Phonon renormalization of the thermoelectric force of an impure metal

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The method of a quantum transport equation is used to deal with the renormalization of the thermoelectric force of an impure metal due to the electron–phonon interaction and the effects of an electron–phonon–impurity interference. The electron–impurity interaction is not assumed to be small and it is included in all orders of perturbation theory. It is shown that the magnitude and sign of the thermoelectric effect depend strongly on the electron–phonon interaction constant and on the amplitude of electron scattering by impurities. In the Born approximation for the electron–impurity interaction and on the assumption of a parabolic energy band the total phonon correction to the impurity thermoelectric force vanishes because of canceling of terms, which differs considerably from the results of previous studies. An investigation is also made of the influence of the electron–magnon interaction and of the electron–magnon–impurity interference on the thermoelectric effect of an impure ferromagnetic metal.

'1. INTRODUCTION

Studies of thermoelectric effects in normal metals are difficult because of the extreme sensitivity of these effects to the characteristics of the electron and phonon spectra, and also to the presence and nature of impurities.[1-3] From the theoretical point of view the greatest problem is that of the phonon renormalization of the thermoelectric force (thermopower) of an impure conductor. It should be pointed out directly that such renormalization is possible because of the pure electron–phonon interaction and also because of an electron–phonon–impurity interference.

The influence of the electron–phonon interaction on the thermoelectric effect of metals is analyzed in Refs. 4 and 5 by the method of linear response in which an important feature is an allowance for the interaction effects in the heat flux operator. However, in our opinion, analytic continuation of the diagrams containing the electron–phonon corrections to the heat flux operator was carried out incorrectly in these investigations. An important role played by electron–phonon–impurity interference processes was first pointed out in Ref. 6. This effect was subsequently investigated by others.[7-9] A semiclassical transport equation was used in Refs. 6–8 so that all the interference processes could not be allowed for. The calculation reported in Ref. 9 was made by the linear response method and the electron–impurity interaction was not assumed to be small, i.e., the non-Born nature of the scattering of electrons by impurities was taken into account. However, the treatment in Ref. 9 was confined to a narrow class of diagrams obtained ignoring corrections to the vertices of the momentum and energy fluxes. Since allowing for the additional diagrams representing the electron–phonon–impurity interference effects may alter not only the numerical value but also the sign of the quantity being calculated, the problem of the influence of the interference processes on the thermoelectric effect of an impurity metal remains unsolved. Moreover, a separate discussion of the electron–phonon interaction and electron–phonon–impurity interference is artificial because the contributions made to the thermoelectric effect by these effects have a simi-

lar structure and, as shown below, largely cancel out one another.

It therefore seemed of interest to analyze all the phonon renormalizations of the thermoelectric force of an impurity metal by the same method. The approach presented in Sec. 2 is based on a quantum transport equation and it makes it possible to allow for the full variety of quantum interference effects. Moreover, it has a number of advantages compared with the linear response method. Firstly, in this approach we have to consider a much smaller number of diagrams (twenty-four diagrams were considered in Ref. 9 and if the vertex corrections are included, then the number of diagrams rises catastrophically). Moreover, in calculations of the electric current due to a temperature gradient there is no question of corrections to the heat flux operator due to the interaction of electrons with phonons and impurities. The Keldysh diagram technique is used in Sec. 3 to construct the effective vertices of the interaction allowing for the non-Born scattering of electrons by impurities. A direct calculation of the corrections to the thermopower of an impure conductor is reported in Sec. 4. In Sec. 5 the method is extended to the case of a ferromagnetic metal when the corrections to the thermoelectric force are due to the electron–magnon interaction and electron–magnon–impurity interference effects. The results obtained differ considerably from the conclusions reached in previous investigations, and this is discussed in the final section.

2. QUANTUM TRANSPORT EQUATION METHOD

We shall calculate the phonon corrections to the thermoelectric force of an impurity metal using the quantum transport equation method developed on the basis of the Keldysh diagram technique in Ref. 10. This model has been used earlier to calculate temperature-dependent corrections to the extrinsic conductivity as a result of the electron–electron[11] and electron–phonon[12] interactions.

In the Keldysh diagram technique[13] the Green's functions of an electron and a phonon, and the electron self-energy are represented by matrices

$$G = \begin{pmatrix} 0 & G^A \\ G^R & G^C \end{pmatrix}, \quad \hat{D} = \begin{pmatrix} 0 & D^A \\ D^R & D^C \end{pmatrix}, \quad \hat{\Sigma} = \begin{pmatrix} \Sigma^C & \Sigma^R \\ \Sigma^A & 0 \end{pmatrix}.$$

(1)

The Green's function of a phonon is

$$D^R(\mathbf{q}, \omega) = (\omega - \omega_q + i0)^{-1} - (\omega + \omega_q + i0)^{-1}.$$ (2)

In the case of equilibrium phonons, we have

$$D^C(\mathbf{q}, \omega) = (2N_\omega + 1)[D^R(\mathbf{q}, \omega) - D^A(\mathbf{q}, \omega)],$$

$$N_\omega = [\exp(\omega/T) - 1]^{-1}.$$ (3)

Without allowance for the interaction with phonons the Green's function of an electron averaged over the impurity positions is

$$G_0{}^R(\mathbf{p}, \varepsilon) = (\varepsilon - \xi_\mathbf{p} + i/2\tau_\varepsilon)^{-1}, \quad \xi_\mathbf{p} = (p^2 - p_F{}^2)/2m,$$ (4)

where τ_ε is the relaxation time of the momentum of an electron with an energy ε scattered by impurities, m is the electron mass, and p_F is the Fermi momentum.

Our aim will be to calculate the electron current which appears under the influence of a temperature gradient ∇T. In accordance with the Onsager principle, the solution of a symmetric problem of finding the heat flux due to an electric field is more complex because the heat flux operator differs from the electric current operator since the former contains corrections due to the interaction.[4,5] An allowance for density gradients in the momentum representation leads naturally to corrections in the form of Poisson brackets[10]:

$$\{A, B\} = \frac{\partial A}{\partial \mathbf{R}} \frac{\partial B}{\partial \mathbf{p}} - \frac{\partial A}{\partial \mathbf{p}} \frac{\partial B}{\partial \mathbf{R}},$$ (5)

where in the presence of a temperature gradient we have $\nabla_\mathbf{R} = (\nabla_\mathbf{R} T)\partial/\partial T$.

To first order in ∇T we find that G^C is described by

$$G^C(\mathbf{p}, \varepsilon) = S(\mathbf{p}, \varepsilon)[G^A(\mathbf{p}, \varepsilon) - G^R(\mathbf{p}, \varepsilon)]$$

$$+ \frac{1}{2}i\{S_0(\varepsilon), G^A(\mathbf{p}, \varepsilon) + G^R(\mathbf{p}, \varepsilon)\},$$ (6)

where under equilibrium conditions we have $S(\mathbf{p},\varepsilon) = S_0(\varepsilon) = -\tanh(\varepsilon/2T)$. Then, the linearized transport equation for the electron distribution function is

$$\mathbf{v} \nabla T \frac{\varepsilon}{T} \frac{\partial S_0}{\partial \varepsilon} = I_{e-imp} + I_{e-ph} + I_{e-ph-imp},$$ (7)

where the collision integrals on the right-hand side represent respectively the electron–impurity and the electron–phonon interactions, and the interference between these interactions. The collision integrals are expressed in terms of the corresponding self-energy part using the formulas

$$I(S) = I^0(S) + \delta I(S), \quad I^0 = -i[\Sigma^C - S(\Sigma^A - \Sigma^R)],$$ (8)

$$\delta I = -i[\delta\Sigma^C - S_0(\delta\Sigma^A - \delta\Sigma^R)] + \frac{1}{2}\{\Sigma^A + \Sigma^R, S_0\},$$ (9)

where $\delta\Sigma$ are corrections in the form of Poisson brackets. Assuming, as in Ref. 10, that the main momentum relaxation mechanism is the scattering by impurities, we shall solve the transport equation by iteration. Without allowance for phonons the nonequilibrium correction to the distribution function is

$$\varphi_0(\mathbf{p}, \varepsilon) = \tau_\varepsilon \mathbf{v} \nabla T \frac{\varepsilon}{T} \frac{\partial S_0(\varepsilon)}{\partial \varepsilon}.$$ (10)

In the first order of perturbation theory for phonons, we find that

$$\varphi_1(\mathbf{p}, \varepsilon) = \tau_\varepsilon[I_{e-ph}(S_0 + \varphi_0) + I_{e-ph-imp}(S_0 + \varphi_0)$$

$$+ \delta_{ph}I_{e-imp}(S_0 + \varphi_0)],$$ (11)

where $\delta_{ph}I_{e-imp}$ is the correction to the impurity collision integral due to the phonon renormalization of the density of states of electrons:

$$\delta_{ph}I_{e-imp}(\mathbf{p}, \varepsilon)$$

$$= \frac{2}{\pi \nu_\varepsilon \tau_\varepsilon} \int \frac{d\mathbf{k}}{(2\pi)^3}[S(\mathbf{k}, \varepsilon) - S(\mathbf{p}, \varepsilon)]\operatorname{Im}[\delta_{ph}G^A(\mathbf{k}, \varepsilon)],$$

(12)

$$\delta_{ph}G^A = (G_0{}^A)^2(\Sigma_{e-ph}^A + \Sigma_{e-ph-imp}^A),$$ (13)

and the renormalized density of states ν_ε is

$$\nu_\varepsilon = \frac{2}{\pi} \int \frac{d\mathbf{p}}{(2\pi)^3} \operatorname{Im} G_0{}^A(\mathbf{p}, \varepsilon) = \frac{m}{\pi^2}[2m(\varepsilon_F + \varepsilon)]^{1/2}, \quad \varepsilon_F = \frac{p_F{}^2}{2m}.$$

The electric current is given by the expression

$$\mathbf{j} = \eta \nabla T = \int \frac{d\mathbf{p}\, d\varepsilon}{(2\pi)^4} \mathbf{v} S(\mathbf{p}, \varepsilon) \operatorname{Im} G^A(\mathbf{p}, \varepsilon),$$ (14)

from which we find that the correction to the thermoelectric force is related both to the correction to the distribution function $\varphi_0(\mathbf{p},\varepsilon)$ of Eq. (10) and to various corrections to the electron density of states:

$$\Delta\eta = \frac{2e}{|\nabla T|} \int \frac{d\mathbf{p}\, d\varepsilon}{(2\pi)^4} \mathbf{v}\mathbf{n}\{\varphi_1 \operatorname{Im} G_0{}^A + \varphi_0 \operatorname{Im}[\delta_{ph}G^A(S_0)]$$

$$+ S_0 \operatorname{Im}[\delta_{ph}G^A(\varphi_0)] + S_0\delta G^A(S_0)\},$$ (15)

where $\mathbf{n}$ is a unit vector directed along ∇T and δG^A is the correction in the form of Poisson brackets:

$$\delta G^A = (G_0{}^A)^2(\delta\Sigma_{e-ph}^A + \delta\Sigma_{e-ph-imp}^A).$$ (16)

3. EFFECTIVE VERTICES

We shall follow Refs. 12 and 13 in the determination of the interaction vertices. The potential of the interaction of an electron with an impurity under screening conditions is

$$U(\mathbf{q}) = -4\pi e^2(Z_{imp} - Z_{ion})/(\varkappa^2 + q^2),$$

$$\varkappa^2 = 4\pi e^2\nu_0, \quad \nu_0 = \nu_\varepsilon(\varepsilon = 0),$$ (17)

where Z_{imp} and Z_{ion} are the valences of the impurity atoms and of the ions in the host lattice. We shall ignore the weak momentum dependence of the potential and assume that the scattering is isotropic. The vertex of the elastic scattering of an electron by an impurity considered in the representation of Eq. (1) corresponds to a matrix $U(\sigma_x)_{ij}$, where σ_x is the familiar Pauli matrix. Without restricting our treatment to the Born approximation, we shall find the total amplitude of the scattering of an electron by an impurity from the equation shown graphically in Fig. 1. In the Keldysh technique the scattering amplitude f_ε has an analytic structure similar to that of the self-energy of Eq. (1), where

$$f_\varepsilon{}^A = (f_\varepsilon{}^R)^* = U(1 + ia_\varepsilon)^{-1}, \quad f_\varepsilon{}^C = S_0(\varepsilon)(f_\varepsilon{}^A - f_\varepsilon{}^R), \quad a_\varepsilon = \frac{1}{2}\pi U\nu_\varepsilon.$$

(18)

$$f_\varepsilon = \{ = \| + \overset{\wedge}{\underset{}{/\!\!/\!\!/}}$$

It should be pointed out that in the case of an isotropic impurity potential the expression for f_ε^C should contain only the equilibrium distribution function $S_0(\varepsilon)$. The relaxation time of the electron momentum is related to the scattering amplitude by

$$\tau_\varepsilon^{-1} = \pi v_\varepsilon n_i |f_\varepsilon^A|^2 = 2Un_i a_\varepsilon (1+a_\varepsilon^2)^{-1}, \tag{19}$$

where n_i is the impurity concentration.

The vertex for the inelastic scattering of an electron by a longitudinal phonon, shown in Fig. 2, is

$$g_{ij}^k = ig_q K_{ij}^k, \quad g_q = {}^2/_3 \varepsilon_F (2MN\omega_{q\lambda})^{-1/2} q e_\lambda,$$

$$K_{ij}^1 = 2^{-1/2}\delta_{ij}, \quad K_{ij}^2 = 2^{-1/2}(\sigma_x)_{ij}, \quad (g_q)^2 = \beta\omega_q/v_0, \tag{20}$$

where e_λ is the phonon polarization vector; M is the mass of an ion; N is the number of unit cells; ε_F is the Fermi energy; β is the dimensionless electron–phonon interaction constant. The system (20) is generally valid only at low values of q; for values $q - q_D$ typical of the present problem (where q_D is the limiting wave vector of the phonon spectrum) the expression for g_q contains an additional factor $\varkappa^2(\varkappa^2 + q^2)^{-1}$. As in the case of the electron–impurity scattering, we shall ignore the weak dependence on q associated with this factor. As shown in Ref. 12, because of the weak screening of transverse electromagnetic fields the corresponding vertex for transverse phonons is somewhat smaller than g_q (with the exception of an unimportant range of very low values of q).

The interaction with transverse phonons appears only because of the inelastic scattering of electrons by impurities, which is described by a vertex γ_{ij}^k (Fig. 2):

$$\gamma_{ij}^k = iU((2MN\omega_{q\lambda})^{-1/2}(\mathbf{p}-\mathbf{p}')e_\lambda K_{ij}^k. \tag{21}$$

It is convenient to carry out partial summation of the diagrams by introducing effective vertices which allow for the electron–phonon–impurity interference. Graphical solutions for the effective vertices are shown in Fig. 3. Assuming that the electrons satisfy an equilibrium distribution, we find that the vertices Λ_{ij}^k are described by the following expressions:
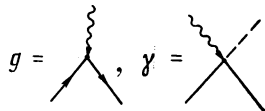

$$g = \overset{\backslash}{\underset{/}{\bigwedge}}, \quad \gamma = \overset{\backslash/}{\underset{/\backslash}{\times}}$$

$$\Lambda_{11}^1 = 2^{-1/2}[1 - S_0(\varepsilon)S_0(\varepsilon+\omega)(f_\varepsilon^A + f_\varepsilon^R)^2/|f_\varepsilon^A|^2]\Lambda, \quad \Lambda_{22}^1 = 2^{-1/2}\Lambda,$$

$$\Lambda_{12}^1 = 2^{-1/2}(1 + f_\varepsilon^R/f_\varepsilon^A)S_0(\varepsilon)\Lambda, \quad \Lambda_{12}^2 = 2^{-1/2}\Lambda f_\varepsilon^R/f_\varepsilon^A,$$

$$\Lambda_{21}^1 = -2^{-1/2}(1 + f_\varepsilon^A/f_\varepsilon^R)S_0(\varepsilon+\omega)\Lambda, \quad \Lambda_{21}^2 = 2^{-1/2}\Lambda f_\varepsilon^A/f_\varepsilon^R,$$

$$\Lambda_{11}^2 = (\Lambda_{12}^1 + \Lambda_{21}^1)^*, \quad \Lambda = ig_q\zeta_0, \tag{22}$$

where the following notation is used:

$$\zeta_n = \frac{1}{\pi v_\varepsilon \tau_\varepsilon} \int \frac{d\mathbf{p}}{(2\pi)^3} G_0^A(\mathbf{p}, \varepsilon) G_0^R(\mathbf{p}+\mathbf{q}, \varepsilon+\omega)x^n, \quad x = \frac{\mathbf{p}\mathbf{q}}{pq}. \tag{23}$$

The calculation of ζ_n is worth a comment. We can represent ζ_0 in the form

$$\zeta_0 = \frac{m}{\pi^2 v_s \tau_\varepsilon} \int_{-\varepsilon_F}^{\infty} d\xi_p\, p \int_{-1}^{+1} dx \left(\varepsilon - \xi_p - \frac{i}{2\tau_\varepsilon}\right)^{-1}$$

$$\cdot \left(\varepsilon + \omega - \xi_p - qvx - \frac{q^2}{2m} + \frac{i}{2\tau_\varepsilon}\right)^{-1}. \tag{24}$$

We shall show later that we are interested in the values such that $q \sim q_D$ and $\varepsilon \sim \omega \sim T$. However, since $\xi_p \ll qv$, integration of each factor in Eq. (24) can be carried out independently (the first factor with respect to ξ_p and the second with respect to x). Assuming $q_D < 2p_F$, we obtain

$$\zeta_0 = \pi(4v_F\tau_\varepsilon q)^{-1}, \quad \zeta_1 = -\frac{q}{2p_F}\zeta_0 - \frac{i}{2v_F\tau_\varepsilon q}. \tag{25}$$

Similarly, we find that

$$\frac{1}{\pi v_\varepsilon \tau_\varepsilon} \int \frac{d\mathbf{p}}{(2\pi)^3} G_0^A(\mathbf{p}, \varepsilon) G_0^A(\mathbf{p}+\mathbf{q}, \varepsilon+\omega) = -\zeta_0,$$

$$\frac{1}{\pi v_\varepsilon \tau_\varepsilon} \int \frac{d\mathbf{p}}{(2\pi)^3} G_0^A(\mathbf{p}, \varepsilon) G_0^A(\mathbf{p}+\mathbf{q}, \varepsilon+\omega)x = \frac{q}{2p_F}\zeta_0 - \frac{i}{2v_F\tau_\varepsilon q}.$$

Inclusion of a nonequilibrium correction to the distribution function φ_0 (p, ε) of Eq. (10), proportional to the temperature gradient, yields the corresponding corrections to the effective vertex Λ_{ij}^k

$$\delta_\varphi \Lambda_{12}{}^1 = -2^{-1/2} \frac{f_\varepsilon{}^R}{f_\varepsilon{}^A} \frac{\varepsilon}{T} \frac{\partial S_0(\varepsilon)}{\partial \varepsilon} \Lambda_\varphi,$$

$$\delta_\varphi \Lambda_{21}{}^1 = -2^{-1/2} \frac{f_\varepsilon{}^A}{f_\varepsilon{}^R} \frac{(\varepsilon+\omega)}{T} \frac{\partial S_0(\varepsilon+\omega)}{\partial \varepsilon} \Lambda_\varphi,$$

$$\delta_\varphi \Lambda_{11}{}^1 = S_0(\varepsilon)\delta_\varphi \Lambda_{21}{}^1 - S_0(\varepsilon+\omega)\delta_\varphi \Lambda_{12}{}^1,$$

$$\delta_\varphi \Lambda_{11}{}^2 = 2^{-1/2}\left[\frac{\varepsilon}{T}\frac{\partial S_0(\varepsilon)}{\partial \varepsilon} + \frac{(\varepsilon+\omega)}{T}\frac{\partial S_0(\varepsilon+\omega)}{\partial \varepsilon}\right]\Lambda_\varphi,$$

$$\Lambda_\varphi = i\pi g_q(4p_F q)^{-1}\mathbf{q}\nabla T. \tag{26}$$

The components of the effective equilibrium vertex Γ_{ij}^k are

$$\Gamma_{22}{}^1 = 2^{-1/2}(\mathrm{pe}_\lambda \,\mathrm{Re}\, f_\varepsilon{}^A + {}^1/_2\mathrm{qe}_\lambda f_\varepsilon{}^A)\Gamma,$$

$$\Gamma_{12}{}^1 = 2^{-1/2}\,\mathrm{Re}\, f_\varepsilon{}^A S_0(\varepsilon)(\mathbf{p}+\mathbf{q})\mathbf{e}_\lambda\Gamma, \quad \Gamma_{11}{}^1 = -(\Gamma_{22}{}^1)^*,$$

$$\Gamma_{21}{}^1 = -2^{-1/2}\,\mathrm{Re}\, f_\varepsilon{}^A S_0(\varepsilon+\omega)\mathrm{pe}_\lambda\Gamma, \quad \Gamma_{11}{}^2 = \Gamma_{12}{}^1 + \Gamma_{21}{}^1,$$

$$\Gamma_{12}{}^2 = -(\Gamma_{21}{}^2)^* = -2^{-1/2}f_\varepsilon{}^R\mathrm{qe}_\lambda\Gamma, \quad \Gamma = 2a_\varepsilon n_i(2MN\omega_{q\lambda})^{-1}. \tag{27}$$

Nonequilibrium corrections to the vertex Γ_{ij}^k are

$$\delta_\varphi \Gamma_{12}{}^1 = -2^{-1/2}f_\varepsilon{}^R \frac{\varepsilon}{T}\frac{\partial S_0(\varepsilon)}{\partial \varepsilon}\Gamma_\varphi, \quad \delta_\varphi \Gamma_{11}{}^2 = \delta_\varphi \Gamma_{12}{}^1 + \delta_\varphi \Gamma_{21}{}^1,$$

$$\delta_\varphi \Gamma_{21}{}^1 = 2^{-1/2}f_\varepsilon{}^A \frac{(\varepsilon+\omega)}{T}\frac{\partial S_0(\varepsilon+\omega)}{\partial \varepsilon}\Gamma_\varphi, \quad \Gamma_\varphi = \tau_\varepsilon g_q q^{-1}\Gamma e_\lambda \nabla T. \tag{28}$$

Calculations indicate that the equilibrium vertex is $L_{ij}^k = 0$ and the nonequilibrium corrections are

$$\delta_\varphi L_{12}{}^1 = 2^{-1/2}\frac{f_\varepsilon{}^R}{f_\varepsilon{}^A}\frac{\varepsilon}{T}\frac{\partial S_0(\varepsilon)}{\partial \varepsilon}L_\varphi,$$

$$\delta_\varphi L_{21}{}^1 = 2^{-1/2}\frac{f_\varepsilon{}^A}{f_\varepsilon{}^R}\frac{(\varepsilon+\omega)}{T}\frac{\partial S_0(\varepsilon+\omega)}{\partial \varepsilon}L_\varphi,$$

$$\delta_\varphi L_{11}{}^1 = S_0(\varepsilon)\delta_\varphi L_{21}{}^1 - S_0(\varepsilon+\omega)\delta_\varphi L_{12}{}^1,$$

$$\delta_\varphi L_{11}{}^2 = -2^{-1/2}\left[\frac{\varepsilon}{T}\frac{\partial S_0(\varepsilon)}{\partial \varepsilon} + \frac{(\varepsilon+\omega)}{T}\frac{\partial S_0(\varepsilon+\omega)}{\partial \varepsilon}\right]L_\varphi,$$

$$L_\varphi = \frac{2}{3}i\varepsilon_F a_\varepsilon e_\lambda \nabla T. \tag{29}$$

These vertices represent phonon absorption. In the case of those associated with phonon emission we have the following relationships which are given below for the specific case of a vertex $\overline{\Lambda}_{ij}^k$:

$$\overline{\Lambda}_{12}{}^1 = -(\Lambda_{21}{}^1)^*, \quad \overline{\Lambda}_{21}{}^1 = -(\Lambda_{12}{}^1)^*, \quad \overline{\Lambda}_{11}{}^1 = -(\Lambda_{11}{}^2)^*,$$

$$\overline{\Lambda}_{22}{}^1 = (\Lambda_{22}{}^1)^*, \quad \overline{\Lambda}_{12}{}^2 = (\Lambda_{21}{}^2)^*, \quad \overline{\Lambda}_{21}{}^2 = (\Lambda_{12}{}^2)^*. \tag{30}$$

The components of the vertices not included in Eqs. (26)–(29) all vanish.

The resultant quantities Λ_{ij}^k, Γ_{ij}^k, and L_{ij}^k make it possible to present in a compact form a number of self-energy diagrams. In some of the diagrams with a complex structure the vertices obtained above can be used as composite blocks.

4. CALCULATION OF THE THERMOELECTRIC FORCE OF AN IMPURE METAL

Without allowance for the electron–phonon interaction the thermoelectric force of a pure metal is given by the expression

$$\eta_1 = 2e\int\frac{d\mathbf{p}\,d\varepsilon}{(2\pi)^4}(\mathbf{v}\mathbf{n})^2\tau_\varepsilon\frac{\varepsilon}{T}\frac{\partial S_0(\varepsilon)}{\partial \varepsilon}\,\mathrm{Im}\,G_0{}^A(\mathbf{p},\varepsilon). \tag{31}$$

Integration with respect to $\mathbf{p}$ identifies the terms that are linear in the frequency ε, so that

$$\eta_1 = c\eta_0, \quad \eta_0 = -{}^2/_9 e\tau_0 p_F T, \quad c = (1+2a_0{}^2)/(1+a_0{}^2),$$

$$a_0 = a_\varepsilon(\varepsilon=0), \quad \tau_0 = \tau_\varepsilon(\varepsilon=0). \tag{32}$$

In Eqs. (31) and (32) an allowance is made for the fact that in the non-Born scattering case the quantity τ_ε can depend in a complex manner on ε via v_ε [see Eq. (19)]. If $a_0 \ll 1$, then η_1 is described by the familiar expression $\eta_1 = \eta_0$.

We shall show later that the characteristic value of the phonon wave vector is $q \sim q_D \sim p_F$. If $ql \gg 1$ ($l = \tau_0 v_F$) the diagrams of the electron self-energy allowing for the electron–phonon interaction in first order of perturbation theory, including fully the electron–impurity interaction, have the form shown in Fig. 4. The diagrams 4–5 in Fig. 4 appear on allowance for the electron–impurity interaction also in the Born approximation (we considered the same diagrams earlier in connection with a calculation of the corrections to the impurity photoconductivity given in Ref. 12). The other diagrams have to be included only in the case of the non-Born scattering of electrons by impurities.

In calculating the contribution of each of the diagrams it is important to consider the following combination of the electron distribution functions and of the Green's phonon function:
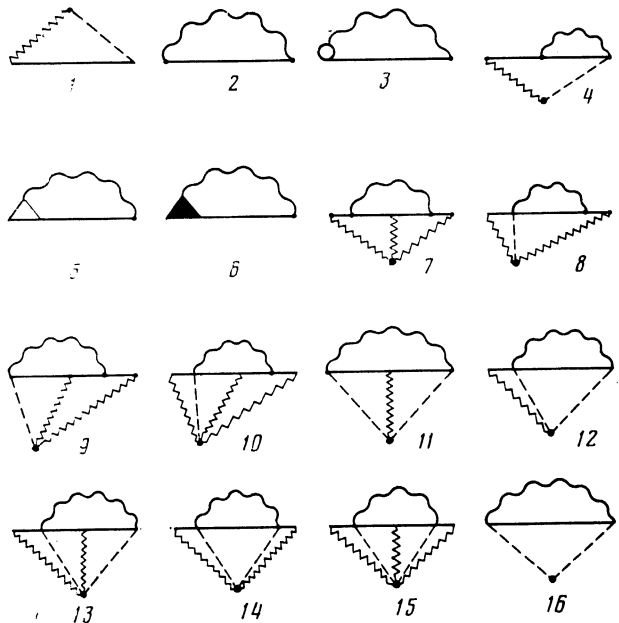

FIG. 4. Diagrams of the self-energy of an electron allowing for the non-Born scattering of the electron on an impurity.

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$$F\left(\frac{\omega_q}{T}\right)=2\int\frac{d\varepsilon\,d\omega}{(2\pi)^2}\,S_0(\varepsilon+\omega)\,\frac{\varepsilon}{T}\,\frac{\partial S_0(\varepsilon)}{\partial\varepsilon}\,\mathrm{Re}\,D^R(\mathbf{q},\omega)=\begin{cases}-\dfrac{2}{3}\dfrac{T}{\omega_q}, & \varepsilon\ll\omega_q\\[2mm]-\dfrac{4}{\pi^2}\dfrac{\omega_q}{T}, & \varepsilon\gg\omega_q,\end{cases}\tag{33}$$

and also the function which is symmetric relative to the above if ε is substituted with $\varepsilon+\omega$. Since the characteristic values are $\varepsilon\sim T$ and $\omega_q\sim\Theta_D$ (Θ_D is the Debye temperature) the condition $\varepsilon\ll\omega_q$ corresponds to temperatures $T\ll\Theta_D$.

The $\Sigma_{e-\mathrm{ph}}$ diagram (2 in Fig. 4) represents corrections to the impurity thermoelectric force due to the usual collision integral I^0, proportional to $\mathrm{Im}\,D^R$, as well as corrections containing the combination of Eq. (33) with $\mathrm{Re}\,D^R$. The former contribution, associated with real thermal phonons, is small for an impurity crystal (in fact, it is equal to the thermoelectric force of a pure semiconductor). The corrections of interest to us appear when the collision integral includes nonlocal terms in the form of Poisson brackets and various corrections to the density of states. The nonlocal part of the collision integral of Eq. (9) is

$$\delta I=-\int\frac{d\mathbf{q}\,d\omega}{(2\pi)^4}\,(g_\mathbf{q})^2\left(\mathbf{v}+\frac{\mathbf{q}}{m}\right)\nabla T\,\frac{\varepsilon}{T}\,\frac{\partial S_0(\varepsilon)}{\partial\varepsilon}$$
$$\cdot\,S_0(\varepsilon+\omega)\,\mathrm{Re}\,D^R(\mathbf{q},\omega)\,\mathrm{Im}[\,G_0^A(\mathbf{p}+\mathbf{q},\varepsilon+\omega)\,]^2.\tag{34}$$

Substituting Eq. (34) in the expression for the current, we shall now represent the contribution made to the thermoelectric force by nonlocal corrections η_2' in the form

$$\eta_2'=-2e\int\frac{d\varepsilon\,d\omega}{(2\pi)^2}\frac{dq\,q^2}{2\pi^2}\,(g_\mathbf{q})^2\tau_\varepsilon S_0(\varepsilon+\omega)\,\frac{\varepsilon}{T}\frac{\partial S_0(\varepsilon)}{\partial\varepsilon}$$
$$\cdot\,\mathrm{Re}\,D^R(\mathbf{q},\omega)P_1(q),\tag{35}$$

where the integral with respect to angular variables represented by the vectors $\mathbf{p}$ and $\mathbf{q}$ is separated:

$$P_1(q)=\int\frac{d\mathbf{p}\,p^2}{2\pi^2}\int\frac{d\Omega_\mathbf{p}\,d\Omega_\mathbf{q}}{(4\pi)^2}\left(\mathbf{v}+\frac{\mathbf{q}}{m}\right)\mathbf{n}(\mathbf{vn})$$
$$\cdot\,\mathrm{Im}\,G_0^A(\mathbf{p},\varepsilon)\,\mathrm{Im}[\,G_0^A(\mathbf{p}+\mathbf{q},\varepsilon+\omega)\,]^2=\frac{m}{12q}.\tag{36}$$

We finally obtain

$$\eta_2'=\tfrac{1}{4}\beta k^2\Psi_1(T)\eta_0,\qquad k=q_D/2p_F,\tag{37}$$

$$\Psi_n(T)=\frac{-3(n+1)}{2q_D^{n+1}}\int_0^{q_D}dq\,q^n\,\frac{\omega_q}{T}\,F\left(\frac{\omega_q}{T}\right).\tag{38}$$

At low temperatures such that $T\ll\Theta_D$, we have $\Psi_n=1$. At arbitrary temperatures the functions of Ψ_n have been calculated before (see, for example, Ref. 5).

The next correction η_2'' originates from the phonon correction to the density of states of electrons, calculated for the equilibrium distribution function $S_0(\varepsilon)$, and it represents the second term on the right-hand side of Eq. (15):

$$\eta_2''=\frac{2e}{|\nabla T|}\int\frac{d\mathbf{p}\,d\varepsilon}{(2\pi)^4}\,\mathbf{vn}\varphi_0$$
$$\cdot\,(\mathbf{p},\varepsilon)\,\mathrm{Im}\{[\,G_0^A(\mathbf{p},\varepsilon)\,]^2\Sigma_{e-ph}^A[\mathbf{p},\varepsilon,S_0(\varepsilon)]\}.\tag{39}$$

As in the calculation of η_2', we shall separate in Eq. (39) the integration with respect to angles and with respect to the electron momentum:

$$\eta_2''=-2e\int\frac{d\varepsilon\,d\omega}{(2\pi)^2}\frac{dq\,q^2}{2\pi^2}\,(g_\mathbf{q})^2\tau_\varepsilon S_0(\varepsilon+\omega)$$
$$\cdot\,\frac{\varepsilon}{T}\frac{\partial S_0(\varepsilon)}{\partial\varepsilon}\,\mathrm{Re}\,D^R(\mathbf{q},\omega)P_2(q),\tag{40}$$

where

$$P_2(q)=\int\frac{d\mathbf{p}}{(2\pi)^3}\frac{d\Omega_\mathbf{q}}{4\pi}\,(\mathbf{vn})^2\,\mathrm{Im}[\,G_0^A(\mathbf{p},\varepsilon)\,]^2$$
$$\cdot\,\mathrm{Im}\,G_0^A(\mathbf{p}+\mathbf{q},\varepsilon+\omega)=\frac{m}{6q}\quad.\tag{41}$$

We finally have $\eta_2''=2\eta_2'$.

One further correction η_2''', associated with the phonon correction to the density of states of electrons in allowance for the nonequilibrium part of the electron distribution function [third term on the right-hand side of Eq. (15)], is

$$\eta_2'''=\frac{2e}{|\nabla T|}\int\frac{d\mathbf{p}\,d\varepsilon}{(2\pi)^4}\,\mathbf{vn}S_0(\varepsilon)$$
$$\cdot\,\mathrm{Im}\{[\,G_0^A(\mathbf{p},\varepsilon)\,]^2\Sigma_{e-ph}^A[\mathbf{p},\varepsilon,\varphi_0(\mathbf{p},\varepsilon)\,]\}.\tag{42}$$

We shall represent η_2''' in the form

$$\eta_2'''=-2e\int\frac{d\varepsilon\,d\omega}{(2\pi)^2}\frac{dq\,q^2}{2\pi^2}\,(g_\mathbf{q})^2\tau_{\varepsilon+\omega}S_0(\varepsilon)\frac{(\varepsilon+\omega)}{T}$$
$$\cdot\,\frac{\partial S_0(\varepsilon+\omega)}{\partial\varepsilon}\,\mathrm{Re}\,D^R(\mathbf{q},\omega)P_3(q),\tag{43}$$

where

$$P_3(q)=\int\frac{d\mathbf{p}}{(2\pi)^3}\frac{d\Omega_\mathbf{q}}{4\pi}\left(\mathbf{vn}+\frac{1}{m}\mathbf{qn}\right)(\mathbf{vn})$$
$$\cdot\,\mathrm{Im}[\,G_0^A(\mathbf{p},\varepsilon)\,]^2\,\mathrm{Im}\,G_0^A(\mathbf{p}+\mathbf{q},\varepsilon+\omega)=\frac{m}{12q}.\tag{44}$$

We then have $\eta_2'''=\eta_2'$.

We can show that the correction to the impurity collision integral due to the phonon renormalization of the density of states [Eq. (12)] does not contribute to the thermopower.

We shall now give the full contribution of the second diagram to the thermoelectric force:

$$\eta_2=\eta_2'+\eta_2''+\eta_2'''=\beta k^2\Psi_1(T)\eta_0.\tag{45}$$

We now consider calculation of the interference diagrams noting that the vertices γ_{ij}^k, Γ_{ij}^k, Λ_{ij}^k, and L_{ij}^k contain an additional small quantity $(\varepsilon_F\tau)^{-1}$, compared with the vertex g_{ij}^k. Therefore, in these diagrams we have to omit all the corrections of the type described by Eqs. (9), (13), and (16) which contain the square of the electron Green's function. It is therefore necessary to consider only those corrections which are associated with the collision integral I^0 of Eq. (8). In the case of diagram 3 in Fig. 4, when only the equilibrium vertices Γ_{ij}^k of Eq. (27) are included, we find

$$I_s^0(\Gamma) = -2 \int \frac{d\omega\, dq}{(2\pi)^4} \Gamma g_q(\mathbf{p+q}) e_\lambda \varphi_0(\mathbf{p}, \varepsilon) S_0(\varepsilon+\omega)$$
$$\cdot \operatorname{Re} f_\varepsilon^A \operatorname{Re} D^R(\mathbf{q}, \omega) \operatorname{Im} G_0^A(\mathbf{p+q}, \varepsilon+\omega); \quad (46)$$

nonequilibrium corrections to the vertices Γ_{ij}^k of Eq. (28) yield

$$I_s^0(\delta_\varphi\Gamma) = -2 \int \frac{d\omega\, dq}{(2\pi)^4} \Gamma_\varphi g_q S_0(\varepsilon+\omega) \frac{\varepsilon}{T} \frac{\partial S_0(\varepsilon)}{\partial \varepsilon}$$
$$\cdot \operatorname{Re} f_\varepsilon^A \operatorname{Re} D^R(\mathbf{q}, \omega) \operatorname{Im} G_0^A(\mathbf{p+q}, \varepsilon+\omega). \quad (47)$$

Substituting the expressions for the collision integrals of Eqs. (46) and (47) into Eq. (15) and using Eq. (11), we obtain $\eta_3 = -\frac{1}{2}\eta_2$. It should be pointed out that in this result the non-Born nature of the impurity scattering is manifested only via the contribution of τ_0 to η_0. This is also true of the fourth diagram $\eta_4 = -\frac{1}{2}\eta_2$. The remaining diagrams make nonzero contributions to the thermoelectric force only if we allow for the non-Born scattering of electrons by impurities.

In the case of the fifth diagram the collision integrals deduced allowing for the equilibrium vertex Λ_{ij}^k of Eq. (23) and for the nonequilibrium correction $\delta_\varphi\Lambda_{ij}^k$ of Eq. (26) are, respectively,

$$I_s^0(\Lambda) = -2i \int \frac{d\omega\, dq}{(2\pi)^4} \Lambda g_q \varphi_0(\mathbf{p}, \varepsilon) \operatorname{Im}\left(\frac{f_\varepsilon^R}{f_\varepsilon^A}\right) S_0(\varepsilon+\omega)$$
$$\cdot \operatorname{Re} D^R(\mathbf{q}, \omega) \operatorname{Im} G_0^A(\mathbf{p+q}, \varepsilon+\omega), \quad (48)$$

$$I_s^0(\delta_\varphi\Lambda) = i \int \frac{d\omega\, dq}{(2\pi)^4} \Lambda_\varphi g_q S_0(\varepsilon) \frac{(\varepsilon+\omega)}{T} \frac{\partial S_0(\varepsilon+\omega)}{\partial \varepsilon}$$
$$\cdot \operatorname{Im}\left(\frac{f_\varepsilon^A}{f_\varepsilon^R}\right) \operatorname{Re} D^R(\mathbf{q}, \omega) \operatorname{Im} G_0^A(\mathbf{p+q}, \varepsilon+\omega). \quad (49)$$

Calculations then yield

$$\eta_5 = \frac{\pi}{4} \frac{a_0}{1+a_0^2} \beta \left[\frac{1}{3} k^3 \Psi_2(T) - k\Psi_0(T)\right] \eta_0. \quad (50)$$

The collision integral for the sixth diagram can be found using the nonequilibrium vertex $\delta_\varphi L_{ij}^k$ of Eq. (29):

$$I_6^0(\delta_\varphi L) = -i \int \frac{d\omega\, dq}{(2\pi)^4} L_\varphi g_q S_0(\varepsilon) \frac{(\varepsilon+\omega)}{T} \frac{\partial S_0(\varepsilon+\omega)}{\partial \varepsilon}$$
$$\cdot \operatorname{Im}\left(\frac{f_\varepsilon^A}{f_\varepsilon^R}\right) \operatorname{Re} D^R(\mathbf{q}, \omega) \operatorname{Im} G_0^A(\mathbf{p+q}, \varepsilon+\omega). \quad (51)$$

The corresponding correction to the thermoelectric force is

$$\eta_6 = -\frac{1}{4} \beta k^2 \frac{a_0^2}{1+a_0^2} \eta_0 \Psi_1(T). \quad (52)$$

The corrections to the impurity thermoelectric force corresponding to the tenth and fifteenth diagrams vanish when we integrate with respect to angles. In calculation of the seventh and ninth diagram it is convenient to use the effective vertex Λ_{ij}^k as the component block and the thirteenth diagram can be calculated using the effective vertex L_{ij}^k. The collision integrals I^0 associated with the eleventh and sixteenth diagrams contain only $\operatorname{Im} D^R$ and, therefore, do not renormalize the impurity thermoelectric force.

We shall now give the final answers:

$$\eta_7 = \frac{\pi}{4} \beta k \frac{a_0^3}{(1+a_0^2)^2} \Psi_0(T) \eta_0,$$

$$\eta_8 = 6\eta_6, \quad \eta_9 = \eta_6, \quad \eta_{12} = \frac{2}{\pi} \beta k a_0 \Psi_0(T) \eta_0, \quad (53)$$

$$\eta_{13} = \frac{2}{\pi} \beta k \frac{a_0^3}{1+a_0^2} \Psi_0(T) \eta_0; \quad \eta_{14} = -\frac{3}{2} \eta_{13}.$$

Transverse phonons contribute only because of the diagrams 12–14 in Fig. 4 and these contain two vertices of inelastic scattering of electrons by impurities. Calculations are made similarly to the case of longitudinal phonons and the final result is obtained from the relevant formula for longitudinal phonons [Eq. (53)] simply by replacing the velocity of sound u_l with u_t.

For comparison with other investigations, we shall express the final result in terms of the renormalization constant λ:

$$\lambda = -\partial \Sigma_{e\text{-}ph}/\partial \varepsilon = \beta k^2. \quad (54)$$

Thus, in the case of one longitudinal and two transverse phonon branches we have the following general formula for the thermoelectric force of an impure metal at temperatures $T \ll \Theta_D$:

$$\frac{\eta}{\eta_0} = \frac{1+2a_0^2}{1+a_0^2} + \lambda\left\{\frac{1}{\pi k}\left(2a_0 - \frac{a_0^3}{1+a_0^2}\right)\left[1 + 2\left(\frac{u_l}{u_t}\right)^2\right]\right.$$
$$\left. + \frac{\pi}{4k} \frac{a_0^3}{(1+a_0^2)^2} - \left(\frac{\pi}{4k} - \frac{\pi k}{12} + 2a_0\right)\frac{a_0}{1+a_0^2}\right\}. \quad (55)$$

We shall consider Eq. (55) for different values of the parameters λ and a_0 in the section headed Conclusions.

5. IMPURE FERROMAGNETIC METAL

The thermoelectric force of an impure ferromagnetic metal is determined by the electron–phonon interaction effects considered above as well as by the electron–magnon interaction and electron–magnon–impurity interference. When the impurity concentration is sufficiently high, the influence of the internal magnetic field can be ignored.

The interaction of electrons with magnons will be described by the s–d exchange Hamiltonian.[15] When only one-magnon processes are allowed for, this Hamiltonian is given by

$$H_{s\text{-}d} = -J\left(\frac{2S}{N}\right)^{1/2} \sum_{\mathbf{pq}} (b_q c_{\mathbf{p+q}\downarrow}^+ c_{\mathbf{p}\uparrow} + b_q^+ c_{\mathbf{p}\uparrow}^+ c_{\mathbf{p+q}\downarrow}), \quad (56)$$

where $c_{\mathbf{p}\uparrow}^+$ is the electron creation operator in which the arrow indicates the spin direction, b_q^+ is the magnon creation operator, J is the exchange integral, N is the number of magnetic atoms with a spin S, and the magnon Green's function and the magnon spectrum are described by

$$D_m^R(\mathbf{q}, \omega) = (\omega - \Omega_q + i0)^{-1}, \quad \Omega_q = Bq^2, \quad B = \Theta_c/p_F^2,$$
$$(57)$$

where the Curie temperature is given by $\Theta_c = dJ^2/\varepsilon_F$ ($d \sim 1$).

The spectrum of electrons in each of the spin subbands is given by the expression $\varepsilon_{\uparrow\downarrow} = p^2/2m \pm JS$. The vertex $\lambda_{\uparrow\downarrow}$ associated with magnon absorption and the vertex $\lambda_{\uparrow\downarrow}$ associated with magnon emission are given by

$$\lambda_{\uparrow\downarrow} = \lambda_{\uparrow\downarrow} = -J(2S/N)^{1/2}.$$

In the case of a ferromagnet we have to allow for the second, fifth, and seventh diagrams in Fig. 4, in which the phonon Green's function is replaced with the magnon function, and the vertices g_q are replaced with $\lambda_{\uparrow\downarrow}$ and $\lambda_{\uparrow\downarrow}$. Calculations similar to those described in Sec. 4 give

$$\frac{\Delta\eta_m}{\Delta\eta_0} = \frac{3ZS}{2d}\left\{ \frac{1}{2}\ln\left(\frac{q_D}{q_0}\right)\Phi_{-1}(T) + \frac{\pi}{8}\frac{p_F}{q_0}\frac{a_0^3}{(1+a_0^2)^2}\Phi_{-2}(T) \right.$$
$$\left. + \frac{\pi}{8}\frac{a_0}{1+a_0^2}\left[\frac{1}{4}\frac{q_D}{p_F}\Phi_0(T) - \frac{p_F}{q_0}\Phi_{-2}(T) \right] \right\}. \quad (58)$$

The relationship $ZN = n_e = p_F^3/3\pi^2$ is used in the above equation; here, n_e is the electron density. The threshold value of the magnon wave vector is $q_0 = 2JS/v_F$ (one-magnon processes are forbidden by the laws of conservation of energy and momentum if $q < q_0$); q_D is the limiting wave vector of the magnon spectrum, identical with the corresponding limiting vector of phonons. The functions Φ_n are described by the relationships

$$\Phi_n(T) = C_n^{-1}\int_{q_0}^{q_D} dq\, q^n\frac{\Omega_q}{T}F\left(\frac{\Omega_q}{T}\right), \quad (59)$$

where

$$C_{-2} = -\frac{2}{3}(q_0)^{-1}, \quad C_{-1} = -\frac{2}{3}[\ln(q_D/q_0)]^{-1}, \quad C_0 = -\frac{2}{3}q_D.$$

The functions Φ_{-1} and Φ_{-2} change significantly at temperatures $T \sim T_0 \sim \Omega_{q_0}$ (for the usual values $J \sim 0.1\varepsilon_F$, we have $T_0 \sim 10$ K). The function Φ_0 corresponds to a characteristic temperature Θ_C. Since the thermoelectric force of an impure ferromagnetic metal is subject to the magnon and phonon renormalizations, its temperature dependence is governed by three characteristic temperatures, T_0, Θ_D, and Θ_C, and can therefore be very complicated.

6. CONCLUSIONS

The main result of the present investigation is the expression (55) for the thermoelectric force of an impure metal, which contains corrections due to the electron–phonon interaction and the electron–phonon–impurity interference effects. The electron–impurity interaction is not assumed to be small (non-Born scattering) and the final answer is expressed in terms of the parameter a_0 of Eq. (18) related to the amplitude f_ε of the scattering of electrons by impurities. Figure 5 shows clearly the behavior of the thermoelectric force when the parameters a_0 and λ are varied in the range of low temperatures $T \ll \Theta_D$.

We can see that both the magnitude and the sign of the thermoelectric effect depend strongly on the amplitude of the scattering of electrons by impurities. We shall note in this connection that the corrections to the impurity thermoelectric force due to the electron–phonon interaction and due to the electron–phonon–impurity interference effects have the same structure [they depend on the same function $F(\omega_q/T)$] and cannot be separated experimentally. In the Born approximation corresponding to $a_0 \ll 1$ the net phonon renormalization of the impurity thermoelectric force in Eq. (55) vanishes. Therefore, inclusion of the non-Born scattering is essential in an analysis of the experimental results.
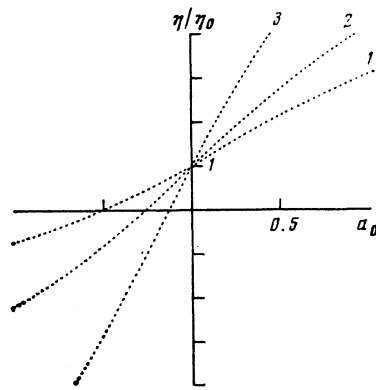

FIG. 5. Dependences of the low-temperature thermopower of a normal metal on the parameter a_0 calculated for different values of the parameter λ: 1) 0.3; 2) 0.5; 3) 1; $k = 0.5$.

In the case of a ferromagnetic metal when the renormalization of the thermoelectric force is both due to phonons Eq. (55)] and due to magnons [Eq. (58)], the results may be quite different from those for a normal metal, as demonstrated at low temperatures in the range $T < T_0$, Θ_D in Fig. 6.

We shall compare the results obtained with the conclusions reached in Refs. 4 and 5, where the linear response method was used to calculate corrections to the impurity thermoelectric force due to the electron–phonon interaction, corresponding to inclusion of the diagram 2 in Fig. 4 in our case. The result obtained in Ref. 5 is in the form $\eta = (1 + 2\lambda)$, which differs somewhat from Ref. 4, where one of the diagrams was omitted. According to the calculations in Sec. 4, if $a_0 \ll 1$, we then obtain $\eta_1 + \eta_2 = (1 + \lambda)\eta_0$. In our opinion this difference is associated with an incorrect analytic continuation in the calculation of diagrams containing the electron–phonon corrections to the heat flux operator in Refs. 4 and 5.

The influence of the electron–phonon interaction on the thermoelectric effect was also considered recently in Ref. 16 using the transport equation method. The integral equations for the vertices of the momentum and energy fluxes obtained in this investigation were analyzed numerically on the assumption that the matrix element of the electron–phonon interaction is independent of q. Although a comparison with the results of Ref. 16 is difficult over the whole temperature
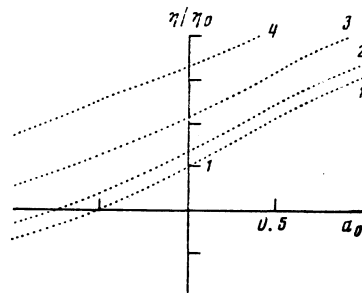

FIG. 6. Dependences of the low-temperature thermopower of a ferromagnetic metal on the parameter a_0 for the following values of the parameter $\lambda_m = 3ZS/2d$: 1) 0; 2) 0.3; 3) 1; 4) 2. In all cases it is assumed that $\lambda = 0.3$.

range, the answer obtained in Ref. 16 for the low-temperature limit $T \ll \Theta_D$ agrees with our results.

As far as the electron–phonon–impurity interference effects are concerned, a direct comparison of Eq. (55) with the results of Ref. 9 shows that the difference is considerable. This difference is due to the fact that only a narrow class of diagrams was considered in Ref. 9. Moreover, there have been several investigations in which incorrect results were obtained not only because of allowance for just some of the interference diagrams, but also because of a nonrigorous approach to the analysis of the electron–phonon–impurity interference effects.[16,17] For example, the contribution of the diagram 5 in Fig. 4 was calculated in Ref. 16 using the Born approximation, whereas the diagram 16 in Fig. 4 was included in Ref. 17; according to our calculations the contributions of these diagrams are zero.

In an analysis of the results of a specific experiment the proposed calculation can be refined by taking into account the exact electron and phonon spectra, and a more realistic model of the electron–impurity interaction potential can be used.

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