Bremsstrahlung in collisions of atomic particles at relativistic velocities

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The frequency and angle dependence of the cross section for bremsstrahlung in collisions of atomic particles at relativistic velocities has been studied. The cross sections obtained have a number of interesting features in comparison with the nonrelativistic case.

INTRODUCTION

The bremsstrahlung occurring in a collision of two atomic particles is due to the polarization of each of them by the static field of the electrons and nucleus of the partner. Here the spectrum of radiated photons is concentrated mainly in the region of characteristic atomic frequencies. The polarization mechanism of bremsstrahlung in atom-atom collisions for fast but not relativistic particle velocities has been discussed in detail in Refs. 1–3.

The purpose of the present work is to study the frequency and angle dependence of the radiation spectrum in atom-atom collisions at relativistic velocities. In the first place, very interesting features of the bremsstrahlung spectrum appear at relativistic velocities of the collision as a consequence of the Doppler effect and the aberration of light, as a result of which there is a change in the frequency and emission angle of the photon emitted by the projectile atom.[4,5] In the second place, retardation in the interaction of a relativistic projectile with a target turns out to be important.[4,6,7] These effects lead to qualitative changes in the bremsstrahlung spectrum in comparison with the nonrelativistic case not only at relativistic velocities $v \sim c$ (c is the velocity of light), but also in the nonrelativistic region $v \ll c$, in collision of identical or almost identical atoms. In fact, complete neglect of relativistic effects leads, for example, to the result that in a system consisting of two identical atoms there is no dipole bremsstrahlung, since no dipole moment is produced by mutual polarization of the atoms in such a system.[1,2]

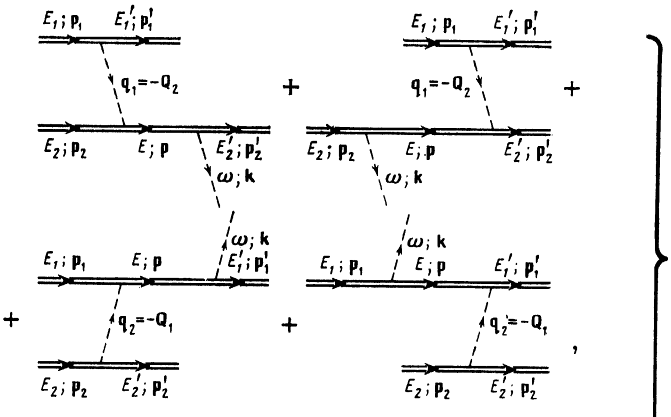
Proximity of the collision velocity v to the velocity of light leads to a substantial dependence of the frequency and angle distribution of the photons on the polarizability of the partners. For example, in the region of high photon frequencies ω, where the polarizability of an atom falls off as $1/\omega^2$, the radiation of the projectile is concentrated mainly in a narrow cone with apex angle $\theta\gamma^{-1}(\gamma = (1 - v^2/c^2)^{-1/2})$ near the direction of motion of the atom. On the other hand, at low frequencies ω when the polarizability of the atom is equal to its static value, the radiation of the projectile occurs mainly in the region of angles $\pi \gg \theta \gg \gamma^{-1}$, where it is close to isotropic. The frequency and angle dependence of the distribution of radiation turns out to be such that for sufficiently large collision velocities there is the possibility in principle of separating the bremsstrahlung of the projectile and the target. This conclusion generalizes the similar result obtained previously[6] in discussion of the bremsstrahlung of a relativistic electron on an atom.

In the present work the main attention is devoted to "elastic" bremsstrahlung, in which the atoms after radiation of a photon remain in the ground state. Of course, it is also possible to have inelastic radiation processes accompanied by ionization or excitation of the atoms. However, as was shown in Refs. 8 and 9, over a wide range of atomic frequencies $\omega_{at} \lesssim \omega \lesssim vR_{at}^{-1}$ (R_{at} is the size of the atom and ω_{at} is its characteristic frequency) the elastic bremsstrahlung process is dominant. The reason for this is that the contributions of individual electrons to the elastic process considered are coherent, as occurs in Rayleigh scattering of light. On the other hand, in inelastic bremsstrahlung processes the contributions of the individual atomic electrons to the total radiation are added incoherently. These processes are analogous to the Raman scattering of light.

1. AMPLITUDE OF THE PROCESS

We shall consider the collision of two atomic particles (for definiteness atom-atom) in which a photon is emitted with momentum $\mathbf{k}$, and polarization e, and the atoms are in the ground states both before and after the process. For description of the bremsstrahlung process at relativistic velocities we shall use the lowest order of perturbation theory in the interaction between the atoms, which reduces to one-photon exchange, and in the interaction of the atoms with the radiation field.

The Feynman diagrams for the bremsstrahlung amplitude have the following form:



$$\mathbf{q}_1=\mathbf{p}_1-\mathbf{p}_1', \quad \mathbf{q}_2=\mathbf{p}_2-\mathbf{p}_2', \quad \mathbf{Q}_1=\mathbf{p}_1-\mathbf{p}_1'-\mathbf{k},$$
$$\mathbf{Q}_2=\mathbf{p}_2-\mathbf{p}_2'-\mathbf{k}, \quad \mathbf{q}_1=-\mathbf{q}_2+\mathbf{k}, \quad \mathbf{q}_1=-\mathbf{Q}_2, \quad \mathbf{q}_2=-\mathbf{Q}_1. \tag{1}$$

The states of the particle are denoted by a pair of letters, the first of which is the total energy of the particle, and the second is its momentum. The total energy of a relativistic

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atom includes its intrinsic energy and its kinetic energy. The subscripts 1 and 2 in the diagrams of Eq. (1) and in the text which follows indicate quantities associated respectively with the projectile atom and the target.

We shall represent the bremsstrahlung amplitude in atom-atom collisions in the form of a sum of two terms:

$$F=F_1+F_2, \tag{2}$$

where F_1 describes the first pair of diagrams in Eq. (1) and F_2 describes the second pair. The analytic expression for the amplitude F_1 has the following form (we use the atomic system of units $|e| = m_e = \hbar = 1$):

$$F_1=J_\mu^{(1)}(\mathbf{q}_1)G^{\mu\nu}(E_1-E_1';\mathbf{q}_1)T_{\lambda\nu}^{(2)}(\omega;\mathbf{k};\mathbf{Q}_2)e^\lambda, \tag{3}$$

where

$$J_\mu^{(1)}(\mathbf{q}_1)=\frac{M_1c^2}{E_1}\left\langle E_1'\left|\sum_{j=1}^{N^{(1)}+1} e_j\gamma_\mu e^{-i\mathbf{q}_1\mathbf{r}_j}\right|E_1\right\rangle,$$

$$T_{\lambda\nu}^{(2)}(\omega;\mathbf{k};\mathbf{Q}_2)=\left(\frac{M_2c^2}{E_2}\right)^2$$

$$\times\sum_E\left\{\frac{\left\langle E_2'\left|\sum_{j=1}^{N^{(2)}+1}e_j\gamma_\lambda e^{-i\mathbf{k}\mathbf{r}_j}\right|E\right\rangle\left\langle E\left|\sum_{j=1}^{N^{(2)}+1}\gamma_\nu e_j e^{-i\mathbf{Q}_2\mathbf{r}_j}\right|E_2\right\rangle}{\omega-(E-E_2')}\right.$$

$$\left.-\frac{\left\langle E_2'\left|\sum_{j=1}^{N^{(2)}+1}e_j\gamma_\nu e^{-i\mathbf{Q}_2\mathbf{r}_j}\right|E\right\rangle\left\langle E\left|\sum_{j=1}^{N^{(2)}+1}e_j\gamma_\lambda e^{-i\mathbf{k}\mathbf{r}_j}\right|E_2\right\rangle}{\omega+(E-E_2)}\right\}.$$

In Eq. (3) integration over the coordinates of the centers of mass of each of the atoms has been carried out. In writing Eq. (3) we have used the following notation: M_1 is the mass of the projectile atom (the first atom), $\{\mathbf{r}_j\}$ ($j = 1, ..., N^{(1)}$) is the set of relative coordinates of the electrons in it, $\mathbf{r}_{N^{(1)}+1}$ $(1)\equiv\mathbf{r}_{\rm nuc}\approx\mathbf{0}$ is the relative coordinate of the nucleus without taking into account its recoil in the collision process, $e_j = -1$ ($j = 1, ..., N^{(1)}$), and $e_{N^{(1)}+1}\equiv Z^{(1)}$. Similar notations are adopted for the target atom (the second atom). The symbol $G^{\mu\nu}(E_1 - E_1'; \mathbf{q}_1)$ stands for the photon Green function, and e^λ is its polarization vector ($\lambda = 0, 1, 2, 3, 4$; γ_λ is the Dirac matrix). The sum in Eq. (3) is carried out over the entire spectrum of positive and negative energies of the target atom in the intermediate state. For convenience in writing the formulas, in the definition of F_1 we have omitted the δ functions which assure conservation of energy and momentum and also the normalization factor of the photon wave function. The normalization factors of the wave functions of the atoms $(M_ic^2/E_i)^{1/2}\approx(M_ic^2E_i')^{1/2}$ ($i = 1, 2$) are included in the definition of $J_\mu^{(1)}$ and $T_{\lambda\nu}^{(2)}$. Here it has been taken into account that in the kinematic region, which will be of interest to us in what follows, the motion of the atoms occurs almost in a straight line, i.e., $E_i - E_i'\ll E_i, |\mathbf{p}_i - \mathbf{p}_i'|\ll|\mathbf{p}_i|$.

The structure of the amplitude F_1 corresponds to the qualitative picture of the process. Actually $J_\mu^{(1)}$ describes the current of the first atom, and $T^{(2)}{}_{\lambda\nu}$ is the dynamic response of the second atom, which leads to emission of a quantum. The interaction between the atoms is mediated by exchange of a photon, the Green function $G^{\mu\nu}$ of which relates $J_\mu^{(1;2)}$ and $J_{\lambda;\nu}^{(1;2)}$.

In calculation of $J_\mu^{(1;2)}$ and $J_{\lambda;\nu}^{(1;2)}$ it is important to have in mind that the characteristic momentum transfers in the collision are small: $|\mathbf{q}_{1,2}|\lesssim 1$. This estimate is easily obtained from simple physical considerations. We shall distinguish two cases.

In a collision of neutral atoms their mutual polarization can occur only when the electron shells of the atoms overlap. Consequently an estimate for the impact parameter is $\rho\approx R_{\rm at}$ ($R_{\rm at}$ is the size of the atom), and for the momentum transfer it is

$$|\mathbf{q}_{1;2}^\perp|\approx 1/\rho\approx 1/R_{\rm at}. \tag{4}$$

Here $q_{1;2}^\perp$ is the momentum component perpendicular to the velocity of the collision.

In collision of an ion with an atom or of an ion with an ion the situation is different. An ion strongly polarizes an atom (or another ion), even when passing at large distances $\rho\gg 1$ from it.[6] Therefore the characteristic momentum transfers are small. As was shown in Ref. 6, they can be estimated in the following way:.

$$|\mathbf{q}_{1;2}^\perp|\approx 1/\rho\infty\omega/v_1\gamma\ll R_{\rm at}^{-1},$$

$$v_1=p_1/M_1,\qquad \gamma=(1-\beta^2)^{-1/2},\qquad \beta=v_1/c. \tag{5}$$

The parallel components of the momentum transfers $q_{1,2}^{\parallel}$ in the bremsstrahlung process are determined by the conservation of energy momentum. In the frame of reference in which the first atom is the projectile and the second is the target, we have

$$|\mathbf{q}_1^{\parallel}|=\omega/v_1,\qquad |\mathbf{q}_2^{\parallel}|=|\mathbf{Q}_1^{\parallel}|\approx(1-\beta\cos\theta)\omega/v_1,$$

where $\theta=\widehat{\mathbf{k};\mathbf{v}_1}$. In what follows, considering the bremsstrahlung process only in the region of frequencies where the polarizabilities of the atoms are especially large and the dipole approximation $R_{\rm at}\omega/c\ll 1$ is applicable, we see that for relativistic velocities $v_1\sim c$ the following inequalities are satisfied:

$$|\mathbf{q}_1^{\parallel}|\approx\omega/v_1\ll 1/R_{\rm at},\qquad |\mathbf{q}_2^{\parallel}|\approx(1-\beta\cos\theta)\omega/v_1\ll 1/R_{\rm at}. \tag{6}$$

From comparison of the inequalities (4) and (6) it follows that in a collision of neutral atoms the parallel component of the momentum transfer actually is unimportant for the bremsstrahlung process, since

$$|\mathbf{q}_{1;2}^{\parallel}|\ll|\mathbf{q}_{1;2}^\perp|\ \text{и}\ \mathbf{q}_{1;2}\approx\mathbf{q}_{1;2}^\perp.$$

On the other hand, in an ion-ion (or ion-atom) collision both momentum transfer components are important since it follows from (5) and (6) that $q_{1;2}\sim q_{1;2}\sim q$.

The estimates (4)–(6) show that in the collision process the atoms or ions move almost without change of velocity: $\delta V_{1;2}\approx|\mathbf{q}_{1;2}|/M_{1;2}\ll 1$. This permits in particular introduction of the rest systems (the proper coordinate system) for the first and second atoms. In these systems the determination of $J_\mu^{(1;2)}$ and $T_{\lambda\nu}^{(1;2)}$ is considerably simplified. Calculating the time and space components of $J_\mu^{(1)}$ in the proper system of the first atom, we obtain

$$J_0^{(1)c}=Z^{(1)}-\left\langle 0_1\left|\sum_{j=1}^{N(1)}\exp(-i\mathbf{q}_1^c\mathbf{r}_j)\right|0_1\right\rangle=Z^{(1)}-W^{(1)}(\mathbf{q}_1^c),$$

$$\mathbf{J}^{(1)c}=-\left\langle 0_1\left|\sum_{j=1}^{N(1)}\exp(-i\mathbf{q}_1^c\mathbf{r}_j)\frac{\hat{\mathbf{p}}_j}{c}\right|0_1\right\rangle=-\frac{\mathbf{q}_1^c}{2c}W^{(1)}(\mathbf{q}_1^c). \quad (7)$$

Here $\hat{\mathbf{p}}_j$ is the momentum operator of the j-th particle of the atom, $|0_1\rangle$ is the ground state of the atom, and $\mathbf{q}_1^c$ is the momentum transfer, which is the spatial part of the 4-vector $q_\mu^c=(0;\mathbf{q}_1^c)$. In derivation of (7) we omitted small corrections of the order $\mathbf{q}_1^c/c\ll 1$, $1/M_1\ll 1$. In (7) we have written out the current components diagonal in the states of the atom, since only they are necessary in what follows.

Knowing $J_\mu^{(1)c}$ in the proper system, it is straightforward to obtain this quantity also in the laboratory system, in which the atom is moving with velocity $\mathbf{v}_1$. Taking into account the small quantity $|\mathbf{J}^{(1)c}|\ll J_0^{(1)c}$ (see Eq. (7)) and the fact that the combination $J_\mu^{(1)}E_1/M_1c^2$ is a 4-vector (see Eq. (3)), we find the following expressions:

$$J_0^{(1)L}=Z^{(1)}-W^{(1)}(q_1^c),$$

$$\mathbf{J}^{(1)L}=(Z^{(1)}-W^{(1)}(\mathbf{q}_1^c))\boldsymbol{\beta},\qquad \mathbf{q}_1^c=\gamma^{-1}\mathbf{q}_1^{\parallel}+\mathbf{q}_1^{\perp}, \quad (8)$$

where γ and β are the same as in Eq. (5). In Eq. (8) and below, the superscripts $\parallel$ and $\perp$ denote the components of the vectors in the laboratory system parallel and perpendicular to the velocity of the atom.

The formulas obtained have a simple meaning. The time component $J_0^{(1)a}$ describes the form factor, deformed as a consequence of the Lorentz contraction, of the charge of the electrons and of the nucleus in the atom which is in its ground state, while the space component $\mathbf{J}^{(1)L}$ describes the current of the charges of the atom due to their translational motion with velocity $\mathbf{v}_1$.

The quantity $T_{\lambda\nu}^{(2)}$ is calculated similarly. In the proper system of the second atom the components of $T_{\lambda\nu}^{(2)c}$ have the following form:

$$T_{\lambda\nu}^{(2)c}=\begin{pmatrix} T_{00}^{(2)}(\omega^c;\mathbf{k}^c;\mathbf{Q}_2^c); & T_{0j}^{(2)}(\omega^c;\mathbf{k}^c;\mathbf{Q}_2^c)\\ T_{i0}^{(2)}(\omega^c;\mathbf{k}^c;\mathbf{Q}_2^c); & T_{ij}^{(2)}(\omega^c;\mathbf{k}^c;\mathbf{Q}_2^c)\end{pmatrix}$$
$$=\begin{pmatrix}\mathbf{k}^c\mathbf{Q}_2^c\alpha^{(2)}(\omega^c;\mathbf{Q}_2^c); & -(\omega^c/c)(\mathbf{k}^c\hat{\beta}^{(2)}(\omega^c;\mathbf{Q}_2^c))_j\\ \dfrac{\omega^c}{c}Q_{2i}\,\alpha^{(2)}(\omega^c;\mathbf{Q}_2^c); & -(\omega^c/c)^2\hat{\beta}_{ij}^{(2)}(\omega^c;\mathbf{Q}_2^c)\end{pmatrix}. \quad (9)$$

Here the subscripts i and j denote the space components of the tensor. The generalized polarizability $\alpha^{(2)}(\omega^c;\mathbf{Q}_2^c)$ is introduced in accordance with the definition

$$\mathbf{Q}_2^c\alpha^{(2)}(\omega^c;\mathbf{Q}_2^c)=i\sum_{n_2}\frac{2\omega_{n_20_2}^c}{(\omega^c)^2-\omega_{n_20_2}^{c\,2}}\mathbf{D}_{0_2n_2}^{(2)}W_{n_20_2}^{(2)}(\mathbf{Q}_2^c), \quad (10)$$

and the polarization tensor $\beta_{ij}^{(2)}$ is defined as

$$\frac{\omega^c}{c}\beta_{ij}^{(2)}(\omega^c;\mathbf{Q}_2^c)=-i\sum_{n_2}\left\{\frac{(\mathbf{D}_{0_2n_2}^{(2)})_i(\mathbf{J}_{n_20_2}^{(2)}(\mathbf{Q}_2^c))_j}{\omega^c-\omega_{n_20_2}^c}-\frac{(\mathbf{J}_{0_2n_2}^{(2)}(\mathbf{Q}_2^c))_j(\mathbf{D}_{n_20_2}^{(2)})_i}{\omega^c+\omega_{n_20_2}^c}\right\}. \quad (11)$$

Here

$$\left.\begin{aligned} W_{n_20_2}^{(2)}(\mathbf{Q}_2^c)&=\left\langle n_2\left|\sum_{j=1}^{N(2)}\exp(-i\mathbf{Q}_2^c\mathbf{r}_j)\right|0_2\right\rangle,\\ \mathbf{J}_{n_20_2}^{(2)}(\mathbf{Q}_2^c)&=\left\langle n_2\left|\sum_{j=1}^{N(2)}\exp(-i\mathbf{Q}_2^c\mathbf{r}_j)\frac{\mathbf{p}_j}{c}\right|0_2\right\rangle\end{aligned}\right\}$$

and $(\mathbf{D}_{n_20_2}^{(2)})_i$ is the i-projection of the matrix element of the dipole-moment operator of the atom. Equations (9)–(11) were obtained in the dipole approximation in the photon momentum $R_{\rm at}\omega^c/c\ll 1$. In the region of small Q_2^c ($Q_2^cR_{\rm at}\ll 1$) the expression for the polarizability is simplified:

$$\alpha^{(2)}(\omega^c;\mathbf{Q}_2^c)\approx\alpha_d^{(2)}(\omega^c),\qquad \beta_{ij}^{2}(\omega^c;\mathbf{Q}_2^c)\approx\delta_{ij}\alpha_d^{(2)}(\omega^c),$$

where $\alpha_d^{(2)}(\omega^c)$ is the dynamic dipole polarizability of the atom.

The expression (9) can be considerably simplified in the kinematic region of interest to us. For the case of collision of neutral atoms one obtains from Eq. (4) and the condition of dipole radiation the estimate $|\mathbf{Q}_2^c|\gg\omega^c/c$. Using this, we find

$$T_{\lambda\nu}^{(2)c}\approx\alpha^{(2)}(\omega^c;\mathbf{Q}_2^c)\begin{pmatrix}\mathbf{k}^c\mathbf{Q}_2^c; & \mathbf{0}_j\\ \dfrac{\omega^c}{c}\mathbf{Q}_{2i}; & \mathbf{0}_{ij}\end{pmatrix}. \quad (12)$$

The symbols $\mathbf{0}_j$ and $\mathbf{0}_{ij}$ denote the components of the zero vector and tensor.

In the case of a collision with participation of an ion, making use of the smallness of the momentum transfers (5), we obtain

$$T_{\lambda\nu}^{(2)c}\approx\alpha_d^{(2)}(\omega^c)\begin{pmatrix}\mathbf{k}^c\mathbf{Q}_2^c; & -(\omega^c/c)\mathbf{k}_j^c\\ (\omega^c/c)\mathbf{Q}_{2i}^c; & -(\omega^c/c)^2\delta_{ij}\end{pmatrix}. \quad (13)$$

Then, noting that the quantity $(E_2/M_2c^2)^2T_{\lambda\nu}^{(2)}$ is a 4-tensor divided by the energy (see Eq. (3)), by means of Lorentz transformations it is straightforward to write the components of $T_{\lambda\nu}^{(2)}$ also in the laboratory system of the second atom, expressing them in terms of the components of $T_{\lambda\nu}^{(2)c}$ (12) and (13) in the proper system, as was done in derivation of $J_\mu^{(1)L}$ (8).

Having constructed $J_\mu^{(1)}T_{\lambda\nu}^{(2)}$ and similarly $J_\mu^{(2)}T_{\lambda\nu}^{(1)}$ in the proper and laboratory systems, we find the amplitudes F_1 and F_2 (see Eq. (2)), which are expressed in terms of these quantities The amplitude F_1 is expressed in terms of the current $J_\mu^{(1)}$ calculated in the laboratory system of the projectile and $T_{\lambda\nu}^{(2)}$ calculated in the proper system of the target. The quantity F_2, on the other hand, involves $J_\mu^{(2)}$ in the proper system of the target atom and $T_{\lambda\nu}^{(1)}$ in the laboratory system of the projectile atom. Omitting the intermediate calculations, we shall write down the amplitudes F_1 and F_2 in the region (4) in a collision of two neutral atoms:

$$F_1=\frac{4\pi(Z^{(1)}-W^{(1)}(q_1^\perp))}{(q_1^\perp)^2}(\mathbf{e}\mathbf{q}_1^\perp)\frac{\omega}{c}\alpha^{(2)}(\omega;\mathbf{q}_1^\perp), \qquad (14)$$

$$F_2=-\frac{4\pi(Z^{(2)}-W^{(2)}(\mathbf{q}_1^\perp))}{(q_1^\perp)^2}\left[\frac{\omega^c}{c}(\mathbf{e}\mathbf{q}_1^\perp)+\gamma(\mathbf{e}\boldsymbol{\beta})(\mathbf{k}\mathbf{q}_1^\perp)\right]$$
$$\times\alpha^{(1)}(\omega^c;\mathbf{q}_1^\perp),\qquad \omega^c=\omega\gamma(1-\beta\cos\theta). \qquad (15)$$

In ion-ion and ion-atom collisions in the region of small momentum transfers (5), F_1 and F_2 are equal to

$$F_1=\frac{4\pi(Z^{(1)}-N^{(1)})}{q_1^2-\omega^2/c^2}\left[\mathbf{e}\mathbf{q}_1-\frac{\omega}{c}\mathbf{e}\boldsymbol{\beta}\right]\frac{\omega}{c}\alpha_d^{(2)}(\omega), \qquad (16)$$

$$F_2=\frac{4\pi(Z^{(2)}-N^{(2)})}{q_2^2}\left[\frac{\omega^c}{c}\mathbf{e}\mathbf{q}_2^\perp+\gamma(\mathbf{e}\boldsymbol{\beta})(\mathbf{k}\mathbf{q}_2^\perp)-\gamma^{-2}\frac{\omega^c}{c}\mathbf{e}\mathbf{q}_1^\parallel\right]\alpha_d^{(1)}(\omega^c). \qquad (17)$$

In the region of nonrelativistic velocities $\beta\ll 1$ equations (15)–(17) go over to the corresponding results of the nonrelativistic theory.[1,2]

The amplitude F_1 given by (14) coincides with its nonrelativistic analog, since it does not depend on β. This agreement is not accidental. The process of collision of the atoms and radiation by them of a quantum occurs at comparatively small distances $R\sim R_{\rm at}$, where the retardation of the electromagnetic interaction and of its transverse component appear only weakly. The binary Coulomb interaction between the particles of the colliding atoms is accompanied mainly by exchange of $\mathbf{q}_1^\perp$. Therefore the polarization of the target atom in the collision process is transverse and does not depend on the choice of the reference system. It follows from this that the amplitude F_2 given by (15) can be obtained from F_1 (14) by transformation to a coordinate system moving with velocity $\mathbf{v}_1$ and subsequent permutation of the atoms. In this transformation only the frequency ω and photon polarization factor **e** are transformed, while the characteristics of the atoms, which enter into Eq. (14), are not affected by the transformation.

The bremsstrahlung process in an ion-ion collision occurs somewhat differently. In this case the collision occurs at large distances, so that at relativistic velocities the retardation in the interaction between the ions and its transverse nature become important. The polarization of each of the ions at large distances is due mainly to the charge of the partner ion $(Z^{(i)}-N^{(i)})$. This explains the reason for the agreement of the amplitude F_1 (16) with the amplitude of the "atomic" bremsstrahlung of a relativistic particle with a charge equal to the charge of the incident ion $(Z^{(1)}-N^{(1)})$, which was obtained in Refs. 6 and 7. The amplitude of the radiation of the projectile (17) can be obtained from (16) by conversion to a coordinate system moving with velocity $\mathbf{v}_1$ and subsequent interchange of the ions. Here, in contrast to the similar conversion in the case of a neutral atom, it is necessary to take into account also the transformation of the longitudinal component of the momentum transfer $\mathbf{q}_1^\parallel$, which produces a longitudinal polarization of the ion (see the third term in the square brackets in Eq. (17)). The magnitude of the longitudinal polarization of the ion in the laboratory system is γ^{-2} times smaller than its transverse polarization. This circumstance must be taken into account in the conversion from (16) to (17).

2. CROSS SECTION FOR THE PROCESS

Using the well known rules[10] relating the amplitude of a process to its cross section, we find

$$\frac{d^5\sigma}{d\omega\,d\Omega\,dq_1\,d\varphi_{q_1}}=\frac{\omega q_1}{(2\pi)^4 c v_1^2}|F_1+F_2|^2. \qquad (18)$$

Here $d\Omega$ is the solid angle of emission of the photon and φ_{q1} is the azimuthal angle of $\mathbf{q}_1$ in the coordinate system with z axis along $\mathbf{v}_1$. It follows from (18) that the bremsstrahlung cross section is made up of three parts:

$$d\sigma=d\sigma^t+d\sigma^p+d\sigma^i, \qquad (19)$$

where $d\sigma^t$ is the cross section for bremsstrahlung of the target, $d\sigma^p$ is the cross section for bremsstrahlung of the projectile, and $d\sigma^i$ is the interference term. Using the amplitudes F_1 and F_2 found in Section 1, we shall carry out the calculation of $d\sigma/d\omega d\Omega$.

A. Atom-atom collision

In an atom-atom collision the cross section (18) integrated over $dq_1 d\varphi_{q1}$ and summed over photon polarizations has the following form:

$$\frac{d\sigma^t}{d\omega\,d\Omega}=\frac{\omega^3(1+\cos^2\theta)}{\pi v_1^2c^3}\times\int_0^\infty\frac{dq_1^\perp}{q_1^\perp}|[Z^{(1)}-W^{(1)}(q_1^\perp)]\alpha^{(2)}(\omega;q_1^\perp)|^2, \qquad (20)$$

$$\frac{d\sigma^p}{d\omega\,d\Omega}=\frac{\omega^3(1+\cos^2\theta^c)}{\pi v_1^2c^3}\left(\frac{\omega^c}{\omega}\right)^2\times\int_0^\infty\frac{dq_1^\perp}{q_1^\perp}|[Z^{(2)}-W^{(2)}(q_1^\perp)]\alpha^{(1)}(\omega^c;q_1^\perp)|^2, \qquad (21)$$

$$\frac{d\sigma^i}{d\omega\,d\Omega}=-\frac{2\omega^3(1+\cos\theta\cos\theta^c)}{\pi v_1^2c^3}\left(\frac{\omega^c}{\omega}\right)\times\int_0^\infty\frac{dq_1^\perp}{q_1^\perp}[Z^{(1)}-W^{(1)}(q_1^\perp)]$$
$$\times[Z^{(2)}-W^{(2)}(q_1^\perp)]\,\mathrm{Re}\{\alpha^{(1)}(\omega^c;q_1^\perp)\alpha^{(2)*}(\omega;q_1^\perp)\}, \qquad (22)$$

$$\omega^c=\omega\gamma(1-\beta\cos\theta);\qquad \cos\theta^c=(\cos\theta-\beta)/(1-\beta\cos\theta).$$

In derivation of these equations it was assumed that each of the atoms in its proper system has a spherically symmetric charge distribution. Equations (20)–(22) show that the main contribution to the bremsstrahlung cross section in collision of a pair of neutral atoms is from the region of momentum transfers $q_1^\perp R_{\rm at}\sim 1$, therby confirming the correctness of the estimate (4). Actually, outside this region either $[Z^{(i)} - (W^{(i)}q_i^\perp)] \ll 1$ or $|\alpha^{(i)}(\omega; q_1^\perp)| \ll 1$.

Comparing $d\sigma^t/d\omega d\Omega$ and $d\sigma^p/d\omega d\Omega$ we see that in $d\sigma^p/d\omega d\Omega$ the effect of aberration appears in a change of the angular dependence of the cross section: $(1+\cos^2\theta) \to (1+\cos^2\theta^c)$, and the Doppler effect appears in the frequency conversion $\omega \to \omega^c$.

The nature of the angular distribution of the radiation of the projectile and of the interference component of the cross section depends strongly on the behavior of its polarizability, as follows from Eqs. (21) and (22).

In the limit of low frequencies $\omega \ll \omega_{at}/\gamma$ such that for all angles $0 \leqslant \theta \leqslant \pi$ the relation $\omega^c \ll \omega_{at}$ is satisfied (ω_{at} denotes the characteristic atomic frequency), the polarizability actually does not depend on ω and θ, but reduces to its static value. In this case the angular distribution of the photons in (21) has the following form:

$$d\sigma^p/d\omega d\Omega \sim (1-\beta\cos\theta)^2 + (\cos\theta - \beta)^2$$

and does not contain the well known behavior of the angular distribution of the radiation of charged particles,[10] in which the photons are emitted preferentially along the direction of motion of the particle in a cone with apex angle $\theta \lesssim \gamma^{-1}$. Comparison of the cross sections (20) and (21) shows that the ratio $\xi = (d\sigma^t/d\omega/d\Omega)/(d\sigma^p/d\omega d\Omega$ is a quantity of order γ^2 in the region of angles $\theta \lesssim \gamma^{-1}$ and $\xi \sim \gamma^{-2}$ for $\theta \gg \gamma^{-1}$. Therefore at small angles $\theta \lesssim \gamma^{-1}$ the bremsstrahlung of the target atom dominates, while in the remaining range of angles $\theta \gg \gamma^{-1}$ the bremsstrahlung of the projectile is more intense.

A special behavior in the angular distribution of the bremsstrahlung of the projectile atom in the small-angle region appears at large frequencies $\omega \gg \omega_{at}\gamma$. In this case actually for all $0 \leqslant \theta \leqslant \pi$ the condition $\omega^c \gg \omega_{at}$ is satisfied and the polarizability $\alpha^{(1)}(\omega^c; q_1^\perp) \sim 1/(\omega^c)^2$, so that in the cross section $d\sigma^p/d\omega d\Omega$ given by (21) one has a behavior $d\sigma^p/d\omega d\Omega \sim 1/\gamma^2(1+\beta\cos\theta)^2$. Taking the ratio ξ of the cross sections (20) and (21), it is straightforward to find that in the region $\theta \lesssim \gamma^{-1}$ the ratio $\xi \sim \gamma^{-2}$, while for angles $\theta \gg \gamma^{-1}$ the ratio $\xi \sim \gamma^2$. Therefore in the radiation of high frequencies in the region small angles $\theta \ll \gamma^{-1}$ the bremsstrahlung of the projectile dominates, while in the large-angle region $\theta \gg \gamma^{-1}$ the radiation of the target dominates. The angular distribution of the bremsstrahlung of a relativistic electron on an atom[6,7] has a similar nature. This agreement is due to the fact that at high frequencies the electrons in the projectile atom can be considered free, and the bremsstrahlung of the projectile can be represented as the coherent radiation of these electrons in the field of the target.

In the general case we can say that if the frequency and angle are chosen so that the following condition is satisfied:

$$(\omega^c/c)^2 |\alpha^{(1)}(\omega^c; q_1^\perp)|^2 \gg |\alpha^{(2)}(\omega; q_1^\perp)|^2; \quad q_1^\perp R_{at} \sim 1, \tag{23}$$

then in this region of $(\omega;\theta)$ the projectile bremsstrahlung will dominate over the target bremsstrahlung. On reversal of the direction of this inequality, dominance of the projectile bremsstrahlung is replaced by dominance of target bremsstrahlung. Therefore we can conclude that at sufficiently high collision velocities one has the possibility in principal of distinguishing the bremsstrahlung of the projectile and the target. This result generalizes the similar conclusion drawn previously[6] in discussion of the bremsstrahlung of a relativistic electron on an atom.

Integrating the cross sections (20) and (21) over $d\Omega$ we obtain expressions for the corresponding spectra:

$$\frac{d\sigma^t}{d\omega} = \frac{16}{3}\frac{\omega^3}{c^3 v_1^2}\int_0^\infty \frac{dq_1^\perp}{q_1^\perp} |[Z^{(1)} - W^{(1)}(q_1^\perp)]\alpha^{(2)}(\omega; q_1^\perp)|^2, \tag{24}$$

$$\frac{d\sigma^p}{d\omega} = \frac{2\omega^3}{c^3 v_1^2}\int_0^\infty \frac{dq_1^\perp}{q_1^\perp}\int_{1/\gamma(1+\beta)}^{\gamma(1+\beta)} dx \left\{\frac{x^2}{\beta\gamma} + \frac{(1-\gamma x)^2}{(\beta\gamma)^3}\right\} \times |[Z^{(2)} - W^{(2)}(q_1^\perp)]\alpha^{(1)}(\omega x; q_1^\perp)|^2, \tag{25}$$

$$\frac{d\sigma^i}{d\omega} = -\frac{4\omega^3}{c^3 v_1^2}\int_0^\infty \frac{dq_1^\perp}{q_1^\perp}\int_{1/\gamma(1+\beta)}^{\gamma(1+\beta)} dx \left\{\frac{x}{\beta\gamma} + \frac{(\gamma - x)(1-\gamma x)}{(\beta\gamma)^3}\right\} \times [Z^{(1)} - W^{(1)}(q_1^\perp)][Z^{(2)} - W^{(2)}(q_1^\perp)] \times \mathrm{Re}\{\alpha^{(1)}(\omega x; q_1^\perp)\alpha^{*(2)}(\omega; q_1^\perp)\}, \quad x = \gamma(1-\beta\cos\theta). \tag{26}$$

An important feature of these expressions is the fact that the spectrum of the projectile bremsstrahlung and the interference term of the cross section at some frequency ω_0 depend on the behavior of the dynamic polarizability of the projectile atom over a very wide range of frequencies from $\omega_0/\gamma(1+\beta)$ to $\omega_0\gamma(1+\beta)$. This in particular leads to the result that the narrow spectral lines which can be observed in the bremsstrahlung spectrum of a stationary atom in the region of the characteristic atomic frequencies $\omega \sim \omega_{at}$ turn out to be smeared in the radiation spectrum of the projectile over a region $\omega_{at}\gamma^{-1} \lesssim \omega \lesssim \omega_{at}\gamma$.

The expressions for the bremsstrahlung spectra (24)–(26) are simplified considerably in the low-frequency region $\omega \ll \omega_{at}\gamma^{-1}$, where the polarizability $\alpha^{(1)}(\omega x; q_1^\perp)$ is approximately equal to its static value $\alpha^{(1)}(0; q_1^\perp)$ for all $x < \gamma(1+\beta)$ and therefore it is possible to calculate in explicit form the integral over the variable x. A simple calculation leads to the following result:

$$\frac{d\sigma^p}{d\omega} = \frac{16}{3}\frac{\omega^3(1+\beta^2)\gamma^2}{c^3 v_1^2}\int_0^\infty \frac{dq_1^\perp}{q_1^\perp}[Z^{(2)} - W^{(2)}(q_1^\perp)]^2 \times (\alpha^{(1)}(0; q_1^\perp))^2, \tag{27}$$

$$\frac{d\sigma^i}{d\omega} = -\frac{32}{3}\frac{\omega^3\gamma}{c^3 v_1^2}\int_0^\infty \frac{dq_1^\perp}{q_1^\perp}[Z^{(1)} - W^{(1)}(q_1^\perp)][Z^{(2)} - W^{(2)}(q_1^\perp)]\alpha^{(1)}(0; q_1^\perp)\alpha^{(2)}(0; q_1^\perp). \tag{28}$$

In this region of ω the spectrum (24) is expressed in the same way as in (27) in terms of the static polarizability. A comparison of (27) with (28) and (24) shows that the bremsstrahlung spectrum of the projectile is γ^2 times more intense than the spectrum of the target bremsstrahlung and is γ times greater than the interference term of the cross section. At high frequencies $\omega \gg \omega_{at}\gamma$ the polarizability is approximately equal to $\alpha^{(1)}(\omega x; q_1^\perp) \approx -W^{(1)}(q_1^\perp)/\omega^2 x^2$ for all x from the interference region in Eqs. (25) and (26), and therefore it is possible to obtain simple expressions for $d\sigma^p/d\omega$ and $d\sigma^i/d\omega$ also in this part of the spectrum:

$$\frac{d\sigma^p}{d\omega}=\frac{16}{3c^3v_1^2\omega}\int_0^\infty\frac{dq_1^\perp}{q_1^\perp}\{W^{(1)}(q_1^\perp)[Z^{(2)}-W^{(2)}(q_1^\perp)]\}^2, \tag{29}$$

$$\frac{d\sigma^i}{d\omega}=-\frac{16}{c^3v_1^2\omega}\left\{\frac{1}{\beta^2\gamma}-\frac{\ln(\gamma(1+\beta))}{(\beta\gamma)^3}\right\}$$

$$\times\int_0^\infty\frac{dq_1^\perp}{q_1^\perp}W^{(1)}(q_1^\perp)W^{(2)}(q_1^\perp)$$

$$\times[Z^{(1)}-W^{(1)}(q_1^\perp)][Z^{(2)}-W^{(2)}(q_1^\perp)]. \tag{30}$$

The expressions (27)–(30) generalize the corresponding formulas of the nonrelativistic theory, which are obtained from (27)–(30) in the limit $\beta\ll 1$. Since $\alpha^{(i)}(0;q_1^\perp)$ and $W^{(i)}(q_1^\perp)$ are real quantities, the complex conjugation symbols have been omitted in (27)–(30). Adding (24), (27), (28), and also (24), (29), and (30), it is possible without complication to obtain an expression for the total spectrum of bremsstrahlung in an atom-atom collision in the regions of high and low frequencies. They give a nonzero result for the collision of identical atoms, which is a direct consequence of the Doppler effect and the aberration of light in the radiation of the projectile atom. For $\omega\ll\omega_{at}\gamma^{-1}$ and sufficiently large γ the radiation of the projectile atom dominates in the combined bremsstrahlung spectrum. In the region $\omega\gg\omega_{at}\gamma$ the spectrum of $d\sigma^p/d\omega$ given by (29) is similar to the target bremsstrahlung spectrum and is γ times greater than the interference part $d\sigma^i/d\omega$ given by (30). The smallness of the interference term of the cross section is due to the fact that the radiation of the projectile in the region $\omega\gg\omega_{at}\gamma$ is concentrated mainly in a narrow range of angles $\theta\lesssim\gamma^{-1}$ where it is γ^2 times more intense than the bremsstrahlung of the target atom, which is weakly anisotropic.

For illustration we shall estimate in order of magnitude $d\sigma^p/d\omega d\Omega$ in the collision of relativistic positronium with xenon in the region $\omega^c\sim 8$ Ry. For this purpose we note that the cross section (21) was expressed in terms of an integral over $dq_1^\perp$ which appears in the nonrelativistic calculation. Making the estimate

$$J^p(\omega^c)=(\omega^c)^3\int_0^\infty\frac{dq_1^\perp}{q_\perp}|(Z_{Xe}^{(2)}-W_{Xe}^{(2)}(q_1^\perp))$$

$$\times\alpha_{Ps}^{(1)}(\omega^c;q_1^\perp)|^2\sim 10^2$$

(see for example the calculations in Refs. 1 and 2), we obtain from (21)

$$\frac{d\sigma^p}{d\omega\, d\Omega}\sim 10^2\frac{(1+\cos^2\theta^c)}{\pi v_1^2c^3}\frac{\omega}{\omega^c}.$$

The cross sections $d\sigma^t/d\omega d\Omega$ given by (20) and $d\sigma^i/d\omega d\Omega$ given by (22) for a collision involving positronium vanish, since positronium is a truly neutral system which does not have a distribute charge, i.e., it has zero form factor.[11]

B. Ion-ion collisions

In Section 1 we found the bremsstrahlung amplitude in ion-ion collisions at relativistic velocities (16) and (17). The cross sections are calculated in the same way as in the atom-atom collision case discussed above. We obtain the following result:

$$\frac{d\sigma^t}{d\omega d\Omega dq}=\frac{(Z_{ion}^{(1)})^2\omega^3|\alpha_d^{(2)}(\omega)|^2q_1}{\pi c^3v_1^2(q_1^2-\omega^2/c^2)^2}\Big\{(q_1^\perp)^2(1+\cos^2\theta)$$

$$+2\sin^2\theta\left(q_\perp^\parallel-\beta\frac{\omega}{c}\right)^2\Big\}, \tag{31}$$

$$\frac{d\sigma^p}{d\omega d\Omega dq}=\frac{(Z_{ion}^{(2)})^2\omega^3}{\pi c^3v_1^2q_2^3}\left(\frac{\omega_c}{\omega}\right)^2|\alpha_d^{(1)}(\omega^c)|^2$$

$$\times\{(q_2^\perp)^2(1+\cos^2\theta^c)+2\gamma^{-2}(q_2^\parallel)^2\sin^2\theta^c\}, \tag{32}$$

where

$$(Z_{ion}^{(i)})=Z^{(i)}-N^{(i)},\quad \sin\theta^c=\gamma^{-1}\sin\theta/(1-\beta\cos\theta).$$

The expressions (31) and (32) obtained for the cross section are valid in the regions $q_1R_{at}\ll 1$, $q_2R_{at}\ll 1$. The first of these expressions coincides with the cross section for the bremsstrahlung of the target atom in a collision with it of a structureless particle with charge $Z_{ion}^{(1)}$, which is natural for the region of small momentum transfers, i.e., large impact parameters of the collision. The structure of (32) is also understandable. The first term in the curly brackets in this expression describes the bremsstrahlung of the projectile, which arises as a consequence of its transverse polarization by the static Coulomb field of the target ion, while the second term is the bremsstrahlung as a result of the longitudinal polarization of the projectile. The result (32) could be obtained in a different way, first transforming $d\sigma^t/d\omega d\Omega dq_1$ to the reference frame in which the projectile atom is at rest, while the target is moving with velocity $\mathbf{v}_1$, and then interchanging the numbers of the atoms. Here it must be taken into account that for a relativistic projectile ion the value of its longitudinal polarizability due to transfer of $\mathbf{q}_2^\parallel$ is γ^{-2} times smaller than its transverse polarizability, which does not depend on the choice of the coordinate system. The expressions (31) and (32) permit one to find with logarithmic accuracy the projectile and target cross sections integrated over q_1 and q_2. Integrating (31) and (32) over

$$q_1^{min}=\omega/v_1,\quad q_2^{min}=(1-\beta\cos\theta)\omega/v_1$$

up to $q_{max}\sim R_{at}^{-1}$ and retaining here only logarithmically large terms, we arrive at the following result:

$$\frac{d\sigma^t}{d\omega d\Omega}=\frac{(Z_{ion}^{(1)})^2\omega^3|\alpha_d^{(2)}(\omega)|^2}{\pi c^3v_1^2}(1+\cos^2\theta)\ln\left(\frac{\gamma v_1}{\omega R_{at}}\right), \tag{33}$$

$$\frac{d\sigma^p}{d\omega d\Omega}=\frac{(Z_{ion}^{(2)})^2\omega^3|\alpha_d^{(1)}(\omega^c)|^2}{\pi c^3v_1^2}\left(\frac{\omega^c}{\omega}\right)^2$$

$$\times(1+\cos^2\theta^c)\ln\left(\frac{\gamma v_1}{\omega^cR_{at}}\right). \tag{34}$$

Knowing the bremsstrahlung amplitudes of the projectile and target, we can find also the interference term of the bremsstrahlung cross section $d\sigma^i/d\omega d\Omega$. In the logarithmic approximation the calculations lead to the following result:

$$\frac{d\sigma^i}{d\omega d\Omega}=-\frac{2\omega^3 Z_{\text{ion}}^{(1)} Z_{\text{ion}}^{(2)}}{\pi c^5 v_1^2}\frac{\omega^c}{\omega}(1+\cos\theta\cos\theta^c)$$
$$\times\operatorname{Re}\{\alpha_d^{(1)}(\omega^c)\alpha_d^{(2)*}(\omega)\}\ln\left(\frac{v_1\gamma}{\omega^c R_{\text{at}}}\right). \quad (35)$$

The structure of the expressions (33)–(35) is similar to the structure of the bremsstrahlung cross sections (20)–(22). Therefore the analysis of the behavior of the frequency and angle dependence of the resulting cross section is completely similar to that carried out above in discussion of bremsstrahlung in an atom-atom collision. It permits one to conclude that the general pattern of bremsstrahlung is similar in the two cases mentioned. The existence of long-range forces in a system consisting of a pair of ions leads to a logarithmic growth of the cross sections, which does not change the general nature of the bremsstrahlung of a system of two colliding particles.

The formula obtained, in application to an ion-atom collision, shows that the bremsstrahlung of the atom in the field of the ion gives a logarithmically large contribution to the cross section for the process. Here the bremsstrahlung of the stationary atom is described by Eq. (33) and that of the moving ion by (34).

C. Collision of identical atomic particles

The expressions obtained in parts A and B of this section can be used to find the bremsstrahlung cross section in the collision of two identical fast but nonrelativistic atomic particles, when $1 \ll v \ll c$. Complete neglect of relativistic corrections in this case leads to a zero result.[1,2] This is due to the fact that the induced dipole moments of identical atoms are equal in magnitude and are oriented in opposite directions, and therefore the total dipole moment of such a system, and consequently also the radiation produced by its change, are equal to zero. Allowance for the Doppler effect and the aberration of light in the radiation of the projectile lifts this forbiddenness of dipole bremsstrahlung. Neglecting all terms of order lower than β^2 in (20)–(22), we obtain for the combined bremsstrahlung cross section (19) the following expression:

$$\frac{d\sigma}{d\omega\, d\Omega}=\frac{\omega^3}{\pi c^5}\int_0^\infty \frac{dq^\perp}{q^\perp}[Z-W(q^\perp)]^2\left\{\cos^2\theta\left|\frac{\partial}{\partial\omega}(\omega\alpha(\omega;q^\perp))\right|^2\right.$$
$$\left.+\left|\alpha(\omega;q^\perp)+\omega\cos^2\theta\frac{\partial}{\partial\omega}\alpha(\omega;q^\perp)\right|^2\right\},\quad \frac{\omega}{v_1}R_{\text{at}}\ll 1. \quad (36)$$

The cross section (36) does not depend on the collision velocity and is β^2 times smaller than the characteristic values of the bremsstrahlung cross sections in collision of different atomic particles. Equation (36) takes into account all relativistic corrections to the amplitude, which are proportional to β. In addition to them, the bremsstrahlung amplitude also contains terms which describe the radiation of quadrupole photons. These terms give a correction to the amplitude of dipole bremsstrahlung of the order $R_{\text{at}}\omega/c \ll 1$. This correction, however, can be neglected in the region where (36) is applicable, $R_{\text{at}}\omega/v_1 \ll 1$, since here its order is much less than β $(R_{\text{at}}\omega/c \ll \beta)$.

Integrating (36) over $d\Omega$, we obtain the bremsstrahlung spectrum in the collision of identical atoms:

$$\frac{d\sigma}{d\omega}=\frac{16}{3}\frac{\omega^3}{c^5}\int_0^\infty\frac{dq^\perp}{q^\perp}[Z-W(q^\perp)]\left\{|\alpha(\omega;q^\perp)|^2\right.$$
$$\left.+\omega\operatorname{Re}\left\{\alpha^*(\omega;q^\perp)\frac{\partial}{\partial\omega}\alpha(\omega;q^\perp)\right\}+\frac{2}{5}\omega^2\left|\frac{\partial}{\partial\omega}\alpha(\omega;q^\perp)\right|^2\right\}. \quad (37)$$

Let us calculate $d\sigma/d\omega d\Omega$ and $d\sigma/d\omega$ in the collision of a pair of identical ions with $\beta \ll 1$. Using the cross sections (33)–(35) or proceeding directly from the amplitudes (16) and (17) found above, we obtain in the logarithmic approximation the following expressions:

$$\frac{d\sigma}{d\omega\, d\Omega}=\frac{Z_{\text{ion}}^2\omega^3}{\pi c^5}\left\{\cos^2\theta\left|\frac{\partial}{\partial\omega}(\omega\alpha_d(\omega))\right|^2\right.$$
$$\left.+\left|\alpha_d(\omega)+\omega\cos^2\theta\frac{\partial}{\partial\omega}\alpha_d(\omega)\right|^2\right\}\ln\frac{v_1}{\omega R_{\text{at}}}, \quad (38)$$

$$\frac{d\sigma}{d\omega}=\frac{16Z_{\text{ion}}^2\omega^3}{3c^5}\left\{|\alpha_d(\omega)|^2+\omega\operatorname{Re}\left\{\alpha_d^*(\omega)\frac{\partial}{\partial\omega}\alpha_d(\omega)\right\}\right.$$
$$\left.+\frac{2}{5}\omega^2\left|\frac{\partial}{\partial\omega}\alpha_d(\omega)\right|^2\right\}\ln\frac{v}{\omega R_{\text{at}}}. \quad (39)$$

In contrast to the similar formulas (36) and (37) for an atom-atom collision, Eqs. (38) and (39) contain a logarithmic dependence on the collision velocity. Its appearance is due to the presence in the system of long-range Coulomb forces.

CONCLUSION

The description of the bremsstrahlung process in collisions of atomic particles at relativistic velocities has been carried out with use of a number of approximations, and it is interesting and important to consider departures from these approximations. For example, study of the bremsstrahlung cross section in the region of high frequencies ω requires rejection of the dipole approximation and allowance for the radiation of quadrupole photons. The corrections to the bremsstrahlung cross section due to retardation and to the transverse nature of the electromagnetic interaction between the particles turn out to be of the same order. At sufficiently high frequencies ω the energy of both the projectile and the target are changed very substantially at the end of the process, so that one cannot assume their motion to be rectilinear. Therefore it is necessary to take into account in different ways the relativistic behavior of the atoms at the beginning and at the end of the process.

The technique used in this work for calculation of the cross section can be applied to description of the bremsstrahlung of other structured relativistic objects: nuclei, clusters, molecules, and so forth. It also permits discussion of the bremsstrahlung process in an arbitrary frame of reference, in which both colliding particles have some relativistic velocity before the interaction. An example of such a frame of reference is the center of mass system of the pair. The technique for calculation of the cross section can be used also in discussion of other processes such as Rayleigh scattering of light by a relativistic atom.

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