Upper critical fields in p-pairing superconductors

I. A. Luk'yanchuk and V. P. Mineev

L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences
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The upper critical field H_{c2} for p-pairing superconductors is determined at arbitrary temperatures, taking into account magnetic effects which originate both from spin and orbital motion. We show that in the absence of spin-orbit interaction the ferromagnetic β-phase has the largest value of $H_{c2}(T)$ of all the p-pairing phases; in addition, we show that the phase transition from metal to p-paired superconductor is split in two in a magnetic field, like the phase transition from the normal to the superfluid state which occurs in liquid ^{3}He. We calculate the critical fields for these two transitions in the Landau-Ginzburg region and describe the characteristics of the resulting superconducting states. We investigate the effects of spontaneous magnetism and of para- and diamagnetic corrections to the function $H_{c2}(T)$ for p-pairing superconductors with a spin-orbit interaction possessing cubic symmetry. For these superconductors, the splitting of the phase transition is found to be absent for arbitrary directions of the external field. We discuss the anisotropy of the upper critical field for superconducting phases with multicomponent order parameters characterized by various representations of the cubic group, and also superconductivity in the paramagnetic limit for superconductors with triplet pairing in the presence of strong spin-orbit interaction.

1. INTRODUCTION

One is the important characteristics of type II superconductors is the magnitude of the upper critical field H_{c2}. Microscopic calculations of H_{c2} for ordinary superconductors with isotropic s-pairing were carried out in Ref. 1–3. The unusual properties of heavy-fermion superconducting compounds,[4–6] which were discovered comparatively recently, provide evidence for the hypothesis that in these materials superconductivity occurs either with anisotropic singlet pairing (in which the spin S of a Cooper pair is a zero) or with triplet ($S=1$) pairing. Because of uncertainties both in the characteristics of the energy spectra and in the form of the pairing interaction for heavy-fermion superconductors, calculation of H_{c2} is possible only at a phenomenological level, within the region of applicability of Landau-Ginzburg theory[7–9], although Scharnberg and Klemm[10,11] carried out microscopic calculations of H_{c2} in the weak-coupling approximation at arbitrary temperatures for the simplest of the triplet-type pairings, i.e., p-pairing. In Ref. 10, the upper critical field was computed for the various isotropic superconducting phases with p-pairing in the absence of spin-orbit interaction. Reference 11 contains a discussion of the problem of calculating the upper critical field in superconductors with a strong spin-orbit interaction possessing cubic symmetry.

A characteristic of superconductors with s-pairing is that their upper critical fields are bounded by the paramagnetic limit $H_p \sim T_c/\mu$ (μ is the magnetic moment of a quasiparticle); this limit is derived from the pair-breaking effect of a magnetic field on the spins of electrons in a Cooper pair.[12–14] In superconductors with p-pairing but no spin-orbit interaction, the paramagnetic limit does not apply because the solutions of the linear integral equation for the order parameter near H_{c2} are phases whose paramagnetic susceptibility coincides with that of the normal metal[10]; hence the field does not disrupt the paired state. In p-superconductors with strong spin-orbit interaction, the action of an external field on the electron spins may or may not give rise to a paramagnetic-limit bound on the upper critical field. In particular, for those phases in which the weight of paired states with zero projection of the total spin onto the direction of the external field is small, there is no paramagnetic suppression of the superconductivity.

Along with the paramagnetic effect, triplet-pairing superconductors exhibit three more types of local magnetic effects which were not considered in Refs. 10, 11 and which influence the value of H_{c2}. As a consequence of the inequality of the densities of states for electrons with spins along and opposite the field and for electrons belonging to Cooper pairs with orbital moments oriented along and opposite the field, it becomes possible for superconducting phases to exist with spontaneous magnetism originating from both spin and orbital momenta. In addition to this effect, an applied field has a depairing effect due to the appearance in the superconducting state of an additional diamagnetic (orbital) susceptibility[15] which bounds the upper critical field at the diamagnetic limit $H_d \sim (T_c/\mu_B)(m^*/m)$ where μ_B is the Bohr magneton and m, m^* are the mass and effective mass of the electrons. This article is devoted to calculating the field H_{c2} microscopically, taking into account all the above-mentioned local magnetic effects for superconductors with p-pairing—both those in which the effects of spin-orbit interaction is negligible (like ^{3}He), and those in which the spin-orbit interaction produces a rigid coupling of the Cooper-pair electron spins in the direction of the crystal axes (like the superconducting heavy-fermion compounds).

An isotropic p-pairing superconductor has several nonferromagnetic phases; in the absence of spin-orbit interaction, the one which has the maximum value of the upper critical field is the so-called polar phase.[10] The spin state of this phase corresponds to an equal-probability combination of states with spins along and opposite the field; therefore,

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the upper critical field of the polar phase is not bounded by the paramagnetic limit. Taking into account the splitting of the Fermi surface for spins along and opposite the field (see the second section of this paper), we find that the transition from the normal to the superconducting state takes place first in only one of the spin states (with spin opposite the field) which make up the polar phase. The resulting phase (the β-phase, which is a spin analog to the A-phase of superfluid ^{3}He) is ferromagnetic; of all the phases of a p-pairing superconductor (in the absence of spin-orbit interaction) this phase has the largest $H_{c2}(T)$.

In the third section of this paper we investigate the problem of superconductivity in the diamagnetic limit, and also the effect of orbital ferromagnetism on the function $H_{c2}(T)$.

For ^{3}He the splitting of the normal-to-superfluid phase transition in a magnetic field is a well-known phenomenon.[16] In contrast to ^{3}He, for a charged Fermi liquid the splitting of this phase transition has a number of peculiarities, whose investigation (in the absence of spin-orbit interaction) is the subject of the fourth section of this paper. In this section, we show that following the formation at $H = H_{c2}$ of a lattice of Abrikosov nuclei of the superconducting β-phase consisting of Cooper pairs with electron spins antiparallel to the field direction, a second phase transition is possible at smaller fields with formation of a lattice of nuclei of the superconducting phase made from pairs of electron spins parallel to the field. The vertices of the second lattice can either coincide with those of the first or be located in the interstices of the latter. In the latter case there are two Abrikosov lattices which are dual to one another, consisting of superconducting pairs with opposite projections of the spin of paired electrons.

The problem of the upper critical field in p-pairing superconductors with strong spin-orbit interaction possessing cubic symmetry was investigated in Ref. 11. It was shown there that it is necessary to seek the superconducting phase with maximal H_{c2} as a linear combination of all nine basis functions for the four irreducible representations of the cubic group which correspond to the case of p-pairing. In connection with this, we note that the phase transition to the superconducting state must be a transition to the phase which has the highest critical temperature (or largest pairing interaction constant)[17,18]; the symmetry of this phase is characterized by one of the representations of the cubic group. Mixing of contributions whose symmetries involve other representations with lower critical temperatures to this phase begins to be significant only when terms $\sim(1 - T/T_c)^2$ are taken into account. Therefore, for $H_{c2} \sim(1 - T/T_c)$, i.e., the Landau-Ginzburg region, and any ratio of interaction constants for two different representations, the one which is important is the one with the largest T_c, as was asserted in Refs. 7, 8.

Since our principal intent in this paper is to investigate how local magnetic effects influence the function $H_{c2}(T)$, we will assume that the pairing interaction constant for the three-dimensional vector representation (denoted by F_1 in the classification of Ref. 17 and T_1 in that of Ref. 18) is large compared with the constants for the other representations. The choice of this representation has interesting implications, in that the upper critical field of the p-pairing phase with F_1 symmetry is isotropic, i.e., does not depend on the orientation of the external field relative to the cubic axes for any given temperature. This property, which obtains only for superconductors with isotropic electronic dispersion relations, distinguishes the F_1 representation from the other multicomponent representations F_2 and E_1 whose critical fields are anisotropic even for the case of an isotropic spectrum.

In the fifth section of this paper we show that the generalized Scharnberg-Klemm phase has the largest field H_{c2} of all the p-pairing phases belonging to the F_1 representation. This phase is ferromagnetic. The corrections to H_{c2} due to spontaneous magnetism act to decrease the value of the upper critical field; in particular, the slope dH_{c2}/dT at $T = T_c$ is decreased. We also find both paramagnetic and diamagnetic corrections to H_{c2}, and show that for a superconducting Fermi gas the paramagnetic corrections are smaller than those due to the spontaneous magnetism. On the other hand, if the electron effective mass is considerably larger than the free-electron mass, the paramagnetic contribution will exceed the contribution due to spontaneous magnetism. For anisotropic superconductors with a strong spin-orbit interaction, the phase transition from the normal to the superconducting state is not split by a magnetic field in any direction. Thus, the assertion made in Ref. 18 that such splitting takes place only for external field directions along one of the cubic symmetry axes is valid only for an uncharged Fermi liquid with a spin-orbit interaction possessing cubic symmetry.

As long as H_{c2} is small compared to the paramagnetic limit H_p, the paramagnetic corrections make only slight changes in the value of the upper critical field. In the opposite case, paramagnetic suppression of triplet superconductivity can either occur or not occur as a function of the weight with which the other representations mix with the basic one with maximum T_c. In the sixth section of this paper we discuss the problem of paramagnetic limits on superconductivity in the presence of strong spin-orbit interaction, when other representations enter into the order parameter. As an example, we discuss mixing of the F_2 representation into a fundamental representation with F_1 symmetry.

In the Conclusion, we assert that we can now interpret the results of measurements of the upper critical field in the cubic-symmetry heavy-fermion superconductor UBe_{13}.[19,20]

2. UPPER CRITICAL FIELD IN THE β-PHASE

In order to determine the critical field, we will solve the linear integral equation for the order parameter which is appropriate for a p-pairing superconductor[10]:

$$\Delta_{\lambda\mu}{}^{i}(\mathbf{R}) = 3gT\sum_{\omega}\int d^3r\,\hat{r}_i\hat{r}_j\bar{G}_{\omega}{}^{\lambda}(r)\bar{G}_{-\omega}{}^{\mu}(r) \cdot \exp\left\{-\mathbf{r}\left[\frac{\partial}{\partial\mathbf{R}} - \frac{2ie}{c}\mathbf{A}(\mathbf{R})\right]\right\}\Delta_{\lambda\mu}{}^{j}(\mathbf{R}). \tag{2.1}$$

Here

$$\Delta_{\lambda\mu}{}^{i} = \begin{pmatrix} \Delta_{\uparrow\uparrow}{}^{i} & \Delta_{\uparrow\downarrow}{}^{i} \\ \Delta_{\downarrow\uparrow}{}^{i} & \Delta_{\downarrow\downarrow}{}^{i} \end{pmatrix} \tag{2.2}$$

is the order parameter matrix

$$\Delta_{\lambda\mu}{}^{i}(\mathbf{R})\hat{r}_i = (i\sigma\sigma_y)_{\lambda\mu}\mathbf{d}_i(\mathbf{R})\hat{r}_i, \tag{2.3}$$

and λ and μ are indices for the spin projections of the pairing particles: $\lambda = \uparrow, \downarrow$ or $+1, -1$; $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ denotes the Pauli matrices and g the interaction constant,

$$\bar{G}_\omega^\lambda(r) = -\frac{\pi N_0}{p_0 r}\exp\left[ip_0^\lambda r \operatorname{sign}\omega - \frac{r|\omega|}{v_0^\lambda}\right] \quad (2.4)$$

is the electron Green's function of the normal metal, $p_0^\lambda = [2m^*(\varepsilon_F - \lambda\mu_B H)]^{1/2}$, $p_0 = p_0^\lambda (H=0)$ is the Fermi momentum, $v_0^\lambda = p_0^\lambda/m^*$, $v_0 = p_0/m^*$, $N_0 = m^* p_0/2\pi^2$ is the density of states at the Fermi surface for a given spin projection, μ_B is the Bohr magneton and $\mathbf{H} = \operatorname{curl}\mathbf{A}$, $\mathbf{A} = (0, Hx, 0)$.

If we neglect the effect of the field on the electron spin, then

$$\bar{G}_\omega(r)\bar{G}_{-\omega}(r) = (\pi N_0/p_0 r)^2 \exp(-2|\omega| r/v_0)$$

and the solution of Eq. (2.1) (Ref. 10) with the maximum value of the upper critical field:

$$\Delta_{\lambda\mu}^i(\mathbf{R}) \sim f_0(\mathbf{R})\hat{z}_i\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}, \quad (2.5)$$

corresponds to the so-called "polar phase", in which states with spin projections ↑↑ and ↓↓ are equally probable. Here $f_0(R)$ is the ground-state eigenfunction of the Schroedinger equation for a particle with mass m and charge $2e$ in a homogeneous magnetic field. The quantity $H_{c2}(T)$ for the polar phase is found from the equation

$$\ln\frac{T_c}{T} = \sum_n\left\{\frac{1}{|2n+1|} - \frac{3T}{N_0}\int d^3r\left(\frac{N_0\pi}{p_0 r}\right)^2 \times \cos^2\theta\exp\left(-\frac{2|\omega_n|r}{v_0} - \frac{|e|Hr_\perp^2}{2c}\right)\right\} \quad (2.6)$$

$$\cos\theta = \hat{\mathbf{r}}\mathbf{H}/H, \quad r_\perp = |[\mathbf{r}\mathbf{H}/H]|,$$

which is conveniently written in the dimensionless variables

$$h = 2H/H_0, \quad \rho = rh^{1/2}/\xi_0, \quad t = T/T_c, \quad (2.7)$$

where $H_0 = \Phi_0/\pi\xi_0^2$, $\Phi_0 = \pi c/|e|$ is the flux quantum, $\xi_0 = v_0/2\pi T_c$ is the coherence length, $T_c = \omega_0\exp(-1/N_0 g)$ is the critical temperature; we use units in which $\hbar = 1$ everywhere. The solution of Eq. (2.6) in the variables (2.7),

$$\ln t = \sum_n\left\{\frac{3t}{4\pi h^{1/2}}\int\frac{d^3\rho}{\rho^2}\cos^2\theta\exp\left(-\frac{\rho t|2n+1|}{h^{1/2}} - \frac{\rho_\perp^2}{4}\right) - \frac{1}{|2n+1|}\right\} \quad (2.8)$$

will be a function $h = f(t)$ which takes on the values[10]

$$f(0) = e^{3/2}/4\gamma \approx 2, \quad f(t\to 1) = 20(1-t)/7\zeta(3). \quad (2.9)$$

In dimensional variables we have

$$H_{c2}^P(T=0) \approx H_0,$$

$$H_{c2}^P(T\to T_c) = 10H_0(1-T/T_c)/7\zeta(3) \quad (2.10)$$

for H_{c2} in the polar phase.

When paramagnetic contributions $\sim\mu_B H$ are included, the integral equations (2.1) for the components $\Delta_{\uparrow\uparrow}$ and $\Delta_{\downarrow\downarrow}$ of the order parameter will have different kernels; therefore, in a neutral homogeneous Fermi liquid the transition to the superfluid state is split in a magnetic field. The equation for the critical temperature coincides with (2.1) if we omit the operator $\exp\{-\mathbf{r}[\partial/\partial\mathbf{R} - (2ie/c)\mathbf{A}(\mathbf{R})]\}$ from this equation, which gives[12]:

$$T_c^\lambda = \omega_0\exp\left[-\frac{1}{gN_0^\lambda}\right] = T_c\left(1 - \frac{\lambda\mu_B H}{2\varepsilon_F}\ln\frac{\omega_0}{T_c}\right), \quad N_0^\lambda = \frac{m^* p_0^\lambda}{2\pi^2}. \quad (2.11)$$

Equation (2.11) implies that for a temperature $T_c^\downarrow > T_c$ pairing first takes place between quasiparticles with spins opposite the field, i.e., the phase transition is not to the polar phase but to the β-phase. The question of a second phase transition for particles with spins along the field is the subject of the fourth section of this paper.

The solution to Eq. (2.1) for the β-phase has the form:

$$\Delta_{\lambda\mu}^i(\mathbf{R}) \sim f_0(\mathbf{R})\hat{z}_i\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}, \quad (2.12)$$

and the corresponding equation for determining $H_{c2}(T)$ is written thus:

$$\ln\frac{T_{cH}}{T} = \sum_n\left\{\frac{1}{|2n+1|} - \frac{3T}{N_{0H}}\int d^3r\left(\frac{\pi N_0}{p_0 r}\right)^2\cos^2\theta \times\exp\left(-\frac{2|\omega_n|r}{v_{0H}} - \frac{|e|Hr_\perp^2}{2c}\right)\right\}. \quad (2.13)$$

Here, $T_{cH} = T_c^\downarrow$, $N_{0H} = N_0^\downarrow$, $v_{0H} = v_0^\downarrow$. The transition to dimensionless units

$$h = 2H/H_{0H}, \quad \rho = rh^{1/2}/\xi_{0H}, \quad t = T/T_{cH}, \quad (2.14)$$

where $H_{0H} = \Phi_0/\pi\xi_{0H}^2$, $\xi_{0H} = v_{0H}/2\pi T_{cH}$, puts Eq. (2.13) in the form (2.8). Its solution therefore will again be a function $h = f(t)$, or in dimensional units

$$H = {}^1/_2 H_{0H} f(T/T_{cH}). \quad (2.15)$$

Because H_{0H} and T_{cH} are functions of H, relation (2.15) is now an equation for determining $H_{c2}(T)$. Taking advantage of expressions (2.9)–(2.11) and (2.13), (2.14), with logarithmic accuracy we obtain the upper critical field for the β-phase:

$$H_{c2}^\beta(T=0) \approx H_0\left(1 + \frac{\mu_B H_0}{\varepsilon_F}\ln\frac{\omega_0}{T_c}\right),$$

$$H_{c2}^\beta(T\to T_c) = \frac{10}{7\zeta(3)}H_0\left(1 + \frac{5}{7\zeta(3)}\frac{\mu_B H_0}{\varepsilon_F}\ln\frac{\omega_0}{T_c}\right)\left(1 - \frac{T}{T_c}\right). \quad (2.16)$$

3. DIAMAGNETIC CORRECTIONS AND THE EFFECT OF SPONTANEOUS ORBITAL MAGNETISM ON THE UPPER CRITICAL FIELD

In Ref. 20 it was pointed out that there are additional limitations on the magnitude of the upper critical field in p-pairing superconductors, which originate from the fact that the Cooper pairs have a spatial structure. The estimates made in Ref. 20 show that the upper critical field for $T = 0$ is limited to the value $H_d \sim (T_c/\mu_B)(m^*/m)$, which we refer to as a diamagnetic limit on the superconductivity. Because usually $H_{c2} \sim \Phi_0/\pi\xi_0^2$, we have $H_{c2}/H_d \sim T_c/\varepsilon_F$ and bounds involving H_d become important only for superconductors with Cooper pairs whose size is on the order of the interatomic spacing. Nevertheless, the local diamagnetism of the Cooper pairs gives rise to important corrections to

H_{c2}. In contrast to Ref. 20, in which the diamagnetic susceptibility was calculated—i.e., the authors investigated the limit $H\to 0$ for arbitrary Δ—a correct calculation of the diamagnetic corrections at arbitrary temperature requires solving the linear integral equation for the order parameter, i.e., studying the limit $\Delta\to 0$ for arbitrary H. In this section we will find the diamagnetic corrections to H_{c2} in the polar phase, and also discuss the effect on H_{c2} of the spontaneous orbital magnetism, which is important for phases having a spontaneous orbital moment. To this end we will write the integral Eq. (2.1) for Δ in a slightly different form which is convenient for including these effects[21]:

$$\Delta^i(\mathbf{R})=\int d^3R'\, K_{ij}(\mathbf{R},\mathbf{R}')\Delta^j(\mathbf{R}'). \qquad (3.1)$$

Here

$$K_{ij}(\mathbf{R},\mathbf{R}')=\lim_{\substack{\mathbf{l}\to\mathbf{R}\\ \mathbf{m}\to\mathbf{R}'}}\frac{3gT}{4p_0^2}\sum_\omega\left(\frac{\partial}{\partial R_i}-\frac{\partial}{\partial l_i}\right)\left(\frac{\partial}{\partial R_j'}-\frac{\partial}{\partial m_j}\right)$$

$$\times G_\omega(\mathbf{R},\mathbf{R}')G_{-\omega}(\mathbf{l},\mathbf{m}). \qquad (3.2)$$

The Green's function G_ω is connected with the Green's function $\overline{G}_\omega$ (2.4) by the relation

$$G_\omega(\mathbf{R},\mathbf{R}')=\exp\left[\frac{ie}{c}\int_{\mathbf{R}'}^{\mathbf{R}}\mathbf{A}(\mathbf{r})\,d\mathbf{r}+i\psi_\omega(\mathbf{R}-\mathbf{R}')\right]\overline{G}_\omega(\mathbf{R}-\mathbf{R}'). \qquad (3.3)$$

In the functions $\overline{G}_\omega(\mathbf{R}-\mathbf{R}')$ and $\Delta^1(\mathbf{R})$ we have omitted the spin indices; this amounts to neglecting paramagnetic effects to simplify the exposition. The correction ψ_ω to the phase of the Green's function in the field has a relative magnitude $\sim(p_0\xi_0)^{-1}$ compared to the first terms in the exponent of (3.3), and is determined by the expression[1]

$$\pm\frac{\partial\psi_{\pm\omega}(\mathbf{r})}{\partial\mathbf{r}}p_0\hat{\mathbf{r}}+\frac{e^2}{4c^2}[\mathbf{H}\mathbf{r}]^2=0. \qquad (3.4)$$

It is not hard to verify that Eq. (2.1) is obtained from (3.1) if we include only the principal contribution in $(p_0\xi_0)^{-1}$, arising from differentiating $\exp(ip_0|\mathbf{R}-\mathbf{R}'|\,\mathrm{sign}\,\omega)$. Let us now consider also the derivative of the phase factor

$$\exp\left[i\frac{e}{c}\int_{\mathbf{R}'}^{\mathbf{R}}\mathbf{A}(\mathbf{r})\,d\mathbf{r}+i\psi_\omega(\mathbf{R}-\mathbf{R}')\right].$$

Then to order $(p_0\xi_0)^{-2}$ we obtain the following expression for the kernel K_{ij}

$$K_{ij}(\mathbf{R},\mathbf{R}')=3gT\sum_\omega\overline{G}_\omega(\mathbf{R}-\mathbf{R}')\overline{G}_{-\omega}(\mathbf{R}-\mathbf{R}')$$

$$\cdot\left\{\frac{(\mathbf{R}-\mathbf{R}')_i(\mathbf{R}-\mathbf{R}')_j}{|\mathbf{R}-\mathbf{R}'|^2}\left(1-\frac{e^2}{2p_0^2c^2}[\mathbf{H}(\mathbf{R}-\mathbf{R}')]^2\right)\right.$$

$$\left.-\frac{ie}{2p_0^2c}e_{ijm}H_m\right\}\exp\left[\frac{2ie}{c}\int_{\mathbf{R}'}^{\mathbf{R}}\mathbf{A}(\mathbf{r})\,d\mathbf{r}\right]. \qquad (3.5)$$

From (3.1) and (3.5) we can make use of the well-known identity[3]

$$\exp\left[\frac{2ie}{c}\int_{\mathbf{R}'}^{\mathbf{R}}\mathbf{A}(\mathbf{r})\,d\mathbf{r}\right]\Delta^j(\mathbf{R}')$$

$$=\exp\left\{-(\mathbf{R}-\mathbf{R}')\left[\frac{\partial}{\partial\mathbf{R}}-\frac{2ie}{c}\mathbf{A}(\mathbf{R})\right]\right\}\Delta^j(\mathbf{R}) \qquad (3.6)$$

to obtain

$$\Delta^i(\mathbf{R})=3gT\int d^3r\sum_\omega\overline{G}_\omega(r)\overline{G}_{-\omega}(r)\left\{\hat{r}_i\hat{r}_j\left(1-\frac{e^2}{2p_0^2c^2}[\mathbf{H}\mathbf{r}]^2\right)\right.$$

$$\left.-\frac{ie}{2p_0^2c}e_{ijm}H_m\right\}\exp\left\{-\mathbf{r}\left[\frac{\partial}{\partial\mathbf{R}}-\frac{2ie}{c}\mathbf{A}(\mathbf{R})\right]\right\}\Delta^j(\mathbf{R}). \qquad (3.7)$$

In contrast to (2.1), Eq. (3.7) contains the effects of local diamagnetism and the spontaneous orbital moment of the Cooper pairs. For the polar phase (2.5), which is the solution with maximum value of H_{c2}, the term $\sim e_{ijm}H_m$ which corresponds to the spontaneous orbital moment (here, to the magnetic orbital moment) of the superfluid Fermi liquid (see Ref. 22) equals zero (for $H\|\hat{z}$). However, for the type A-phase solution[10] this term gives a nonzero correction to H_{c2} (see below).

The equation for H_{c2} corresponding to (3.7) has the following form for the polar phase:

$$\ln\frac{T_c}{T}=\sum_n\left\{\frac{1}{|2n+1|}-\frac{3T}{N_0}\int d^3r\left(\frac{\pi N_0}{p_0r}\right)^2\cos^2\theta\right.$$

$$\left.\cdot\left(1-\frac{e^2H^2r_\perp^2}{2p_0^2c^2}\right)\exp\left(-\frac{2|\omega_n|r}{v_0}-\frac{|e|Hr_\perp^2}{2c}\right)\right\} \qquad (3.8)$$

Passing to dimensionless units, we obtain in analogy with (2.7)

$$\ln t=\sum_n\left[\frac{3t}{4\pi h^{1/2}}\int\frac{d^3\rho}{\rho^2}\cos^2\theta\left(1-\frac{h\rho_\perp^2}{8\xi_0^2p_0^2}\right)\right.$$

$$\left.\times\exp\left(-\frac{\rho t|2n+1|}{h^{1/2}}-\frac{\rho_\perp^2}{4}\right)-\frac{1}{|2n+1|}\right], \qquad (3.9)$$

from which we have at $T=0$ (see with (2.9))

$$h=e^{8/3}e^{-\alpha h}/4\gamma\approx 2e^{-\alpha h}. \qquad (3.10)$$

Here $\alpha=1/2p_0^2\xi_0^2$. Thus, the value of the upper critical field in the polar phase equals

$$H_{c2}^p(T=0)=H_0(1-2\alpha). \qquad (3.11)$$

The spatial structure of the Cooper pairs gives rise to effects which contribute to H_{c2} for any type of pairing in addition to s-pairing. As one might expect, this effect will be important only for $T_c\sim\varepsilon_F$. The relative diamagnetic decrease in H_{c2} is of order $\alpha\sim(\mu_BH_0/\varepsilon_F)(m/m^*)$. At the same time, the paramagnetic increase in H_{c2} due to the separation of the Fermi surfaces for particles with spins along and parallel to the field (which was found in the previous section) is of order $(\mu_BH_0/\varepsilon_F)\ln(\omega_0/T_c)$. Therefore, diamagnetic effects need not be considered in calculating the upper critical field in the β-phase.

It is also appropriate to discuss renormalization of H_{c2} for the solution which has the A–phase structure (see Ref. 10):

$$\Delta_{\alpha\beta}{}^i\sim f_0(\mathbf{R})(-\hat{\mathbf{x}}+i\hat{\mathbf{y}})_i\begin{pmatrix}-1&0\\0&1\end{pmatrix}, \qquad (3.12)$$

where this renormalization is due to the spontaneous orbital moment of the Cooper pairs. Such a contribution to H_{c2} arises, e.g., in the Scharnberg-Klemm phase which has the maximum H_{c2} in a cubic crystal with strong spin-orbit interaction (see Sec. 5). Here we discuss the effect of the spontaneous orbital moment in its pure form, again omitting the spin indices, or in other words neglecting the spin paramagnetism; taking this latter effect into account would give the A_1-phase as a solution to the integral equation for the order parameter, which possesses a higher H_{c2} compared to the A-phase (3.12).

Omitting the term $\sim[\mathbf{Hr}]^2$ in (3.7), which is small compared to the terms $-(ie/2p_0^2c)e_{ijm}H_m$, which now give the principal contribution, we write for H_{c2}

$$\ln\frac{T_{cH}}{T}=\sum_n\left\{\frac{1}{|2n+1|}-T\left[N_0\left(1+\frac{3}{2}\frac{\mu_BH}{\varepsilon_F}\frac{m}{m^*}\right)\right]^{-1}\right.$$

$$\cdot\int d^3r\left(\frac{\pi N_0}{p_0r}\right)^2\left(\frac{3}{2}\sin^2\theta+\frac{3}{2}\frac{\mu_BH}{\varepsilon_F}\frac{m}{m^*}\right)$$

$$\left.\cdot\ \exp\left(-\frac{2|\omega|r}{v_0}-\frac{|e|Hr_\perp^2}{2c}\right)\right\},\tag{3.13}$$

where T_{eH} s determined from the equation

$$1=gT_{cH}\int d^3r\left(\frac{3}{2}\sin^2\theta+\frac{3}{2}\frac{\mu_BH}{\varepsilon_F}\frac{m}{m^*}\right)\bar{G}_\omega(r)\bar{G}_{-\omega}(r),\tag{3.14}$$

whose solution is in essence

$$T_{cH}=T_c\left(1+\frac{3}{2}\frac{\mu_BH}{\varepsilon_F}\frac{m}{m^*}\ln\frac{\omega_0}{T_c}\right).\tag{3.15}$$

Solving (3.13) to within logarithmic corrections, we obtain

$$H_{c2}^A(T=0)=\frac{e^{5/3}}{8\gamma}H_0\left(1+\frac{3e^{5/3}}{8\gamma}\frac{\mu_BH_0m}{\varepsilon_Fm^*}\ln\frac{\omega_0}{T_c}\right),$$

$$H_{c2}^A(T\to T_c)$$

$$=\frac{5}{7\zeta(3)}H_0\left(1+\frac{15}{14\zeta(3)}\frac{\mu_BH_0m}{\varepsilon_Fm^*}\ln\frac{\omega_0}{T_c}\right)\left(1-\frac{T}{T_c}\right).\tag{3.16}$$

Thus, inclusion of the spontaneous orbital moment slightly increases H_{c2} in the A-phase. The rise in H_{c2} takes place because the orbital moment of the A-phase (3.12), being a solution to the linear integral equation (3.7) for $H=H_{c2}^A$, is directed antiparallel to the external field: $\mathbf{l}=-[\hat{\mathbf{x}}\hat{\mathbf{y}}]=-\hat{\mathbf{z}}$.

4. SPLITTING OF THE PHASE TRANSITION IN A MAGNETIC FIELD

It is well-known that the liquid ^{3}He phase transition from the normal phase to the superfluid A-phase splits in two in a magnetic field. As the temperature falls, the first phase transition takes place to the A_1-phase, corresponding to pairing of particles with magnetic moments oriented along the field (spins opposite the field), followed by a phase transition to the A-phase in which particles with magnetic moments opposite the field also pair. For a charged Fermi liquid with p-pairing, the maximum H_{c2} corresponds to the polar phase; it is therefore necessary to investigate for this specific case the question of whether the transition from normal state to p-pairing superconductor splits in a magnetic field.

Let us write the order parameter (2.3) in the form

$$\mathbf{d}_i={}^1/_2\hat{z}_i[a(\hat{\mathbf{x}}-i\hat{\mathbf{y}})+b(\hat{\mathbf{x}}+i\hat{\mathbf{y}})]\tag{4.1}$$

(the field direction, as usual, is along the z-axis). From (2.2), we conclude that $a=\Delta_{\downarrow\downarrow}^i z_i$, $b=\Delta_{\uparrow\uparrow}^i z_i$ are the coordinate-dependent pairing amplitudes for states with spin projections $+1$ and -1. The free energy of a superconductor with order parameter (4.1) in the Landau-Ginzburg regime can be written as[23]

$$\mathscr{F}=\int dV\left\{-\frac{1}{2}(\alpha+g_1H)|a|^2-\frac{1}{2}(\alpha-g_1H)|b|^2\right.$$

$$+\frac{\beta}{2}(|a|^4+|b|^4)$$

$$\left.+\gamma|a|^2|b|^2+\frac{K_1}{2}(|D_ia|^2+|D_ib|^2)\right\},\tag{4.2}$$

where

$$\alpha={}^1/_3N_0(1-T/T_c),\quad g_1={}^1/_3N_0'\mu_B\ln(\omega_0/T_c),\quad N_0'=dN_0/d\varepsilon,$$

and the coefficients β and γ are expressed in terms of the standard Landau-Ginzburg expansion coefficients as follows:

$$\beta={}^1/_2(\beta_2+\beta_3+\beta_4),\quad\gamma={}^1/_2(\beta_2+\beta_3+\beta_4+2\beta_1+2\beta_5).$$

In the weak-coupling approximation

$$-2\beta_1=\beta_2=\beta_3=\beta_4=-\beta_5=7\zeta(3)N_0/120\pi^2T_c^2,$$

$$K_1=[7\zeta(3)/240\pi^2]N_0(v_0/T_c)^2,$$

so that $\gamma=0$. Furthermore,

$$D_ia=[-i\nabla_i-(2e/c)A_i]a,\quad\mathbf{A}=(0,Hx,0)$$

and we assume that a and b are independent of the z-coordinate, which is along the magnetic field.

By varying Eq. (4.2) we obtain a system of Landau-Ginzburg equations:

$$-{}^1/_2(\alpha+g_1H)a+\beta|a|^2a+\gamma|b|^2a+{}^1/_2K_1D_i^2a=0,\tag{4.3}$$

$$-{}^1/_2(\alpha-g_1H)b+\beta|b|^2b+\gamma|a|^2b+{}^1/_2K_1D_i^2b=0.\tag{4.4}$$

Discarding the nonlinear terms in (4.3), we obtain a linear equation whose smallest eigenvalue determines the upper critical field for a transition from the normal state ($a=0$, $b=0$) to the superconducting state (β-phase) with $a\neq0$, $b=0$:

$$H_{c2}^{\downarrow}=H_{c2}^{\beta}=H_{c2}^{P}(1+\nu),\tag{4.5}$$

which naturally coincides with (2.16). Here

$$H_{c2}^P=\alpha/4K_1m\mu_B,\quad\nu=g_1/4K_1m\mu_B,\tag{4.6}$$

H_{c2}^P is the upper critical field for the polar phase (2.10).

As the field H decreases to a value slightly smaller than H_{c2}^β, it becomes necessary to solve the nonlinear equation (4.3) with $b=0$ along with the Maxwell equation which is obtained from (4.2) by variation with respect to the vector potential $\mathbf{A}$ (for $b=0$). The solution to this system[24] consists of a planar lattice of β-phase nuclei ($a\neq0$, $b=0$) with period $\sim(K_1/\alpha)^{1/2}$, where

$$|a(\mathbf{r})|^2=(1-H/H_{c2}^\beta)|f(\mathbf{r})|^2,\tag{4.7}$$

and $f(\mathbf{r})$ is a doubly-periodic function which does not depend on H. As the field is further decreased, a phase transition can take place to a state with $b \neq 0$. In this case, a lattice appears which is made up of nuclei of the $\Delta_{\uparrow\uparrow}$-phase)$a = 0$, $b \neq 0$). Because the energy of this interaction $\gamma \int dV |a|^2 |b|^2$ must take on its smallest value, for $\gamma > 0$ the $\Delta_{\uparrow\uparrow}$-phase nuclei which appear will be repelled by the preexisting lattice of $\Delta_{\downarrow\downarrow}$-phase nuclei, and will form a lattice dual to the latter. If no other phases occur as the field is further reduced (for example, structural transitions in the lattice or a transition to the A-phase, which may turn out to be energetically more favorable at low fields), then all the evidence suggests that for $H \approx H_{c1}$ the structure of two dual lattices with $a \neq 0$, $b = 0$ and $a = 0$, $b \neq 0$ goes over to a nonsingular vortex structure made up of the ferromagnetic β-phase with order parameter

$$\mathbf{d}_i = \Delta(T)(\mathbf{e}_1 + i\mathbf{e}_2)\hat{z}_i, \qquad (4.8)$$

where $\mathbf{e}_1$, $\mathbf{e}_2$ are unit vectors with $\mathbf{e}_1 \cdot \mathbf{e}_2 = 0$, and the direction of spontaneous magnetic moment $\mathbf{M} = \mathbf{e}_1 \times \mathbf{e}_2$ forms a vector triad with them similar to the triad formed by the orbital moment $\mathbf{l}$ for the lattice of nonsingular vortices in rotational ^{3}He-A (Ref. 25). If, however, $\gamma < 0$ holds, a new $\Delta_{\uparrow\uparrow}$-phase appears which replaces the $\Delta_{\downarrow\downarrow}$-phase nuclei, i.e., once more the transition will be split; however, a lattice of nuclei of the real-valued polar phase forms below the second phase transition.

In order to find the field $H_{c2}^{\uparrow}$ which corresponds to the appearance of the $\Delta_{\uparrow\uparrow}$-phase, we write the equation which comes from linearizing (4.4) in b in the form

$$\frac{1}{2m} D_i^2 b + \frac{\gamma}{Km} |a|^2 b = \frac{\alpha - g_1 H}{2Km} b. \qquad (4.9)$$

This is the Schroedinger equation for a particle with mass m and charge $2e$ in a magnetic field and acted on by a potential $\gamma |a(\mathbf{r})|^2 / K_1 m$. Treating this potential as a perturbation and using (4.7), we find the smallest eigenvalue of this equation:

$$\frac{\alpha - g_1 H}{2K_1 m} = 2\mu_B H + \frac{\gamma}{K_1 m}\left(1 - \frac{H}{H_{c2}^{\beta}}\right) I, \qquad (4.10)$$

where

$$I = \int |f(\mathbf{r})|^2 |b(\mathbf{r})|^2 d^3\mathbf{r} \Big/ \int |b(\mathbf{r})|^2 d^3\mathbf{r}. \qquad (4.11)$$

The solution to (4.10) in the notation (4.6) has the form

$$H_{c2}^{\uparrow} = H_{c2}^{P}[1 - \nu(\alpha + 2\gamma I)/(\alpha - 2\gamma I)]. \qquad (4.12)$$

Using (4.5) and (4.12), we also obtain the magnitude of the transition splitting:

$$\delta H_{c2} = H_{c2}^{\downarrow} - H_{c2}^{\uparrow} = [2\nu\alpha/(\alpha - 2\gamma I)] H_{c2}^{P}. \qquad (4.13)$$

Equations (4.12) and (4.13) apply when $\gamma \ll \alpha/2I$. In the region $\gamma \sim \alpha/2I$, and more so for $\gamma > \alpha/2I$, it is not permissible to use perturbation theory to find the lowest eigenvalue of Eq. (4.9). We will leave the question of what the splitting of the transition is in this region for a future investigation.

A change in the sign of γ, due, e.g., to the effect of a pressure P, as we have already shown, can lead to a structural transition from the two-sublattice phase to a lattice of polar-phase nuclei. Because the value of I is different on both sides of this transition (i.e., for $\gamma > 0$ and $\gamma < 0$), there will be a kink in the function $H_{c2}^{\uparrow}(P)$ (at fixed temperature) at the point $\gamma(P) = 0$.

5. UPPER CRITICAL FIELD IN SUPERCONDUCTORS WITH CUBIC SYMMETRY

Let us now discuss the temperature behavior of the upper critical field for cubic-symmetry superconductor crystals which possess a center of inversion in the presence of spin-orbit interaction. We limit ourselves to the case of an order parameter linear in $\hat{r}$, where $\hat{r}$ is the direction of a radius vector joining the particles in a Cooper pair (this is analogous to p-pairing). In such a case, the order parameter is written[11] as a linear combination of basis functions $d_{\alpha i}^{p} r_i$ of the four irreducible representations of the O_h group[17,18]:

$$\Delta_{\lambda\mu}^{i}(\mathbf{R})\hat{r}_i = \eta^{p}(\mathbf{R}) g_{\lambda\mu}^{\alpha} d_{\alpha i}^{p} \hat{r}_i. \qquad (5.1)$$

Here $g_{\lambda\mu}^{\alpha} = (i\sigma_\alpha \sigma_y)_{\lambda\mu}$, $\eta^{p}(\mathbf{R})$ is an expansion coefficient; the index p labels the basis functions.

As we have already noted in the Introduction, since our primary intent is to investigate the effects of local magnetism, we retain in the linear combination (5.1) only those terms which correspond to an expansion in basis functions belonging to the three-dimensional vector representation F_1:

$$d_{\alpha i}^{p} = 2^{-1/2} e_{pts} \hat{x}_\alpha^{t} \hat{x}_i^{s}. \qquad (5.2)$$

Here, $\hat{x}_\alpha^t = (\hat{x}_\alpha, \hat{y}_\alpha, \hat{z}_\alpha)$ are unit basis vectors; the indices α and i label their projections onto the coordinate axes directed along the cube edges. Thus, we will assume that the critical temperature of the phase whose order parameter is a linear combination of the functions (5.2) belonging to F_1 is large compared to the critical temperatures for phases with order parameters belonging to the other representations.

The analogue of Eqs. (2.1) and (3.7) for the coefficients $\eta^p(\mathbf{R})$, which play the role of order parameters, has the form

$$\eta^{p}(\mathbf{R}) = \frac{3}{2} gT \sum_{\omega} \int d^3r\, d_{\beta i}^{p} d_{\alpha j}^{q} \left\{ \hat{r}_i \hat{r}_j \left(1 - \frac{e^2}{2p_0^2 c^2} [\mathbf{Hr}]^2\right) - \frac{ie}{2p_0^2 c} e_{ijm} H \right\} g_{\mu\lambda}^{*\beta} g_{\lambda\mu}^{\alpha} \overline{G}_{\omega}^{\lambda}(r) \overline{G}_{-\omega}^{\mu}(r) \cdot \exp\left\{ -\mathbf{r}\left[\frac{\partial}{\partial \mathbf{R}} - \frac{2ie}{c}\mathbf{A}(\mathbf{R})\right]\right\} \eta^{q}(\mathbf{R}). \qquad (5.3)$$

Here we understand by the combination $g_{\lambda\mu}^{\alpha} \overline{G}_{\omega}^{\lambda}(r) \overline{G}_{-\omega}^{\mu}(r)$ not contraction but rather element-by-element multiplication according to the indices λ and μ. As in the case of (2.1), we will solve Eq. (5.3) under the assumption of an isotropic electron spectrum, i.e., using expression (2.4) for the Green's function $\overline{G}_{\omega}^{\lambda}(r)$. In this case, it is not difficult to verify that for the basis functions (5.2) the representation F_1 of Eq. (5.3) determines H_{c2} independent of orientation of the external field relative to the crystal symmetry axes. Actually, the form of Eq. (5.3) does not change under the transformations $\hat{x}^t = R_{tv} \hat{x}'^{v}$, $\eta^p = R_{pa} \eta'^{a}$, etc., which give rise to a transition from a basis $\hat{x}^t$ to a basis $\hat{x}'^{v}$ and from solutions η^p to solutions η'^{a}, where the latter are related to the former by the three-dimensional rotation matrix R^{-1}. Isotropy of H_{c2} for a multicomponent order parameter is a characteristic only of the F_1 representation. The field H_{c2} for the other multicomponent representations E and F_2 is anisotropic even for the case of an isotropic electron spectrum.

If we do not consider the effect of a magnetic field on the electron spin (i.e., ignore the indices λ and μ of the Green's function) and omit the contribution of the spontaneous orbital moment and diamagnetism, Eq. (5.3) is significantly simplified:

$$\eta^p(\mathbf{R}) = 3gT\sum_\omega \int d^3r\,\Lambda_{pq}\bar{G}_\omega(r)\bar{G}_{-\omega}(r) \cdot \exp\left\{-\mathbf{r}\left[\frac{\partial}{\partial\mathbf{R}} - \frac{2ie}{c}\mathbf{A}(\mathbf{R})\right]\right\}\eta^q(\mathbf{R}). \quad (5.4)$$

Here Λ_{pq} is a matrix consisting of a product of the basis functions (5.2):

$$\Lambda_{pq} = d_{\alpha i}{}^p\hat{r}_i d_{\alpha j}{}^q\hat{r}_j = {}^1/_2(\delta_{pq} - \hat{r}_p\hat{r}_q). \quad (5.5)$$

Solving Eq. (5.4) (see Ref. 10 and Appendix A) shows that the phase with maximum H_{c2} has an order parameter of the form

$$\eta^p d^p_{\alpha i}\hat{r}_i \sim \{\beta_0 f_0(\mathbf{R})[-\hat{z}_\alpha(\hat{r}_y - i\hat{r}_x) + (\hat{y}_\alpha - i\hat{x}_\alpha)\hat{r}_z] - (\alpha_0 + \gamma_0 - 2)f_2(\mathbf{R})[-\hat{z}_\alpha(\hat{r}_y + i\hat{r}_x) + (\hat{y}_\alpha + i\hat{x}_\alpha)\hat{r}_z]\}. \quad (5.6)$$

In Appendix A, we show how to determine the coefficients α_0, β_0, λ_0; $f_N(\mathbf{R})$ is an eigenfunction of the Schroedinger equation for particles with mass m and charge $2e$ in a magnetic field $\mathbf{A}(\mathbf{R}) = (0, Hx, 0)$. The order parameter (5.6) can also be usefully written in spinor form:

$$\Delta_{\lambda\mu}(\hat{\mathbf{r}}) = g_{\lambda\mu}{}^\alpha\eta^p d_{\alpha i}{}^p\hat{r}_i \sim [\beta_0 f_0(\mathbf{R})\Delta_{\lambda\mu}{}^- - (\alpha_0 + \gamma_0 - 2)f_2(\mathbf{R})\Delta_{\lambda\mu}{}^+], \quad (5.7)$$

$$\Delta^-_{\lambda\mu} = \begin{pmatrix} 2i\hat{r}_z & -(\hat{r}_y - i\hat{r}_x) \\ -(\hat{r}_y - i\hat{r}_x) & 0 \end{pmatrix}, \quad (5.8a)$$

$$\Delta^+_{\lambda\mu} = \begin{pmatrix} 0 & -(\hat{r}_y + i\hat{r}_x) \\ -(\hat{r}_y + i\hat{r}_x) & 2i\hat{r}_z \end{pmatrix}. \quad (5.8b)$$

The phase which results [in analogy with the isotropic case[10] we will call it the Scharnberg-Klemm (SK) phase] is a linear combination of two complex vector phases with order parameters $\sim\Delta^-_{\lambda\mu}(\hat{r})$ and $\sim\Delta^+_{\lambda\mu}(\hat{r})$. Each of the complex vector phases in its turn is a linear combination of the A-phase and the β-phase. The complex vector phases possess spontaneous magnetism, which contains both a spin (the β-phase) and an orbital (the A-phase) component. The spontaneous magnetic moments of the $\Delta^+_{\lambda\mu}$ and $\Delta^-_{\lambda\mu}$ phases are oriented along and opposite the external field, respectively.

It is important to point out the solutions (5.6)–(5.8) to Eq. (5.4) corresponding to the maximum value of H_{c2}, are unique—i.e., combinations of the form (5.7) with oppositely directed spontaneous moments in each of the terms

$$\Delta_{\lambda\mu} \sim [\beta_0 f_0(\mathbf{R})\Delta_{\lambda\mu}{}^+ - (\alpha_0 + \gamma_0 - 2)f_2(\mathbf{R})\Delta_{\lambda\mu}{}^-],$$

are no longer solutions to Eq. (5.4). In this respect the phase transition to the superconducting state in the presence of strong spin-orbit interaction differs from the transition in the absence of spin-orbit interaction discussed in Sec. 4. In the latter case, without including the effects of spontaneous magnetism the two independent ampltudes $\Delta^i_{\uparrow\uparrow}$ and $\Delta^i_{\downarrow\downarrow}$ appear at the same value $H = H^P_{c2}$. Including spontaneous magnetism removes the degeneracy of the $\Delta^i_{\uparrow\uparrow}$ and $\Delta^i_{\downarrow\downarrow}$ states, and leads to the appearance at $H = H^\beta_{c2}$ first of the β-phase with $\Delta^i_{\downarrow\downarrow} \neq 0$, $\Delta^i_{\uparrow\uparrow} = 0$, and then at lower fields of an additional state with $\Delta^i_{\uparrow\uparrow} \neq 0$, $\Delta^i_{\downarrow\downarrow} = 0$. The absence of degenerate solutions for $H = H^{\rm SK}_{c2}$ in superconductors with strong spin-orbit interactions implies also the absence of a splitting of the phase transition for arbitrary directions of the external field. Thus, the assertion of the authors of Ref. 17—that the splitting in a magnetic field of the phase transition from the Fermi liquid to the superfluid state in the presence of anisotropic spin-orbit interactions takes place only for field directions along the cubic-symmetry crystal axes—is correct only for an uncharged Fermi liquid.

The upper critical field of the SK phase (5.6)–(5.8) (see Appendix A) equals

$$H^{\rm SK}_{c2}(T=0) = aH_0, \qquad H^{\rm SK}_{c2}(T\to T_c) = bH_0(1 - T/T_c), \quad (5.9)$$

where

$$a = \frac{\exp({}^7/_6 + 3^{1/2})}{8\gamma} \qquad b = \frac{20(9 + 38^{1/2})}{291\zeta(3)}, \quad H = \frac{\Phi_0}{\pi\xi_0{}^2}. \quad (5.10)$$

In order to include the effects of local magnetism, it is necessary first to solve Eq. (5.3). The solution to this equation is again the SK phase:

$$\Delta_{\lambda\mu}(\hat{\mathbf{r}}) \sim [\beta_0{}^{\uparrow\downarrow}f_0(\mathbf{R})\Delta_{\lambda\mu}{}^- - (\alpha_0{}^{\uparrow\downarrow,\,-1} + \gamma_0{}^{\uparrow\uparrow} - 2)f_2(\mathbf{R})\Delta_{\lambda\mu}{}^+], \quad (5.11)$$

in which other new functions figure into the coefficients: $\alpha_0^{\uparrow\downarrow,-1}$, $\beta_0^{\uparrow\downarrow}$, $\gamma_0^{\uparrow\uparrow}$ (see Appendix B); $\Delta^-_{\lambda\mu}$, $\Delta^+_{\lambda\mu}$ are determined by Eq. (5.8). Calculation of H_{c2} gives four types of corrections to $H^{\rm SK}_{c2}$ (5.9), (5.10) due to local magnetic effects (see Appendix B). For $T = 0$ we have

$$\delta H^{\rm SK}_{c2}/H^{\rm SK\ III\kappa}_{c2} = h_1 + h_2 + h_3 + h_4, \quad (5.12)$$

For $T \to T_c$,

$$\delta H^{\rm SK}_{c2}/H^{\rm SK\ III\kappa}_{c2} = h_1 + h_2. \quad (5.13)$$

Here,

$$h_1 \sim -(\mu_B H_0/\varepsilon_F)\ln(\omega_0/T_c), \quad (5.14)$$

$$h_2 \sim -(\mu_B H_0/\varepsilon_F)(m/m^*)\ln(\omega_0/T_c), \quad (5.15)$$

$$h_3 \sim -(\mu_B H_0/\varepsilon_F)(m^*/h), \quad (5.16)$$

$$h_4 \sim -(\mu_B H_0/\varepsilon_F)(m/m_*) \quad (5.17)$$

are corrections due to spin ferromagnetism, orbital ferromagnetism, paramagnetism and diamagnetism, respectively; m and m^* are the mass the effective mass of the electron. It is immediately clear from expressions (5.14)–(5.17) that in a Fermi gas where $m^* \approx m$, the corrections h_1 and h_2 due to spontaneous magnetism are always large compared to the para- and diamagnetic corrections.

Inclusion of the spontaneous magnetization decreases $H_{c2}(T)$ in the SK phase. This happens because the spontaneous magnetism increases the weights of the state Δ^+ (see (5.11)) with magnetic moment oriented along the field compared to the weights of the Δ^- state with oppositely oriented moment. The pure Δ^+ state which appears in phase (5.11) corresponds to a smaller value of H_{c2} [its amplitude is $\sim f_2(R)$] than the pure state Δ^- [its amplitude is $\sim f_0(R)$]; therefore, when we include the spontaneous magnetism the field $H^{\rm SK}_{c2}$ becomes smaller. Thus, the slope

dH_{c2}/dT as $T\to T_c$ will by no means always increase because of the spontaneous magnetism, contrary to the assertion made in Ref. 17.

For heavy-fermion compounds, where $m^*/m\sim 10^2 - 10^3$, at $T=0$ the largest correction is h_3, which is caused by spin paramagnetism. The estimate (5.16) for h_3 can be rewritten as $h_3\sim -(H_0/H_p)^2$, where $H_p\sim T_c/\mu_B$ is the paramagnetic limit. For such a large ratio m^*/m it is even possible that $H_0/H_p\approx(m^*/m)(T_c/\varepsilon_F)>1$, and expression (5.12) for $H_{c2}^{SK}(T=0)$ ceases to be valid, because the paramagnetic limit comes into play to bound the upper critical field. In such cases we must consider the possibility that the order parameter, which has the symmetry of the F_1 representation, will be mixed with contributions from other representations with lower critical temperatures.

6. PARAMAGNETIC BOUNDS FOR TRIPLET SUPERCONDUCTIVITY

For definiteness let us discuss the situation where the state with the highest critical temperature T_c, which belongs to the symmetry representation F_1, is mixed with a state with symmetry representation F_2 and a critical temperature $\tilde{T}_c<T_c$. The order parameter can again be written in the form of the linear combination (5.1):

$$\Delta_{\lambda\mu}{}^i(\mathbf{R})\hat{r}_i=\eta^p(\mathbf{R})g_{\lambda\mu}{}^\alpha d_{\alpha i}{}^p\hat{r}_i. \quad (6.1)$$

Here, $p=1,2,3,4,5,6$. For $p=1,2,3$,

$$d_{\alpha i}{}^p=2^{-1/2}e_{pts}\hat{x}_\alpha{}^t\hat{x}_i{}^s \quad (6.2)$$

are the basis functions of the F_1 representation, while for $p=4,5,6$,

$$d_{\alpha i}{}^4=2^{-1/2}(\hat{y}_\alpha\hat{z}_i+\hat{z}_\alpha\hat{y}_i),\quad d_{\alpha i}{}^5=2^{-1/2}(\hat{z}_\alpha\hat{x}_i+\hat{x}_\alpha\hat{z}_i),$$
$$d_{\alpha i}{}^6=2^{-1/2}(\hat{x}_\alpha\hat{y}_i+\hat{y}_\alpha\hat{x}_i) \quad (6.3)$$

are the F_2 representation basis functions.

If we neglect contributions from the orbital ferromagnetism and diamagnetism, Eq. (5.3) has the form

$$\frac{\eta^p(\mathbf{R})}{g^p}=\frac{3}{2}T\sum_\omega\int d^3r\, d_{\beta i}{}^p d_{\alpha j}{}^q\hat{r}_i\hat{r}_j g_{\mu\lambda}{}^{*\beta}g_{\lambda\mu}{}^\alpha\bar{G}_\omega{}^\lambda(r)$$
$$\cdot\bar{G}_{-\omega}{}^\mu(r)\exp\left\{-\mathbf{r}\left[\frac{\partial}{\partial\mathbf{R}}-\frac{2ie}{c}\mathbf{A}(\mathbf{R})\right]\right\}\eta^q(\mathbf{R}). \quad (6.4)$$

For $p=1$–3, the quantity $g^p=g$ determines $T_c=\omega_0\exp(-1/N_0g)$, while for $p=4, 5, 6$ the quantity $g^p=\tilde{g}$ determines $\tilde{T}_c=\omega_0\exp(-1/N_0\tilde{g})$. Equation (6.4) is solved in a way analogous to Eqs. (5.3), (5.4) (see Appendices A and B). For convenience in doing this we pass to the variables

$$\tilde{\eta}^p=[2^{-1/2}(\eta_1+i\eta_2),\ 2^{-1/2}(\eta_4-i\eta_5),$$
$$2^{-1/2}(\eta_1-i\eta_2),\ 2^{-1/2}(\eta_4+i\eta_5),\ \eta_3,\ \eta_6]$$

and expand them in the complete set of states $f_N(\mathbf{R})$:

$$\tilde{\eta}^p(\mathbf{R})=\sum_N A_N{}^p f_N(\mathbf{R}).$$

Rather than present the extremely tedious calculations, we will describe the results qualitatively.

The upper critical field as defined by Eq. (6.4) is anisotropic, i.e., depends on the direction of the external field relative to cubic axes. For $T_c>\tilde{T}_c$ this anisotropy is not significant as $T\to T_c$, where with accuracy $\sim 1-T/T_c$ the field H_{c2} is determined only by contributions from the F_1 representation, for which H_{c2} is isotropic. At low temperatures the anisotropy of H_{c2} is felt to the extent that the contribution with symmetry F_2 is mixed in.

The upper critical fields corresponding to the phases with symmetries F_1 and F_2 for $T=0$ without including paramagnetic renormalization are estimated to be [see (5.9)] $H_0\sim\Phi_0T_c^2/v_0^2$ and $\tilde{H}_0\sim\Phi_0\tilde{T}_c^2/v_0^2$. The paramagnetic bound on the upper critical field can be expressed in various forms depending on the relative magnitudes of the characteristic fields H_0, $\tilde{H}_0$ and $H_p\sim T_c/\mu_B$.

In the case $H_p\gg H_0>\tilde{H}_0$ discussed in the previous section, the paramagnetism gives only small corrections to the magnitude of the upper critical field, which is basically determined by the quantity H_0.

When the inequality $H_0>H_p>\tilde{H}_0$ holds, the upper critical field becomes much smaller than H_0. Its value is determined by competition between paramagnetism and the mixing of representations. In particular, when the stronger inequality $H_0\gg H_p\gg\tilde{H}_0$ holds, the paramagnetism suppresses the phase with symmetry F_1 but preserves the phase with symmetry F_2. The upper critical field is thus determined essentially by the quantity H_p, i.e., in superconductors with triplet pairing the strong spin-orbit interaction can lead to an important paramagnetic bound on the superconductivity.

Finally, for $H_0>\tilde{H}_0\gg H_p$, the paramagnetic bounds again become irrelevant. The solution to Eq. (6.4) turns out to be close to the β-phase, for which there is no paramagnetic limit, while the upper critical field satisfies $H_{c2}\sim H_0$.

7. CONCLUSION

The experimental behavior of the upper critical field in UBe_{13}, a superconducting heavy-fermion compound with cubic symmetry, is characterized by a number of peculiarities. First of all, for temperatures close to T_c the upper critical field does not depend on the direction of the external field relative to the cubic axes; the anisotropy of H_{c2} begins to appear only as the temperature decreases.[19] Secondly, for temperatures near zero the upper critical field of this material exceeds the paramagnetic limit by an order of magnitude.[20] Finally, the decrease in the field $H_{c2}(T)$ is practically linear over the entire temperature interval from $T/T_c\approx 0.1$ to $T/T_c\approx 0.9$; only near T does this decrease in H_{c2} become more rapid.[20]

Symmetry considerations[7] imply that any anisotropy in H_{c2} as $T\to T_c$ which is not connected with an anisotropic effective mass (which does not occur in cubic superconductors) is necessarily a sign of superconductivity with a multicomponent order parameter. The absence of this kind of anisotropy, it would seem, unambiguously indicates that for UBe_{13} the transition to the superconducting state in a magnetic field in the limit $T\to T_c$ takes place into one of the phases whose symmetries are characterized by the one-dimensional representations A_1 and A_2. The authors of Ref. 4 prefer to characterize UBe_{13} by the representation A_2, which implies that the thermoelectric power does not have an activation form.[16] However, if the cubic anisotropies of the spectrum and interactions are for some reason sufficiently small, the isotropic behavior of H_{c2} as $T\to T_c$ is not inconsistent with phases which possess the symmetries of the three-dimensional vector representations (see Ref. 5). The anisotropy of H_{c2} observed in UBe_{13} as the temperature is lowered is then naturally explained in terms of the admixture of other representations with anisotropic values of H_{c2}. As we saw from the example of a mixture of the representations F_1 and F_2 (Sec. 6), the upper critical field in a superconductor with triplet pairing, even in the presence of strong spin-orbit interactions, need not be subject to bounding by the paramagnetic limit. As regards the remarkably linear behavior of H_{c2} over a large temperature interval, while this behavior is explained by mixing of representations by paramagnetism,[11] it could also be a result of some dependence of the pairing interaction on magnetic field which is unknown at this time (see also Ref. 26).

In conclusion, one of the authors (V. P. M.) is grateful to G. E. Volovik and L. P. Gor'kov for some healthy scepticism in relation to p-pairing superconductors with isotropic dispersion laws which they expressed during discussions of the results of this paper, and also to N. E. Alekseevskiĭ for discussing the experimental situation and for kindly sending reprints of papers on UBe_{13}, and D. I. Khomski for useful exchanges of information relating to the properties of superconducting compounds with heavy fermions.

APPENDIX A

Solution of Eq. (5.4):

Following Ref. 10, we transform to the variables

$$\hat{r}^{\pm}=2^{-1/2}(\hat{r}_x\pm i\hat{r}_y) \text{ and } \hat{r}_z \tag{A1}$$

and functions

$$\tilde{\eta}^p=(\eta^+,\ \eta^-,\ \eta^3),\quad \eta^{\pm}=2^{1/2}(\eta^1\pm i\eta^2). \tag{A2}$$

Let us also define the raising and lowering operators

$$a^{\pm}=(c/4|e|H)^{1/2}(\Pi_x\pm i\Pi_y) \tag{A3}$$

for the set of eigenfunctions $f_N(R)$ of the Schroedinger equation for particles with mass m and charge $2e$ in a magnetic field $\mathbf{A}=(0,\ Hx,\ 0)$. Here $\mathbf{\Pi}=(\Pi_x,\ \Pi_y,\ \Pi_z) = -i(\partial/\partial\mathbf{R})-(2e/c)\mathbf{A}(\mathbf{R})$.

Assuming that η^p does not depend on the coordinate z along the field direction, and expanding

$$\tilde{\eta}^p(\mathbf{R})=\sum_{N=0}^{\infty}A_N{}^pf_N(\mathbf{R}) \tag{A4}$$

in the complete set $f_N(R)$, we obtain for the coefficients A_N^p from (5.4) the infinite system of algebraic linear equations

$$K_{N-2}^{pq}A_{N-2}^q+(L_N{}^{pq}-\delta_{pq})A_N{}^q+M_N{}^{pq}A_{N+2}^q=0, \tag{A5}$$

where

$$K_N^{pq}=\begin{pmatrix}0 & \beta_N/2 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},$$

$$L_N^{pq}=\begin{pmatrix}(\alpha_N+\gamma_N)/2 & 0 & 0\\ 0 & (\alpha_N+\gamma_N)/2 & 0\\ 0 & 0 & \alpha_N\end{pmatrix},$$

$$M_N^{pq}=\begin{pmatrix}0 & 0 & 0\\ \beta_N/2 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}. \tag{A6}$$

The functions

$$\alpha_N=\alpha_N(H,\ T),\quad \beta_N=\beta_N(H,\ T),\quad \gamma_N=\gamma_N(H,\ T)$$

are determined by the equations[10]

$$\alpha_N=\frac{3}{2}gT\sum_\omega\int d^3r\sin^2\theta\bar{G}_\omega(r)\bar{G}_{-\omega}(r)\cdot\exp\left(-\frac{|e|Hr_\perp{}^2}{2c}\right)L_N\left(\frac{|e|Hr_\perp{}^2}{c}\right),$$

$$\beta_N=\frac{3}{2}gT\sum_\omega\int d^3r\sin^2\theta\bar{G}_\omega(r)\bar{G}_{-\omega}(r)\exp\left(-\frac{|e|Hr_\perp{}^2}{2c}\right)\cdot\sum_{n=0}^{N}\frac{(-r_\perp{}^2|e|H/c)^{n+1}}{(n+2)!}[(N+1)(N+2)]^{1/2}C_N{}^n,$$

$$\gamma_N=3gT\sum_\omega\int d^3r\cos^2\theta\bar{G}_\omega(r)\bar{G}_{-\omega}(r)\cdot\exp\left(-\frac{|e|Hr_\perp{}^2}{2c}\right)L_N\left(\frac{|e|Hr_\perp{}^2}{c}\right). \tag{A7}$$

Here $L_N(x)$ are Laguerre polynominals, $r_\perp=r\sin\theta$. The determinant of the system (A5) will be zero if one of the following conditions hold:

$$\begin{gathered}\alpha_N+\gamma_N=2,\\ (\alpha_N+\gamma_N-2)(\alpha_{N+2}+\gamma_{N+2}-2)-\beta_N{}^2=0,\\ \alpha_N=1.\end{gathered} \tag{A8}$$

It can be verified that the maximum value of $H_{c2}(T)$ is determined by the equation

$$[\alpha_0(H,\ T)+\gamma_0(H,\ T)-2][\alpha_2(H,\ T)+\gamma_2(H,\ T)-2]-[\beta_0(H,\ T)]^2=0, \tag{A9}$$

which corresponds to a solution of the system (A5):

$$\tilde{\eta}^1=-C(\alpha_0+\gamma_0-2)f_2(\mathbf{R}),\quad \tilde{\eta}^2=C\beta_0f_0(\mathbf{R}), \tag{A10}$$

where C is an arbitrary constant. The vector and spinor notation for the order parameter (A10) are presented in the main text of the article [formulae (5.6)–(5.8)].

Combining Eq. (A9) and the equation for the critical temperature

$$\alpha_0(0,\ T_c)=\gamma_0(0,\ T_c)=\alpha_2(0,\ T_c)=\gamma_2(0,\ T_c)=1, \tag{A11}$$

we can write the equation $H_{c2}(T)$ in a form which contains no logarithmic divergences:

$$\begin{gathered}\{\alpha_0(H,\ T)+\gamma_0(H,\ T)-\alpha_0(0,\ T)-\gamma_0(0,\ T)-[\alpha_0(0,\ T_c)\\ +\gamma_0(0,\ T_c)-\alpha_0(0,\ T)-\gamma_0(0,\ T)]\}\{\alpha_2(H,\ T)+\gamma_2(H,\ T)\\ -\alpha_2(0,\ T)-\gamma_2(0,\ T)-[\alpha_2(0,\ T_c)+\gamma_2(0,\ T_c)-\alpha_2(0,\ T)\\ -\gamma_2(0,\ T)]\}-[\beta_0(H,\ T)]^2=0.\end{gathered} \tag{A12}$$

Making use of (A7), we rewrite Eq. (A12) in dimensionless variables (2.7):

$$\left\{\ln t-\sum_n\left[\frac{t}{4\pi h^{1/2}}\int\frac{d^3\rho}{\rho^2}\frac{3}{4}(\sin^2\theta+2\cos^2\theta)\cdot\exp\left(-\frac{\rho}{\alpha_n}-\frac{\rho_\perp^2}{4}\right)-\frac{1}{|2n+1|}\right]\right\}\left\{\ln t-\sum_n\left[\frac{t}{4\pi h^{1/2}}\int\frac{d^3\rho}{\rho^2}\frac{3}{4}(\sin^2\theta+2\cos^2\theta)\cdot\left(1-\rho_\perp^2+\frac{\rho_\perp^2}{8}\right)\exp\left(-\frac{\rho}{\alpha_n}-\frac{\rho_\perp^2}{4}\right)-\frac{1}{|2n+1|}\right]\right\}$$

$$-\frac{3^2}{2^7}\left[\sum_n\frac{t}{4\pi h^{1/2}}\int\frac{d^3\rho}{\rho^2}\sin^2\theta\exp\left(-\frac{\rho}{\alpha_n}-\frac{\rho_\perp^2}{4}\right)\rho_\perp^2\right]^2=0,$$

$$\alpha_n=\frac{h^{1/2}}{t|2n+1|}. \tag{A13}$$

Equation (A13) determines the upper critical field in phase (A10) for arbitrary temperatures. The solutions (A13) for $T=0$ and $T\to T_c$ are given by formulae (5.9) and (5.10).

APPENDIX B

Solution to Eq. (5.3):

From Eq. (5.3) we obtain a system of algebraic equations for the expansion coefficients A_N^p of the order parameter (A4) in the basis functions $f_N(\mathbf{R})$:

$$\mathcal{K}_{N-2}^{pq}A_{N-2}^q+(\mathcal{L}_N^{pq}-\delta_{pq})A_N^q+\mathcal{M}_N^{pq}A_{N+2}^q=0, \tag{B1}$$

where

$$\mathcal{K}_N^{pq}=\begin{pmatrix}0 & \beta_N^{\uparrow\downarrow}/2 & 0\\ 0&0&0\\0&0&0\end{pmatrix},$$

$$\mathcal{L}_N^{pq}=\begin{pmatrix}\frac{1}{2}(\alpha_N^{\uparrow\downarrow,+1}+\gamma_N^{\downarrow\downarrow}) & 0 & 0\\ 0 & \frac{1}{2}(\alpha_N^{\uparrow\downarrow,-1}+\gamma_N^{\uparrow\uparrow}) & 0\\ 0&0&\frac{1}{2}(\alpha_N^{\uparrow\uparrow,+1}+\alpha_N^{\downarrow\downarrow,-1})\end{pmatrix},$$

$$\mathcal{M}_N^{pq}=\begin{pmatrix}0&0&0\\ \beta_N^{\uparrow\downarrow}/2&0&0\\0&0&0\end{pmatrix}. \tag{B2}$$

The functions

$$\alpha_N^{\lambda\mu,\nu}=\alpha_N^{\lambda\mu,\nu}(H,T),\quad \beta_N^{\lambda\mu}=\beta_N^{\lambda\mu}(H,T),\quad \gamma_N^{\lambda\mu}=\gamma_N^{\lambda\mu}(H,T)$$

are determined by the expressions

$$\alpha_N^{\lambda\mu,\nu}=3gT\sum_\omega\int d^3r\frac{1}{2}\left[\sin^2\theta\left(1-\frac{e^2H^2r_\perp^2}{2p_0^2c^2}\right)+\frac{\nu\mu_BH}{\varepsilon_F}\frac{m}{m^*}\right]\frac{1}{2}[\bar G_\omega^\lambda(r)\bar G_{-\omega}^\mu(r)+\bar G_\omega^\mu(r)\bar G_{-\omega}^\lambda(r)]\cdot\exp\left(-\frac{|e|Hr_\perp^2}{2c}\right)L_N\left(\frac{|e|Hr_\perp^2}{c}\right),$$

$$\beta_N^{\lambda\mu}=3gT\sum_\omega\int d^3r\frac{\sin^2\theta}{2}\left(1-\frac{e^2H^2}{2p_0^2c^2}r_\perp^2\right)\cdot\frac{1}{2}[\bar G_\omega^\lambda(r)\bar G_{-\omega}^\mu(r)+\bar G_\omega^\mu(r)\bar G_{-\omega}^\lambda(r)]\exp\left(-\frac{|e|Hr_\perp^2}{2c}\right).$$

$$\cdot\sum_{n=0}^N\frac{(-|e|Hr_\perp^2/c)^{n+1}}{(n+2)!}[(N+1)(N+2)]^{1/2}C_N^n, \tag{B3}$$

$$\gamma_N^{\lambda\mu}=3gT\sum_\omega\int d^3r\cos^2\theta\left(1-\frac{e^2H^2}{2p_0^2c^2}r_\perp^2\right)\cdot\frac{1}{2}[\bar G_\omega^\lambda(r)\bar G_{-\omega}^\mu(r)+\bar G_\omega^\mu(r)\bar G_{-\omega}^\lambda(r)]\exp\left(-\frac{|e|Hr_\perp^2}{2c}\right)\cdot L_N\left(\frac{|e|Hr_\perp^2}{c}\right).$$

The indices λ and μ take on the values ↑, ↓ or +1, −1. The index $\nu=\pm1$.

Setting to zero the determinant of the system (B1), we obtain a set of relations analogous to the relations (A8). The maximum value of H_{c2} is determined by the equation

$$(\alpha_0^{\uparrow\downarrow,-1}+\gamma_0^{\uparrow\uparrow}-2)(\alpha_2^{\uparrow\downarrow,+1}+\gamma_2^{\downarrow\downarrow}-2)-(\beta_0^{\uparrow\downarrow})^2=0 \tag{B4}$$

[the analog to Eq. (A9)], which corresponds to solution of the system (B1):

$$\eta^1=-C(\alpha_0^{\uparrow\downarrow,-1}+\gamma_0^{\uparrow\uparrow}-2)f_2(\mathbf{R}),$$

$$\eta^2=C\beta_0^{\uparrow\downarrow}f_0(\mathbf{R}), \tag{B5}$$

i.e., taking into account the overdetermination of the functions α, β and γ, we again obtain phase (A10).

Along with the functions $\alpha_N^{\lambda\mu,\nu}(H,T)$, $\beta_N^{\lambda\mu}(H,T)$, $\gamma_N^{\lambda\mu}(H,T)$, we determine the functions

$$\tilde\alpha^{\lambda\mu,\nu}(H,T)=3gT\sum_\omega\int d^3r\frac{1}{2}\left[\sin^2\theta\left(1-\frac{e^2H^2r_\perp^2}{2p_0^2c^2}\right)+\frac{\nu\mu_BH}{\varepsilon_F}\frac{m}{m^*}\right]\frac{1}{2}[\bar G_\omega^\lambda(r)\bar G_{-\omega}^\mu(r)+\bar G_\omega^\mu(r)\bar G_{-\omega}^\lambda(r)],$$

$$\tilde\beta^{\lambda\mu}(H,T)=0,$$

$$\tilde\gamma^{\lambda\mu}(H,T)=3gT\sum_\omega\int d^3r\cos^2\theta\left(1-\frac{e^2H^2r_\perp^2}{2p_0^2c^2}\right)\cdot\frac{1}{2}[\bar G_\omega^\lambda(r)\bar G_{-\omega}^\mu(r)+\bar G_\omega^\mu(r)\bar G_{-\omega}^\lambda(r)]. \tag{B6}$$

With a goal of eliminating the logarithmic divergences in Eq. (B4), we determine the temperatures T'_H and T''_H as solutions to the equations

$$\tilde{\alpha}^{\uparrow\downarrow,+1}(H, T_H')+\tilde{\gamma}^{\downarrow\downarrow}(H, T_H')-2=0,$$

$$\tilde{\alpha}^{\uparrow\downarrow,-1}(H, T_H'')+\tilde{\gamma}^{\uparrow\uparrow}(H, T_H'')-2=0, \qquad \text{(B7)}$$

from which we obtain

$$T_H' = T_c\left[1+\left(1+3\frac{m}{m^*}\right)\frac{\mu_B H}{4\varepsilon_F}\ln\frac{\omega_0}{T_c}\right],$$

$$T_H'' = T_c\left[1-\left(1+3\frac{m}{m^*}\right)\frac{\mu_B H}{4\varepsilon_F}\ln\frac{\omega_0}{T_c}\right]. \qquad \text{(B8)}$$

Combining (B4) and (B8), we obtain an equation for $H_{c2}(T)$ which does not contain any logarithmic divergences

$$\begin{aligned}&\{\alpha_0^{\uparrow\downarrow,-1}(H,T)+\gamma_0^{\uparrow\uparrow}(H,T)-\tilde{\alpha}^{\uparrow\downarrow,-1}(H,T)-\tilde{\gamma}^{\uparrow\uparrow}(H,T)\\&-[\tilde{\alpha}^{\uparrow\downarrow,-1}(H,T_H'')+\tilde{\gamma}^{\uparrow\uparrow}(H,T_H'')-\tilde{\alpha}^{\uparrow\downarrow,-1}(H,T)-\tilde{\gamma}^{\uparrow\uparrow}(H,T)]\}\\&\cdot\{\alpha_2^{\uparrow\downarrow,+1}(H,T)+\gamma_2^{\downarrow\downarrow}(H,T)-\tilde{\alpha}^{\uparrow\downarrow,+1}(H,T)-\tilde{\gamma}^{\downarrow\downarrow}(H,T)\\&-[\tilde{\alpha}^{\uparrow\downarrow,+1}(H,T_H')+\tilde{\gamma}^{\downarrow\downarrow}(H,T_H')-\tilde{\alpha}^{\uparrow\downarrow,+1}(H,T)-\tilde{\gamma}^{\downarrow\downarrow}(H,T)]\}\\&-[\beta_0^{\uparrow\downarrow}(H,T)]^2=0.\end{aligned} \qquad \text{(B9)}$$

Equation (B9) determines H_{c2} in the phase (B5), taking into account the effects of paramagnetism, diamagnetism and spontaneous spin and orbital magnetism at arbitrary temperatures. The qualitative results of solving this equation for the case of small renormalization of H_{c2} at $T = 0$ and for $T \to T_c$ are given by Eqs. (5.12)–(5.17).

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