Lifetime and switching-current distribution of the resistive state of a Josephson junction

V. I. Mel'nikov

L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR
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The Fokker-Planck equation is solved for the distribution function of the phase and the voltage across a high-Q Josephson junction. An exact expression is derived for the lifetime of the resistive state of the junction. If the current through the junction is reduced slowly and linearly with time, the distribution of the currents of the stochastic switching of the junction between the resistive state and the superconducting state is Gaussian with a half-width on the order of $eT/RC\omega$, where e is the electron charge, T is the temperature, and R, C, and ω are the resistance, capacitance, and Josephson frequency of the junction. A measure of the dispersion of this distribution is derived as a function of the position of the distribution maximum for currents on the order of $I_c/RC\omega$ ($RC\omega \gg 1$, where I_c is the critical current of the junction).

1. INTRODUCTION

In the resistive model of a point Josephson junction[1] the current through the junction, I, is the sum of the supercurrent $I_c \sin\varphi$ and the normal current $V/R + CdV/dt$, where I_c is the critical current, φ is the phase difference between the order parameters, V is the voltage across the junction, R is the resistance of the junction in its normal state, and C is its capacitance. Here V and φ are connected by the Josephson relation, so the system of equations for these properties.

$$CdV/dt+V/R+I_c\sin\varphi=I, \quad d\varphi/dt=2eV,$$

is equivalent to the single equation

$$\frac{d^2\varphi}{dt^2}+\frac{1}{RC}\frac{d\varphi}{dt}+\frac{2e}{C}(I_c\sin\varphi-I)=0, \tag{1}$$

which describes the motion of a particle with friction in a sloping periodic potential. We consider a high-Q junction, for which the friction coefficient $1/RC$ is small in comparison with the small-oscillation frequency at $I = 0$:

$$\omega=(2eI_c/C)^{1/2}.$$

Our results will then hold under the condition $\beta \gg 1$, where $\beta \equiv RC\omega$. It can be seen from Eq. (1) that as the current is increased slowly from $I = 0$ there is no voltage across the junction ($d\varphi/dt = 0$) as long as the potential has minima ($I < I_c$). For $I > I_c$ the junction goes into its resistive state, with $d\varphi/dt \neq 0$, and a nonzero average voltage $V(I)$ appears across the junction. This average voltage is determined by the condition that the energy dissipated by friction is offset by the energy acquired from the slope of the potential. This condition is not tied to the existence of potential minima, so as the current decreases the resistive state may persist, even if I falls below I_c. The voltage across the junction remains nonzero as the current is lowered to

$$I=4I_c/\pi\beta.$$

At lower currents, the state of the junction with $d\varphi/dt = 0$ is the only one possible. For $I > I_c$, there can thus be only a resistive state, while for $I < 4I_c/\pi\beta$ there can be only a superconducting state; at intermediate currents, both of these states are stable. This effect is familiar as hysteresis in the I-V characteristic of a Josephson junction. Switching between stable states can be accomplished, as pointed out above, by increasing or reducing the current I beyond the hysteresis interval. When thermal fluctuations are taken into account, we see that switching between the two branches of the voltage-current characteristic are possible even at constant I. The probability for such switching events is small as long as the energy barrier between the two states, I_c/e, is large in comparison with the temperature T. This condition gives us a second large parameter of the problem: $\gamma = I_c/eT$. Under the conditions $\beta \gg 1$ and $\gamma \gg 1$, we can take up the question of calculating the lifetime of the superconducting and resistive states.[2]

The lifetime of the superconducting state is determined by the rate at which the Brownian particle modeling the state of the junction leaves the deep potential well. This lifetime was calculated in Ref. 3, where the partial probabilities for the trapping of an escaping particle in neighboring potential wells (i.e., the probabilities for a jump of $2\pi n$, where n is an integer, in the phase of the junction) were also found. To determine the lifetime of the resistive state we need to know the distribution of the particles which move high above the barriers. This problem was solved in the exponential approximation by Vollmer and Risken.[4] In the present paper we offer a complete solution for the lifetime of the resistive state, taking the approach of Ref. 3. We use the result to find the distribution of the currents at which the junction switches from the resistive state to the superconducting state for the case in which the current I decreases very slowly and linearly over time. The results show that this distribution is Gaussian. We find a measure of the dispersion of this distribution as a function of the position of its peak.

2. DISTRIBUTION FUNCTION AT HIGH ENERGIES

We begin with a discussion of the I-V characteristics. For a high-Q junction, currents $I \gtrsim I_c/\beta \ll I_c$ are important. To first order we can thus ignore the effect of the slope and the dissipation and assume that the total energy

$$\varepsilon=\frac{I_c}{e}\left[\frac{1}{4\omega^2}\left(\frac{d\varphi}{dt}\right)^2-\frac{1}{2}(\cos\varphi+1)\right] \tag{2}$$

is conserved. We can then find the functional dependence

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$\varphi(t)$ for a given ε. For the time-averaged voltage across the junction we then find

$$V=\frac{1}{2e}\left\langle\frac{d\varphi}{dt}\right\rangle=\frac{\pi\omega}{2ep(\varepsilon e/I_c)}, \qquad (3)$$

where

$$p(x)=\int_0^{\pi/2}(x+\sin^2\varphi)^{-1/2}\,d\varphi. \qquad (4)$$

The rate of energy dissipation is

$$\frac{d\varepsilon}{dt}=-\frac{1}{4e^2R}\left(\frac{d\varphi}{dt}\right)^2,$$

so when φ changes by 2π the energy decreases by an amount

$$\delta(\varepsilon)=\frac{1}{4e^2R}\int_0^{2\pi}\frac{d\varphi}{dt}\,d\varphi=\frac{4I_c}{e\beta}\,r\left(\frac{\varepsilon e}{I_c}\right), \qquad (5)$$

where

$$r(x)=\int_0^{\pi/2}(x+\sin^2\varphi)^{1/2}\,d\varphi. \qquad (6)$$

The energy ε is described as a function of the current by

$$r(\varepsilon e/I_c)=\pi\beta I/4I_c. \qquad (7)$$

This equation relates the energy dissipation to the slope of the potential. Equations (3)–(7) parametrically determine the time-averaged I-V characteristic of the junction. Equation (7) has a solution for ε only at $I>4I_c/\pi\beta$, so we have $V(I)=0$ as long as the condition $I<4I_c/\pi\beta$ holds; thereafter, $V(I)$ increases sharply and quickly assumes an ohmic (linear) dependence.[2] An important point is that the characteristic values $\varepsilon\sim I_c/e$ correspond to the largest of the energy scales of the problem.

At a nonzero temperature the current I in Eq. (1) must be supplemented with a fluctuation $I_f(t)$ with a correlation function

$$\langle I_f(t)I_f(t')\rangle=(2T/R)\delta(t-t').$$

The state of the junction in the presence of fluctuations is described by a Fokker-Planck equation for the distribution function $f(t,\varphi,\dot\varphi)$:

$$\frac{\partial f}{\partial t}+\dot\varphi\frac{\partial f}{\partial\varphi}-\frac{2e}{C}(I_c\sin\varphi-I)\frac{\partial f}{\partial\dot\varphi}=\frac{1}{RC}\frac{\partial}{\partial\dot\varphi}\left(\frac{4e^2T}{C}\frac{\partial f}{\partial\dot\varphi}+\dot\varphi f\right), \qquad (8)$$

where $\dot\varphi\equiv d\varphi/dt$. We wish to find solutions of the form

$$f(t,\varphi,\dot\varphi)=f(\varphi,\dot\varphi)\exp(-t/\tau), \qquad (9)$$

which are periodic in φ. We show below that under the condition $I>4I_c/\pi\beta$ the parameter τ which we have introduced here is determined unambiguously by the conditions of the problems and thus gives the lifetime of the resistive state.

Substituting (9) into (8), and transforming from the variables φ, $\dot\varphi$ to φ, ε, we find

$$-\frac{f}{\tau\dot\varphi(\varepsilon,\varphi)}+\frac{\partial f}{\partial\varphi}+\frac{I}{2e}\frac{\partial f}{\partial\varepsilon}=\frac{1}{4e^2R}\frac{\partial}{\partial\varepsilon}\left[\dot\varphi(\varepsilon,\varphi)\left(T\frac{\partial f}{\partial\varepsilon}+f\right)\right], \qquad (10)$$

where $\dot\varphi(\varepsilon,\varphi)$ is determined from (2) and is positive, since only particles with positive velocities are present in the region $\varepsilon\gg T$. The right side of (10) is small because the dissipation is small, and the last term on the left side is small because the slope of the potential is small. The first term on the left side is even smaller, since the lifetime is exponentially large. If we ignore these terms we find $\partial f/\partial\varphi=0$. In the next approximation, we follow the customary procedure of integrating (10) over the interval $(0, 2\pi)$. The integral of the leading term, $\partial f/\partial\varphi$, vanishes identically because f is periodic in φ, and in the other integrals we can assume f to be independent of φ. As a result we find the equation

$$\mu f+\frac{d}{d\varepsilon}\left\{\delta(\varepsilon)T\frac{df}{d\varepsilon}+[\delta(\varepsilon)-\pi I/e]f\right]=0, \qquad (11)$$

where $\mu\equiv2p(\varepsilon e/I_c)/\omega\tau$. The term μf distinguishes this equation from that derived previously[4] for the static case. If we ignore the term μf, we find that the solution of this equation is

$$f=\exp\left\{-\int_0^{\varepsilon}\frac{d\varepsilon'}{T}\left[1-\frac{\pi I}{e\delta(\varepsilon')}\right]\right\}+\frac{C_1}{\delta(\varepsilon)-\pi I/e}, \qquad (12)$$

where C_1 is a constant. The solution (12) does not hold in a small neighborhood of the energy $\tilde\varepsilon$ which satisfies Eq. (7). In this neighborhood we use the expansion

$$\delta(\varepsilon)-\pi I/e=\Delta(\varepsilon-\tilde\varepsilon),\quad \Delta=\delta'(\tilde\varepsilon)=2p(\tilde\varepsilon e/I_c)/\beta,$$

and we set $\delta(\varepsilon)=\delta(\tilde\varepsilon)=\pi I/e$. The replacement

$$f=\psi\exp[-e\Delta(\varepsilon-\tilde\varepsilon)^2/4\pi IT]$$

and the switch to the variable $x\equiv(\varepsilon-\tilde\varepsilon)(e\Delta/\pi IT)^{1/2}$ lead to the equation

$$\psi''+(1/2+\mu/\Delta-x^2/4)\psi=0,$$

i.e., a Schrödinger equation for an oscillator whose energy exceeds the ground-state energy by $\mu/\Delta\ll1$. The function $f(\varepsilon)$ is normalizable if $\psi(x)$ decays as $x\to\infty$. A nonzero value of μ/Δ would then mean that ψ contains an exponentially growing term at large negative x:

$$\psi=\exp(-x^2/4)+((2\pi)^{1/2}\mu/\Delta x)\exp(x^2/4),\quad x<0,\quad |x|\gg1.$$

Comparison of this expression with (12) yields the constant C_1. In the region $\tilde\varepsilon\gg\varepsilon\gg T$, where we can now set

$$\delta(\varepsilon)=\delta(0)=4I_c/e\beta,$$

but in which we do not yet have to allow for the change in $f(\varepsilon)$ due to the reflection of particles from the potential barriers, we find

$$f(\varepsilon)=\exp\left(-\frac{\zeta\varepsilon}{T}\right)+\frac{\pi\mu\beta}{4I_c\zeta}\left(\frac{2eIT}{\Delta}\right)^{1/2}\times\exp\left\{-\int_0^{\tilde\varepsilon}\frac{d\varepsilon}{T}\left[1-\frac{\pi I}{e\delta(\varepsilon)}\right]\right\}, \qquad (13)$$

where $\zeta=1-\pi\beta I/4I_c$. We will use this expression below as a boundary condition on the function describing the distribution of particles at energies $\varepsilon\sim T$ near the crest of the barrier. Here we have $\zeta<0$, so the first term in (13) increases exponentially with the energy.

3. DISTRIBUTION FUNCTION NEAR THE TOP OF THE BARRIER

We can draw the following picture of the decay of the resistive state. As the Brownian particle representing the state of the junction moves above the periodic sequence of potential barriers, it eventually strikes one of them, is reflected, and—after a relaxation time, and with a nonzero probability—is trapped by one of the potential wells. The time scale of the inverse process—the escape of the particle from the well—is large, proportional to $\exp(I_c/eT)$, so that process can be ignored. The particle flux in energy space is determined by the second term in expression (13), so in order to determine μ we need to determine the structure of the distribution function at an energy at the level of the barrier, with $|\varepsilon| \sim T$. In this energy region, we introduce the functions $f^R(\varepsilon)$ and $f^L(\varepsilon)$ to describe the particles whose velocities are directed to the right and to the left. We write the periodicity condition for the transition from one barrier to another as the system of integral equations[3]

$$f^R(\varepsilon)=\int_{-\infty}^{\infty} g(\varepsilon-\varepsilon'-\pi I/e)\,[f^R(\varepsilon')\theta(\varepsilon')+f^L(\varepsilon')\theta(-\varepsilon')]\,d\varepsilon',$$

$$f^L(\varepsilon)=\int_{-\infty}^{\infty} g(\varepsilon-\varepsilon'+\pi I/e)\,[f^R(\varepsilon')\theta(-\varepsilon')+f^L(\varepsilon')\theta(\varepsilon')]\,d\varepsilon'. \tag{14}$$

We see that the function $f^R(\varepsilon)$ is made up of particles which have moved above the neighboring barrier with energies $\varepsilon' > 0$ [the term with $f^R(\varepsilon')$] and particles which have been reflected from the same barrier and which have energies $\varepsilon' < 0$ [the term with $f^L(\varepsilon')$]. The kernel of the integral equations (14).

$$g(\varepsilon-\varepsilon')=(4\pi\delta T)^{-1/2}\exp[-(\varepsilon-\varepsilon'+\delta)^2/4\delta T],$$

is the Green's function of Eq. (11) in the case $\mu = 0, I = 0$. It describes a Gaussian spreading of the distribution function due to the friction and the fluctuations when a particle is displaced by 2π. Argument shifts $\pm \pi I/e$ allow for the fact that energy is acquired or lost when a particle undergoes a displacement of 2π to the right or left. Since the range over which the energy varies is small, $|\varepsilon| \sim T$, we set $\delta(\varepsilon) = \delta(0) = \delta$ in Eqs. (14). After taking one-sided Fourier transforms,

$$\varphi_\pm(\lambda)=\int f(\varepsilon)\theta(\pm\varepsilon)\exp(i\lambda\varepsilon/T)\,d\varepsilon,$$

we put the system (14) in the form

$$\varphi_+{}^R+\varphi_-{}^R=g_+(\varphi_+{}^R+\varphi_-{}^L),\quad \varphi_+{}^L+\varphi_-{}^L=g_-(\varphi_-{}^R+\varphi_+{}^L), \tag{15}$$

where

$$g_\pm(\lambda)=\exp[-\delta\lambda^2/T-i\lambda(\delta\mp\pi I/e)/T].$$

Solving (15) for $\varphi_-{}^R$ and $\varphi_-{}^L$, and forming the difference $\varphi = \varphi^R - \varphi^L$, we find a Wiener-Hopf equation for the new function:

$$\varphi_+(\lambda)=G(\lambda)\varphi_-(\lambda), \tag{16}$$

where

$$G(\lambda)=(1-g_+g_-)/(1-g_+)(1-g_-). \tag{17}$$

The resulting equation relates the function φ_+, which is analytic in the upper λ half-plane, to the function φ_-, which is analytic in the lower half-plane. Our problem differs from that solved in Ref. 3 in that there is no equilibrium population of the potential minima; there is only a particle flux in energy space due to the relaxation of the particles which are moving high above the barriers [the second term in (13)].

What are the conditions under which Eq. (16), should be solved? The presence of a certain number of particles with a Boltzmann distribution function $\propto \exp(-\varepsilon/T)$, in the potential wells would correspond to a pole $\varphi_-(\lambda)$ at point $\lambda = -i$. In the absence of such particles, $\varphi_-(-i)$ would be finite, and by virtue of the condition $G(-i) = 0$ the following condition would hold:

$$\varphi_+(-i)=0. \tag{18}$$

In turn, asymptotic expression (13) shows that $\varphi_+(\lambda)$ has poles at the points $\lambda = -i\zeta$ and $\lambda = 0$. If we define $\varphi_+(\lambda)$ in such a way that we have

$$\varphi_+(\lambda)\approx iT/(\lambda+i\zeta),\quad |\lambda+i\zeta|\ll 1 \tag{19}$$

[this case corresponds to the amplitude of the first term in (13)], then we could use the residue $\varphi_+(\lambda)$ at the point $\lambda = 0$ to find the unknown parameter μ. A solution of Eq. (16) satisfies conditions (18) and (19) as

$$\varphi_+(\lambda)/G_+(\lambda)=-G_-(\lambda)\varphi_-(\lambda)=iT/(\lambda+i\zeta)G_+(-i\zeta), \tag{20}$$

if we use the following expression for the factorization $G(\lambda) = G_+(\lambda)G_-(\lambda)$:

$$G_\pm(\lambda)=\exp\Big[\pm\int_{-\infty}^{\infty}\frac{d\lambda'}{2\pi i}\frac{\ln G(\lambda')}{\lambda'-\lambda\mp i0}\Big]. \tag{21}$$

Condition (18) would then in fact hold, by virtue of the condition $G_+(-i) = 0$ [a change in the sign of the imaginary part of λ in (21) would take a factor $G(\lambda)$ out of the coefficient of the exponential function], and the validity of condition (19) is obvious from the way (20) is written. Formally, the problem has now been solved. It remains to find explicit expressions for $G_+(\lambda)$. We return to the expression for $G(\lambda)$ in the form of a product, (17), and we transform the integrals corresponding to the various factors, introducing a shift along λ' in order to make the expressions in the logarithm real. We find

$$G_+(\lambda)=\frac{\Phi_2(1-2i\lambda,1)}{\Phi_1(\xi-2i\lambda,\xi)\Phi_1(\zeta-2i\lambda,\zeta)}, \tag{22}$$

where $\xi = 1 + \pi\beta I/4I_c$ and $\zeta = 1 - \pi\beta I/4I_c$. The function $\Phi_n(x,y)$ is determined by the expression

$$\Phi_n(x,y)=\exp\Big\{\int_0^{\pi/2}\frac{dz}{\pi}\ln\Big[1-\exp\Big(-\frac{n\gamma}{\beta}(x^2\,\mathrm{tg}^2 z+y^2)\Big)\Big]\Big\}. \tag{23}$$

We recall that $\gamma = I_c/eT$ and $\beta = RC\omega$. Relation (22) is suitable for calculating $G_+(-i\zeta)$ if we use the condition $\zeta < 0$. To calculate $G_+(\lambda)$ near $\lambda = 0$ we need to consider the region $\mathrm{Im}\lambda < -\zeta/2$. In this case the shift of the integration contour for the factor $1 - g_-(\lambda)$ leads to an intersection of the singular point $\lambda' = \lambda$ and to a change in the form of $G_+(\lambda)$:

$$G_+(\lambda)=\frac{\Phi_2(1-2i\lambda,1)\Phi_1(\zeta-2i\lambda,\zeta)}{\Phi_1(\xi-2i\lambda,\xi)[1-g_-(\lambda)]}. \tag{24}$$

We see that $G_+(\lambda)$ has a pole at $\lambda = 0$. Substituting expression (2) for $G_+(-i\zeta)$ and expression (24) for $G_+(\lambda)$ into (20), and taking the limits $\lambda \to 0$, we find

$$\frac{i\lambda\varphi_+(\lambda)}{T}\Big|_{\lambda=0}=\frac{T}{\zeta^2\delta}\frac{\Phi_2(\xi-2\zeta,\xi)\Phi_1{}^2(\zeta,\zeta)\Phi_1(\xi-2\zeta,\xi)}{\Phi_2(1,1)\Phi_1(\xi,\xi)}$$

The right side of this expression must be the same as the second term in (13) if we replace ζ by $|\zeta|$. Making use of the relationship between μ and τ, we write the final result for τ as a function of the reduced current $\eta = \pi\beta I/4I_c$, in the form of parametric relations:

$$\tau(\eta)=2\pi(RC/\omega)^{1/2}B(\eta,\gamma/\beta)\exp[\gamma A(\eta)],$$

$$A=\int_0^x [r(x)/r(y)-1]dy, \quad r(x)=\eta,$$

$$B=\left[\frac{4\gamma\eta p(x)}{\pi\beta}\right]^{1/2}\frac{(\eta-1)\Phi_2(2\eta-1,1)\Phi_1(1+\eta,1+\eta)}{\Phi_2(1,1)\Phi_1^2(1-\eta,1-\eta)\Phi_1(3\eta-1,1+\eta)}. \quad (25)$$

The function $p(x)$ and $r(x)$ are defined by expressions (4) and (6); the functions $\Phi_n(x,y)$ are defined by (23). Figure 1 shows the functional dependence $A(\eta)$, and Fig. 2 shows a family of curves of B versus η for various values of the ratio $\gamma/\beta = I_c/(eTRC\omega)$.

4. DISTRIBUTION OF SWITCHING CURRENTS

We assume that the current I decreases very slowly and linearly with time:

$$dI/dt=-s, \quad s\ll\omega I_c.$$

The normalized probability that the junction will switch from the resistive state to the superconducting state at the current I is given by the expression

$$P(I)=[s\tau(I)]^{-1}\exp\left\{-\int_I^\infty [s\tau(I_1)]^{-1}dI_1\right\}.$$

This expression simplifies dramatically in the limit $s\to 0$. We will now show that the distribution $P(I)$ is Gaussian, and we will find a measure of its dispersion as a function of the position of the peak, $I = I_m$.

Working in the eikonal approximation, and using (25), we can write

$$P(I)\propto\exp\left\{-\int_I^\infty [s\tau(I_1)]^{-1}dI_1-\gamma A(\pi\beta I/4I_c)\right\}.$$

The extremum of $P(I)$ is reached at a current I_m such that the following relation holds:

$$[s\tau(I_m)]^{-1}=(\gamma\pi\beta/4I_c)A'(\pi\beta I_m/4I_c). \quad (26)$$

This equation implicitly determines the functional dependence $I_m(s)$. To calculate a measure of the dispersion of the switching current it is sufficient to differentiate the term

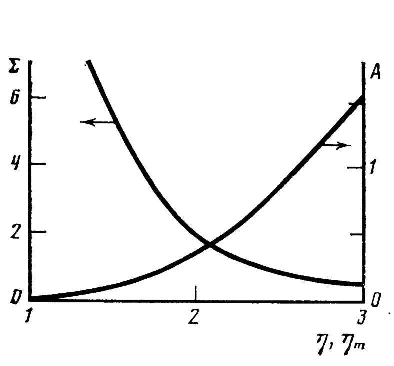

FIG. 1. The functions $A(\eta)$ and $\Sigma(\eta_m)$.

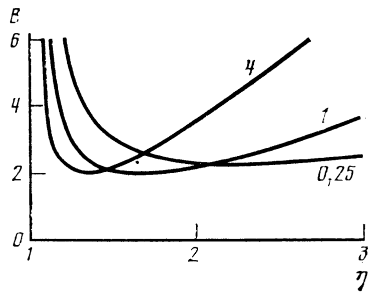

FIG. 2. The coefficient of the exponential function, B, versus the reduced current $\eta = \pi I/(4I_cRC\omega)$ for various values of the parameter $\gamma/\beta = I_c/(eTRC\omega)$.

$[s\tau(I)]^{-1}$, since the relative contribution of the second term is small, on the order of the parameter $\gamma^{-1}\ll 1$. Again differentiating only the exponential functional which appears in $1/\tau$, we find

$$-\frac{d}{dI}\frac{1}{s\tau(I)}=\frac{1}{s\tau(I)}\frac{\pi\gamma\beta}{4I_c}A'\left(\frac{\pi\beta I}{4I_c}\right).$$

This expression, taken at $I = I_m$, where we can use (26) to replace the factor $[s\tau(I_m)]^{-1}$, determines the dispersion of the switching current. Transforming to the distribution of the reduced current, $P(\eta)$, through the substitution $P(\eta)d\eta = P(I)dI$, we find

$$P(\eta)=\gamma\Sigma^{-1/2}(\eta_m)\exp[-\gamma^2(\eta-\eta_m)^2/\Sigma(\eta_m)], \quad (27)$$

where Σ, the measure of the dispersion, is given parametrically as a function of the peak position η_m:

$$\Sigma=\left[\int_0^x r^{-1}(y)dy\right]^{-2}, \quad r(y)=\int_0^{\pi/2}(y+\sin^2\varphi)^{1/2}d\varphi,$$

$$r(x)=\eta_m.$$

5. CONCLUSION

The problem of the lifetime of the resistive state of a Josephson junction has a long history, and its solution has been the subject of several papers.[4-8] In the eikonal approximation, which is sufficient for calculating the switching-current distribution, (27), a solution has been found by Vollmer and Risken.[4] In our opinion, the approach suggested by Ben Jakob *et al.*[5] is incorrect. The results found in Refs. 6–8 differ from our own results and also from the results of Ref. 4.

Calculating the coefficient of the exponential function in expression (25) for the lifetime of the resistive state will require solving a Fokker-Planck equation with two variables in the case at hand, in which the friction is small, this equation can be reduced to a system of Wiener-Hopf equations. This method was used in Ref. 9 to find an exact solution of the Kramers problem[10] of the lifetime of a Brownian particle in a deep potential well. In Ref. 3, the fluctuational I-V characteristic, the lifetime of the superconducting state, and the partial probabilities for phase jumps of a high-Q Josephson junction were calculated. The Kramers problem has also been solved for the case with quantum-mechanical effects.[11] That approach has made it possible to find the fluctuational

I-V characteristic of a Josephson junction for the case with Nyquist noise and tunneling through the potential barriers.[12]

Editor's note: G. Iche and P. Noziéres [J. de Phys. **37**, 1313 (1976); **40**, 225 (1979)] previously pointed out the possibility of passing from the Fokker-Planck equation to a system of integral equations for the energy distribution function in the limit of vanishing small dissipation.

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