Radiative corrections to the Landau damping of intense plasma waves

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It is shown that the collective effects due to the quasiadiabatic, quasilinear increase in the fluctuation-induced electromagnetic field of a system of charged particles are important for the radiative corrections to the Landau damping of intense plasma waves. The high-energy particles produce power-law spectra as they interact with the electromagnetic fluctuations. Under conditions when most particles are nonrelativistic ($p \ll m$), the dominant contribution to the radiative corrections to the Landau damping is made by the generation from a small number of relativistic particles ($p \sim m$) with power-law spectra at $p \gg m$. In this approximation the radiative corrections, like the Landau damping itself, are proportional to the derivative of the particle distribution, and also contain the small quantity $(8e^2/3\pi\hbar c)(\ln 2 - 11/24)$. In the next order in $p^2/m^2 \ll 1$ the nonrelativistic particles make a substantial contribution to the radiative corrections; these corrections are not analytic in p^2/m^2 [they contain $(p^2/m^2)\ln(p^2/m^2)$], and also depend on the resonance-particle distribution function itself. If the effects determined by the derivatives of the particle distribution function are severely suppressed in the quasilinear-relaxation processes, then the part of the radiative corrections that is determined by the distribution function can exceed the normal Landau damping.

1. INTRODUCTION

The damping of the electrostatic oscillations in a system of charged particles was first considered by Landau.[1] He showed that the initial perturbations attenuate asymptotically like $\exp(-\gamma_{\mathbf{q}}^L t)$, where $\mathbf{q}$ is the wave vector and $\gamma_{\mathbf{q}}^L$ is the $\mathbf{q}$-dependent Landau damping constant.

We consider here the problem in its more general formulation, when the damped oscillations are sufficiently intense, and are capable of changing the particle distribution.[2] The Landau damping is then determined not by the initial particle distribution $\Phi_{\mathbf{p}}(0)$, but by the instantaneous distribution $\Phi_{\mathbf{p}}(t)$

$$\gamma_{\mathbf{q}}^L(t) = 4\pi^2 e^2 \left(q^2 \frac{\partial \varepsilon(\omega, \mathbf{q})}{\partial \omega}\bigg|_{\omega=\omega_{\mathbf{q}}} \right)^{-1} \times \int \delta(\omega_{\mathbf{q}} - \mathbf{q}\mathbf{v}) \left(\mathbf{q} \frac{\partial \Phi_{\mathbf{p}}(t)}{\partial \mathbf{p}} \right) \frac{d\mathbf{p}}{(2\pi)^3}. \quad (1)$$

The equation governing the variation in time of $\Phi_{\mathbf{p}}(t)$ is simplest in the case of random oscillations[2]:

$$\frac{d\Phi_{\mathbf{p}}(t)}{dt} = \pi e^2 \int |\varphi_{\mathbf{q}}(t)|^2 \mathbf{q} \frac{\partial}{\partial \mathbf{p}} \delta(\omega_{\mathbf{q}} - \mathbf{q}\mathbf{v}) \mathbf{q} \frac{\partial}{\partial \mathbf{p}} \Phi_{\mathbf{p}}(t) \, d\mathbf{q}. \quad (2)$$

Here $\varphi_{\mathbf{q}}(t)$ is the spatial Fourier component of the potential in which the oscillations occur; $\mathbf{q}$ and $\omega_{\mathbf{q}} = -\omega_{-\mathbf{q}}$ are the wave vector and frequency, respectively, of the oscillations; $\Phi_{\mathbf{p}}(t)$ describes the fluctuation-averaged distribution of the particles over the momenta $\mathbf{p}$; and $\varepsilon(\omega, \mathbf{q})$ is the real part of the longitudinal permittivity. The normalization of $\Phi_{\mathbf{p}}$ and of the correlation function of the potential are given by the relations

$$\Phi_{\mathbf{p}} = \langle f_{\mathbf{p}} \rangle, \quad \int \Phi_{\mathbf{p}} \frac{d\mathbf{p}}{(2\pi)^3} = n, \quad f_{\mathbf{p}} = \Phi_{\mathbf{p}} + \delta f_{\mathbf{p}},$$

$$\langle \varphi_{\mathbf{q}}(t) \varphi_{\mathbf{q}'}(t) \rangle = |\varphi_{\mathbf{q}}(t)|^2 \delta(\mathbf{q} + \mathbf{q}'). \quad (3)$$

Equations (1) and (2) have a simple physical meaning. They describe the variation of both the correlation function for the oscillation-field distribution and for the particle distribution as effects of induced Cherenkov emission and absorption of waves by particles.[3]

We consider hereafter precisely the case of sufficiently intense oscillations, for which allowance for the t-dependence of $\Phi_{\mathbf{p}}$ is important.

The quantity $\gamma_{\mathbf{q}}^L$ describes the damping of the correlation function $W_q(t)$ defined by the relation

$$W_q(t) = \frac{1}{2\pi} \int \left\langle \varphi_{\mathbf{q}'}\left(t - \frac{\tau}{2}\right) \varphi_{\mathbf{q}}\left(t + \frac{\tau}{2}\right) \right\rangle e^{i\omega\tau} \, d\tau \, d\mathbf{q}',$$

$$q = \{\mathbf{q}, \omega\}, \quad (4)$$

or by its integral

$$W_{\mathbf{q}}(t) = \int W_q(t) \, d\omega = |\varphi_{\mathbf{q}}(t)|^2.$$

Using the kinetic equation for the fluctuational part $\delta f_{\mathbf{p}}$ of the distribution function:

$$\frac{\partial}{\partial t} \delta f_{\mathbf{p},\mathbf{q}}(t) + i\mathbf{q}\mathbf{v} \delta f_{\mathbf{p},\mathbf{q}}(t) - ie\varphi_{\mathbf{q}}(t) \mathbf{q} \frac{\partial}{\partial \mathbf{p}} \Phi_{\mathbf{p}}(t) = 0 \quad (5)$$

and the Poisson equation

$$\exp(-i\omega_{\mathbf{q}} t) \, \varepsilon\left(\omega_{\mathbf{q}} + i \frac{\partial}{\partial t}, \mathbf{q} \right) \exp(i\omega_{\mathbf{q}} t) \varphi_{\mathbf{q}}(t) = \frac{4\pi e}{\mathbf{q}^2} \int \delta f_{\mathbf{p},\mathbf{q}}(t) \frac{d\mathbf{p}}{(2\pi)^3}, \quad (6)$$

as well as the relations

$$\varepsilon(\omega_{\mathbf{q}}, \mathbf{q}) = 0, \quad \left| \frac{1}{\varphi_{\mathbf{q}}} \exp(-i\omega_{\mathbf{q}} t) \frac{\partial}{\omega_{\mathbf{q}} \partial t} \exp(i\omega_{\mathbf{q}} t) \varphi_{\mathbf{q}} \right| \ll 1,$$

$$\left| \frac{1}{\Phi_{\mathbf{p}}} \frac{1}{\omega_{\mathbf{q}}} \frac{\partial}{\partial t} \Phi_{\mathbf{p}} \right| \ll 1$$

and

$$\frac{\sin(\omega_{\mathbf{q}} - \mathbf{q}\mathbf{v}) t}{\omega_{\mathbf{q}} - \mathbf{q}\mathbf{v}} \xrightarrow[t \to \infty]{} \pi \delta(\omega_{\mathbf{q}} - \mathbf{q}\mathbf{v}), \quad (7)$$

we obtain

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$$dW_{\mathbf{q}}(t)/dt=2\gamma_{\mathbf{q}}^{L}(t)W_{\mathbf{q}}(t),$$

where $\gamma_{\mathbf{q}}^{L}(t)$ is given by the relation (1). This result is in line with Landau's ideal[1] [cf. the $t\to\infty$ limits in (7) and Ref. 1], but is naturally more general (see Refs. 3–5).

In the present paper we consider the problem of first-order—in $e^2/\hbar c$—radiative corrections to the damping of high-intensity plasma oscillations described in the zeroth approximation by the relations (1) and (2). We shall thus be dealing with the radiative corrections for the fluctuation-averaged quantities $W_{\mathbf{q}}(t)$ and $\Phi_{\mathbf{p}}(t)$.

There is no reason to think that, for such average characteristics, the radiative corrections can be reduced to the radiative corrections to the probabilities for induced Cherenkov radiation emission by the individual particles in the external stray fields $\varphi_{\mathbf{q}}$. Indeed, there are at least three features here.

First when the true microdistribution $f_{\mathbf{p}}$ is split up into a regular averaged $\Phi_{\mathbf{p}}(t)$, and a random $\delta f_{\mathbf{p}}$, distribution, the random component describes oscillations, and, thus, it is as if part of the particle motion is connected not with the particles (i.e., with $\Phi_{\mathbf{p}}$), but with the oscillations.

Second, good examples are known which show that the cross sections for the processes in the equations for the "dressed" particles (described by the fluctuation-averaged distribution $\Phi_{\mathbf{p}}$) differ essentially from the cross sections for the "bare" particles. The presence of the polarization jacket (Debye screening) for the quasiparticles described by $\Phi_{\mathbf{p}}$ is important. As is well known, this leads to a situation in which the collision integral (which contains $\Phi_{\mathbf{p}}$) obtained through averaging over the fluctuations of the single-particle distribution contains in the denominator the square of the permittivity[5] (screening of the field of the colliding particles). The most striking example is the manifestation of the new plasma-wave-scattering mechanism, namely, transition scattering,[6] and the new mechanism for radiation emission during collisions,[7] namely, transition bremsstrahlung (see Ref. 8). The radiation-emission and scattering processes in a homogeneous medium are connected with fluctuations, and averaging over these fluctuations leads to new scattering and radiation-emission mechanisms that can radically change even the orders of magnitude of the cross sections for the corresponding processes.[8] Entering into the equations for the fluctuation-averaged quantities describing the scattering and bremsstrahlung-emission processes is $\Phi_{\mathbf{p}}$ with modified cross sections.

Thirdly, the ground state of the system $\Phi_{\mathbf{p}}(t)$ is nonstationary in time, and this can lead to additional effects in the radiative corrections because of the dependence of the permittivity on the time. All these arguments indicate that the collective effects in the radiative corrections to (1) and (2) can be important. The problem of the following analysis is to compute such collective effects.

Below, by the term "averaging over the fluctuations" we shall mean averaging over the quantum fluctuations and the zero-point oscillations.[9] We shall give the relativistic quantum generalizations for (1) and (2), and take account of the effects that occur in many-particle systems, and are described in the averaged equations by terms containing the additional parameter $e^2/\hbar c$. Further, the final formulas will be expanded in powers of $\hbar q/p$ ($\mathbf{q}$ is the wave vector of the resonance fields) up to the first nonvanishing term. This presupposes the quasiclassicality of the resonance fields $\varphi_{\mathbf{q}}$, and are of greatest interest for applications.

Participating in the radiative corrections are the virtual quanta of the electromagnetic field. We shall denote the wave vector and momentum of these quanta by $\mathbf{k}$ and $\hbar\mathbf{k}$, respectively, and set $\mathbf{p}'=\mathbf{p}+\hbar\mathbf{k}$. The value of $\mathbf{p}'$ is arbitrary. It is nevertheless assumed that the magnitude of $\mathbf{k}$ is so large that the contribution of the macroscopic mass renormalization[10] can be neglected: the transverse permittivity $\varepsilon^t(\omega,\mathbf{k})$ is close to unity. But because of the nonstationarity of $\Phi_{\mathbf{p}}(t)$, we must take account of the derivatives $\partial\varepsilon^t(\omega,\mathbf{k})/\partial t$, which leads to one of the collective effects in the radiative corrections (an effect which would not occur for the individual particles).

Another collective effect is the "exchange" interaction, in which a virtual photon is absorbed "not by that" particle which emitted it: The photons can transfer energy from several particles to one of them.

Allowance for the radiative corrections leads to a situation in which the longitudinal-wave packets described in the zeroth approximation by the potential $\varphi_{\mathbf{q}}$ also envelop themselves with a jacket. This jackets corresponds to fluctuating electromagnetic fields, and, consequently, the wave packet is not strictly classical, but strictly longitudinal waves. Therefore, dW/dt should describe the total change in the field energy.

It is easy to use the fact that, for random waves, the change in the energy is equal to the sum of the contributions of the individual harmonics. For the Landau damping (1) it is sufficient to know the change in the longitudinal energy W^l, whereas for the radiative corrections we must know the change in the total energy. For (1) we obtain

$$\frac{dW^l}{dt}=\frac{d}{dt}\int d\omega W_q(t)\,d\mathbf{q}\,\frac{q^2\omega_{\mathbf{q}}}{8\pi}\frac{\partial\varepsilon(\omega,\mathbf{q})}{\partial\omega}\Bigg|_{\omega=\omega_{\mathbf{q}}}$$

$$=\pi e^2\int|\varphi_{\mathbf{q}}|^2\delta(\omega_{\mathbf{q}}-\mathbf{q}\mathbf{v})\left(\mathbf{q}\frac{\partial\Phi_{\mathbf{p}}}{\partial\mathbf{p}}\right)d\mathbf{q}\frac{d\mathbf{p}}{(2\pi)^3}. \qquad (8)$$

It is significant that $(\partial\varepsilon(\omega,\mathbf{q})/\partial\omega)_{\omega=\omega_{\mathbf{q}}}$ has dropped out of this expression. The physical meaning of this derivative is quite clear: The absorption rate is determined by the resonance particles, while the dispersion (i.e., the $\omega=\omega_{\mathbf{q}}$ dependence) is determined by entirely different (specifically, the nonresonance) particles.

Below we shall assume that $\omega_{\mathbf{q}}$ is determined by a given classical subsystem, the investigation of which is beyond the scope of the present calculation, and that the influence of the resonance particles on $\omega_{\mathbf{q}}$ is negligibly weak. This gives us ground to neglect also the influence of the radiative corrections on $\omega_{\mathbf{q}}$. The problem is to make allowance in (8) (with dW/dt on the left-hand side instead of dW^l/dt) for the effects that are of the order of $e^2/\hbar c$ and linear in $|\varphi_{\mathbf{q}}|^2$. This is, $|\varphi_{\mathbf{q}}|^2$. This is, in fact, sufficient for the computation of the radiative corrections to the Landau damping.

To obtain the Landau damping (1) from (8), it is sufficient to use the general expression for the energy W^l:

$$W^l=\int\frac{q^2|\varphi_{\mathbf{q}}|^2}{8\pi}\frac{\partial\varepsilon(\omega,\mathbf{q})}{\partial\omega}\Bigg|_{\omega=\omega_{\mathbf{q}}}d\mathbf{q}. \qquad (9)$$

In view of the additivity of (8) and (9) with respect to the contributions of the various harmonics (i.e., with respect to the terms with different $\mathbf{q}$), we can set $|\varphi_{\mathbf{q}}|^2 = |\varphi_{\mathbf{q}0}|^2\delta(\mathbf{q}-\mathbf{q}_0)$, and, by dividing $dW_{\mathbf{q}0}/dt$ by $W_{\mathbf{q}0}$ obtain $2\gamma_{\mathbf{q}_0}$. From (8) and (9) we thus obtain (1) for $\mathbf{q}=\mathbf{q}_0$.

Let us choose here this simplest method of representing the radiative-correction results obtained independently in the present paper and those obtained by other methods.

In the subsequent discussion we use the system of units with $\hbar = c = 1$.

2. THE BASIC RELATIONS

Let us set forth the general computational scheme that allows us to find a simple relativistic quantum-mechanical generalization for, and the radiative corrections to, the Landau damping.

The general Wigner spinor-field density matrix operator, defined by the relation[9]

$$\hat{f}_{\alpha,\beta,\mathbf{p},\mathbf{k}}(t) = {}^1/_2(\hat{\Psi}^+_{\beta,\mathbf{p}-\mathbf{k}/2}(t)\,\hat{\Psi}_{\alpha,\mathbf{p}+\mathbf{k}/2}(t) - \hat{\Psi}_{\alpha,\mathbf{p}+\mathbf{k}/2}(t)\,\hat{\Psi}^+_{\beta,\mathbf{p}-\mathbf{k}/2}(t)), \qquad (10)$$

can be expressed in terms of the free-particle density matrix operator $\hat{f}^{(0)}_{\alpha,\beta,\mathbf{p},\mathbf{q}}$ with the aid of the S matrix:

$$\hat{f}_{\alpha,\beta,\mathbf{p},\mathbf{q}} = \hat{S}^+(t)\,\hat{f}^{(0)}_{\alpha,\beta,\mathbf{p},\mathbf{q}}S(t), \qquad (11)$$

where α and β are the spinor indices and $\hat{\Psi}_{\alpha,\mathbf{p}}(t)$ is the spinor field operator in the momentum representation. The spatial component of the Fourier operator for the current density (the $\gamma_\mu = \{\gamma,\beta\}$ are the Dirac matrices)

$$\hat{j}_{\mu,\mathbf{k}}(t) = e\int \operatorname{Sp} i\beta\gamma_\mu \hat{f}_{\mathbf{p},\mathbf{k}}(t)\,\frac{d\mathbf{p}}{(2\pi)^3} \qquad (12)$$

will enter into the S-matrix interaction Lagrangian:

$$\hat{L}(t) = -\int \hat{j}^{(0)}_\mu(t,\mathbf{r})\hat{A}^{(0)}_\mu(t,\mathbf{r})\,d\mathbf{r}, \qquad S = T\exp\left(-i\int \hat{L}\,dt\right). \qquad (13)$$

For the description of the noninteracting $\hat{A}_\mu^{(0)}$ fields, we use the Coulomb gauge. The longitudinal fields are described by the classical subsystem, and satisfy the equation $\varepsilon(\omega,\mathbf{q})=0$, where ε is the longitudinal permittivity of the classical subsystem. We denote the potential of the longitudinal fields by $\varphi_{\mathbf{q}}$ and the corresponding Lagrangian by $\hat{L}_\varphi$. The quantum subsystem of resonance particles is described by the operators $\hat{f}^{(0)}_{\mathbf{p},\mathbf{q}}$ and the zero-point-oscillation field operators $\hat{A}_{\mathbf{k}}^{(0)}(t)$ in L_A. We have

$$\hat{f}^{(0)}_{\alpha,\beta,\mathbf{p},\mathbf{k}}(t) = {}^1/_2(\hat{\Psi}^{+(0)}_{\alpha,\mathbf{p}-\mathbf{k}/2}(t)\,\hat{\Psi}^{(0)}_{\beta,\mathbf{p}+\mathbf{k}/2}(t) - \hat{\Psi}^{(0)}_{\beta,\mathbf{p}+\mathbf{k}/2}(t)\,\hat{\Psi}^{+(0)}_{\alpha,\mathbf{p}-\mathbf{k}/2}(t)), \qquad (14)$$

$$\hat{\Psi}^{(0)}_{\alpha,\mathbf{p}}(t) = \sum_{\mu,\lambda} u^{\mu,\lambda}_{\alpha,\mathbf{p}}\hat{a}^{\mu,\lambda}_{\mathbf{p}}\exp(-i\lambda\varepsilon_{\mathbf{p}}t), \qquad \varepsilon_{\mathbf{p}} = (\mathbf{p}^2+m^2)^{1/2}, \qquad (15)$$

where $\lambda = \pm 1$ is the sign of the energy, $\mu = \pm 1$ is the sign of the spin component along $\mathbf{p}$, the $u^{\mu,\lambda}_{\alpha,\mathbf{p}}$ are bispinors, and the $\hat{a}^{\mu,\lambda}_{\mathbf{p}}$ are the particle annihilation (creation) operators. The operator $\hat{L}_\varphi$ describes the interaction with the classical field:

$$L_\varphi = \sum_{\mu,\lambda,\mu',\lambda'} e\int \frac{d\mathbf{p}'\,d\mathbf{q}''}{(2\pi)^3}\, u^{\mu,\lambda+}_{\alpha,\mathbf{p}'+\mathbf{q}''/2}u^{\mu',\lambda'}_{\alpha,\mathbf{p}'-\mathbf{q}''/2}\varphi_{\mathbf{q}''}(t) \times\frac{1}{2}(\hat{a}^{+\mu,\lambda}_{\mathbf{p}'+\mathbf{q}''/2}\hat{a}^{\mu',\lambda'}_{\mathbf{p}'-\mathbf{q}''/2} - \hat{a}^{\mu',\lambda'}_{\mathbf{p}'-\mathbf{q}''/2}\hat{a}^{+\mu,\lambda}_{\mathbf{p}'+\mathbf{q}''/2}) \times\exp[i\lambda\varepsilon_{\mathbf{p}'+\mathbf{q}''/2}t - i\lambda'\varepsilon_{\mathbf{p}'-\mathbf{q}''/2}t], \qquad (16)$$

while $\hat{L}_A$ describes the electromagnetic fluctuations:

$$\hat{L}_A = -ie\sum_{\mu,\lambda,\mu',\lambda'}\int \frac{d\mathbf{p}'d\mathbf{k}}{(2\pi)^3}\, u^{+\mu,\lambda}_{\alpha,\mathbf{p}'+\mathbf{k}}u^{\mu'\lambda'}_{\beta,\mathbf{p}'}\,\gamma^t_{\alpha,\beta,\mathbf{k}}\hat{A}^{(0)(\rho)}_{\mathbf{k}} \times\frac{1}{2}(\hat{a}^{+\mu,\lambda}_{\mathbf{p}'+\mathbf{k}}\hat{a}^{\mu',\lambda'}_{\mathbf{p}'} - \hat{a}^{\mu',\lambda'}_{\mathbf{p}'}\hat{a}^{+\mu,\lambda}_{\mathbf{p}'+\mathbf{k}}) \times\exp[-i|\mathbf{k}|\rho t + i\lambda\varepsilon_{\mathbf{p}'+\mathbf{k}}t + i\lambda'\varepsilon_{\mathbf{p}'}t], \qquad (17)$$

where

$$\rho = \pm 1, \quad \hat{A}^{(0)(+1)}_{\mathbf{k}} = \hat{A}^{(0)}_{\mathbf{k}}, \quad \hat{A}^{(0)(-1)}_{\mathbf{k}} = \hat{A}^{(0)+}_{\mathbf{k}},$$

$$\gamma^t_{\mathbf{k}} = \gamma - \mathbf{k}(\mathbf{k}\gamma)/k^2$$

and the vacuum average is not equal to zero only for the following combination of the operators $\hat{A}^{(0)}_{\mathbf{k}}$ and $\hat{A}^{(0)+}_{\mathbf{k}'}$:

$$\langle \hat{A}^{(0)}_{\mathbf{k},i}\hat{A}^{(0)+}_{\mathbf{k}',j}\rangle = \frac{\delta(\mathbf{k}+\mathbf{k}')}{4\pi^2|\mathbf{k}|}\left(\delta_{ij} - \frac{k_ik_j}{k^2}\right). \qquad (18)$$

For the ensemble of particles under consideration here, $\hat{L}_A$ also describes all the fluctuations of the electromagnetic fields in the particle system, which are proportional to the particle distribution function $\Phi_{\mathbf{p}}$.

In operator form, the relations written down above describe any fluctuations in any particle system. Let us consider systems of particles that, in the absence of interaction, are described by the distribution function (occupation numbers) $\Phi^{(0)}_{\mathbf{p}}$. Let us assume that the particles are unpolarized, and that there are no antiparticles, i.e., that

$$\langle \hat{a}_{\mathbf{p}}^{+,\mu,1}\hat{a}^{\mu',1}_{\mathbf{p}+\mathbf{q}}\rangle = {}^1/_2\Phi^{(0)}_{\mathbf{p}}\delta_{\mu,\mu'}\delta(\mathbf{q}). \qquad (19)$$

The invariant quantity used in the relativistic calculations in the general case is the charge density per electron charge e, i.e., in accordance with (12),

$$\Phi_{\mathbf{p}}(t) = \int \operatorname{Sp}\langle f_{\mathbf{p},\mathbf{q}'}(t)\rangle d\mathbf{q}'. \qquad (20)$$

From (19) and the relations

$$\sum_\mu u^{+\mu,\lambda}_{\alpha,\mathbf{p}}\cdot u^{\mu,\lambda}_{\beta,\mathbf{p}} = \lambda(\Lambda_{\mathbf{p}}{}^\lambda\beta)_{\alpha,\beta}, \qquad \Lambda_{\mathbf{p}}{}^\lambda = \frac{m - i\gamma\mathbf{p} + \lambda\beta\varepsilon_{\mathbf{p}}}{2\varepsilon_{\mathbf{p}}}, \qquad (21)$$

we easily find that

$$\Phi^{(0)}_{\mathbf{p}}(t) = \int \operatorname{Sp}\langle f^{(0)}_{\mathbf{p},\mathbf{q}'}(t)\rangle d\mathbf{q}' = \Phi^{(0)}_{\mathbf{p}}.$$

The averaging in (20) is over the vacuum of the electromagnetic fluctuations (electromagnetic waves are not emitted), the statistical particle ensemble, in accordance with (19), and the statistical ensemble of random classical fields $\varphi_{\mathbf{q}}$.

Since in the classical $\varphi_{\mathbf{q}}$ field the resonance particles could, on account of the quasilinear acceleration (2), acquire infinite energy the formulation of the problem should be as follows: The $\varphi_{\mathbf{q}}$ field is switched on adiabatically at $t=0$, and the asymptotic behavior at $t\to\infty$ is investigated. Only the resonances of the type (7) are considered; in the quantum-mechanical case these resonances have the form

$$\frac{\sin(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}-\mathbf{q}} - \omega_{\mathbf{q}})}{\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}-\mathbf{q}} - \omega_{\mathbf{q}}} \xrightarrow[t\to\infty]{} \pi\delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}-\mathbf{q}} - \omega_{\mathbf{q}}). \qquad (22)$$

The pair-production resonances containing $\varepsilon_{\mathbf{p}} + \varepsilon_{\mathbf{p}+\mathbf{q}}$

$-\omega_p = 0$, are discarded on account of the classical nature of the field, and the scattering resonances containing $\varepsilon_p - \varepsilon_{p-q-k} - \omega_q + |\mathbf{k}| = 0$ are neglected because these conditions cannot be fulfilled simultaneously with the condition (22) for Cherenkov resonance in a plasma (this would have led to the condition $\varepsilon_{p-q} - \varepsilon_{p-q-k} + |\mathbf{k}| = 0$, which is not fulfilled for free particles).

Since a jacket of electromagnetic fluctuations grows around the longitudinal-wave packets, we consider the time variation of the total mean energy $\langle W\rangle$ of the field. The calculation is carried out with allowance for only the terms linear in $|\varphi_q|^2$:

$$\frac{d}{dt}\langle W\rangle = \int d\mathbf{q}\, G_q |\varphi_q|^2 \omega_q. \tag{23}$$

The relation (23) is the definition of G_q. The quantity G_q is sought up to terms of first order in $e^2/\hbar c$, i.e., ($\hbar = c = 1$)

$$G_q = G_q^L + e^2 (G_q^{R(l)} + G_q^{R(t)}). \tag{24}$$

In the radiative corrections we can, in the first approximation, now separate the longitudinal- and electromagnetic-field fluctuation effects $G_q^{R(l)}$ and $G_q^{R(t)}$, respectively.

In the general case G_q is a functional of $\Phi_q(t)$. Without allowance for the radiative corrections, we find, in accordance with (8), that

$$G_q^L = \pi e^2 \int \delta(\omega_q - \mathbf{q}\mathbf{v}) \left(\mathbf{q}\frac{\partial \Phi_p(t)}{\partial \mathbf{p}}\right) \frac{d\mathbf{p}}{(2\pi)^3}. \tag{25}$$

Below we shall take account of only the linear functional dependence of G_q on $\Phi_p(t)$. We can use the fact that the functional $G_q^R = G_q^{R(l)} + G_q^{R(t)}$ in the radiative corrections can be found on the set $\Phi_p^{(0)}$, whereas G_q^L should be found on the more correct set that takes account of that part of the temporal dependence $\Phi_p(t)$ which corresponds to a relative order of $e^2/\hbar c$. The resulting equations, into which $\Phi_p(t)$ enters through both G_q^L and G_q^R, correspond to the Dyson summation of the irreducible diagrams.

On account of the assumption, made above, that the dispersion is determined by the classical subsystem, the relation (23), together with (9), allows us to write down the relations

$$\gamma_q = \gamma_q^L + \gamma_q^R, \qquad \gamma_q^L = \frac{4\pi e^2 G_q^L}{q^2 (\partial \varepsilon(\omega, \mathbf{q})/\partial\omega)_{\omega=\omega_q}}, \tag{26}$$

$$\gamma_q^R = \gamma_q^{R(l)} + \gamma_q^{R(t)} = \frac{4\pi e^2}{q^2 (\partial \varepsilon(\omega, \mathbf{q})/\partial\omega)_{\omega=\omega_q}} (G_q^l + G_q^t). \tag{27}$$

This method can be tried out in the calculation of the linear Landau damping in the general case of relativistic particles and in the case when allowance is made for the quantum effects. The operator equation

$$\exp(-i\omega_q t) \left.\frac{\partial \varepsilon(\omega, \mathbf{q})}{\partial \omega}\right|_{\omega=\omega_q} i\frac{\partial}{\partial t}\hat{\varphi}_q(t) \exp(i\omega_q t)$$

$$= \mathrm{Sp}\frac{4\pi e}{\mathbf{q}^2}\int \delta\hat{f}_{p,q}(t)\frac{d\mathbf{p}}{(2\pi)^3}, \tag{28}$$

obtained from (6), can be used to derive an expression for the rate of variation of the longitudinal-field energy (taking account of the fact that $\mathbf{q}' \to -\mathbf{q}$ and $(\partial\varepsilon(\omega, -\mathbf{q})/\partial\omega)_{\omega=\omega_{-q}} = (-\partial\varepsilon(\omega,\mathbf{q})/\partial\omega)_{\omega=\omega_q}$):

$$\frac{d}{dt}\langle W\rangle = \frac{e}{2i}\int\frac{d\mathbf{p}\,d\mathbf{q}'}{(2\pi)^3}\omega_q\{\langle\hat{\varphi}_{q'}(t)\delta\hat{f}_{p,q}(t) - \delta\hat{f}_{p,q'}(t)\hat{\varphi}_q(t)\rangle\}. \tag{29}$$

If we introduce for S an index (i) corresponding to the exponent L in (13), then, to obtain the linear Landau damping, it is sufficient to take into account in (29) only the linear term $S^{(1)}$ with the Lagrangian L_φ [see (16)], i.e.,

$$\delta\hat{f}_{p,q}^{(1)}(t) = \hat{S}^{(1)+}(t)\delta\hat{f}_{p,q}^{(0)}(t) + \delta\hat{f}_{p,q}^{(0)}(t)\hat{S}^{(1)}(t)$$

$$= -\delta\hat{f}_{p,q}^{(0)}(t)\, i\int_0^t \hat{L}_\varphi\, dt' + i\int_0^t \hat{L}_\varphi\, dt'\,\delta\hat{f}_{p,q}(t). \tag{30}$$

Using (22), (19), and (16), we obtain

$$\frac{d}{dt}\langle W\rangle = \frac{\pi e^2}{2}\int\frac{d\mathbf{q}\,d\mathbf{p}}{(2\pi)^3}\omega_q|\varphi_q|^2\delta(\varepsilon_p - \varepsilon_{p-q} - \omega_q)$$

$$\times\, \mathrm{Sp}\,\Lambda_p^+\beta\Lambda_{p-q}^+\beta(\Phi_p - \Phi_{p-q}) = \int |\varphi_q|^2\omega_q G_q^L d_q. \tag{31}$$

The expression obtained for the linear Landau damping with the aid of (31), (23), and (26) corresponds to a relativistic quantum generalization of (1), and coincides with the expression found earlier in Ref. 10 by the Green-functions method for many-particle systems. The difference is that (31) is suitable for nonequilibrium distributions. If $\mathbf{q} \ll \mathbf{p}$ the relation (31) gives, when account is taken of the fact that

$$\mathrm{Sp}\,\Lambda_p^+\beta\Lambda_{p-q}^+\beta \approx \mathrm{Sp}\,\Lambda_p^+\beta = 2$$

$$\text{and}\quad (\Phi_p - \Phi_{p-q}) \approx \left(\mathbf{q}\frac{\partial}{\partial\mathbf{p}}\right)\Phi_p,$$

the same result as (25).

3. THE RADIATIVE CORRECTIONS DUE TO THE VARIATION OF THE LONGITUDINAL-FIELD ENERGY

Since the calculations are tedious, and their basic scheme has already been expounded, we shall give the results, emphasizing the most important physical aspects.

In the calculations we should take into account the fact that

$$\delta\hat{f}_{p,q}^{(3)}(t) = \delta\hat{f}_{p,q}^{(0)}(t)\hat{S}^{(3)}(t) + \hat{S}^{+(3)}(t)\delta\hat{f}_{p,q}^{(0)}(t)$$

$$+ \hat{S}^{+(2)}(t)\delta\hat{f}_{p,q}^{(0)}(t)\hat{S}^{(1)}(t)$$

$$+ \hat{S}^{+(1)}(t)\delta\hat{f}_{p,q}^{(0)}(t)\hat{S}^{(2)}(t), \tag{32}$$

retaining the terms linear in $\hat{L}_\varphi$ and quadratic in $\hat{L}_A$. We should also take into account the renormalization counterterms by standard methods,[11] the renormalization of Φ_p, and the radiative modification of the functional G_q^L (which was discussed above).

The final result, expanded in powers of $q/p \ll 1$ up to the first nonvanishing term, has the form

$$G_q^{R(l)} = -2\pi^2 e^2 \int \frac{1}{2}\delta(\omega_q - \mathbf{q}\mathbf{v})$$

$$\times\, \mathbf{q}\frac{\partial}{\partial\mathbf{p}}(R_{p,p'}\Phi_p - R_{p',p}\Phi_{p'})\frac{d\mathbf{p}\,d\mathbf{p}'}{(2\pi)^6}, \tag{33}$$

where

$$R_{p,p'} = -\frac{2}{|\mathbf{p}-\mathbf{p}'|}\left\{\frac{\mathrm{Sp}\,\Lambda_p^+\gamma_i^t\Lambda_{p'}^+\gamma_i^t}{(\varepsilon_{p'} - \varepsilon_p + |\mathbf{p}-\mathbf{p}'|)^2}\right.$$

$$\left. - \frac{\mathrm{Sp}\,\Lambda_p^+\gamma_i^t\Lambda_{p'}^-\gamma_i^t}{(\varepsilon_{p'} + \varepsilon_p + |\mathbf{p}-\mathbf{p}'|)^2}\right\}, \tag{34}$$

$$\boldsymbol{\gamma}'=\boldsymbol{\gamma}-\frac{((\mathbf{p}-\mathbf{p}')\boldsymbol{\gamma})(\mathbf{p}-\mathbf{p}')}{(\mathbf{p}-\mathbf{p}')^2}. \tag{35}$$

The expression (33) contains an integral taken over the virtual momenta $\mathbf{k}$ (see (17)) or $\mathbf{p}'=\mathbf{p}+\mathbf{k}$.

Let us discuss the limits of applicability of (33). Since we have found the terms linear in $\Phi_\mathbf{p}$, (33) takes account of the change that occurs in the electrostatic-field energy as a result of the electromagnetic fluctuations connected with the presence of particles, i.e., that additional part of the zero-point fluctuations which is proportional to $\Phi_\mathbf{p}$. The expression (33) describes the appearance and absorption of virtual photons in the many-particle system when the emitted virtual photon can be absorbed by any of the particles of the system. The radiation damping force due to the emission of real photons is ignored in (33). It is assumed that $\mathbf{k}\gg\omega_\mathbf{q}$. The emission of real photons is possible when $k\sim\omega_\mathbf{q}$ in the case of nonrelativistic particles and when $k\sim\omega_\mathbf{q}\varepsilon_\mathbf{p}{}^2/m^2$ in the case of ultrarelativistic particles (the process of scattering of the $\varphi_\mathbf{q}$ fields into electromagnetic waves on the particles). But those particles which participate in the resonance interaction, i.e., for which $\omega_\mathbf{q}=\mathbf{qv}$ (see (33)), cannot participate in the scattering. Therefore, the effects of the radiation damping force (which moreover, are more often than not weak) can be described by the standard methods, and are simply combined with the radiative corrections. Since $k\sim m$ in (33), and $k\gg\omega_\mathbf{q}$, the condition of applicability of (33) is

$$\omega_\mathbf{q}\ll m, \tag{36}$$

which is usually fulfilled in the cases of practical interest.

The expression (33) describes the possibility of the absorption of a virtual photon by any of the particles of the system, and therefore takes account of the collective effect due to the presence of the particle system. From this it follows that the result (33) should differ from what would be obtained if we took account of only the radiative corrections to the probabilities for induced Cherenkov emission and absorption of the longitudinal $\varphi_\mathbf{q}$ waves by the individual particles, and then found the change in the energy $\langle W\rangle$ with allowance made for both the transitions from the states and the transitions from the states $\Phi_{\mathbf{p}-\mathbf{q}}$ and the transitions from the $\Phi_\mathbf{p}$ states, i.e., if we multiplied the the probability by $(\Phi_\mathbf{p}-\Phi_{\mathbf{p}-\mathbf{q}})\omega_\mathbf{q}$ and integrated over all $\mathbf{p}$. The difference should correspond to the loss of the collective effect in which the virtual photon is absorbed by any particle of the system, and not by only a particle that figures in the computation of the probability. Of course, in this case the indistinguishability of the particles is taken into account in the correct calculation through averaging over the fluctuations, as was done in the derivation of (33).

It is natural that no collective effects of the indicated type will occur in the first-order approximation, i.e., in the linear Landau damping (without allowance for the radiative corrections). This is confirmed by direct computation. Taking $\hat{S}^{(1)}$ and and $\hat{L}_\varphi$ into account, we obtain, in accordance with the rules of quantum electrodynamics,[11] the probability for a $\mathbf{p}-\mathbf{q}\to\mathbf{p}$ change in the particle state in the random field $\varphi_\mathbf{q}$ (the second angle brackets in (37) correspond to statistical averaging over the classical random fields $\varphi_\mathbf{q}$):

$$w_\mathbf{p}\underset{t\to\infty}{=}\frac{d}{dt}\langle|\langle\mathbf{p}|S^{(1)}|\mathbf{p}-\mathbf{q}\rangle|^2\rangle$$

$$=\frac{\pi e^2}{2}\int|\varphi_\mathbf{q}|^2\,d\mathbf{q}\,\delta(\varepsilon_\mathbf{p}-\varepsilon_{\mathbf{p}-\mathbf{q}}-\omega_\mathbf{q})\operatorname{Sp}\Lambda_\mathbf{p}{}^+\beta\Lambda^+_{\mathbf{p}-\mathbf{q}}\beta \tag{37}$$

and hence the expression

$$\frac{d}{dt}\langle W\rangle=\int\omega_\mathbf{q}w_\mathbf{p}(\Phi_\mathbf{p}-\Phi_{\mathbf{p}-\mathbf{q}})\frac{d\mathbf{p}}{(2\pi)^3}, \tag{38}$$

which coincides with (31).

According to the standard rules,[11] the radiative corrections to (37) are given by the expression

$$\frac{d}{dt}\langle M^{(1)}M^{(3)+}+M^{(3)}M^{(1)+}\rangle,$$

where $M^{(1)}=\langle\mathbf{p}|S^{(1)}|\mathbf{p}-\mathbf{q}\rangle$ takes account of L_φ, while $M^{(3)}=\langle\mathbf{p}|S^{(3)}|\mathbf{p}-\mathbf{q}\rangle$ takes accont of one L_φ and two L_A. Allowance for the renormalizing counterterms, the balance equation (38), and the expansion in powers of $\mathbf{q}$ leads to a result different from (33), although it contains the same quantity $R_{\mathbf{p},\mathbf{p}'}$. Let us denote the corresponding $G_\mathbf{q}$ quantity that does not take account of the collective effects by $G_\mathbf{q}^{R(\text{indiv})}$. We have

$$G_\mathbf{q}^{R(\text{indiv})}=-2\pi^2e^2\int\delta(\omega_\mathbf{q}-\mathbf{qv})\Phi_\mathbf{p}\mathbf{q}\frac{\partial}{\partial\mathbf{p}}R_{\mathbf{pp}'}\frac{d\mathbf{p}\,d\mathbf{p}'}{(2\pi)^6}. \tag{39}$$

A comparison of (33) and (39) reveals the striking fact that (33), as it were, takes account of the exchange effect (the half-difference between the two terms, which coincide up to the interchange $\mathbf{p}\rightleftarrows\mathbf{p}'$). It is natural that the correct expression (33) can be obtained only through averaging over the fluctuations. The existence of operators that do not commute with δf, and take account of such fluctuations can be seen even from the expansion (32) [the terms with L_A on either side of δf always lead to the relation $\Phi_{\mathbf{p}+\mathbf{k}}=\Phi_{\mathbf{p}'}$, which does not occur in (34)].

We must draw attention to two other circumstances. The relation (39) contains an infrared divergence at $\mathbf{p}-\mathbf{p}'\to0$ ($\mathbf{k}\to0$), whereas this divergence does not occur in (33). Its elimination by the standard methods[11] in (39) gives rise to large logarithms $\ln(m/\omega_\mathbf{q})$, which actually do not occur in the correct expression (33). The presence of only the $\mathbf{q}\partial/\partial\mathbf{p}$ derivative of $R_{\mathbf{pp}'}$ in (39) is natural, since the renormalization requires the subtraction of the expression for it at $\mathbf{q}=0$ from (3), and the degree of accuracy allowed in the $\mathbf{q}$ expansion will be exceeded if we take account of the difference between $\mathbf{p}+\mathbf{q}$ and $\mathbf{p}$ in the expression for $\Phi_\mathbf{p}$. But the first term in (33) contains the derivative $(\mathbf{q}\partial/\partial\mathbf{p})\Phi_\mathbf{p}$ as well. This is due to the fact that (39) describes the total effect, while (33) describes, in the case of a many-particle system, only that part of the effect which is connected with the change that occurs in the longitudinal-field energy as a result of the electromagnetic fluctuations, whereas the change $G_\mathbf{q}^{R(t)}(t)$ in the energy of the electromagnetic fluctuations of the many-particle system should also be taken into account.

4. CHANGE INDUCED IN THE ENERGY OF THE ELECTROMAGNETIC FLUCTUATIONS BY INTENSE RESONANCE ELECTROSTATIC FIELDS

The indicated change occurs only in a many-particle system, and in the presence of sufficiently strong electrostatic fields. It describes an additional collective effect inseparable from the one considered in (33). In the absence of real photons, the energy of the fluctuation field will vary because of the fact that, in a system of particles, it depends on Φ_p, and Φ_p varies quasilinearly in time in accordance with (2). Because $k \gg \omega_q$, we can assert that this variation should be quasiadiabatic. The specific content of this assertion can be established by analogy with the adiabatic variation of the real quanta of electromagnetic radiation.

Let $N_k \gg 1$, where N_k is the number of quanta of the electromagnetic waves with $k \gg \omega_q$. The variation of the energy of such waves in the presence of the quasilinear acceleration (2) is considered in the nonquantum limit $k \ll p$ in Ref. 12. We can generalize the result obtained in Ref. 12 to the case of arbitrary $\mathbf{k}$ and arbitrary Φ_p, i.e., to the general quantum relativistic case. Let us show how this can be done. Let us first note that, in Ref. 12, it is shown (for $k \ll p$) that the variation of Φ_p in time causes [in the case of the standard description of the quasilinear equation (2) containing the instantaneous $\Phi_p(t)$] the permittivity for the transverse waves to acquire an imaginary part,

$$\operatorname{Im} \varepsilon^t = \frac{\partial}{\partial \omega} \frac{\partial}{\partial t} \operatorname{Re} \varepsilon^t,$$

that differs by a factor of two from the one found in Ref. 13, and does not guarantee the conservation of the photon number N_k. But there arises an additional $\delta(\omega_q - \mathbf{q}\cdot\mathbf{v})|\Phi_p|^2$-containing imaginary part of the nonlinear response, that, in combination with the imaginary part due to the nonstationarity, guarantees the conservation of the adiabatic invariant, namely, the photon number N_k:

$$\frac{dW^t}{dt} = \int 2N_k{}^t \frac{d\omega_k(t)}{dt} \frac{d\mathbf{k}}{(2\pi)^3}. \tag{40}$$

Let us generalize this result to the quantum relativistic case for $N_k \gg 1$, and then find its $N_k = 0$ analog, which is of interest to us here. Let us use the equation for $\hat{\mathbf{A}}_k(t)$ (the Coulomb gauge):

$$k^2 \hat{\mathbf{A}}_k(t) + \frac{\partial^2}{\partial t^2} \hat{\mathbf{A}}_k(t) = 4\pi \hat{\mathbf{j}}_k{}^t, \quad \hat{\mathbf{j}}_k{}^t = \hat{\mathbf{j}}_k - \frac{\mathbf{k}(\mathbf{k}\hat{j}_k)}{\mathbf{k}^2}, \tag{41}$$

where $\hat{\mathbf{j}}_k(t)$ is given by (12). From (41) we obtain the energy conservation law in the form

$$\begin{aligned} \frac{d}{dt}\langle W^t \rangle = \frac{d}{dt} \int \frac{d\mathbf{k}\, d\mathbf{k}'}{8\pi} \Big[\langle \hat{\mathbf{A}}_{k'}(t) \hat{\mathbf{A}}_k(t) \rangle_A \mathbf{k}^2 \\ + \Big\langle \frac{\partial \hat{\mathbf{A}}_{k'}(t)}{\partial t} \frac{\partial \hat{\mathbf{A}}_k(t)}{\partial t} \Big\rangle_A \\ - 4\pi \int_{-\infty}^{t} \Big\langle \frac{\partial \hat{\mathbf{A}}_{k'}(t')}{\partial t'} \hat{\mathbf{j}}_k{}^t(t') + \hat{\mathbf{j}}_k{}^t(t') \frac{\partial \hat{\mathbf{A}}_{k'}(t')}{\partial t'} \Big\rangle_A dt' \Big] \\ = 4\pi \int \frac{d\mathbf{k}\, d\mathbf{k}'}{8\pi} \Big\langle \frac{\partial \hat{\mathbf{A}}_{k'}(t)}{\partial t} \hat{\mathbf{j}}_k{}^t(t) + \hat{\mathbf{j}}_k{}^t(t) \frac{\partial \hat{\mathbf{A}}_{k'}(t)}{\partial t} \Big\rangle_{\text{nonl}}, \end{aligned} \tag{42}$$

where we have taken account of the fact that $\mathbf{k}' = -\mathbf{k}$, and have set $\exp[i(\mathbf{k}+\mathbf{k}')\mathbf{r}] = 1$. In (42)

$$\langle \hat{L} \rangle_A = \langle \hat{S}_A{}^+ \hat{L}^{(0)} \hat{S}_A \rangle, \quad \langle L \rangle_{\text{nonl}} = \hat{S}^+ L^{(0)} \hat{S} \rangle - \langle \hat{S}_A \hat{L}^{(0)} \hat{S}_A \rangle,$$

where $\hat{S}_A$ takes account of only $\hat{L}_A$, while $\hat{S}$ takes account of both $\hat{L}_A$ and $\hat{L}_\varphi$. The time-integrated term on the left-hand side of (42), a term which is proportional to A^2, gives the contribution of the linear permittivity to the energy of the fluctuating fields, since it takes account of the presence of the particle ensemble.

Let us first find the left-hand side of (41) for $N_k \gg 1$, when $\langle \hat{A}^+ \hat{A} \rangle \approx \langle \hat{A} \hat{A}^+ \rangle \propto N_k$. We have

$$W^t_{N\gg 1} = \int \frac{N_k\, d\mathbf{k}}{(2\pi)^3} \Big\{ \omega \frac{\partial}{\partial \omega} \omega \varepsilon^t + \omega \Big\}_{\omega = |\mathbf{k}|}, \tag{43}$$

where the obtained transverse permittivity has the form

$$\begin{aligned} \varepsilon^t(\omega, \mathbf{k}) = 1 + \delta\varepsilon^{t(+)}(\omega, \mathbf{k}) + \delta\varepsilon^{t(-)}(\omega, \mathbf{k}) + \delta\varepsilon^{t(+)}(-\omega, \mathbf{k}) \\ + \delta\varepsilon^{t(-)}(-\omega, \mathbf{k}), \end{aligned} \tag{44}$$

$$\delta\varepsilon^{t(\pm)}(\omega, \mathbf{k}) = -\frac{\pi e^2}{\omega^2} \int \frac{\Phi_p\, d\mathbf{p}}{(2\pi)^3} \frac{\operatorname{Sp} \Lambda_p{}^+ \gamma_i{}^t \Lambda^{\pm}_{p-k} \gamma_i{}^t}{\omega - \varepsilon_p \pm \varepsilon_{p-k}},$$

$$\gamma_k{}^t = \gamma - \mathbf{k}(\mathbf{k}\gamma)/\mathbf{k}^2.$$

It is easy to verify (after taking the traces) that the expression (44) for ε^t coincides with the one found in Ref. 10. The reason why $\omega = |\mathbf{k}|$ figures in (43) is that the result was obtained within the framework of perturbation theory with the aid of the S matrix for the case of waves with $k \gg \omega_q$, when ε^t is close to unity, and only the term linear in is taken into account. Therefore, the deviation of ω_k from $|\mathbf{k}|$ is small, and (43) takes account of this deviation in first order perturbation theory, expressing the corresponding corrections in terms of the zeroth approximation, in which $\omega_k = |\mathbf{k}|$.

Because of the quasilinear [according to (2)] variation of ε^t in time, we find from (43) that

$$\begin{aligned} \frac{dW^{t,L}_{N\gg 1}}{dt} = \int \frac{d\mathbf{k}}{(2\pi)^3} N_k \omega \frac{\partial}{\partial \omega} \omega \frac{\partial \varepsilon^t}{\partial t} \Big|_{\omega = |\mathbf{k}|} \\ = \int \frac{N_k\, d\mathbf{k}}{(2\pi)^3} \Big\{ -\omega \frac{\partial \varepsilon^t}{\partial t} + \frac{\partial}{\partial \omega} \frac{\partial}{\partial t} \omega^2 \varepsilon^t \Big\}_{\omega = |\mathbf{k}|} \end{aligned} \tag{45}$$

The superscript L on $W^t_N \gg 1$ is there to indicate the fact that this variation is governed by the variation of the linear ε^t. The explicit expression of (45) in terms of $|\varphi_q|^2$ can be obtained with the use of (2).

As in Ref. 12, we must also take account of the nonlinear contribution, which, in the present case, is given by the right member of (42). A fairly tedious computation yields the result that the obtained $dW^{\text{nonl}}_{N\gg 1}/dt$ is exactly canceled out by the second term in the last expression in (45). Thus,

$$\begin{aligned} \frac{dW^t_{N\gg 1}}{dt} = \frac{dW^{tL}_{N\gg 1}}{dt} + \frac{dW^{t\,\text{nonl}}_{N\gg 1}}{dt} \\ = \int \frac{N_k\, d\mathbf{k}}{(2\pi)^3} \Big(-\omega \frac{\partial \varepsilon^t(\omega, \mathbf{k})}{\partial t} \Big)_{\omega = |\mathbf{k}|}. \end{aligned} \tag{46}$$

This is precisely the sought generalization of the result obtained in Ref. 12. Indeed, on account of the equality $(\omega_k(t))^2 \varepsilon^t(\omega_k(t), \mathbf{k}, t) = \mathbf{k}^2$, we have

$$\frac{\partial}{\partial \omega} \omega^2 \varepsilon^t \Big|_{\omega = \omega_k} \frac{d\omega_k(t)}{dt} + \omega^2 \frac{\partial \varepsilon^t}{\partial t} \Big|_{\omega = \omega_k} = 0,$$

and, on account of the fact that $\varepsilon^t \approx 1$,

$$2 \frac{d\omega_k(t)}{\partial t} = -\omega \frac{\partial \varepsilon^t(\omega, \mathbf{k}, t)}{\partial t} \Big|_{\omega = |\mathbf{k}|},$$

which gives in the general case the relation (40).

Let us now generalize this result to the case in which only the averages of $\hat{A}\hat{A}^+$ are nonzero. According to (44), both the intermediate [$\delta\varepsilon^{t(+)}$ in (44)] and the intermediate negative [$\delta\varepsilon^{t(-)}$ (44)] energies contribute to the permittivity. Now since the express ion for the energy W contains the $\hat{A}\hat{A}^+$, and not the $\hat{A}^+\hat{A}$, combinations, the calculation shows that the expression will contain $\delta\varepsilon^{t(+)}(|\mathbf{k}|,\mathbf{k})$ and $\delta\varepsilon^{t(-)}(-|\mathbf{k}|,\mathbf{k})$, i.e., only energy denomiantors having the same form ($|\mathbf{k}|+\varepsilon_{\mathbf{p}+\mathbf{k}}-\varepsilon_{\mathbf{p}}$ and $|\mathbf{k}|+\varepsilon_{\mathbf{p}+\mathbf{k}}+\varepsilon_{\mathbf{p}}$) as those contained in (34) (in (34) $|\mathbf{k}|=|\mathbf{p}-\mathbf{p}'|$]. Apart from this, we must take into account the renormalization terms in the first two terms in the left member of (42), since the renor malization of the mass gives rise to effects of the same order of magnitude, and is due to $\langle\hat{A}\hat{A}^+\rangle$. In the present case we take account of the renormalization term that makes a contribution proportional to $\Phi_{\mathbf{p}}$. Let us give the result for the part connected with the positive intermediate energies $W_+^{t,L}$ after the expansion in powers of $q\ll p$:

$$\frac{dW_+^{t,L}}{dt}=\frac{e^4}{8\pi}\int\frac{d\mathbf{k}}{|\mathbf{k}|}\frac{\mathrm{Sp}\,\Lambda_{\mathbf{p}}^+\gamma_i{}^t\Lambda_{\mathbf{p}+\mathbf{k}}^+\gamma_i{}^t}{(|\mathbf{k}|+\varepsilon_{\mathbf{p}+\mathbf{k}}-\varepsilon_{\mathbf{p}})^2}\{(\varepsilon_{\mathbf{p}+\mathbf{k}}-\varepsilon_{\mathbf{p}}+|\mathbf{k}|)\}$$
$$\times|\varphi_{\mathbf{q}}|^2\,d\mathbf{q}\,\mathbf{q}\frac{\partial}{\partial\mathbf{p}}\delta(\omega_{\mathbf{q}}-\mathbf{q}\mathbf{v})\,\mathbf{q}\frac{\partial}{\partial\mathbf{p}}\Phi_{\mathbf{p}}\frac{d\mathbf{p}}{(2\pi)^3}. \quad (47)$$

In deriving (47), we used the quasilinear equation (2) to write down the specific expression for $\partial\varepsilon^t/\partial t$. The index L in (47) indicates that the nonlinear effects described by the right member of (42) have been ignored. They give for $dW_+^{t,\mathrm{nonl}}/dt$, as a result of rather tedious calculations an expression that reduces to the form

$$\frac{dW_+^{t,\mathrm{nonl}}}{dt}=-\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{\partial}{\partial\omega}\omega^2\frac{\partial}{\partial t}\delta\varepsilon^{t(+)}(\omega,\mathbf{k})\Big|_{\omega=|\mathbf{k}|}. \quad (48)$$

This result has been written in such a form as to bring out the analogy with the last term in (45), a term which is also canceled out by the nonlinear term. The relation (47) can also be written in a form in which the term that cancels out (48) [this is the term with $|\mathbf{k}|$ in the curly brackets in (47)] is separated out:

$$\frac{dW_+^{tL}}{dt}=\frac{e^4}{8\pi}\int\frac{d\mathbf{k}}{|\mathbf{k}|}\frac{\mathrm{Sp}\,\Lambda_{\mathbf{p}}^+\gamma_i{}^t\Lambda_{\mathbf{p}+\mathbf{k}}^+\gamma_i{}^t}{(|\mathbf{k}|+\varepsilon_{\mathbf{p}+\mathbf{k}}-\varepsilon_{\mathbf{p}})^2}(\varepsilon_{\mathbf{p}+\mathbf{k}}-\varepsilon_{\mathbf{p}})$$
$$\times|\varphi_{\mathbf{q}}|^2\,d\mathbf{q}\frac{d\mathbf{p}}{(2\pi)^3}\mathbf{q}\frac{\partial}{\partial\mathbf{p}}\delta(\omega_{\mathbf{q}}-\mathbf{q}\mathbf{v})\,\mathbf{q}\frac{\partial}{\partial\mathbf{p}}\Phi_{\mathbf{p}}$$
$$+\int\frac{d\mathbf{k}}{(2\pi)^3}\left(\frac{\partial}{\partial\omega}\omega^2\frac{\partial\varepsilon^{t(+)}(\omega,\mathbf{k})}{\partial t}\right)_{\omega=|\mathbf{k}|}. \quad (49)$$

Similarly, we can find $dW_-^t/dt=dW_-^{t,L}/dt+dW_-^{t\,\mathrm{nonl}}/dt$, and, from a comparison with (23) and (24), the sought $G_{\mathbf{q}}^t$ (with $\mathbf{p}'=\mathbf{p}+\mathbf{k}$):

$$G_{\mathbf{q}}^{R(t)}=-\pi^2e^2\int\frac{d\mathbf{p}\,d\mathbf{p}'}{(2\pi)^6}(\varepsilon_{\mathbf{p}'}-\varepsilon_{\mathbf{p}})R_{\mathbf{p},\mathbf{p}'}\mathbf{q}\frac{\partial}{\partial\mathbf{p}}\delta(\omega_{\mathbf{q}}-\mathbf{q}\mathbf{v})\,\mathbf{q}\frac{\partial\Phi_{\mathbf{p}}}{\partial\mathbf{p}}. \quad (50)$$

Let us note that this expression, like (33), does not possess an infrared divergence (at $\mathbf{p}\to\mathbf{p}'$) on account of the factor ($\varepsilon_{\mathbf{p}'}-\varepsilon_{\mathbf{p}}$). In (50) $R_{\mathbf{p},\mathbf{p}'}$ is given by the relations (34) and (35). The total contribution to the radiative corrections is, according to (24) and (27), given by the sum of (50) and (33). By integrating by parts [i.e., by transferring to the left the first $\mathbf{q}(\partial/\partial\mathbf{p})$ derivative in (50)], we find that the final result contains only derivatives of $R_{\mathbf{p},\mathbf{p}'}$:

$$G_{\mathbf{q}}^{R(l)}+G_{\mathbf{q}}^{R(t)}=\frac{e^2\pi^2}{\omega_{\mathbf{q}}}\int\frac{d\mathbf{p}\,d\mathbf{p}'}{(2\pi)^6}\delta(\omega_{\mathbf{q}}-\mathbf{q}\mathbf{v})$$
$$\times\left\{\left[\varepsilon_{\mathbf{p}'}\mathbf{q}\frac{\partial R_{\mathbf{p},\mathbf{p}'}}{\partial\mathbf{p}}-\varepsilon_{\mathbf{p}}\mathbf{q}\frac{\partial R_{\mathbf{p},\mathbf{p}'}}{\partial\mathbf{p}}\right]\mathbf{q}\frac{\partial\Phi_{\mathbf{p}}}{\partial\mathbf{p}}-\omega_{\mathbf{q}}\right.$$
$$\left.\times\left[\Phi_{\mathbf{p}}\mathbf{q}\frac{\partial R_{\mathbf{p},\mathbf{p}'}}{\partial\mathbf{p}}-\Phi_{\mathbf{p}'}\mathbf{q}\frac{\partial R_{\mathbf{p}',\mathbf{p}}}{\partial\mathbf{p}}\right]\right\}. \quad (51)$$

The expressions obtained from (51) and (27) for the radiative corrections to the Landau damping differ qualitatively from the expression (5) for the Landau damping itself in that they contain, besides terms proportional to the derivative of the distribution function, terms proportional to the distribution function itself. Often, the quasilinear relaxation can lead to the decrease of the derivatives of the distribution function, which, without allowance for the radiative corrections, were limited to the pair-collision quantities.[2] Such quasilinear relaxation should have no effect on that part of the radiative corrections which does not depend on the derivatives of the particle distribution function. If most of the particles are nonrelativistic ($p\ll m$), then estimates and calculations show that the terms proportional to $\Phi_{\mathbf{p}}$ in (51) are nonanalytic when expanded in powers of $p^2/m^2\ll 1$, and their ratio to the derivative dependent terms in (51) is of the order of $|(p^2/m^2)\ln(p^2/m^2)|$ in smallness. This means that the part of the radiative corrections that is determined by $\Phi_{\mathbf{p}}$ can be greater than the linear damping (1) (and, consequently, that part of (51) which depends on $\mathbf{q}\partial\Phi_{\mathbf{p}}/\partial\mathbf{p}$, and contains the additional small factor $e^2/\hbar c$), if, roughly speaking,

$$\frac{1}{N_d}\ll\frac{e^2}{\hbar c}\frac{p^2}{m^2}\left|\ln\frac{p^2}{m^2}\right|, \quad (52)$$

where N_d is the number of particles in the Debye medium that determine the frequency of pair collisions.

5. RADIATIVE KINETICS OF PARTICLES

The energy of the fluctuational electromagnetic fields of quite high frequencies (i.e., with $|\mathbf{k}|\sim m$ or even with $|\mathbf{k}|\gg m$) increases, according to (50), quasiadiabatically in time because of the quasilinear particle acceleration (2). The presence of such increasing—in time—electromagnetic fluctuations leads to the possibility of large energy transfers to some particles. This changes the distribution $\Phi_{\mathbf{p}}$. The variation of $\Phi_{\mathbf{p}}(t)$ is described by the radiative corrections to Eq. (2). The equation for $\Phi_{\mathbf{p}}(t)$ can be obtained by differentiating with respect to the time the relation (20), in which allowance is made for the terms quadratic in $\hat{L}_\varphi$ and $\hat{L}_A$, as well for as the renormalization counterterms:

$$\frac{d\Phi_{\mathbf{p}}^R}{dt}\underset{t\to\infty}{\equiv}\frac{d}{dt}\mathrm{Sp}\int d\mathbf{q}'\{\langle\delta\hat{f}_{\mathbf{p}'\mathbf{q}'}^{(0)}(t)\hat{S}^{(4)}(t)+\hat{S}^{+(4)}(t)\delta\hat{f}_{\mathbf{p},\mathbf{q}'}^{(0)}\rangle$$
$$+\langle\hat{S}^{+(1)}(t)\delta\hat{f}_{\mathbf{p},\mathbf{q}'}^{(0)}(t)\hat{S}^{(3)}(t)+\hat{S}^{+(3)}(t)\delta\hat{f}_{\mathbf{p},\mathbf{q}'}^{(0)}(t)\hat{S}^{(1)}(t)\rangle$$
$$+\langle\hat{S}^{+(2)}(t)\delta\hat{f}_{\mathbf{p},\mathbf{q}'}^{(0)}(t)\hat{S}^{(2)}(t)\rangle\}. \quad (53)$$

Allowance must be made in the calculations for the renormalization of $\Phi_{\mathbf{p}}$ in the quasilinear equation. As a result of tedious calculations we obtain[14,15]:

$$\frac{d\Phi_{\mathbf{p}}^R}{dt}=\pi^2e^4\int|\varphi_{\mathbf{q}}|^2\frac{d\mathbf{q}\,d\mathbf{p}'}{(2\pi)^3}\left\{R_{\mathbf{p}',\mathbf{q}}\mathbf{q}\frac{\partial}{\partial\mathbf{p}'}\delta(\omega_{\mathbf{q}}-\mathbf{q}\mathbf{v}')\,\mathbf{q}\frac{\partial\Phi_{\mathbf{p}'}}{\partial\mathbf{p}'}\right.$$
$$-R_{\mathbf{p},\mathbf{p}'}\mathbf{q}\frac{\partial}{\partial\mathbf{p}}\delta(\omega_{\mathbf{q}}-\mathbf{q}\mathbf{v})\,\mathbf{q}\frac{\partial\Phi_{\mathbf{p}}}{\partial\mathbf{p}}$$
$$\left.+\mathbf{q}\frac{\partial}{\partial\mathbf{p}}\delta(\omega_{\mathbf{q}}-\mathbf{q}\mathbf{v})\,\mathbf{q}\frac{\partial}{\partial\mathbf{p}}(R_{\mathbf{p},\mathbf{p}'}\Phi_{\mathbf{p}}-R_{\mathbf{p}',\mathbf{p}}\Phi_{\mathbf{p}'})\right\}. \quad (54)$$

It is easy to see that the energy conservation law is obeyed, namely that the change that occurs in the particle energy as a result of the radiative corrections, and is given by (54), is equal to minus the field-energy change given by (51). Equation (54) can also be obtained by methods closer to those usually used to derive Eqs. (1) and (2). For the operator $\hat{f}_{\mathbf{p},\mathbf{k}}$ [see (10)], by using the equation of motion and the commutation relations for the one-time operators $\hat{\Psi}$, we can obtain[14]

$$\frac{\partial \hat{f}_{\mathbf{p},\mathbf{k}}(t)}{\partial t}+i\beta(i\gamma(\mathbf{p}+\mathbf{k}/2)+m)\hat{f}_{\mathbf{p},\mathbf{k}}(t)-\hat{f}_{\mathbf{p},\mathbf{k}}(t) \times(-i\gamma(\mathbf{p}-\mathbf{k}/2)+m)i\beta=e\int d\mathbf{k}_1\,d\mathbf{k}_2\,\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2) \times[\hat{f}_{\mathbf{p}+\mathbf{k}_2/2,\mathbf{k}_1}\beta\hat{A}_{\mathbf{k}_2}(t)-\beta\hat{A}_{\mathbf{k}_2}(t)\hat{f}_{\mathbf{p}-\mathbf{k}_2/2,\mathbf{k}_1}(t)], \quad \hat{A}_{\mathbf{k}}(t)=\gamma_\mu\hat{A}_{\mathbf{k},\mu}(t). \tag{55}$$

The procedure for deriving (54) from (55) is similar to the procedure for deriving (1) and (2) in the classical theory. We average Eq. (55) over the fluctuations, introduce $\langle\hat{f}_{\mathbf{p},\mathbf{k}}\rangle$ and $\delta\hat{f}_{\mathbf{p},\mathbf{k}}=\hat{f}_{\mathbf{p},\mathbf{k}}-\langle\hat{f}_{\mathbf{p},\mathbf{k}}\rangle$, and construct perturbation theory in terms of the fields φ and A [see (16) and (17)] both in Eq. (55) and in the Maxwell equations with the current (12). We take account of the terms quadratic in φ and A, the renormalization terms, and the fact that, in the zeroth approximation, the particle distribution varies quasilinearly in time in accordance with (2). The latter is extremely important, since it allows us to take into account by another method the role of the quasiadiabatic variation of the fluctuations discussed in Sec. 4 of the present paper. We should, in deriving the kinetic equation (54), use for the mean fluctuations the following relation, which is easily proved with the aid of (19), (21), and (10):

$$\delta\hat{f}^{(0)}_{\mathbf{p},\mathbf{k}}(t)=\sum_{\lambda,\lambda'}\delta\hat{f}^{(0)\lambda,\lambda'}_{\mathbf{p},\mathbf{k}}\exp[i\lambda\varepsilon_{\mathbf{p}+\mathbf{k}/2}t-i\lambda'\varepsilon_{\mathbf{p}-\mathbf{k}/2}t],$$

$$\langle\delta\hat{f}^{(0)++}_{\alpha,\beta,\mathbf{p},\mathbf{k}}\,\delta\hat{f}^{(0)++}_{\alpha',\beta',\mathbf{p}',\mathbf{k}'}\rangle={}^1/_2\Phi_{\mathbf{p}-\mathbf{k}/2}(\overset{+}{\Lambda}_{\mathbf{p}+\mathbf{k}/2}\beta)_{\alpha,\beta'} \times(\overset{+}{\Lambda}_{\mathbf{p}-\mathbf{k}/2}\beta)_{\alpha',\beta}\delta(\mathbf{p}-\mathbf{p}')\delta(\mathbf{k}+\mathbf{k}'), \tag{56}$$

$$\langle\delta\hat{f}^{(0)+-}_{\alpha,\beta,\mathbf{p},\mathbf{k}}\,\delta\hat{f}^{(0)-+}_{\alpha',\beta',\mathbf{p}',\mathbf{k}'}\rangle={}^1/_2\Phi_{\mathbf{p}+\mathbf{k}/2}(\overset{+}{\Lambda}_{\mathbf{p}+\mathbf{k}/2}\beta)_{\alpha,\beta'} \times(\overset{-}{\Lambda}_{\mathbf{p}-\mathbf{k}/2}\beta)_{\alpha',\beta}\delta(\mathbf{p}-\mathbf{p}')\delta(\mathbf{k}+\mathbf{k}').$$

In the perturbation theory in (55), $\delta f^{(0)}_{\mathbf{p},\mathbf{k}}$ arises as the zeroth approximation in the absence of fields. The structure of the relation in (56) that contains the indices α,β' and α',β', and also the presence of a Φ that depends on the shifted momentum $\mathbf{p}\pm\mathbf{k}/2$, indicate the presence of exchange effects and the fact that their consideration will give rise to $\Phi_{\mathbf{p}'}$. This explains why the equation for the radiative corrections should contain $\Phi_{\mathbf{p}'}=\Phi_{\mathbf{p}+\mathbf{k}}$ [see the first and last terms in (54)].

The relation (56) is a generalization of the well-known classical relation for the δf in (5):

$$\delta f^{(0)}_{\mathbf{p},\mathbf{q}}(t)=\delta f^{(0)}_{\mathbf{p},\mathbf{q}}\exp(-i\mathbf{q}\mathbf{v}t), \quad \langle\delta f^{(0)}_{\mathbf{p},\mathbf{q}}\,\delta f^{(0)}_{\mathbf{p}',\mathbf{q}'}\rangle=\Phi_{\mathbf{p}}\delta(\mathbf{p}-\mathbf{p}')\delta(\mathbf{q}+\mathbf{q}'), \tag{57}$$

with the aid of which we can easily obtain the Landau-Balescu pair-collision integral from (5) (see, for example, Ref. 5). The collision can also be realized through virtual-field exchange between different particles.

The derivation of (54) from (55) with allowance for (56) sets off from a somewhat different standpoint the question of the occurrence of high-energy particles in (54). As has been pointed out, a virtual photon considered in the calculation of the radiative corrections is absorbed in the particle system, i.e., can effect energy exchange between different particles, in particular, between particles that differ greatly in their energies. In contrast to the pair collisions, which are proportional to $\Phi_{\mathbf{p}}\Phi_{\mathbf{p}'}$ here the first order—the Cherenkov interaction—and, consequently, the effects described by the product of this interaction and the virtual-photon exchange, which are linear in $\Phi_{\mathbf{p}}$ and $\Phi_{\mathbf{p}'}$, do not vanish. Notice that the exchange terms containing $\Phi_{\mathbf{p}'}$ in (54) do not contain renormalizations, since the exchange occurs between "different" particles, and there are no self-energy parts in the graphs.

The terms containing $\Phi_{\mathbf{p}'}$ in (54) can exceed by far not only the remaining terms in (54), but also the quasilinear acceleration (2), if the number $\Phi_{\mathbf{p}}$ of particles is many-orders-of-magnitude smaller than $\Phi_{\mathbf{p}'}$, i.e., if the exchange processes describe the production of a very small number of particles of very high energies. For the computation of the spectrum of the high-energy particles, it is sufficient to know the asymptotic form of the first term in (54) at $p\gg p'$, which requires knowledge of $R_{\mathbf{p},\mathbf{p}'}$. The general expression for $R_{\mathbf{p},\mathbf{p}'}$ has, after the evaluation of the traces in (34), the form

$$R_{\mathbf{p},\mathbf{p}'}=-\frac{2}{|\mathbf{p}-\mathbf{p}'|\varepsilon_{\mathbf{p}}\varepsilon_{\mathbf{p}'}} \times\left\{\frac{m^2-\varepsilon_{\mathbf{p}}\varepsilon_{\mathbf{p}'}+(\mathbf{p}(\mathbf{p}-\mathbf{p}'))(\mathbf{p}'(\mathbf{p}-\mathbf{p}'))/(\mathbf{p}-\mathbf{p}')^2}{(\varepsilon_{\mathbf{p}'}+|\mathbf{p}-\mathbf{p}'|-\varepsilon_{\mathbf{p}})^2} -\frac{m^2+\varepsilon_{\mathbf{p}}\varepsilon_{\mathbf{p}'}+(\mathbf{p}(\mathbf{p}-\mathbf{p}'))(\mathbf{p}'(\mathbf{p}-\mathbf{p}'))/(\mathbf{p}-\mathbf{p}')^2}{(\varepsilon_{\mathbf{p}'}+\varepsilon_{\mathbf{p}}+|\mathbf{p}-\mathbf{p}'|)^2}\right\}. \tag{58}$$

The asymptotic form obtained for $R_{\mathbf{p},\mathbf{p}'}$ from (58) at $p\gg p'$, $p'\ll m$ under conditions of isotropic particle distribution has the form (the bar denotes averaging over the angles)

$$\bar{R}_{\mathbf{p}',\mathbf{p}}=\frac{4p'^2}{3\varepsilon_{\mathbf{p}}p^3(p+\varepsilon_{\mathbf{p}})}\left(1+\frac{p}{2(\varepsilon_{\mathbf{p}}+p)}\right). \tag{59}$$

This gives for the isotropic relativistic-particle distribution the universal power-law spectrum

$$\frac{d}{dt}\Phi_{\mathbf{p}}p^2\propto\frac{1}{p^3}. \tag{60}$$

Similar results are obtained in Ref. 15 for the spectrum of particles of zero spin (the equation for $\Phi_{\mathbf{p}}$ in this case coincides in form with (54), the difference being that $R_{\mathbf{p},\mathbf{p}'}$ is given by a different expression, although it has the same asymptotic form (60)). For the anisotropic case (58) yields the spectrum $1/p^2$, instead of (60). The indicate power-law spectra are close to the observed cosmic-ray spectra.[16]

Let us show that the above-described effect of power-law spectrum generation is, in the present approximation, entirely due to that part of the radiative corrections to the Landau damping which is connected with the quasiadiabatic variation of the fluctuational electromagnetic fields (Sec. 4). To obtain the rate of acquisition of energy by the fast ($p\gg p'$, $p'\ll m$) particles, we must multiply the first term in (54), which is just the term that describes this generation, by the kinetic energy of the fast particles ($\varepsilon_{\mathbf{p}}-m$), and integrate over p [the index 1 in (61) indicates the fact that the first term in (54) is considered]:

$$\frac{d}{dt}\int(\varepsilon_{\mathbf{p}}-m)\left(\frac{d\Phi_{\mathbf{p}}^{R}}{dt}\right)_{1}\frac{d\mathbf{p}}{(2\pi)^{3}}=\pi^{2}e^{4}\int|\varphi_{\mathbf{q}}|^{2}d\mathbf{q}\frac{d\mathbf{p}\,d\mathbf{p}'}{(2\pi)^{6}}\times\mathbf{q}\frac{\partial}{\partial\mathbf{p}'}(\varepsilon_{\mathbf{p}}-m)R_{\mathbf{p}',\mathbf{p}}\delta(\omega_{\mathbf{q}}-\mathbf{q}\mathbf{v}')\mathbf{q}\frac{\partial\Phi_{\mathbf{p}'}}{\partial\mathbf{p}'}. \quad (61)$$

To prove the assertion made above, it is sufficient to turn to the relations (50), (23), and (24), make the substitution $\mathbf{p}\rightleftarrows\mathbf{p}'$ in (50), and take account of the fact that most of the particles are nonrelativistic: $\varepsilon_{\mathbf{p}'}\approx m$. It can then be seen that (50) yields (61) with the opposite sign.

Thus, we have obtained an interpretation of the acceleration effects as being due to the quasilinear pumping of the high-frequency electromagnetic fluctuations. It is naturally connected with the idea of exchange-governed acceleration. Using the approximation (59) in (23), (24), and (50), we find that, in this approximation, the radiative corrections are entirely due to the acceleration of the particles, and that

$$\gamma_{\mathbf{q}}^{R}=\gamma_{\mathbf{q}}^{L}\frac{8e^{2}}{3\pi\hbar c}\left(\ln 2-\frac{11}{24}\right)\approx 1.46\cdot 10^{-3}\gamma_{\mathbf{q}}^{L}. \quad (62)$$

The last numerical estimate in (62) is for $e^2/\hbar c = 1/137$. The quantity $\gamma_{\mathbf{q}}^{R}/\gamma_{\mathbf{q}}^{L}$ gives the relative fraction of the energy, that is transferred for the generation of fast particles.

[1]L. D. Landau, Zh. Eksp. Teor. Fiz. **11**, 592 (1941).

[2]A. A. Vedenov, E. P. Velikhov, and R. Z. Sagdeev, Yad. Sintez **1**, 82 (1962); *ibid.*, Suppl. Part 2, 465 (1962); W. E. Drummond and P. Pines, Nucl. Fusion Suppl. Part 3, 1049 (1962).

[3]V. N. Tsytovich, *Nonlinear Effects in Plasma*, Plenum, New York, 1970; *Theory of Turbulent Plasma*, Plenum, New York, 1977.

[4]B. B. Kadomtsev, in: Problems of Plasma Theory, Vol. 4 [in Russian], Atomizdat, Moscow, 1964, p. 258.

[5]A. G. Sitenko, *Fluctuations & Non-Linear Wave Interactions in Plasmas*, Pergamon, Elmsford, N. Y., 1982.

[6]V. L. Ginzburg and V. N. Tsytovich, Zh. Eksp. Teor. Fiz. **65**, 1818 (1973) [Sov. Phys. JETP **38**, 909 (1974)].

[7]V. N. Tsytovich, Tr. Fiz. Inst. Akad. Nauk SSSR **66**, 176 (1973); A. V. Akopyan and V. N. Tsytovich, Zh. Eksp. Teor. Fiz. **71**, 166 (1976) [Sov. Phys. JETP **44**, 87 (1976)].

[8]V. L. Ginzburg, and V. N. Tsytovich, Transition Radiation and Transition Scattering [in Russian], Nauka, Moscow, 1984.

[9]V. N. Tsytovich, Preprint No. 90, Institute of General Physics, Academy of Sciences of the USSR, 1986.

[10]V. N. Tsytovich Zh. Eksp. Teor. Fiz. **40**, 1775 (1961) [Sov. Phys. JETP **13**, 1249 (1961)].

[11]E. M. Lifshitz and L. P. Pitaevskiĭ, *Relativistic Quantum Theory*, Parts 1 & 2, Pergamon, Oxford, 1974.

[12]S. B. Isakov, V. S. Krivitskiĭ, and V. N. Tsytovich, Zh. Eksp. Teor. Fiz. **90**, 933 (1986) [Sov. Phys. JETP **63**, 545 (1986)].

[13]L. P. Pitaevskiĭ, Zh. Eksp. Teor. Fiz. **39**, 1450 (1960) [Sov. Phys. JETP **12**, 1008 (1961)].

[14]V. N. Tsytovich, Physica Scripta **2/1**, 54 (1982); **2/2**, 567 (1982).

[15]V. N. Tsytovich, Zh. Eksp. Teor. Fiz. **87**, 1105 (1984) [Sov. Phys. JETP **60**, 631 (1984)]; Preprint No. 186, Fiz. Inst. Akad. Nauk SSSR, Moscow, 1983.

[16]V. N. Tsytovich, in Proc. School and Workshop on Plasma Astrophysics, ESA Varenna, 1984, p. 141; ESA Sukhumi, 1986, p. 217.

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