(+)}(\mathbf{p})+m_e}{2E_p}\left[\frac{1-(2\pi)^3f^{(+)}(E_p)/2}{E_p+\mu+p_0-i\delta}\right.$$

$$\left.+\frac{(2\pi)^3f^{(+)}(E_p)/2}{E_p+\mu+p_0+i\delta}\right]; \tag{4}$$

$$f^{(\mp)}(E_p) = \frac{2}{(2\pi)^3}\{\exp[(E_p\mp\mu)/T]+1\}^{-1}$$

are the equilibrium distributions of electrons and positrons with chemical potentials $\pm\mu$ and temperature T; $Q_\mu^{(\pm)}(\mathbf{p}) = \{\mp E_p,\mathbf{p}\}$ are the 4-momenta of the electrons and holes ($E_p^2-\mathbf{p}^2=m_e^2$); m_e, M_W, and M_Z are the rest masses of the electron, the W boson, and the Z boson; $\nu_L = [(1-\gamma_5)/2]\nu$ is the bispinor of left-handed neutrinos.

We do not consider here the matrix elements of diagrams (c) and (d), which together with the vacuum part of diagram (b) [see Eqs. (3) and (4)] must be taken into account in obtaining the vacuum anomalous magnetic moment of a Dirac neutrino of mass m_ν, $\Delta\mu^{\rm vac} = |e|F_2^{vac}(0)/2m_\nu$, which is equal to[17]

$$\Delta\mu^{vac} = \frac{3m_\nu m_e G_F}{4\pi^2\sqrt{2}}\mu_B. \tag{5}$$

Here $\mu_B = |e|/2m_e$ is the Bohr magneton and m_ν is the mass of the Dirac neutrino.

As will be seen below, in the same lowest (in G_F) approximation we obtain from the Lamb diagram (b), in addition to the contribution due to tensor polarization of the medium and corresponding to the existence of the electric charge (1), a new contribution corresponding to an induced magnetic moment of the normal (Dirac) type.

Indeed, integrating in Eq. (2) over the component p_0'' and calculating the traces of the γ matrices, we obtain in the approximation of small momentum transfers ($|q|^2\ll M_Z^2$) a contribution to the electromagnetic 4-current of the neutrino

$$I_\mu^{(a)} = -\frac{e(1-4\xi)G_F}{\sqrt{2}e^2}\bar{\nu}_L(p')\gamma^\rho\nu_L(p)\Pi_{\rho\mu}(\omega,\mathbf{k}). \tag{6}$$

The polarization symmetric tensor of statistical quantum electrodynamics $\Pi_{\rho\mu}(\omega,k)$, which is equal to

$$\Pi_{\rho\mu}=e^2\int\frac{d^3p''}{E_{p''}E_{p''-k}}\{[Q_\rho^{(-)}(p'')Q_\mu^{(-)}(p''-k)$$
$$+Q_\mu^{(-)}(p'')Q_\rho^{(-)}(p''-k)+(m_e^2-(Q^{(-)}(p'')Q^{(-)}(p''-k)))g_{\mu\rho}]$$
$$\times\frac{f^{(-)}(E_{p''})-f^{(-)}(E_{p''-k})}{E_{p''}-E_{p''-k}-\omega}+[Q_\rho^{(+)}(p'')Q_\mu^{(+)}(p''-k)$$
$$+Q_\mu^{(+)}(p'')Q_\rho^{(+)}(p''-k)+(m_e^2-Q^{(+)}(p'')Q^{(+)}(p''-k))g_{\mu\rho}]$$
$$\times\frac{f^{(+)}(E_{p''})-f^{(+)}(E_{p''-k})}{E_{p''}-E_{p''-k}+\omega}+[Q_\rho^{(+)}(p'')Q_\mu^{(-)}(p''-k)$$
$$+Q_\mu^{(+)}(p'')Q_\rho^{(-)}(p''-k)$$
$$+(m_e^2-(Q^{(+)}(p'')Q^{(-)}(p''-k)))g_{\mu\rho}]$$
$$\times\frac{2/(2\pi)^3-f^{(+)}(E_{p''})-f^{(-)}(E_{p''-k})}{E_{p''}+E_{p''-k}+\omega}$$
$$+[Q_\rho^{(-)}(p'')Q_\mu^{(+)}(p''-k)+Q_\mu^{(-)}(p'')Q_\rho^{(+)}(p''-k)$$
$$+(m_e^2-(Q^{(-)}(p'')Q^{(+)}(p''-k)))g_{\mu\rho}]$$
$$\times\frac{2/(2\pi)^3-f^{(+)}(E_{p''-k})-f^{(-)}(E_{p''})}{E_{p''}+E_{p''-k}-\omega}\Bigg\},\qquad(7)$$

has been calculated previously, for example, in Ref. 15, and is completely determined by the longitudinal $\varepsilon_l(\omega,k)$ and transverse $\varepsilon_{tr}(\omega,k)$ dielectric permittivities of the medium.

More precisely, in the rest system of the isotropic medium as a whole ($\mathbf{\Omega}=\mathbf{V}=0$, where Ω_μ is the unit 4-vector of the velocity of the medium, $\Omega^2=1$) the representation $\Pi_{\rho\mu}=\Pi_{\rho\mu}^{(l)}+\Pi_{\rho\mu}^{(tr)}$, is valid, where

$$\Pi_{\rho\mu}^{(l)}=-q^2(\varepsilon_l-1)e_\rho e_\mu,\qquad \Pi_{00}^{(tr)}=\Pi_{0i}^{(tr)}=\Pi_{i0}^{(tr)}=0,$$
$$\Pi_{ij}^{(tr)}=-\omega^2(\varepsilon_{tr}-1)(\delta_{ij}-k_ik_j/k^2).\qquad(8)$$

In the same lowest approximation in G_F we obtain similarly from the matrix element (3) another contribution to the neutrino 4-current:

$$I_\mu^{(b)}=\frac{2G_Fe}{\sqrt{2}e^2}[\bar\nu_L(p')\gamma^\rho\nu_L(p)]\Pi_{\rho\mu}(\omega,\mathbf{k})$$
$$+\frac{iG_Fe}{\sqrt{2}e^2}\varepsilon_{\mu\nu\rho\sigma}[\bar\nu_L(p')\gamma_5\gamma^\nu\nu_L(p)]A^{\rho\sigma}(\omega,\mathbf{k}),\qquad(9)$$

where $\varepsilon_{\mu\nu\rho\sigma}$ is the unit completely antisymmetric Levi-Civita tensor, and the antisymmetric tensor $A_{\rho\sigma}$ is equal to

$$A_{\rho\sigma}=e^2\int\frac{d^3p}{E_{p''}E_{p''-k}}\Big\{[Q_\rho^{(-)}(p'')Q_\sigma^{(-)}(p''-k)$$
$$-Q_\sigma^{(-)}(p'')Q_\rho^{(-)}(p''-k)]\frac{f^{(-)}(E_{p''})-f^{(-)}(E_{p''-k})}{E_{p''}-E_{p''-k}-\omega}$$
$$+[Q_\rho^{(+)}(p'')Q_\sigma^{(+)}(p''-k)-Q_\sigma^{(+)}(p'')Q_\rho^{(+)}(p''-k)]$$
$$\times\frac{f^{(+)}(E_{p''})-f^{(+)}(E_{p''-k})}{E_{p''}-E_{p''-k}+\omega}+[Q_\rho^{(+)}(p'')Q_\sigma^{(-)}(p''-k)$$
$$-Q_\sigma^{(+)}(p'')Q_\rho^{(-)}(p''-k)]\frac{2/(2\pi)^3-f^{(+)}(E_{p''})-f^{(-)}(E_{p''-k})}{E_{p''}+E_{p''-k}+\omega}$$
$$+[Q_\rho^{(-)}(p'')Q_\sigma^{(+)}(p''-k)-Q_\sigma^{(-)}(p'')Q_\rho^{(+)}(p''-k)$$
$$\times\frac{2/(2\pi)^3-f^{(+)}(E_{p''-k})-f^{(-)}(E_{p''})}{E_{p''}+E_{p''-k}-\omega}\Big\}.\qquad(10)$$

Calculation of the tensor (10) in the form $A_{\rho\sigma}=A(q_\rho\Omega_\sigma-q_\sigma\Omega_\rho)$ corresponding to conservation of the 4-current (9) ($q^\mu I_\mu^{(b)}=0$) leads to a value of the coefficient A equal to (in the rest system of the medium as a whole $\Omega_\mu=\delta_{\mu0}$)

$$A(\omega,k)=\frac{2\pi e^2q^2}{k^3}\int_0^\infty p\,dp[f^{(-)}(E_p)-f^{(+)}(E_p)]$$
$$\times\left\{\ln\left|\frac{(2pk-q^2)^2-4E_p{}^2\omega^2}{(2pk+q^2)^2-4E_p{}^2\omega^2}\right|+\frac{\omega}{2E_p}\ln\left|\frac{q^4-4(pk-E_p\omega)^2}{q^4-4(pk+E_p\omega)^2}\right|\right\}.\qquad(11)$$

It is not difficult to show that the vacuum contribution of the tensor (10) to the second term of the current (9) vanishes. In the rest system of the medium as a whole this term has only a 3-vector part:

$$I_i=-\frac{2ieG_FA(\omega,k)}{2^{1/2}4\pi\alpha}e_{ikl}k_l\nu_L{}^+\Sigma_k\nu_L,\qquad(12)$$

which corresponds to interaction of the spin of a left-handed neutrino

$$\Sigma=\begin{pmatrix}\sigma&0\\0&\sigma\end{pmatrix}$$

with a transverse field [see also Eq. (17) below]. It is obvious that the 3-current (12) does not interact with a longitudinal plasmon, the radiation of which was discussed in Ref. 1. The difference of the distribution functions $[f^{(-)}(E_p)-f^{(+)}(E_p)]$, which corresponds to opposite contributions of electrons and positrons to the electromagnetic current (12), distinguishes the new dispersion characteristic (11) from the well-known expressions for $\varepsilon_{l,tr}(\omega,k)$,[15] in which the contributions of electrons and positrons are summed. Let us make clear the physical meaning of the current (12).

Let the neutrino momentum be directed along the z axis (the spin is directed opposite to the momentum if there is no neutrino mass, $m_\nu=0$). Then, taking into account that the quantity (11) is proportional to $f^{(-)}(E_p)-f^{(+)}(E_p)$, i.e., that the contributions of the induced $e^\pm$ charges to the current (12) are opposite, we arrive at the conclusion that the electrons and positrons of the medium (the ions in a plasma) are rotated in the same direction. As a result (see Eq. (12)) the azimuthal current $\mathbf{I}=(0,0,I_\varphi^{(-)}+I_\varphi^{(+)})$ in vacuum or in a hot quantum gas ($f^{(-)}(E_p)=f^{(+)}(E_p)$; $\mu=0$) is absent ($I_\varphi^{(-)}=-I_\varphi^{(+)}$. We recall that for current compensation, $\Sigma_a\mathbf{I}_a=0$, it is necessary either that charges of one sign (with identical current density vn) move in opposite directions—the neutralization of an electron beam by the induced reverse current of the electrons of the background plasma occurs in this way—or that, as in the present case, charges of opposite sign must move in the same direction, and again one requires the equality $|I^{(+)}|=|I^{(-)}|$.

In a real medium the circular current I_i is nonzero and creates an induced neutrino magnetic moment[16]

$$\mu=\frac{1}{2}\int d^3r[\mathbf{rI}].$$

The value of this moment can be obtained by means of Eqs. (11) and (12);

$$\mu_\nu^{\text{ind}}=\frac{2^{1/2}G_FA(0,0)m_e}{4\pi\alpha}\mu_B.\qquad(13)$$

The magnetic moment (13) would have to be called normal

(Dirac) in the sense that its coupling with the induced (effective) neutrino charge (1)

$$\mu_\nu^{\text{ind}} = \frac{e_\nu^{\text{ind}}}{2m_e}\frac{2}{1+4\xi} \tag{13'}$$

is canonical in the special case $\theta_W = 30°$ when there is no contribution of diagram 1a, i.e., the weak vector charge of the electron is equal to zero. The only difference is in the replacement of the neutrino mass m_ν by the mass m_e of the electrons which are attracted to the neutrino.

It is important to mention that, in contrast to the case of the anomalous magnetic moment,[17] the magnetic moment (13) is due to the current (12), which does not change the helicity of the neutrino. In this sense this confirms the analogy noted above with the normal magnetic moment of the electron, for which in the absence of radiative corrections ($\mu_{\text{anom}}^{(e)} = 0$) the helicity in an external magnetic field is also conserved. Finally, in contrast to Ref. 13, in interaction of the current of an ultrarelativistic neutrino (12) with a magnetic field $\mathbf{B}_0$ only the longitudinal part of the field $\mathbf{pB}_0/p$ enters.

Note the large value of the induced magnetic moment (13) in comparison with the vacuum moment (5) in the case of real media such as metals or a degenerate ultrarelativistic gas.[16] In the latter case the quantity (13), which is $\sim 10^{-10}$ μ_B, is comparable with the values of anomalous magnetic moment which are obtained in extended models with right-handed currents.[12,17]

If the neutrino has a Dirac mass $m_\nu \neq 0$, then in the electromagnetic vertex Γ_μ it is necessary to take into account also the vacuum contribution (5). Then, separating the bispinor contributions $\nu = \nu_L + \nu_R$, assuming (6) and (9), and taking into account (12), we finally obtain in lowest order in G_F the electromagnetic vertex of a neutrino in an isotropic medium:

$$\Gamma_\mu = \frac{G_F(1+\gamma_5)}{2^{1/2}8\pi\alpha}\left[(1+4\xi)\gamma^\rho\Pi_{\rho\mu}(\omega,\mathbf{k}) + \delta_{\mu i}A(\omega,k)2ie_{ikl}k_l\gamma_5\gamma_k\right] + \frac{i\Delta\mu^{vac}}{e}\sigma_{\mu\nu}q^\nu. \tag{14}$$

The terms in square brackets in Eq. (14) correspond to interaction with the medium of left-handed neutrinos in the standard model of weak interactions used. At the same time this contribution, in contrast to the last term in (14), does not disappear in the limit $m_\nu \to 0$.

3. νe SCATTERING IN A DISPERSIVE MEDIUM

Let us consider the S-matrix element

$$\langle p_1p_2|S-1|p_3p_4\rangle = -\frac{(2\pi)^4 i\delta^{(4)}(p_1+p_2-p_3-p_4)}{4(E_1E_2E_3E_4)^{1/2}}\langle p_1p_2|T|p_3p_4\rangle,$$

which corresponds to the diagrams of Fig. 2.

We neglect here the contribution of diagrams corresponding to radiative corrections (RC) to the electron current (and to the Compton correction with two-boson exchange)[18] and assuming (as is confirmed by the subsequent calculations) that the main contribution is from the pole diagram 2a.

Without writing out the well known expression for the Born amplitude (Fig. 2b), we consider only the contribution of diagram 2a, for which we have

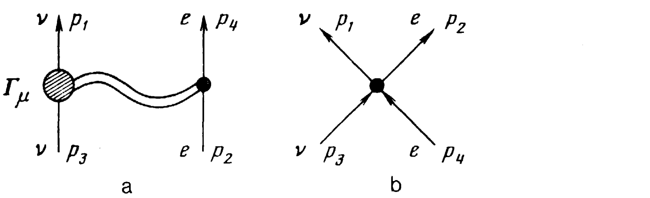

FIG. 2. Feynman diagrams of low-energy νe scattering: (a) scattering through a plasmon with inclusion of the neutrino electromagnetic vertex (14); (b) the point approximation of νe scattering in the standard model of electroweak interactions.

$$\langle p_1p_2|T^a|p_3p_4\rangle = \frac{e^2}{(4\pi)^{1/2}}\bar{\nu}(\mathbf{p}_1)\Gamma^\mu\nu(\mathbf{p}_3)D_{\mu\nu}\bar{u}(\mathbf{p}_2)\gamma^\nu u(\mathbf{p}_4). \tag{15}$$

Here the Green function of the photon in an isotropic medium (plasmon) in the Feynman gauge is[15]

$$D_{\mu\nu}(\omega,\mathbf{k}) = -\frac{g_{\mu\nu}-q_\mu q_\nu/q^2}{\omega^2\varepsilon_{tr}-k^2} + \frac{\omega^2}{k^2}\left(\frac{1}{q^2\varepsilon_l} - \frac{1}{\omega^2\varepsilon_{tr}-k^2}\right)\left[\frac{q_\mu q_\nu}{q^2} + \frac{q^2}{\omega^2}\delta_{\mu0}\delta_{\nu0} - \frac{q_\mu\delta_{\nu0}+q_\nu\delta_{\mu0}}{\omega}\right]. \tag{16}$$

In this case we use the form of the plasmon Green function in the rest system of the medium as a whole with replacement of the 4-velocity of the medium $\Omega_\mu \to \delta_{\mu0}$ and with use of the Doppler formulas $\omega = (q'\Omega)$, $k = [q'^2 - (q'\Omega)^2]^{1/2}$, where $q'_\mu = (\omega',\mathbf{k}')$ is the 4-vector of the plasmon in the system of reference moving with the medium ($\mathbf{V}\neq 0$).

In vacuum ($\varepsilon_l = \varepsilon_{tr} = 1$) with inclusion of Eqs. (8), (11), and (14) the matrix element (15) describes the previously studied[10] interaction of the anomalous (vacuum) magnetic moment of the neutrino with an electron. The contribution of vacuum polarization would be the next (in α) correction to the vacuum interaction of the magnetic moment (5) with an electron.

The matrix element (15) finally has the form

$$\langle p_1p_2|T^a|p_3p_4\rangle = \frac{e^2}{(4\pi)^{1/2}}\left\{\frac{F_\parallel(\omega,k)}{k^2\varepsilon_l(\omega,k)}[\bar{\nu}_L(\mathbf{p}_1)\gamma_0\nu_L(\mathbf{p}_3)][\bar{u}(\mathbf{p}_2)\gamma_0 u(\mathbf{p}_4)] + \frac{F_\perp(\omega,k)}{\omega^2\varepsilon_{tr}-k^2}\left(\delta_{ik}-\frac{k_ik_k}{k^2}\right)[\bar{\nu}_L(\mathbf{p}_1)\gamma_i\nu_L(\mathbf{p}_3)][\bar{u}(\mathbf{p}_2)\gamma_k u(\mathbf{p}_4)] + \frac{2G_FA(\omega,k)ie_{ikl}k_l}{2^{1/2}4\pi\alpha(\omega^2\varepsilon_{tr}-k^2)}[\bar{\nu}_L(\mathbf{p}_1)\gamma_5\gamma_k\nu_L(\mathbf{p}_3)][\bar{u}(\mathbf{p}_2)\gamma_i u(\mathbf{p}_4)] + \frac{\Delta\mu^{vac}(i\bar{\nu}_R\sigma_{\mu\nu}q^\nu\nu_L + \text{H.c.})}{e(\omega^2\varepsilon_{tr}-k^2)}[\bar{u}(\mathbf{p}_2)\gamma^\mu u(\mathbf{p}_4)]\right\}. \tag{17}$$

Here we have made use of the separation of the polariztion tensor (8) with the notation

$$F_\parallel(\omega,k) = G^V q^2(\varepsilon_l-1)/4\pi\alpha 2^{1/2} \tag{18}$$

for the electric form factor of the neutrino introduced in Ref. 1 and

$$F_\perp(\omega,k) = \frac{G^V\omega^2(\varepsilon_{tr}-1)}{4\pi\alpha 2^{1/2}} \tag{19}$$

for the transverse part, which has not been used previously but which is formally present in the same electromagnetic structure of the neutrino. The form factor (19) can be obtained from the quantum kinetic equation for the neutrino[7] in the same way as the form factor (18). The further calculations of the cross section are standard. Simultaneously with the calculations of the average cross section we calculate the average loss of a massless neutrino.

For νe scattering the averaging of the contribution of the Born diagram (Fig. 2b) is given by the well known formula ($m_\nu = 0$)

$$\langle\sigma_B{}^{\nu e}\rangle = \frac{2G_F{}^2}{\pi n_0}\int d^3p_4 f^{(e)}(E_4)\int_0^\infty \frac{p_2{}^2\,dp_2}{E_2}\left[1-\frac{(2\pi)^3}{2}f^{(e)}(E_2)\right]$$
$$\times\int\frac{d\cos\theta_{\mathbf{p}_2,\mathbf{p}_3+\mathbf{p}_4}}{|\mathbf{p}_3+\mathbf{p}_4-\mathbf{p}_2|}\delta(|\mathbf{p}_3+\mathbf{p}_4-\mathbf{p}_2|+E_2-E_3-E_4)\left\{g_L{}^2(p_1p_2)\right.$$
$$\left.+\frac{g_R{}^2(p_2p_3)(p_1p_4)}{(p_3p_4)}-\frac{m_e{}^2g_Rg_L(p_1p_3)}{(p_3p_4)}\right\}, \tag{20}$$

where

$$f^{(e)}(E) = \frac{2}{(2\pi)^3}\left[\exp\left(\frac{E-\mu}{T}\right)+1\right]^{-1} \tag{21}$$

is the equilibrium distribution function of electrons in the target in the laboratory system, which coincides with the rest system of the electron gas as a whole; $n_0 = \int d^3pf^{(e)}(E)$ is the density of electrons in the material. For electron antineutrinos it is necessary in Eq. (20) to make the replacement $g_L \rightleftarrows g_R$.

We shall not write out the expressions for the average energy loss of neutrinos $\langle dE^{(\nu)}/dl\rangle$, which are calculated with a formula such as (20) without division by the density n_0, but with inclusion in the integrand of the energy transferred to the electrons $(E_2 - E_4)$ (see Table I).

The contributions to the total cross section of interference $(T^{(a)+}T^{(b)} + T^{(b)+}T^{(a)}$ and also of all transverse interactions in the amplitude (17) turn out to be less than the contribution of the first term in (17), which corresponds to a "Coulomb" interaction with exchange of a longitudinal plasmon (see Fig. 2a). Compare, for example, the contributions of the first and last (vacuum) terms in the νe scattering amplitude (17). In the standard model the ratio of these contributions turns out to be small for two reasons. First, the ratio of the anomalous magnetic moment (5) to the value of the form factor (18) is small, i.e., $\Delta\mu^{\rm vac}|q|/e_\nu^{\rm ind}\sim am_\nu|q|r_D^2 \ll 1$. Here the induced neutrino charge in the medium $e_\nu^{\rm ind}$ is given by Eq. (1). In addition, both the Debye radius ($r_D \sim \mathrm{keV}^{-1}$ for metals) and the magnitude of the dispersion of the weakly damped Langmuir waves $kr_D \ll 1$ are rather small. Second, the main reason is associated with the pole nature of the longitudinal interaction in an isotropic medium, in which in the first term of (17) there is a resonance of the transferred energy $\omega = E_2 - E_4$ with the frequency of longitudinal oscillations ($\mathrm{Re}\,\varepsilon_l = 0$). The remaining terms, which correspond to the transverse interaction, do not have poles.[5)]

We note that in an anisotropic medium, in particular in a magnetoactive plasma, this situation can change since for extraordinary transverse waves the refractive index $n = k/\omega_{tr} = \varepsilon_{tr}^{1/2}$ can be greater than unity.[6] However, even in this case the contributions of the medium associated with the transverse form factor (19) and the induced magnetic moment (13) remain the principal ones.

Here we give the final expression for the indicated leading contribution to the average cross section for electromagnetic νe scattering obtained by means of the first term in (17) (again $m_\nu = 0$):

$$\langle\sigma_{em}\rangle = \frac{(G^V)^2}{4\pi n_0}\int_{-\infty}^{\infty} d^3p_4 f^{(e)}(E_4)\int_0^\infty \frac{p_2{}^2\,dp_2}{E_2}\left[1-\frac{(2\pi)^3}{2}f^{(e)}(E_2)\right]$$
$$\times\int\frac{d\cos\theta_{\mathbf{p}_2,\mathbf{p}_3+\mathbf{p}_4}}{|\mathbf{p}_3+\mathbf{p}_4-\mathbf{p}_2|}\delta(|\mathbf{p}_3+\mathbf{p}_4-\mathbf{p}_2|+E_2-E_3-E_4)$$
$$\times\frac{[2E_2E_4+q^2/2][2E_3(E_3+E_4-E_2)+q^2/2]}{(p_3p_4)}\frac{\mathrm{Re}\,\varepsilon_l(\omega,k)-1}{k^4|\mathrm{Im}\,\varepsilon_l|}$$
$$\times q^4\delta(\mathrm{Re}\,\varepsilon_l). \tag{22}$$

Here $f^{(e)}(E_4)$ is the distribution function (21) and $\mathrm{Im}\,\varepsilon_l$ is the imaginary part of the dielectric permittivity.

Calculations of the cross sections (20) and (22) and of the corresponding average loss values were carried out for various media. We give below the results for a degenerate nonrelativistic plasma of metals ($p_{F_e} \ll m_e$), where, first of all, there is an increase of the energy loss of neutrinos which

TABLE I. Energy loss of neutrinos in an isotropic plasma of metals ($p_F \ll m_e$); $y = m_e\omega_p/2p_FE_3$; $F_3 = E_\nu$ is the energy of the incident neutrinos.

Range of E_3	$\langle dE_B/dl\rangle$	$\langle dE_{em}/dl\rangle$	$\frac{\langle dE_{em}/dl\rangle}{\langle dE_B/dl\rangle}$
$p_F \ll m_e \ll E_3$	$\frac{G_F^2}{3\pi^3}E_3^2 m_e p_F^3\left(g_L^2+\frac{g_R^2}{6}\right)$	$\frac{(G^V)^2}{3\pi^2}p_F^3\frac{\omega_p^3 m_e}{\nu}$	$22\frac{\omega_p}{\nu}\left(\frac{\omega_p}{E_3}\right)^2 \sim 10^{-7}$
$p_F \ll E_3 \ll m_e$	$\frac{4G_F^2}{3\pi^3}\frac{E_3^4p_F^3}{m_e}\left(g_L^2+g_R^2-\frac{4}{3}g_Lg_R\right)$	$\frac{(G^V)^2}{3\pi^2}\frac{p_F^3\omega_p^3 m_e}{\nu}$	$17\left(\frac{m_e\omega_p}{E_3^2}\right)^2\frac{\omega_p}{\nu}$
$E_3 \ll p_F \ll m_e$	$\frac{4G_F^2}{3\pi^3}\frac{E_3^4p_F^3}{m_e}\left(g_L^2+g_R^2-\frac{4}{3}g_Lg_R\right)$	$\frac{(G^V)^2}{2\pi^2}\frac{m_e\omega_p^3}{\nu}E_3^2p_F \times 4y(1-y)$	$\frac{\omega_p}{\nu}200y^3(1-y)\sim 10^3$

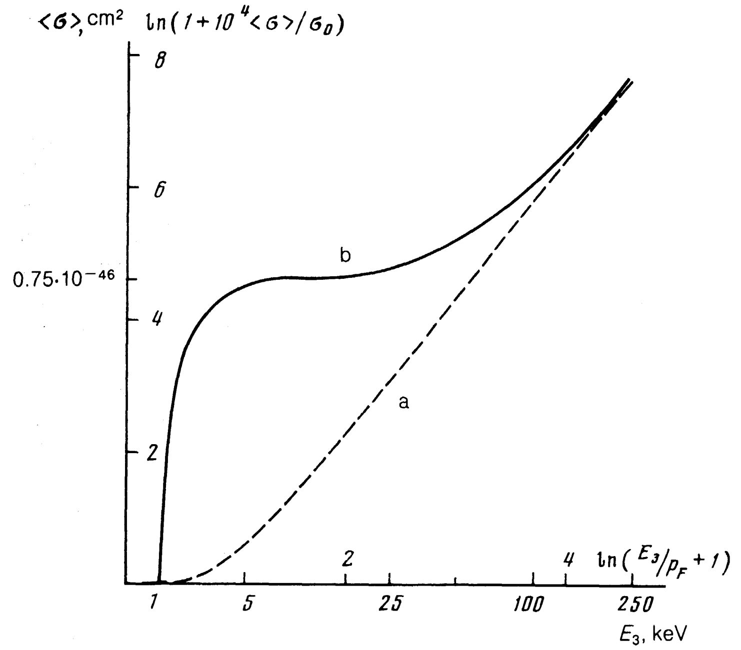

FIG. 3. Cross section for low-energy νe scattering in an isotropic medium: (a) the Born cross section (20); (b) the total cross section, with inclusion of (22).

have electromagnetic properties (14) and, second, there is the possibility of carrying through to completion analytical calculations.

In Fig. 3 we have shown on a logarithmic scale curves of the dependence on the energy E_3 of the incident neutrino for the following quantities: 1) the total cross section for νe scattering in a metal obtained by means of (20) and (22) without inclusion of interference of diagrams (a) and (b) in Fig. 2; 2) the Born cross section (20) with averaging over the same state of the electrons of the medium.

In the table for various energy ranges of the incident neutrino energy E_3 we have given analytical expressions for the neutrino energy loss in a metal calculated both by means of the amplitude (17) and a formula similar to (22) ($\langle dE_{em}^{(\nu)}/dl\rangle$), and for the Born result ($\langle dE_B/dl\rangle$) with averaging over the same state of the electrons of the medium considered. The ratio of the loss values is given in the last column of the table.

4. DISCUSSION OF RESULTS

It is not difficult to see from Fig. 3 that with increase of the DeBroglie wavelength for neutrinos $\lambda_\nu = E_3^{-1}$, when the latter becomes comparable with the average distance between the particles of the medium $n_0^{-1/3} \sim p_F^{-1}$, there is a resonance enhancement of the cross section for νe scattering and accordingly an increase of the neutrino energy loss (see the table). This enhancement of the interaction is due to excitation in the medium of collective oscillations: first at the Langmuir frequency ω_p, and then, for neutrinos of lower energies, at the ion-sound frequency ω_i. The corresponding Čerenkov resonance condition for the energy transfer $\omega = E_2^{(e)} - E_4^{(e)} = E_3^{(\nu)} - E_1^{(\nu)}$, which has the form $\omega \approx \omega^{(l)}$, where $\omega^{(l)}$ is the characteristic frequency of longitudinal excitations of the medium, imposes definite kinematic restrictions on the energy and momentum of the recoil electron. It is easy to see that the kinetic energy of the recoil electron $W^{(e)} = E_2^{(e)} - m_e$ with allowance for the degenerate state of the electrons in a metal ($0 \leqslant p_4 \leqslant p_F$; $p_2 \geqslant p_F$) is restricted to the interval

$$\max\{\omega^{(l)}, p_F^2/2m_e\} \leqslant W^{(e)} \leqslant \omega^{(l)} + p_F^2/2m_e. \tag{23}$$

The electron scattering angle ($\theta_{\mathbf{p}_2\mathbf{p}_3}$) relative to a neutrino with momentum $\mathbf{p}_3$ is fixed, for example, in the energy range $p_F \ll E_3^{(\nu)} \ll m_e$ by the requirement

$$|\cos\theta_{\mathbf{p}_2\mathbf{p}_3}| \leqslant [1 - \omega_{p_e}/W^{(e)}]^{1/2},$$

which signifies scattering of electrons at large angles and excludes forward scattering in a medium ($\omega_p \neq 0$).

In this range of neutrino energy the differential cross section for the electromagnetic channel of νe scattering (the diagram of Fig. 2a)

$$\frac{\partial^2\langle\sigma_{em}\rangle}{\partial W^{(e)}\partial\cos\theta_{p_2p_3}} = \frac{(G^V)^2}{\pi^2 n_0}\frac{m_e^2\omega_p^2}{2\nu}(2m_e W^{(e)})^{1/2} \tag{24}$$

is at least three orders of magnitude greater than the Born differential cross section determined by means of the matrix element of the diagram (b) of Fig. 2.

Large electron scattering angles are determined by small momentum transfers to the electrons from the neutrinos [see Eq. (23)] i.e., by the peripheral interaction of the colliding particles with exchange of a longitudinal plasmon. Since the neutrino electric form factor (18) already is proportional to the square q^2 of the 4-momentum transfer, the differential cross section (24) naturally differs from the standard Coulomb cross section in a different dependence on the energy transfer W($W^{1/2}$ instead of W^{-2}).

For numerical estimates of the cross sections and energy loss we set the ratio ω_p/ν equal to about 30, as follows for a metallic plasma in the case of copper in the isotropic approximation and under normal conditions. Actually the Langmuir frequency $\omega_{p_e} = 5.65\cdot 10^4 n_e^{1/2}$ ($n_e \sim 8\cdot 10^{22}$) is about 1.6.10[16] Hz, while the frequency of electron-ion collisions ($\nu = \nu_{ei}$) can be determined from the static ($\omega \ll \nu$) electrical conductivity of copper under normal conditions ($\sigma = \omega_{p_e}^2/\nu$, from which we obtain $\nu \sim 5\cdot 10^{14}$ Hz.

However, this ratio can be substantially increased with reduction of the temperature, when the frequency of electron-phonon collisions in a metal decreases in proportion to the temperature T.

A separate article will be devoted to investigation of the influence of the anisotropic structure of a crystal on the cross section for νe scattering, and also of the influence of an external magnetic field.

On turning on an external magnetic field we can expect both an increase of the induced magnetic moment (13) and appearance of poles of the Green function of transverse plasmons, in contrast to the isotropic case discussed here with a dominant role of longitudinal plasmons in electromagnetic νe scattering. The most appropriate experiment in νe scattering at low energies may be detection of recoil electrons in crystals (at low temperatures) irradiated by a flux of antineutrinos from a tritium source. In a thermal reservoir the heating of the sample by recoil electrons by an amount of the order of 10^{-3} K is real only in the case of an increase in the cross section for electromagnetic νe scattering which is still larger than in the case of an isotropic medium studied here.

In conclusion we mention that in vacuum the electromagnetic vertex (14) for neutrinos disappears in the limit[6)] $q_2 \to 0$ (the electric charge in vacuum disappears, $e_\nu^{med} \to 0$). Therefore in calculation of the RC in low-energy νe scattering in Ref. 18 only the RC to the electron current (and two-boson Compton exchange) were taken into account. In a medium, on the other hand, a dominant role at low energies is played by the vertex (14), the contribution of which is already nonvanishing for soft photons in a medium with $q^2 \neq 0$ (plasmons).

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[3)]Equation (13') serves as an independent determination of the "charge" (1).

[4)]Here for definiteness we can indicate a normalization of the wave function of the longitudinal plasmon $N_l = (q^2 \partial \operatorname{Re} \varepsilon_l / \partial \omega)_{\omega = \omega_l}$ (ε_l is the longitudinal permittivity) and its polarization $e_\mu = (k, \omega \hat{\mathbf{k}})/\sqrt{q^2}$ (see Refs. 1 and 2).

[5)]In an isotropic cold plasma the spectrum of transverse waves $\omega_{tr} = (\omega_p^2 + k^2)^{1/2} > k$ corresponds to a refractive index $n = k/\omega_{tr} = \varepsilon_{tr}^{1/2} < 1$. Therefore the propagator $[\omega^2 \varepsilon_{tr} - k^2]^{-1}$ does not vanish for any values of the space-like 4-vector q_u ($q^2 < 0$). For scattering forward here there are no singularities in the vacuum (the propagator is equal to $\omega_p^{-2} \neq 0$). However, for longitudinal waves ($\omega_l = \omega_p$) a Cerenkov resonance with neutrinos is possible, $\omega \approx \mathbf{kc}$ ($\omega = E_3(\mathbf{p}) - E_1(\mathbf{p} - \mathbf{k})$, $|\mathbf{k}| \ll |\mathbf{p}|$) since the phase velocity of a longitudinal wave ($\omega = \omega_l$) can be less than (or equal to) the velocity of light ($\omega_l / k = v_{ph} \leqslant 1$).

[6)]The permittivities are exactly equal to unity: $\varepsilon_l = \varepsilon_{tr} = 1$, since in the limit $q^2 \to 0$ the QED polarization operator disappears, $P(0) = P'(0) = 0$. In addition, $m_\nu = 0$.

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