Hydrodynamical theory of collective motions of nuclear matter in interaction of nuclei with higher-energy hadrons

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It is shown that in passage of a particle through nuclear matter, two types of collective motion arise: a shock wave (the Mach cone) and a jet which is formed in a narrow region directly behind the traveling particle. The medium behind the Mach cone remains cold, while the temperature in the region of the jet reaches values of the order 70 MeV.

1. INTRODUCTION

In nuclear physics in recent years a continuously increasing role has been played by processes with large transfer of energy and momentum to a nucleus. Such processes arise, for example, in the collision of heavy ions of high energy. Here a large number of degrees of freedom are excited and the statistical and macroscopic properties of the nuclear systems become of primary importance. One of the manifestations of the macroscopic properties of nuclear systems is the possibility of the appearance in nuclear matter of collective motions of a hydrodynamical type, in particular shock waves[1] and jet flows.[2] As a result the nuclear matter may turn out to be at high densities and high temperatures, which will permit study of the equation of state of nuclear matter under these conditions and investigation of the possibility of phase transitions in it.

The present work is devoted to a discussion of the passage of a nuclear particle through matter and to investigation of the collective motions of a hydrodynamical type which arise in it. This problem apparently was investigated for the first time by Glassgold *et al.*,[1] who used an acoustic approach to demonstrate the possibility of occurrence of shock waves in nuclear matter. In Ref. 1 it was assumed that the hydrodynamical flow arising is isentropic.

Khangulyan[3] obtained a system of hydrodynamical equations which describes the behavior of nuclear matter on passage through it of a particle and which avoids the assumption that the hydrodynamical flow which arises is isentropic. It has the form

$$\frac{\partial \rho}{\partial t}+\operatorname{div}\rho\mathbf{V}=0,$$
$$\rho\left[\frac{\partial \mathbf{V}}{\partial t}+(\mathbf{V}\nabla)\mathbf{V}\right]=-\nabla p-\frac{m(\mathbf{V}-\mathbf{u}_0)}{\tau}\delta(\mathbf{x}-\mathbf{u}_0 t), \qquad (1)$$
$$\frac{\partial}{\partial t}\left(\frac{\rho\mathbf{V}^2}{2}+\rho\varepsilon\right)=-\operatorname{div}\left[\rho\mathbf{V}\left(\frac{\mathbf{V}^2}{2}+w\right)\right]-\left[\frac{m(\mathbf{V}^2-\mathbf{u}_0^2)}{2\tau}+\frac{m(\mathbf{V}-\mathbf{u}_0)^2}{\tau}\alpha\right]\delta(\mathbf{x}-\mathbf{u}_0 t),$$

where ρ, $\mathbf{V}$, p, and $\mathbf{u}_0$ are respectively the density, mass velocity, hydrodynamical pressure of the nuclear medium, and velocity of the incident particle, ε and w are the internal energy and thermal function of a unit mass, and τ and α are certain parameters, generally speaking phenomenological, which describe the transfer of energy and momentum from the incident particle to the medium. In obtaining the system of equations (1) the author of Ref. 1 neglected the loss of energy and momentum by the incident particle.

The hydrodynamical system of equations (1) differs from the hydrodynamical models[4] widely used in description of heavy-ion collisions by the presence in it of sources in the Euler equation and in the energy equation (terms proportional to $\delta(\mathbf{x}-\mathbf{u}_0 t)$. They are proportional to the difference $\mathbf{V}-\mathbf{u}_0$, for if the incident particle is moving with the hydrodynamical velocity, i.e., if $\mathbf{u}_0=\mathbf{V}$, then it will not transfer either energy or momentum to the medium. The phenomenological parameter τ is the characteristic time of transfer of momentum from the incident particle to the medium; the δ function expresses the law of motion of the incident particle. There are two sources in the energy equation. One of them, namely $m(\alpha/\tau)(\mathbf{V}-\mathbf{u}_0)^2\delta(\mathbf{x}-\mathbf{u}_0 t)$, is scalar relative to both rotations and Galilean transformations. The presence of the other term, $(m/2\tau)(\mathbf{V}^2-\mathbf{u}_0^2)\delta(\mathbf{x}-\mathbf{u}_0 t)$, which changes under Galilean transformations, is due to the source in the Euler equation. Indeed, it can be shown[3] that introduction of a source into the Euler equation automatically requires introduction of a source into the energy transport equation, so that the covariance of the system of hydrodynamical equations relative to Galilean transformations is preserved. If we assume that nuclear matter is a simple classical nucleon gas, then $\alpha\equiv 0$.[3] When inelastic processes such as pion production, excitation of nucleon resonances, and so forth are taken into account, we have the coefficient $\alpha\neq 0$. In what follows for estimates we shall everywhere set $\alpha=0$.

All derivations given in the present work refer to hadron-nucleus collisions. For application of the results to a nucleus-nucleus interaction it is necessary to perform averaging over the momentum distribution of the nucleons in the incident nucleus, assuming in the simplest case that each nucleon of the incident nucleus interacts independently.

In the present work the system of hydrodynamical equations (1) will be discussed in the acoustical approximation and stationary collective motions of hydrodynamical type of the nuclear matter will be studied. In Sec. 2 we have obtained by the Fourier transformation method a solution of the linearized system of hydrodynamical equations (1) and have shown that on passage of a nuclear particle through nuclear matter there arise in it two forms of collective motion of hydrodynamical type. First, we have a jet collective motion, the investigation of which is carried out in Sec. 3. Second, collective motions of the shock-wave type arise, which are discussed in Sec. 4. Section 5 is devoted to calculation of the temperature of the medium.

2. THE ACOUSTIC APPROXIMATION

We shall consider the system of hydrodynamical equations (1) and shall write it in a coordinate system attached to the incident particle, i.e., we shall carry out a Galilean trans-

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formation: $\mathbf{x}\to\mathbf{x}'-\mathbf{u}_0 t$, $t\to t'=t$. Then the system (1) will take the form

$$\frac{\partial\rho}{\partial t}+\operatorname{div}\rho\mathbf{V}=0,$$

$$\rho\left[\frac{\partial\mathbf{V}}{\partial t}+(\mathbf{V}\nabla)\mathbf{V}\right]=-\nabla p-\frac{m\mathbf{V}}{\tau}\delta(\mathbf{x}), \qquad (2)$$

$$\frac{\partial}{\partial t}\left(\rho\frac{\mathbf{V}^2}{2}+\rho\varepsilon\right)=-\operatorname{div}\left[\rho\mathbf{V}\left(\frac{\mathbf{V}^2}{2}+w\right)\right]-\frac{m\mathbf{V}^2}{2\tau}(1+2\alpha)\delta(\mathbf{x}).$$

We note that in a coordinate system connected with the incident particle (the direction of the velocity of which coincides with the x axis), the system of equation (2) must be supplemented by boundary conditions at infinity, namely that the medium at $x\to+\infty$ has a constant velocity $\mathbf{u}=-\mathbf{u}_0$.

In what follows we shall discuss stationary solutions of the hydrodynamical system (2), i.e., we shall assume that the hydrodynamical characteristics do not depend on t. This means that the solution of the system (1) will depend on time in the form of the combination $\mathbf{x}-\mathbf{u}_0 t$.

It is necessary to add to the system of equations (2) an equation of state which closes the system. In writing down the equation of state of nuclear matter we shall proceed from the assumption of an ideal Fermi gas of nucleons moving in some averaged potential.[5] Here we shall assume that the effective mass of the nucleons does not depend on the density of particles: $m^*=m^*(\rho)=m$ (m is the mass of a free nucleon). The equation of state for $T=0$ can be written in the form $\varepsilon=\varepsilon_c(\rho)$. Then

$$(d\varepsilon_c/d\rho)_{\rho=\rho_0}=0, \qquad (3)$$

where $\rho_0=mn_0$ is the equilibrium density of nuclei and $n_0=0.17\,\mathrm{F}^{-3}$. The velocity of sound in nuclear matter,[6] which is determined by its compressibility $K(s^2=K/9m$ for $K=210\pm30$ MeV, $s=0.15c)$, is related to the second derivative of the internal energy with respect to density:

$$s^2=\rho^2(d^2\varepsilon_c/d\rho^2)_{\rho=\rho_0}=(dp_c/d\rho)_{\rho=\rho_0}, \qquad (4)$$

where $p_c(\rho)=\rho^2(d\varepsilon_c/d\rho)$ and $p_c(\rho_0)=0$ for $\rho=\rho_0$.

For $T\neq0$ the entropy per nucleon is nonzero. In what follows, as a model for calculation of the thermal motion we shall take

$$\varepsilon=\varepsilon_{\mathrm{id}}(\rho,s)+\varepsilon_c(\rho)-\varepsilon_0(\rho),$$

where $\varepsilon_{\mathrm{id}}(\rho,s)$ is the internal energy of an ideal Fermi gas as a function of the density and entropy and $\varepsilon_0(\rho)$ is the energy of an ideal Fermi gas at $T=0$. Since the effective mass of the nucleons does not depend on the density, we have $(\rho^2\,\partial\varepsilon_{\mathrm{id}}/\partial\rho)_s=(2/3)\rho\varepsilon_{\mathrm{id}}$ and then it is possible to calculate the pressure

$$p=\left(\rho^2\frac{\partial\varepsilon}{\partial\rho}\right)_s=\frac{2}{3}\rho(\varepsilon_{\mathrm{id}}-\varepsilon_0)+p_c(\rho);$$

therefore $\varepsilon(\rho,p)$—the internal energy as a function of ρ and p—in this model has the form

$$\varepsilon(\rho,p)={}^3/_2[p-p_c(\rho)]/\rho+\varepsilon_c(\rho). \qquad (5)$$

Then the thermal function of a unit mass is written as

$$w(\rho,p)={}^5/_2p/\rho-{}^3/_2p_c(\rho)/\rho+\varepsilon_c(\rho). \qquad (6)$$

Since in what follows we shall be interested in the acoustic approximation of the hydrodynamics equations, we shall write out the change of the internal energy and the thermal function for small changes of $\rho'=\rho-\rho_0$ and $p'=p-p_0$ near the point $\rho=\rho_0$ and $p_0=0$. From Eqs. (5) and (6), the condition (3), and the definition of the velocity of sound in nuclear matter (4), it follows[1)] that

$$\varepsilon(\rho,p)=\varepsilon(\rho_0,p_0=0)+\frac{3}{2}\frac{p'}{\rho_0}-\frac{3}{2}\frac{s^2}{\rho_0}\rho', \qquad (7)$$

$$w(\rho,p)=w(\rho_0,p_0=0)+{}^5/_2p'/\rho_0-{}^3/_2s^2\rho'/\rho_0. \qquad (8)$$

Now, using Eq. (8), we can linearize the system of hydrodynamical equations (2), assuming that the perturbation of the medium which arises is small. Let $\rho=\rho_0+\rho'$, $p=p_0+p'$, $\mathbf{V}=\mathbf{u}+\mathbf{v}'$, where ρ_0, $\mathbf{u}$, and p_0 are the unperturbed values of the quantities. Then ρ', p', and $\mathbf{v}'$ satisfy the following system of equations:

$$\rho_0\operatorname{div}\mathbf{v}'+\mathbf{u}\operatorname{grad}\rho'=0,$$

$$\rho_0(\mathbf{u}\nabla)\mathbf{v}'=-\nabla p'-\frac{m\mathbf{u}}{\tau}\delta(\mathbf{x}), \qquad (9)$$

$$\rho_0\mathbf{u}\left\{(\mathbf{u}\nabla)\mathbf{v}'+\frac{5}{2}\frac{1}{\rho_0}\nabla p'-\frac{3}{2}\frac{s^2}{\rho_0}\nabla\rho'\right\}=-\frac{m\mathbf{u}^2}{2\tau}(1+2\alpha)\delta(\mathbf{x}).$$

To obtain the system of equations (9) it is necessary to use the well known procedure[9] of expanding in small quantities p', ρ', $\mathbf{v}'$ to terms of fourth order. In the expansion of the source terms, which are proportional to δ functions, it is necessary to take into account that they are quantities of the first order of smallness.

For solution of the linear system of equations (9) we shall use the method of Fourier transformation, i.e., we shall represent ρ', p', $\mathbf{v}'$ in the form of Fourier integrals:

$$\rho'=\int a_{\mathbf{k}}e^{i\mathbf{kx}}\frac{d^3k}{(2\pi)^3},\qquad p'=\int b_{\mathbf{k}}e^{i\mathbf{kx}}\frac{d^3k}{(2\pi)^3},$$

$$\mathbf{v}'=\int \mathbf{c}_{\mathbf{k}}e^{i\mathbf{kx}}\frac{d^3k}{(2\pi)^3}. \qquad (10)$$

Then the system of equations (9) is converted into an algebraic system of equations for the Fourier transforms $a_{\mathbf{k}}$, $b_{\mathbf{k}}$, and $c_{\mathbf{k}}$, the solution of which has the form

$$a_{\mathbf{k}}=-i\frac{m}{\tau}\left\{-\frac{1}{3}(1-2\alpha)\frac{\mathbf{u}^2}{s^2\mathbf{uk}}+\frac{1}{s^2}\left[s^2+\frac{1}{3}(1-2\alpha)\mathbf{u}^2\right]\frac{\mathbf{ku}}{(\mathbf{ku})^2-\mathbf{k}^2s^2}\right\},$$

$$b_{\mathbf{k}}=-i\frac{m}{\tau}\left[s^2+\frac{1}{3}(1-2\alpha)\mathbf{u}^2\right]\frac{\mathbf{ku}}{(\mathbf{ku})^2-\mathbf{k}^2s^2}, \qquad (11)$$

$$\mathbf{c}_{\mathbf{k}}=i\frac{m}{\tau\rho_0}\left[s^2+\frac{1}{3}(1-2\alpha)\mathbf{u}^2\right]\frac{\mathbf{k}}{(\mathbf{ku})^2-\mathbf{k}^2s^2}+i\frac{m\mathbf{u}}{\tau\rho_0\mathbf{uk}}.$$

From the expressions (11) one obtains the relation between the coefficients $a_{\mathbf{k}}$ and $b_{\mathbf{k}}$ of the following form:

$$a_{\mathbf{k}}=i\frac{m}{3\tau}(1-2\alpha)\frac{\mathbf{u}^2}{s^2\mathbf{uk}}+\frac{b_{\mathbf{k}}}{s^2}, \qquad (11')$$

which will be used more than once in what follows.

To obtain a solution of the system of equations (9) it is necessary to perform an inverse Fourier transformation. We shall discuss this procedure in the case of p'. For this purpose we shall use cylindrical coordinates:

$$d^3k=dk_{\|}d^2\mathbf{k}_\perp, \quad \mathbf{kx}=k_{\|}x+\mathbf{k}_\perp\mathbf{r}_\perp, \quad \mathbf{k}^2=k_{\|}^2+\mathbf{k}_\perp^2,$$

where $k_{\|}$ is the component of the vector $\mathbf{k}$ along $\mathbf{u}$, and $\mathbf{k}_\perp$ is the component of $\mathbf{k}$ perpendicular to $\mathbf{u}$. Then we can write

$$p'=i\frac{m}{\tau}\frac{s^2+{}^1/_3\mathbf{u}^2(1-2\alpha)}{\mathbf{u}^2-s^2}\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^2} \times\exp(i\mathbf{k}_\perp\mathbf{r}_\perp)\int\frac{dk_{\|}}{2\pi}\frac{k_{\|}|\mathbf{u}|}{k_{\|}^2-a^2\mathbf{k}_\perp^2}\exp(ik_{\|}x), \tag{12}$$

where $a^2=s^2/(\mathbf{u}^2-s^2)$. In writing down the expression (12) we have taken into account that $\mathbf{ku}=-k_{\|}|\mathbf{u}|$. The integrand has two poles at the points $\pm ak_\perp$. In view of causality these poles must be displaced from the real axis $k_{\|}$ to the lower half plane. Indeed, since the velocity of the incident particle $\mathbf{u}_0$, and correspondingly also the velocity of the incident flux in the coordinate system connected with the particle, are greater than the velocity of sound $s(|\mathbf{u}_0|>s)$, then the perturbation which arises from the incident particle can be manifested in the flow only downstream, and it is this which determines the displacement of the poles from the axis of integration. We note that in writing out the equations of hydrodynamics with viscosity (see Sec. 3) the poles $\pm ak_\perp$ automatically turn out to be in the lower complex half plane of $k_{\|}$. As a result we have

$$p'=\frac{m}{\tau}\left[s^2+\frac{1}{3}\mathbf{u}^2(1-2\alpha)\right]\frac{|\mathbf{u}|}{\mathbf{u}^2-s^2}\frac{I_0}{2\pi}\theta(-x), \tag{13}$$

where $\theta(-x)$ is a step function and

$$I_0=\int_0^\infty dk_\perp k_\perp\cos(xk_\perp a)J_0(k_\perp r_\perp). \tag{13'}$$

In obtaining Eq. (13) we made use of the integral representation of the Bessel function for $J_n(x)$, in order to carry out the integration over the polar angle in the $\mathbf{k}_\perp$ plane.

For calculation of the integral I_0 it is necessary to use the integral representation of the Bessel function and, changing the order of integration, it is easily possible to carry out the integration with respect to $dk_\perp$. In the remaining integral, which contains an integration over the angle $\varphi(0\leqslant\varphi\leqslant2\pi)$, we go over to integration over $z=e^{i\varphi}$, in which case the integration contour is a circle of unit radius. For $a|x|>r_\perp$ one of the poles will lie inside the integration contour, and the other outside it. For $a|x|=r_\perp$ they lie on the integration contour and compress it, and therefore at this point I_0 has a singularity. As a result of integration of (13') we have

$$I_0=\begin{cases}-|x|a(x^2a^2-\mathbf{r}_\perp^2)^{-3/2}, & a|x|>r_\perp\\ 0, & a|x|<r_\perp\end{cases} \tag{13''}$$

From the expressions (13) it follows that the perturbation from the incident particle is propagated in the Mach cone,[1] the angle of which is 2β, where

$$\sin\beta=s/u. \tag{14}$$

For determination of the pressure p' on the Mach cone (for $a|x|=r_\perp$) it is necessary in the hydrodynamics equations to take into account dissipative processes, in particular the viscosity.

Using the expression for $a_\mathbf{k}$, we obtain

$$\rho'=\frac{m}{\tau}\left\{\left[s^2+\frac{1}{3}\mathbf{u}^2(1-2\alpha)\right]\frac{|\mathbf{u}|}{2\pi s^2(\mathbf{u}^2-s^2)}I_0\theta(-x)\right.$$
$$\left.+\frac{i}{6\pi}(1-2\alpha)\frac{\mathbf{u}^2}{s^2}\int_{-\infty}^{+\infty}\frac{dk_{\|}}{|\mathbf{u}|k_{\|}}\exp(ik_{\|}x)\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^2}\exp(i\mathbf{k}_\perp\mathbf{r}_\perp)\right\}. \tag{15}$$

As the result of integration of (15) we have

$$\rho'=\frac{p'}{s^2}-\frac{1}{3}(1-2\alpha)\frac{|\mathbf{u}|}{s^2}\frac{m}{\tau}\delta^{(2)}(\mathbf{r}_\perp)\theta(-x), \tag{16}$$

where $\delta^2(\mathbf{r}_\perp)$ is a two-dimensional δ function. From the expressions (13) and (16) it follows that in addition to the well known collective motion of the hydrodynamical type—shock waves (the Mach cone),[1] an additional collective excitation of nuclear matter occurs in passage through it of a particle. This excitation is described by the second term in the expression (16) and is a hydrodynamical motion of the jet type. In order to show this, let us obtain an expression for $v'(\mathbf{x})$.

Using the expression (11) for $c_\mathbf{k}$, we can see that

$$\mathbf{v}'=i\frac{m}{\tau\rho_0}\left[s^2+\frac{1}{3}(1-2\alpha)\mathbf{u}^2\right]\mathbf{I}-\frac{m}{\tau\rho_0}\frac{\mathbf{u}}{|\mathbf{u}|}\delta^{(2)}(\mathbf{r}_\perp)\theta(-x), \tag{17}$$

where

$$\mathbf{I}=\int\frac{d^3k}{(2\pi)^3}e^{i\mathbf{kx}}\frac{\mathbf{k}}{(\mathbf{ku})^2-\mathbf{k}^2s^2}. \tag{17'}$$

For calculation of the integral $\mathbf{I}$ we shall make use of the fact that the problem has cylindrical symmetry around the x axis. Then it is possible to decompose the integral $\mathbf{I}$ into unit vectors $\mathbf{n}_u$ and $\mathbf{n}_\perp$ (where $\mathbf{n}_u=\mathbf{u}/|\mathbf{u}|$, $\mathbf{n}_\perp=\mathbf{r}_\perp/|\mathbf{r}_\perp|$, and $\mathbf{r}_\perp$ is the vector in the plane perpendicular to $\mathbf{u}$):

$$\mathbf{I}=A\mathbf{n}_u+B\mathbf{n}_\perp. \tag{18}$$

We than have

$$A=\mathbf{n}_u\mathbf{I}=iI_0\theta(-x)/2\pi(\mathbf{u}^2-s^2), \tag{19}$$

where I_0 is determined by Eq. (13'). Correspondingly, carrying out the integration, we can see that B is given by the equality

$$B=\mathbf{n}_\perp\mathbf{I}=iI_1\theta(-x)/2\pi s(\mathbf{u}^2-s^2)^{1/2}, \tag{20}$$

where

$$I_1=\int_0^\infty dk_\perp k_\perp J_1(k_\perp r_\perp)\sin(k_\perp xa). \tag{21}$$

The calculation of the integral I_1 is similar to that of the integral I_0:

$$I_1=\begin{cases}r_\perp(x^2a^2-\mathbf{r}_\perp^2)^{-3/2}, & a|x|>r_\perp\\ 0, & a|x|<r_\perp\end{cases}. \tag{22}$$

As in the case of the integral I_0, the integral I_1 has a singularity at $a|x|=r_\perp$. Collecting together the expressions (17)–(20), we finally have

$$\mathbf{v}'=\left\{-\frac{m}{2\pi\rho_0\tau}\left[s^2+\frac{1}{3}(1-2\alpha)\mathbf{u}^2\right]\frac{1}{\mathbf{u}^2-s^2}\left[I_0\mathbf{n}_u\right.\right.$$
$$\left.\left.+\frac{(\mathbf{u}^2-s^2)^{1/2}}{s}I_1\mathbf{n}_\perp\right]-\frac{m}{\rho_0\tau}\mathbf{n}_u\delta^{(2)}(\mathbf{r}_\perp)\right\}\theta(-x). \tag{23}$$

Equations (13), (16), and (23) give the solution of the posed problem of determining the hydrodynamical behavior of nuclear matter on passage of a particle through it. Here, as we have already mentioned, in the medium there is a characteristic Mach cone associated with the shock wave. In addition to the Mach cone there is an additional characteristic feature: presence of the δ-function terms ρ' and $\mathbf{v}'$. These terms describe the phenomenon of separation of the jet (or of a separated streamline),[9] i.e., the flow is characterized by a tangential break along the stream line, which extends from $x=0$ to $x=-\infty$. Along this stream line the condition of potentiality of the flow is violated. In fact, using the expression for $\mathbf{c_k}$ (11), we can easily calculate

$$\operatorname{rot}\mathbf{v}'=-\frac{m}{\tau\rho_0}\int\frac{d^3k}{(2\pi)^3}\frac{1}{\mathbf{uk}}e^{i\mathbf{kx}}[\mathbf{ku}]$$
$$=-\frac{m}{\tau\rho_0}[\mathbf{xu}]\left\{\frac{1}{\mathbf{xu}+x|\mathbf{u}|}\delta^{(3)}(\mathbf{x})+\frac{2|\mathbf{u}|}{(\mathbf{xu})^2-\mathbf{x}^2\mathbf{u}^2}\theta(-x)\delta^{(2)}(\mathbf{r}_\perp)\right\}, \qquad (24)$$

i.e., rot $\mathbf{v}'$ is nonzero along the separated streamline.

The description of the behavior of nuclear matter under the influence of a passing particle, obtained in this section in the acoustic approximation, is discontinuous. There is a discontinuity at the Mach cone and a discontinuity associated with the phenomenon of jet separation. This is due to the fact that our treatment is based on the equations of ideal hydrodynamics. Taking into account dissipative processes of viscosity and heat conduction will lead to smearing of these discontinuites and to formation of regions of finite size in which the condition of potentiality will be violated. As a result there will be formed a jet flow—a collective motion of nuclear matter.

A similar investigation of the behavior of nuclear matter was carried out previously (see for example Ref. 1). However, in writing down and solving the corresponding system of equations, it was assumed that the flow was potential over the entire region. This is equivalent to the assumption that the equation for the energy contained a source with $\alpha=0.5$. For all remaining values of α, as was shown in Sec. 3, the potentiality of the flow is violated.

3. INVESTIGATION OF JET FLOW

For investigation of the behavior of the discontinuous solutions near the singularities, we shall go over from ideal hydrodynamics to viscous hydrodynamics. We shall introduce into the equation of hydrodynamics a shear viscosity, neglecting heat conduction. This transition is accomplished by a well known method.[9] For this purpose in the momentum conservation law it is necessary instead of the momentum flux density tensor of an ideal liquid $\Pi_{ik}=p\delta_{ik}+\rho V_iV_k$ to write $\Pi_{ik}-\sigma'_{ik}$ (where σ'_{ik} determines the irreversible viscous transport of momentum[9]), and in the energy equation it is necessary to add to the heat flow, which is due to simple transport of mass, $Q_i=\rho V_i(w+\mathbf{V}^2/2)$, a term which takes into account friction: $Q_i-V_k\sigma'_{ik}$. Then the system of hydrodynamical equations will take the form

$$\frac{\partial\rho}{\partial t}+\operatorname{div}\rho\mathbf{V}=0, \qquad (25)$$

$$\rho\left(\frac{\partial V_i}{\partial t}+V_k\frac{\partial V_i}{\partial x_k}\right)=-\frac{\partial p}{\partial x_i}+\frac{\partial\sigma_{ik}'}{\partial x_k}-\frac{m(V_i-u_{0i})}{\tau}\delta(\mathbf{x}-\mathbf{u}_0t),$$
$$\frac{\partial}{\partial t}\rho\left(\varepsilon+\frac{\mathbf{V}^2}{2}\right)=-\frac{\partial}{\partial x_k}\left\{\rho V_k\left(w+\frac{\mathbf{V}^2}{2}\right)-V_i\sigma_{ik}'\right\}-\left[\frac{m(\mathbf{V}^2-\mathbf{u}_0^2)}{2\tau}+\frac{m(\mathbf{V}-\mathbf{u}_0)^2}{\tau}\alpha\right]\delta(\mathbf{x}-\mathbf{u}_0t),$$

where

$$\sigma_{ik}'=\eta\left(\frac{\partial V_i}{\partial x_k}+\frac{\partial V_k}{\partial x_i}-\frac{2}{3}\delta_{ik}\frac{\partial V_l}{\partial x_l}\right),$$

and η is the coefficient of shear viscosity. As previously, we shall supplement this system by an equation of state (7) and (8). Rewriting the resulting system of equations in the coordinate system connected with the incident particle, and linearizing it as was done for the system (1), we obtain a system of equations in the acoustic approximation with inclusion of viscosity:

$$\rho_0\operatorname{div}\mathbf{v}'+\mathbf{u}\operatorname{grad}\rho'=0,$$
$$\rho_0u_k\frac{\partial v_i'}{\partial x_k}+\frac{\partial p'}{\partial x_i}-\eta\frac{\partial}{\partial x_k}\left(\frac{\partial v_i'}{\partial x_k}+\frac{\partial v_k'}{\partial x_i}-\frac{2}{3}\delta_{ik}\frac{\partial v_l'}{\partial x_l}\right)=-\frac{mu_i}{\tau}\delta(\mathbf{x}), \qquad (26)$$
$$\rho_0u_k\left(u_i\frac{\partial v_i'}{\partial x_k}+\frac{5}{2\rho_0}\frac{\partial p'}{\partial x_k}-\frac{3s^2}{2\rho_0}\frac{\partial\rho'}{\partial x_k}\right)-\eta\left[u_i\frac{\partial}{\partial x_k}\left(\frac{\partial v_i'}{\partial x_k}+\frac{\partial v_k'}{\partial x_i}\right)-\frac{2}{3}u_i\frac{\partial}{\partial x_i}\left(\frac{\partial v_l'}{\partial x_l}\right)\right]=-\frac{m\mathbf{u}^2}{2\tau}(1+2\alpha)\delta(\mathbf{x}).$$

We shall again seek a solution of the linear system (26) by the method of Fourier transformation (10). For the Fourier transform we have an algebraic systems of equations, the solution of which has the form

$$a_\mathbf{k}=-i\frac{m}{\tau}\left\{-\frac{1}{3}(1-2\alpha)\frac{\mathbf{u}^2}{s^2\mathbf{uk}}+\frac{\rho_0s^2\mathbf{uk}+{}^1/_3(1-2\alpha)\rho_0\mathbf{u}^2(\mathbf{uk})-{}^4/_9i(1-2\alpha)\eta\mathbf{k}^2\mathbf{u}^2}{s^2[\rho_0(\mathbf{uk})^2-\rho_0\mathbf{k}^2s^2-{}^4/_3i\eta\mathbf{k}^2\mathbf{ku}]}\right\},$$
$$b_\mathbf{k}=-i\frac{m}{\tau}[{}^1/_3(1-2\alpha)\rho_0\mathbf{u}^2\mathbf{uk}+\rho_0s^2\mathbf{uk}-{}^4/_9i(1-2\alpha)\eta\mathbf{k}^2\mathbf{u}^2][\rho_0(\mathbf{uk})^2-\rho_0\mathbf{k}^2s^2-{}^4/_3i\eta\mathbf{k}^2\mathbf{ku}]^{-1}, \qquad (27)$$
$$\mathbf{c_k}=i\frac{m\mathbf{u}}{\tau}\frac{1}{\rho_0\mathbf{uk}-i\eta\mathbf{k}^2}+i\frac{m\mathbf{k}}{\tau}\frac{s^2+{}^1/_3(1-2\alpha)\mathbf{u}^2}{\rho_0(\mathbf{uk})^2-\rho_0s^2\mathbf{k}^2-{}^4/_3i\eta\mathbf{k}^2\mathbf{ku}}+i\frac{m\mathbf{k}}{\tau}\frac{i\eta[\mathbf{k}^2s^2+{}^1/_3(\mathbf{ku})^2]}{[\rho_0\mathbf{uk}-i\eta\mathbf{k}^2][\rho_0(\mathbf{uk})^2-\rho_0s^2\mathbf{k}^2-{}^4/_3i\eta\mathbf{k}^2\mathbf{ku}]}.$$

From the expressions (27) it can be seen that on taking into account viscosity the relation (11′) between the coefficients $a_\mathbf{k}$ and $b_\mathbf{k}$ is preserved, and therefore the coupling between ρ' and p' is given by Eq. (16). Therefore viscosity does not smear the singularity in the density distribution.

The coefficient $\mathbf{c_k}$ in (27) consists of three terms: the first term describes the jet collective flow; the second term, which has characteristic sound poles, is a Mach shock cone; and the last term represents the influence of the jet on the flow in general. For determination of the motion inside the jet it is necessary to carry out inverse transformations in the first term in the expression for $\mathbf{c_k}$. Introducing variables of integration $k_\parallel$ and $\mathbf{k}_\perp$, we can represent the expression for $\mathbf{v}'_{\text{jet}}$ in the form

$$\mathbf{v}'_{\rm jet} = -\frac{m\mathbf{u}}{\tau\eta}\int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2}\exp(i\mathbf{k}_\perp\mathbf{r}_\perp)$$
$$\times\int_{-\infty}^{+\infty}\frac{dk_\parallel}{2\pi}\frac{\exp(ik_\parallel x)}{k_\parallel{}^2 - i\rho_0|\mathbf{u}|k_\parallel/\eta + \mathbf{k}_\perp{}^2}. \quad (28)$$

The integrand has poles at the points

$$k_\parallel = (i/2\eta)\,[\rho_0|\mathbf{u}| \pm (\rho_0{}^2\mathbf{u}^2 + 4\eta^2\mathbf{k}_\perp{}^2)^{1/2}].$$

Here, taking into account that $k_\perp \geqslant 0$, we find that one pole lies in the uppper half plane (with the plus sign), and the second lies in the lower half plane, and both of them are on the imaginary axis, so that, closing the integration contour either in the upper half plane (for $x > 0$) or in the lower half plane (for $x < 0$), we find that

$$\mathbf{v}'_{\rm jet} = -\frac{m\mathbf{u}}{2\pi\tau}\exp\left(-\frac{\rho_0|\mathbf{u}|x}{2\eta}\right)$$
$$\times\int k_\perp\, dk_\perp J_0(k_\perp r_\perp)\,[\rho_0{}^2\mathbf{u}^2 + 4\eta^2\mathbf{k}_\perp{}^2]^{-1/2}$$
$$\times\exp\left[\frac{-|x|\,(\rho_0{}^2\mathbf{u}^2+4\eta^2\mathbf{k}_\perp{}^2)^{1/2}}{2\eta}\right]. \quad (29)$$

In Eq. (29) the integration over the polar angle in the $k_\perp$ plane is carried out by means of the Bessel integral representation for the Bessel functions. We finally obtain

$$\mathbf{v}'_{\rm jet} = -\frac{m\mathbf{u}}{4\pi\eta\tau}\frac{1}{|\mathbf{r}|}\exp\left[-\frac{\rho_0|\mathbf{u}|\,(x+|\mathbf{r}|)}{2\eta}\right], \quad (30)$$

where $\mathbf{r}^2 = \mathbf{r}_\perp^2 + x^2$.

The expression (30) gives the velocity distribution in the jet. From the mathematical point of view we have obtained a sequence of functions which gives a representation of the product of the functions $\delta^{(2)}(\mathbf{r}_\perp)\theta(-x)$:

$$\theta(-x)\,\delta^{(2)}(\mathbf{r}_\perp) = \lim_{\eta\to 0}\left\{\frac{\rho_0|\mathbf{u}|}{4\pi\eta|\mathbf{r}|}\exp\left[-\frac{\rho_0|\mathbf{u}|\,(x+|\mathbf{r}|)}{2\eta}\right]\right\}. \quad (31)$$

Actually, $\int \mathbf{v}'_{\rm jet} d^2 r_\perp$ for $x < 0$ coincides with the case of ideal hydrodynamics in which the velocity of the jet is given by a two-dimensional δ function (23), and for $x > 0$ it dies out exponentially in a mean free path.

Therefore taking velocity into account leads to a finite size of the jet in the expression for the velocity. Let us estimate the transverse dimension of the jet for a fixed value of x. From (30) we have

$$\rho_0|\mathbf{u}|\,(x+|\mathbf{r}|)/2\eta \sim 1, \quad (32)$$

from which

$$\delta_\perp = \frac{2\eta}{\rho_0|\mathbf{u}|}\left(1-\frac{x\rho_0|\mathbf{u}|}{\eta}\right)^{1/2}. \quad (33)$$

By making use of the phenomenological theory of transport coefficients, according to which $\eta = \rho\lambda\bar{c}/2$ (where λ is the mean free path and $\bar{c}$ is the average velocity of motion of the particles, and taking into account that $\bar{c}\sim s$), it is possible to show that around the incident particle a layer with characteristic dimensions λ arises, and behind the particle a jet is formed, the transverse dimensions of which are macroscopic:

$$\delta \approx 2(\eta|x|/\rho_0|\mathbf{u}|)^{1/2} \sim (|x|\lambda)^{1/2} \gg \lambda \quad (34)$$

for $|x| \gg \lambda$ and $x < 0$.

4. INVESTIGATION OF THE FLOW IN THE MACH CONE

We shall turn now to discussion of the behavior of nuclear matter near the Mach cone. For this purpose we shall return again to the expressions (27) and shall discuss how the pressure p' behaves in the case when there is viscosity in nuclear matter. According to (10) and (27), we can write

$$p' = i\frac{m}{\tau}\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^2}\exp(i\mathbf{k}_\perp\mathbf{r}_\perp)\int_{-\infty}^{+\infty}\frac{d\xi}{2\pi}\exp(ik_\perp x\xi)\frac{|\mathbf{u}|s^2}{\mathbf{u}^2-s^2}$$
$$\times\frac{(1-2\alpha)\mathbf{u}^2\xi/s^2 + 3\xi + i(1-2\alpha)\beta + (1-2\alpha)\beta\xi^2}{3(i\beta a^2\xi^3 + \xi^2 + i\beta a^2\xi - a^2)}, \quad (35)$$

where we have introduced the dimensionless quantity

$$\beta = {}^4/_3\eta|\mathbf{u}|k_\perp/\rho_0 s^2. \quad (36)$$

In addition, in writing out Eq. (35) we have gone over from the variable $k_\parallel$ to ξ, which is given by the equality $k_\parallel = \xi k_\perp$. On the assumption that $\beta a^2 \ll 1$, the poles of the integrand as a function of $k_\perp$ can be found by the method of successive approximations. As a result we have

$$\xi_{1,2} = \pm a - i\beta a^2(a^2+1)/2, \quad \xi_3 = -i/\beta a^2. \quad (37)$$

Two poles lie in the lower half plane, and one in the upper half plane. Therefore in integration over ξ for $x < 0$, when the integration contour must be closed in the lower half plane, two poles are effective:

$$p' = \frac{m}{2\pi\tau}|\mathbf{u}|a^2\int_0^\infty k_\perp dk_\perp[A\cos(k_\perp ax) + \beta B\sin(k_\perp ax)]$$
$$\times J_0(k_\perp r_\perp)\exp(-\gamma k_\perp{}^2), \quad (38)$$

where we have introduced the notation

$$A \equiv 1 + {}^1/_3(1-2\alpha)\mathbf{u}^2/s^2, \quad {}^1/_2 xa^2(a^2+1)\beta k_\perp = -\gamma k_\perp{}^2,$$
$$B \equiv {}^1/_2 Aa(3a^2+1) + {}^1/_3(1-2\alpha)(a^2+1),$$

and for $x < 0$ we have $\gamma > 0$. In the case $x > 0$ when the integration contour must be closed in the upper half plane, one pole contributes to the integral:

$$p' = \frac{m}{2\pi\tau}a^2|\mathbf{u}|\left[1+\frac{1}{3}(1-2\alpha)\right]$$
$$\times\int_0^\infty k_\perp dk_\perp J_0(k_\perp r_\perp)\exp\left\{-\frac{k_\perp x}{\beta a^2}\right\}. \quad (39)$$

Since the poles (37) were obtained in the first order in βa^2, the integration over ξ in these expressions was carried out with accuracy to βa^2. The structure of the singularities of the integrand of (38) in a finite part of the complex $k_\perp$ plane is the same as in the case of ideal hydrodynamics; see Eq. (13'). Inclusion of viscosity leads only to an additional exponential damping (a factor $\exp(-\gamma k_\perp^2)$). Therefore in what follows in investigation of the behavior of the hydrodynamical characteristics of the medium near the Mach cone for $x < 0$ we shall make two assumptions. First, taking into account that $\beta \ll 1$ (since $\beta a^2 \ll 1$, $u\sim s$), we shall discard in the square brackets the term $\beta B\sin(k_\perp ax)$. Second, we shall assume that the additional exponential damping has the form $\exp(-\gamma k_\perp)$, and not $\exp(-\gamma k_\perp^2)$. This is apparently permissible since this substitution does not change the structure of the singularities of the integrand in a finite part of the complex plane and, one can hope, will not qualitatively change the behavior of p'. It can be said that instead of a shear viscosity we are introducing some mathematical viscosity which in Fourier space leads to such an exponential damping. Therefore for $x<0$ p' will take the form

$$p'=\frac{m}{2\pi\tau}|\mathbf{u}|a^2\left[1+\frac{1}{3}(1-2\alpha)\frac{\mathbf{u}^2}{s^2}\right]I_0^\gamma, \tag{40}$$

where

$$I_0^\gamma=\int_0^\infty k_\perp dk_\perp J_0(k_\perp r_\perp)\cos(k_\perp ax)\exp(-\gamma k_\perp). \tag{41}$$

The further calculation of this integral can be carried out by standard methods. As a result we obtain

$$I_0^\gamma=-\frac{i}{2}\left[\frac{a|x|+i\gamma}{[r_\perp^2-(a|x|+i\gamma)^2]^{3/2}}-\frac{a|x|-i\gamma}{[r_\perp^2-(a|x|-i\gamma)^2]^{3/2}}\right]. \tag{42}$$

In the expression (42), the radical function is defined in such a way that the cut is directed along the negative semiaxis, and therefore it is possible to obtain finally an expression for the pressure p' in the following form:

$$p'=\frac{m}{2^{3/2}\pi\tau}\left[1+\frac{1}{3}(1-2\alpha)\frac{\mathbf{u}^2}{s^2}\right]\frac{|\mathbf{u}|a^2}{|z_0|^3}$$
$$\times\{a|x|[|z_0|-(r_\perp^2-a^2x^2+\gamma^2)]^{1/2}[|z_0|+2(r_\perp^2-a^2x^2+\gamma^2)]$$
$$-\gamma[|z_0|+(r_\perp^2-a^2x^2+\gamma^2)]^{1/2}[|z_0|-2(r_\perp^2-a^2x^2+\gamma^2)]\}, \tag{43}$$

where

$$|z_0|=[(r_\perp^2-a^2x^2+\gamma^2)^2-4a^2x^2\gamma^2]^{1/2}.$$

A distinctive feature of this expression is the existence of two shock waves following each other in the medium, in accordance with the general postulates of the theory in the case of nonplanar waves.[9,10] Indeed, the behavior of p' has the same form as the dot-dash curve in the figure: it can be seen that in the transition from the unperturbed region to the perturbed region at fixed $x(x<0)$ first there is a region of crowding—a region where $p'>0$, beyond which there is a region of rarefaction ($p'<0$) and there is a point at which the rarefaction is maximal. As a result of the effect of gradual distortion of the profile, this point will lag those located behind it, and therefore an ambiguity results. This feature of the behavior of the hydrodynamical characteristics of the medium is manifested, in the transition to ideal hydrodynamics, in the appearance of a singularity at the point $r_\perp=a|x|$. As $\eta\to 0$ we have $I_0^\gamma\to I_0$, where

$$I_0=\begin{cases}-a|x|(a^2x^2-r_\perp^2)^{-3/2}, & r_\perp<a|x|,\\ \varepsilon^{-1/2} & r_\perp=a|x|,\quad \varepsilon\to 0,\\ 0, & r_\perp>a|x|.\end{cases} \tag{44}$$

After everything that has been said, it is easy to write down also an expression for the density ρ'. Using the expression for the Fourier transform of ρ' (27), we can easily show that

$$\rho'=\rho'_{\rm jet}+\tilde\rho', \tag{45}$$

where $\rho'_{\rm jet}$ has the same form as in the case of an ideal liquid (16), whereas

$$\tilde\rho'=p'/s^2. \tag{46}$$

Let us turn now to discussion of $\mathbf{v}'$. Using the expression for the Fourier transform of $\mathbf{v}'$ (27), $\mathbf{v}'$ can be written in the form of the sum of two terms: $\mathbf{v}'=\mathbf{v}'_{\rm jet}+\tilde{\mathbf{v}}'$. The expression for $\mathbf{v}'_{\rm jet}$ was obtained in the preceding section; see Eq. (30). To obtain $\tilde{\mathbf{v}}$, we shall use all of the same approximations which were used in calculation of the pressure p'. As a result we have (for $x<0$)

$$\tilde{\mathbf{v}}'=-\frac{m}{2\pi\tau\rho_0}\,\frac{s^2+{}^1/_3(1-2\alpha)\mathbf{u}^2}{\mathbf{u}^2-s^2}\left[\mathbf{n}_uI_0^\gamma+\frac{(\mathbf{u}^2-s^2)^{1/2}}{s}\mathbf{n}_\perp I_1^\gamma\right]. \tag{47}$$

where I_0^γ is given by Eq. (41) and I_1^γ has the form

$$I_1^\gamma=\int_0^\infty k_\perp dk_\perp \exp(-\gamma k_\perp)J_1(k_\perp r_\perp)\sin(k_\perp ax). \tag{48}$$

Calculation of I_1^γ is similar to that of I_0^γ:

$$I_1^\gamma={}^1/_2 ir_\perp\{[r_\perp^2-(a|x|+i\gamma)^2]^{-3/2}+[r_\perp^2-(a|x|-i\gamma)^2]^{-3/2}\}. \tag{49}$$

As $\gamma\to 0$ we obtain $I_1^\gamma\to I_1$, where

$$I_1=\begin{cases}r_\perp(a^2x^2-r_\perp^2)^{-3/2}, & a|x|>r_\perp\\ -\infty, & a|x|=r_\perp.\\ 0, & a|x|<r_\perp\end{cases} \tag{50}$$

In Fig. 1 we have shown the dependence on $r_\perp$ for fixed $x(x<0)$ of the x component of the hydrodynamical velocity $\mathbf{v}'$. It is measured in units of the velocity of sound s, and distances are measured in characteristic units of length $l_*=\eta/\rho_0 s$. In the figure we have shown three curves: the dot-dash curve describes the behavior of the x component of the velocity $\tilde{\mathbf{v}}'$, the dashed curve describes the jet velocity

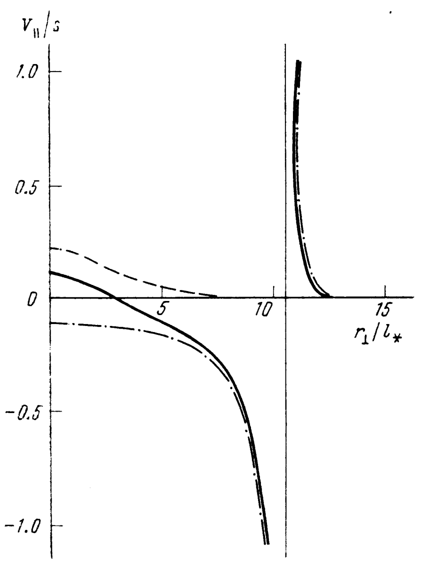

FIG. 1. Dependence of the x components of the velocities $\mathbf{v}'$ (dot-dash), $\mathbf{v}'_{\rm jet}$ (dashed), and their sum (solid curve) on $r_\perp$ for fixed x.

$\mathbf{v}'_{\rm jet}$, and finally, the solid curve is the sum of these curves. It should be mentioned that in addition to the change of sign of the x component of the velocity in the region of the Mach cone (at $r_\perp = a|x|$) the x component of the velocity changes sign at $r_\perp \approx 3$. This last change of sign is due to the presence of the jet.

In concluding this section it is necessary to mention that Eqs. (43)–(47) solve the problem of the behavior of the hydrodynamical characteristics of nuclear matter in the Mach cone. From the discussion of this and the preceding section it follows that taking into account dissipative processes removes all uncertainties of the hydrodynamical characteristics which exist in the case of ideal hydrodynamics.

5. CALCULATION OF THE TEMPERATURE OF THE NUCLEAR MEDIUM

The hydrodynamical characteristics obtained for the medium can be used to determine the change of temperature of nuclear matter produced by a particle passing through it. On the one hand, the change of the internal energy of the nuclear matter is expressed in terms of the change of the hydrodynamical characteristics of the matter by Eq. (7). On the other hand, the change of the internal energy of the nuclear matter can be represented in the form

$$\Delta\varepsilon=\Delta\varepsilon_x+\Delta\varepsilon_T, \tag{51}$$

where $\Delta\varepsilon_x$ is the change of the internal energy as the result of compression at $T=0$. From the condition (3) it follows that in the acoustic approximation $\Delta\varepsilon_x = 0$.

The quantity $\Delta\varepsilon_T$ is the change of the internal energy of a unit mass as the result of heating. According to the equation of state, this quantity is the same for nuclear matter and for an ideal Fermi gas[8]:

$$\Delta\varepsilon_T={}^3/_8\pi^2\widetilde{K}^2(T/\varepsilon_F)^2, \tag{52}$$

where ε_F is the Fermi limiting energy and $\widetilde{K}$ is the compressibility of an ideal Fermi gas at $T=0$. Introducing the notation $\varepsilon_0 = (3/5)\varepsilon_F$ and combining the expressions (51) and (52), we obtain[2)]

$$T^2=(10/3\pi)^2\varepsilon_0^2(p'-s^2\rho')/\widetilde{K}^2\rho_0.$$

Using the expression for the coupling between p' and ρ' (16) we obtain for the temperature the following expression:

$$T^2=\xi\varepsilon_0^2(1-2\alpha)\frac{m|\mathbf{u}|}{3\widetilde{K}^2\rho_0\tau}\delta^{(2)}(\mathbf{r}_\perp)\theta(-x), \tag{53}$$

where $\xi = 1.14$–1.15. Thus we see that the temperature of the medium changes only as the result of the presence of a δ function in the expression for the density, i.e., in the region of the jet, while in the Mach cone it remains unchanged. In order to estimate the temperature of the jet—the hot region of space inside the Mach cone, it is necessary somehow to introduce instead of $\delta^{(2)}(\mathbf{r}_\perp)$ the dimensions of the jet. From the preceding discussion it is evident that taking into account dissipative processes leads to a smearing of the δ-function term. However, inclusion of viscosity leads to a smearing of this term only in the velocity. For smearing of the δ-function term in the density it is necessary to take into account thermal conduction. Since thermal conduction and viscosity are determined by the same microscopic mechanism, we have $\nu\sim\chi$ (ν and χ are respectively the kinematic viscosity and the thermal conductivity of the medium), and the characteristic dimensions of the jet should be the same in order of magnitude with inclusion of viscosity and with inclusion of thermal conductivity, and therefore the characteristic size of the jet δ given by (34) can be used to determine the temperature of the jet. Then, making use of the fact that $\delta^{(2)}(\mathbf{r}_\perp) = \delta(\mathbf{r}_\perp)/\pi r_\perp$, we can rewrite the expression (53) in the form

$$\left(\frac{T}{\varepsilon_0}\right)^2=\frac{\xi(1-2\alpha)|\mathbf{u}|m}{6\pi\widetilde{K}^2\rho_0\tau r_\perp}\left(\frac{\rho_0|\mathbf{u}|}{\eta|x|}\right)^{1/2}\theta(\delta-r_\perp), \tag{54}$$

which is an equation for determination of the jet temperature, since the shear viscosity coefficient is a function of the temperature. In the region $T=30$–150 MeV this function has the form[11]

$$c\eta_s=\alpha_s T^{1/2}\exp(\beta_s T),$$

where $\alpha_s = 7.8\cdot10^{-2}$ GeV$\cdot$F^{-2} and $\beta_s = 11.8$ GeV^{-1}. Using this approximation for the shear viscosity coefficient and solving Eq. (54) by the method of successive approximations, for the condition that

$$1.3<T/\varepsilon_0\ll9/2\beta\varepsilon_0\approx17,$$

we obtain an expression for the jet temperature.

$$T=\varepsilon_0\frac{\Delta}{1+{}^2/_9\beta\varepsilon_0\Delta}\theta(\delta-r_\perp),$$

where

$$\Delta=\left[\frac{\xi}{6\pi}\left(\frac{m}{\tau\alpha_s\varepsilon_0^{1/2}}\right)^{1/2}\frac{\gamma_0^{3/2}}{\tilde{\gamma}^2}(n_0r_\perp^2|x|)^{-1/2}\right]^{4/9},$$

$\widetilde{K}^2=\tilde{\gamma}^2c^2$, and γ_0 is the velocity of the incident particle in units of c. Then the temperature of the jet with variation of the incident-particle energy from 100 MeV up to 1 GeV is

$$T=(2.2\text{–}3.3)\varepsilon_0\approx80\text{–}50\ \text{MeV}$$

with $r_\perp = 1$ F and $|x| = 1$–10 F.

6. CONCLUSIONS

Studies of the collective motion of nuclear matter, carried out in this work and based on linear hydrodynamical equations, have shown that two forms of collective motion of the hydrodynamical type arise.

First, there is a collective motion of the shock-wave type which is manifested in the presence of the Mach cone in the hydrodynamical characteristics of the medium. Here it is necessary to mention especially that behind the Mach cone the temperature of the medium is not increased, and the nuclear matter remains cold. Second, there is a jet collective motion which is located inside the Mach cone and the transverse dimensions of which are much smaller than those of the Mach cone. In the jet the temperature is increased and reaches about 70 MeV, i.e., a strongly heated region of the medium arises. Phenomena of this type can occur in the interaction of energetic protons with nuclei. In particular, effects of this type apparently can be explained by the "explosion" of nuclei in their interaction with protons.[12]

V. M. Galitskiĭ took part in setting up the problem and solving it in the initial stage. Any deficiencies which may exist in the final version of the work are the responsibility of the other two authors.

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[1)] In Ref. 2 the equation of state of an ideal Fermi gas $\varepsilon = 3p/2\rho$ was used as the equation of state. For an internal energy ε and a thermal function w for small changes of pressure and density p' and ρ', in this case the same relations (7) and (8) exist but with the compressibility of Ref. 7 in an ideal Fermi gas at $T=0$, $s^2 = \widetilde{K}^2$ ($\widetilde{K}^2 = (\partial p_0/\partial \rho)_{\rho=\rho_0} = 5p_0(\rho_0)/3\rho_0$, where $p_0(\rho)$ is the pressure of an ideal Fermi gas at $T=0$ (Ref. 8)) and with a coefficient 9/10 in front of the last term in the expression (7).

[2)] In Ref. 2 this expression was obtained in the case in which the equation of state of an ideal Fermi gas was used as the equation of state. In this case $\Delta\varepsilon_x = (3/5)\widetilde{K}^2\rho'/\rho_0$.

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