Deep focusing of an atomic beam in the Angstrom region by laser radiation

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(Submitted 30 March 1987)
Zh. Eksp. Teor. Fiz. **94**, 140–150 (January 1988)

A potential field whose action is similar to that of the objective lens in electron optics is found for an atomic beam regarded as de Broglie waves. The main properties of the focusing potential field are determined: the maximum resolution, the focal length, and the aberrations. The distribution of the atom density in the focal plane is calculated. The feasibility of producing an atomic microscope is discussed.

1. INTRODUCTION

Microscopy with spatial resolution in the angstrom region makes use of electrons,[1] x-ray photons,[2] and neutrons.[3] We call attention in the present article to the possibility of using beams of neutral atoms for the very same purpose. The key to implementation of this idea is the possibility of focusing an atomic beam into a spot several Å in size by using a laser field of definite configuration, power, and frequency. This possibility of deep focusing of an atomic beam is a logical step forward in laser methods of controlling the motion of neutral atoms, methods actively developed in the last few years.[4–7]

The main idea can be easily understood by considering the analogy between optics and, say, electron optics.

To obtain an image in any electron-optical instrument it is necessary to determine the distribution of the electron-current density in the image plane. The connection between the current-density distributions in the object plane and in the image plane can be established by classical mechanics. This approach neglects diffraction effects, which become essential for maximum resolution. The construction of the image can be considered from the standpoint of wave mechanics. In this approach it is necessary to consider the wave function of the employed particles, which in fact determines completely the current-density distribution. This problem was considered many times for electron microscopy.[1] It was shown that a complete analogy exists between the solutions of the image construction problem in optical and electron-optical instruments. In the case of the microscope, the problem reduces in essence to a determination of the current density in the focal plane of the objective.

In the present paper we consider an atomic beam in the form of de Broglie waves, and find next a potential field whose action is similar to the action of an objective lens in optics or electron optics. This is followed by calculation of the field distribution in the focal plane of the objective, using the above analogy between optics and electron optics, and determination of the characteristics of the objective, i.e., of the potential field, viz., the focal length, the relative aperture, the aberrations, and also the possibility of its realization by modern laser techniques.

2. LASER OBJECTIVE LENS FOR AN ATOMIC MICROSCOPE

In the theory of the optical image,[8] the ideal objective is a transparency having the following phase transmission function:

$$T(x,\ y)=\exp\left[-ik(x^2+y^2)/2f\right], \qquad (1)$$

where $k = 2\pi/\lambda$ and f is the focal length of the objective. A light beam passing through such a transparency acquires an additional phase shift equal to $k(x^2 + y^2)/2f$.

Our task is to find a potential field in which the phase shift of the wave function (of the de Broglie wave) is also determined by Eq. (1), where $k = 2\pi/\lambda_B$. We know that if the de Broglie wavelength λ_B is short compared with the characteristic dimensions that determine the conditions of our problem, the properties of the system are close to classical. The wave function of an atom is given in the semiclassical approximation by[9]

$$\psi[C/(p(z))^{1/2}]\exp\left[(i/\hbar)\int p(z)\,dz\right], \qquad (2)$$

where C is a constant, $p(z) = [2M(E - U(z))]^{1/2}$ the momentum of the atom, M its mass, E the total energy, and $U(z)$ the potential energy of the atom. We consider hereafter motion of an atom in a quasiresonant laser field, i.e., $\gamma \ll \Delta = \omega_L - \omega_0$, $\Delta \approx \gamma(I/I_s)^{1/2}$, where 2γ is the homogeneous width of the atom's resonant absorption line, ω_L the frequency of the laser radiation, ω_0 the frequency of the atomic transition, I the laser-emission intensity, and I_s the saturation intensity of the atomic transition.

The potential energy in a laser radiation field is given by[10]

$$U=\frac{\hbar\Delta}{2}\ln(1+p), \qquad (3)$$

where

$$p=\frac{I}{I_s}\frac{\gamma^2}{4}\Big/\left(\frac{\gamma^2}{4}+\Delta^2\right)$$

is the saturation parameter of the atomic transition. The phase shift of the wave function (1) on the potential (3) is

$$\Delta\varphi=\Delta\varphi_n-\Delta\varphi_0$$
$$=\frac{1}{\hbar}\int p^n(z)\,dz-\frac{1}{\hbar}\int p^0(z)\,dz=\frac{1}{\hbar\vartheta}\int U(z)\,dz, \qquad (4)$$

where $\Delta\varphi_n$ and $\Delta\varphi_0$ are respectively the phase shifts with and without allowance for the field, and ϑ_z is the atom velocity along the z axis.

We can easily imagine a laser field in which the phase shift of the wave function takes the form of the argument of

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the exponential in Eq. (1). Such a field should have a saturation perameter with the following dependence on the transverse coordinate:

$$p(x, y)=p(\rho)=\exp(\rho^2/\rho_0^2)-1. \qquad (5)$$

In fact, using expressions (3)–(5), we obtain

$$\Delta\varphi=\text{const}\,\rho^2. \qquad (6)$$

It is impossible to produce a laser field with a saturation parameter in the form (5), since the intensity increases without limit with increase of ρ, and the total power is infinite. One might attempt to produce a field in which the intensity is a maximum at the center and which also meets the condition (6). In such a field, however, the atoms will move predominantly in these regions with maximum intensity and this, as will be shown below, leads in the quasiresonant state, in view of momentum diffusion, to smearing of the image. From among the presently known laser-field configurations, the TEM_{01}^* transverse laser mode near the beam propagation axis comes closest to the ideal field (5). The radiation intensity in the TEM_{01}^* mode is given by[11] (Fig. 1)

$$I(r,z)=4I_0\frac{w_0^2}{w^2(z)}\frac{2r^2}{w^2(z)}\exp\left(-\frac{2r^2}{w^2(z)}\right), \qquad (7)$$

where w_0 is the radius of the laser-beam neck in the $z=0$ plane, $w^2(z)=w_0^2(1+z^2/z_R^2)$ is the dimension of the beam in the z plane, $z_R=(\pi/\lambda)w_0^2$ is the Rayleigh length, and $I_0=P_0/2\pi w_0^2$, and P_0 is the radiation power.

Consider now the motion of the atoms along the z axis, which coincides with the propagation axis of the laser beam (7). For paraxial optics ($\rho=r/w(z)\ll 1$) and when the condition $p<1$ is met, the expression for the phase shift of the de Broglie wave in a laser field of form (7), with the exponential expanded up to fourth-order terms in ρ, takes the form

$$\Delta\varphi=\frac{\pi^2\Delta z_R\alpha\rho^2}{\vartheta_z}\left[1-\frac{1}{2}\left(1-\frac{\alpha}{2}\right)\rho^2+\frac{3}{16}(1-4\alpha)\rho^4\right], \qquad (8)$$

where $\alpha=P_0\gamma^2/\pi I_s\Delta^2 w_0^2$.

The phase shift is found to be proportional to the square of the radius. The terms proportional to ρ^2 and ρ^4 in the square brackets can be neglected in first-order approximation ($\rho\ll 1$). In this approximation the field (7) acts on the atomic beam as an ideal objective with resolution determined by the de Broglie wavelength and by the numerical aperture. The terms in the square brackets describe spherical aberrations of fourth and sixth order, respectively.

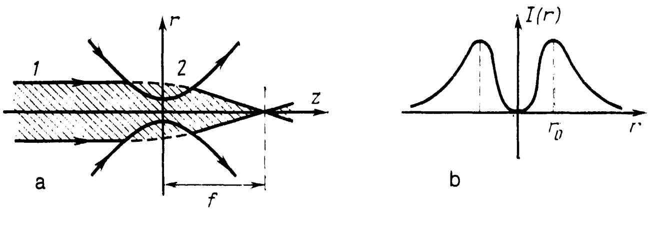

FIG. 1. Laser-field configuration for the focusing of an atomic beam, and the de Broglie wavelength; a—locations of the laser atom beam; b—transverse profile of the intensity of the laser-field TEM_{01}^* mode.

From a comparison of the phase shift (8) with expression (1) for the transmissivity of an ideal objective we obtain the focal length of the atomic lens:

$$f=\hbar\left(\lambda\frac{\Delta}{P_0}\right)(w_0^2)\left(\frac{I_s}{\gamma^2 M\lambda_B^2}\right). \qquad (9)$$

The expression in the first parentheses is determined by the parameters of the laser beam, that in the second by its caustic, and that in the third by the atomic parameters. We can write for the focal length in terms of the longitudinal velocity of the beam atoms the expression

$$f=\frac{1}{2\pi h}\left(\lambda\frac{\Delta}{P_0}\right)(w_0^2)\left(\frac{I_s M\vartheta_z^2}{\gamma^2}\right). \qquad (9')$$

3. RESOLUTION OF A LASER OBJECTIVE LENS FOR AN ATOM BEAM

The resolution of a laser objective lens is determined mainly by the following four factors: 1) the diffraction of the atom by the aperture that limits the transverse dimension of the atom beam; 2) the momentum diffusion, due to spontaneously reradiated photons; 3) the chromatic aberration due to the strong dependence of the focal length on the atom velocity; 4) the spherical aberration produced when a real laser field is used. Let us examine the influences of these factors on the resolution:

a) Diffusion aberration

When an atom interacts with quasiresonant radiation, the latter can absorb and spontaneously re-emit, the laser-field photons. Assuming spherical symmetry of the emitted spontaneous photons, the change of the transverse momentum of the atom is equal to

$$\Delta p_\perp=(2N/3)^{1/2}\hbar k=\hbar\Delta k_B^\perp, \qquad (10)$$

where $\Delta k_B^\perp$ is the change of the transverse component of the de Broglie wave vector. If the diameter of the atom beam is $2d$, we obtain for the diffractive transverse range of values of the wave vector

$$\Delta k_B^0\approx 1/2a.$$

The diffusion aberration can be neglected under the condition

$$\Delta k_B^\perp\leqslant\Delta k_B^0. \qquad (11)$$

Equation (11) is equivalent to

$$N\leqslant(3\lambda_B/4a). \qquad (11')$$

The rate of the spontaneous decays is determined by the relation

$$\beta=\pi\gamma p(x, y), \qquad (12)$$

where

$$p(x,y)=\left(\frac{4P_0}{\pi w_0^2 I_s}\right)\left(\frac{\gamma}{2\Delta}\right)^2\left(\frac{2r^2}{w^2(z)}\right)$$

is the saturation parameter of the atomic transition. The number of photons reradiated during the interaction time t_{int} is

$$N=\int_{t_{int}}\beta\,dt=\frac{1}{\vartheta_z}\int_{L_L}\beta\,dz, \qquad (13)$$

where L_L is the interaction length of the atoms with the laser fields.

From (11)–(13) we get the condition under which the diffusion aberration can be neglected:

$$\Delta^2\geqslant\frac{8\pi^4P_0\gamma^3w_0^2}{3I_s\vartheta_z\lambda^3}\frac{r^4}{w_0^4}. \qquad (14)$$

The estimate of the role of diffusion can be approached somewhat differently. Momentum diffusion leads to distortion of the de Broglie wavefront. This decreases in turn the intensity of the diffraction spot in the image plane. If the de Broglie wavefront distortions are neligible, the new intensity is given by[12]

$$i=i_0(1-k_B^2\overline{\Delta\varphi^2})^2, \qquad (15)$$

where i_0 is the intensity at the center of the diffraction pattern if the wavefront distortion is neglected, and $\overline{\Delta\varphi^2}$ is the mean squared deviation of the wavefront from spherical at the objective aperture. Let furthermore the wavefront radius past the objective be R (Fig. 2). The momentum diffusion leads to a change of the wavefront component and accordingly to a local change of the wavefront radius and to defocusing. It is easy to obtain the following relation between the defocusing Δf and the change Δk_B of the wave vector (see Fig. 2):

$$\Delta f=\frac{f^2}{r}\frac{\Delta k_B}{k_B}. \qquad (16)$$

The local distortion of the wave phase is determined in turn by the expression[12]

$$\Delta\varphi_{dif}=\frac{2\pi}{\lambda_B}\Delta f\left(\frac{r^2}{2R^2}\right). \qquad (17)$$

Using expressions (10) and (13) for Δk_B we determine the local and mean squared phase shifts:

$$\Delta\varphi_{dif}=\left(\frac{\pi P_0\gamma^3z_R}{3w_0^2I_s\Delta^2\vartheta_z}\right)^{1/2}kr^2, \qquad (18')$$

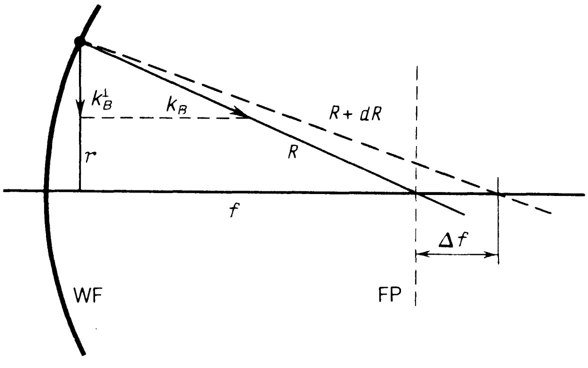

FIG. 2. Illustrating the determination of the aberrations of a "laser objective" for an atomic beam; WF—wavefront, FP—focal plane.

$$\overline{\Delta\varphi_{dif}^2}=\Delta\varphi_{dif}^2/5. \qquad (18'')$$

According to the Rayleigh "quarter-wave" criterion, a qualitative description can be obtained if the mean squared phase shift does not exceed $\lambda/4$.[13] This criterion leads to the following condition on the laser and atomic parameters:

$$\Delta^2>\frac{2}{3}\frac{\pi^4P_0\gamma^3w_0^2}{I_s\vartheta_z\lambda^3}\left(\frac{a^4}{w_0^4}\right). \qquad (19)$$

If its right-hand side is averaged over the beam aperture, inequality (14) coincides with the corresponding inequality (19), i.e., the two approaches lead to the same result.

b) Chromatic aberration

The influence of the chromatic aberration can be estimated in the same way as the influence of the diffusion, from its distortion of the wave front. Using the expression (9′) for the focal length and expression (17) for the defocusing, we obtain the mean squared phase shift due to the nonmonochromaticity of the atom beam:

$$\overline{\Delta\varphi^2}=\frac{9k_Ba^4}{80f^2\vartheta_z^2}(\Delta\vartheta_z)^2. \qquad (20)$$

If the Rayleigh quarter-wave criterion for the image quality is again assumed, the following constraint is imposed on the beam nonchromaticity:

$$\Delta\vartheta_z\leqslant\frac{\sqrt{32}}{6\pi}\vartheta_z\frac{f}{a^2}. \qquad (21)$$

c) Spherical aberrations

It is seen from expression (8) for the phase shift that the influence of the spherical aberration on the diffraction pattern can be eliminated in two ways. First, one can decrease the diameter of the atom beam. This method, however, enhances the role of diffraction and decreases the beam intensity at the center of the diffraction spot. Second, the parameters of the laser radiation can be chosen such as to reduce this type of aberration to a minimum. It follows from (8), for example, that aberrations of fourth order are eliminated at $\alpha=2$. The laser and atomic parameters must satisfy then the condition [see (8)]

$$\alpha=P_0\gamma^2/\pi I_s\Delta^2w_0^2=2. \qquad (22)$$

We must verify that this new condition does not contradict the condition $p<1$ used for the series expansion of the potential (3). The saturation parameter can be expressed in terms of a as follows:

$$p=\frac{I}{I_s}\frac{\gamma^2}{4\Delta^2}=0.37\alpha\frac{I(a)}{I_{max}}, \qquad (23)$$

where $I(a)$ is the intensity of the laser field at a distance from the beam axis, and I_{max} is the maximum intensity at the distance $r=w_0/2^{1/2}$. The parameter p becomes substantially smaller than unity even at $a=w_0/4$.

Simultaneous allowance for the restrictions imposed by the spherical and diffusion aberrations [Eqs. (14) and (22)] leads to the inequality

$$\frac{4\pi^5\gamma\lambda}{3}\frac{N^4}{\vartheta_z}<1, \qquad (24)$$

where $N = a/\lambda$. It is seen from this relation that for thermal atomic beams ($\vartheta_z = 10^4$–10^5 cm/s) the diameter of the atomic lens should not differ greatly from the laser-emission wavelength. For example, at $a = \lambda$ the a near-diffration resolution can be obtained at $\vartheta_z = 1.2\cdot 10^5$ cm/s.

The laser field and the atomic beam are subject to one more constraint. In ordinary optics, the image produced by a lens is determined in the Fresnel approximation. This approximation is valid under the condition[8]

$$\left(\frac{\lambda}{z}\right)^{1/4} \geqslant \frac{d+\rho}{z}, \tag{25}$$

where z is the distance from the lens on which the diffraction field is considered, d is the lens diameter, and ρ is the image radius. For our case this condition becomes

$$f \geqslant \lambda_B^{-1/4}. \tag{26}$$

For $\lambda_B = 10^{-8}$ cm and $a = w_0/4$ the lower bounds of the focal length are 1) $f \gtrsim 1.4\lambda$ for $w_0 = \lambda$, 2) $f \gtrsim 28\lambda$ for $w_0 = 10\lambda$, and 3) $f \gtrsim 616\lambda$ for $w_0 \gtrsim 10^2\lambda$.

In the considered laser-beam configuration, the thickness of the lens is strictly speaking infinite, so that account must be taken of the influence of the laser dimension along the z axis on the parameters of the lens. The phase shift on the potential of the laser field along the z axis over a length from $-z_i$ to $+z_i$ is [see Eq. (4)]:

$$\Delta\varphi = \text{const}\, \rho^2 \operatorname{arc\,tg} z_i. \tag{27}$$

If a limit $z_i = 5z_R$ is chosen, the phase shift over a length $10z_R$ differs from the total phase shift (8) by only 10%, and this is manifested only by like change of the focal length of the lens.

We have used so far a classical description of the laser field. Let us estimate the influence of quantum fluctuations on the focusing of an atom beam.

Single-mode laser radiation with stabilized frequency and stabilized intensity is well described by the mode of the electromagnetic field excited to a coherent state. The electric field fluctuations in a coherent state are independent of the field amplitude and are given by the expression[14]

$$\Delta E = (\hbar\omega/2\varepsilon_0 V)^{1/2}, \tag{28}$$

where ε_0 is the dielectric constant of vacuum and V is the volume of the field mode.

Expression (3) for the laser-field potential can be expressed in terms of the electric field strength in the form

$$U = 2\hbar\Delta \frac{\gamma^2}{\Delta^2}\left(\frac{\mu^2 E_s^2}{2\hbar^2\gamma^2}\right)\frac{E^2}{E_s^2}, \tag{29}$$

where μ is a matrix element of the dipole moment of the atom, $E_s = \hbar\omega/\varepsilon_0\tau\sigma c$ is strength of the electric field that saturates the atomic transition, and σ is the atomic-transition absorption cross section. The expression in the parentheses is the atomic-transition saturation parameter and is equal to unity at $E = E_s$.

The additional atom-wave-function phase change due to quantum fluctuations of the magnetic field can be determined by replacing E in (2) by the field fluctuations (28):

$$\delta(\Delta\varphi) = \gamma^2\tau\sigma c/2\pi\vartheta_z\Delta w_0^2. \tag{30}$$

To prevent the quantum fluctuations from strongly distorting the atom-density distribution in the focal plane of the atomic lens, expression (30) must not exceed $\pi/2$.[12]

We determine its value for the case when the laser-field constriction is a minimum: $w_0 = \lambda$. The phase-shift change (30) takes then the form

$$\delta(\Delta\varphi) = \frac{1}{4\pi^2}\frac{c}{\vartheta_z}\frac{\gamma}{\Delta} \tag{30'}$$

It follows from the form of (30') that the quantum fluctuations of the field become negligible when the ratio of the laser-field frequency detuning to the homogeneous width of the transition line becomes equal to the ratio of the speed of light to the atom velocity. We have already spelled out [see (14)] the constraint on the detuning due to the momentum diffusion. At $w_0 = \lambda$ this constraint yields $\Delta \gtrsim 1.3\cdot 10^{-11}$ s^{-1}. At this frequency detuning, the phase shift due to quantum fluctuations becomes equal to $\delta(\Delta\varphi) \leqslant 0.3$, or substantially less than $\pi/2$. For the case $w_0 > \lambda$ the influence of the quantum fluctuations of the field is even more insignificant.

4. ATOM-BEAM DENSITY DISTRIBUTION IN THE FOCAL PLANE

The phase shift of the de Broglie wave for the considered real atomic lens can be represented by the sum

$$\Delta\varphi = \frac{k_B}{2f} r^2 (\alpha_{\text{dif}} + \alpha_{\text{chr}} + \alpha_{\text{sph}}), \tag{31}$$

where

$$\alpha_{\text{dif}} = \left(\frac{\vartheta_z w_0^2 I_s}{3\lambda\gamma P_0}\right)^{1/2}, \tag{31'}$$

$$\alpha_{\text{chr}} = \frac{3}{2}\frac{\Delta\vartheta_z}{\vartheta_z}, \tag{31''}$$

$$\alpha_{\text{sph}} = -\left(1 - \frac{\alpha}{2}\right)\frac{\rho^2}{2} + (1-4\alpha)\frac{3}{16}\rho^4 \tag{31'''}$$

are the terms that determine the contributions of the diffusion, chromatic and spherical aberrations.

To calculate the atom density distribution in the focal plane we use the Kirchhoff diffraction method,[8] according to which the diffraction field in image space is given, accurate to an inessential phase factor, by the integral

$$i(z,\rho) = \frac{2\pi i}{\lambda z}\int_0^a \rho' f(\rho') \exp\frac{ik_B\rho'^2}{2z} \times \exp -\frac{ik_B\rho'^2}{2f} J_0\left(k_B\frac{\rho\rho'}{z}\right) d\rho', \tag{32}$$

where $f(\rho')$ is the field distribution at the entrance to the lens and J_0 is a Bessel function of zero order. To take into account the aberrations in the integral (32), the argument of the second exponential is replaced by (31). Correct allowance for the combined influence of the individual aberrations is a complicated task[1,13] and we determine separately the influence of the individual aberrations on the atom density distribution in the focal plane.

To satisfy the thin-lens condition (26), we assume that the focal length is $f = 5z_0$; then, using (9) and (22), we find

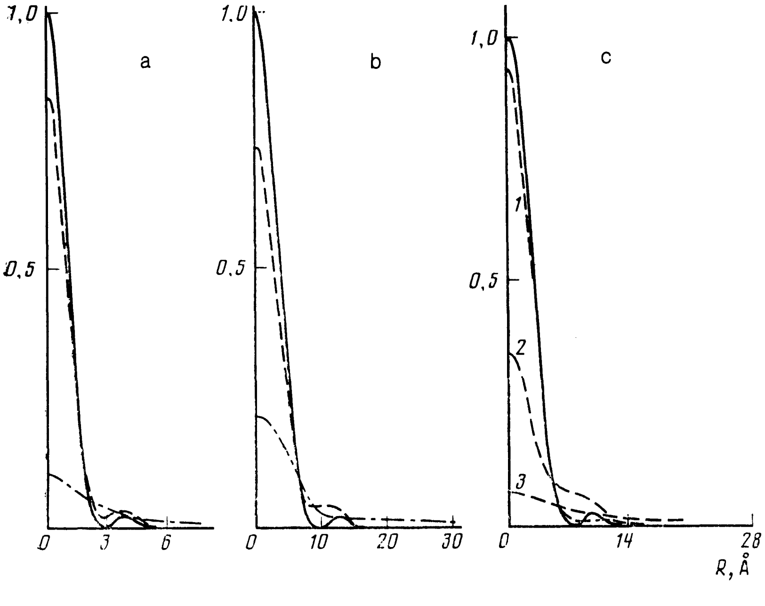

FIG. 3. Distribution of the atomic-beam density in the focal planes: the solid curve corresponds to the aberration-free case, the dashed to allowance for chromatic aberration, the dash-dot to allowance for spherical aberration, and the dotted one to allowance for diffusion aberration, all for the case $w_0 = \lambda$, $f = 15.6\lambda$, $a = w_0/4$: a—$P_0 = 1$ W, $\vartheta_z = 2.2\cdot 10^5$ cm/s, $\Delta\vartheta_z/\vartheta_z = 10^{-3}$; b—$P_0 = 10^{-2}$ W, $\vartheta_z = 6.9\cdot 10^4$ cm/s, $\Delta\vartheta_z/\vartheta_z = 2\cdot 10^{-3}$; c—$P_0 = 10^{-2}$ W, $\vartheta_z = 6.9\cdot 10^4$ cm/s (1—$\Delta V/V = 10^{-3}$, 2—$4\cdot 10^{-3}$, 3—10^{-2}).

the atom longitudinal velocity that makes such a focal length possible:

$$\vartheta_z = \left(200\pi^5 \frac{\gamma^2 w_0^2 \vartheta_r^2 P_0}{I_s \lambda^2}\right)^{1/4}, \qquad (33)$$

where ϑ_r is the atom recoil velocity. This expression must be substituted in (31′)–(31‴) to determine the values of the aberrations. Next, assume that a plane de Broglie wave be incident on the atomic lens and we are interested in the distribution in the focal plane. As expected from (24), for thermal atomic beams near-diffraction pattern can be obtained for a laser-beam neck $w_0 \sim \lambda$. At values $w_0 > 10\lambda$ the spherical and diffusion aberrations become very large, and the requirements on the chromaticity of the atomic beam become very stringent ($\Delta\vartheta/\vartheta \leqslant 10^{-4}$).

Figure 3 shows the distribution of the atom density in the focal plane for $w_0 = \lambda$. The solid curve corresponds to the aberration-free case, the dashed to a distribution with allowance for chromatic aberrations, the dash-dot to a distribution with allowance for spherical aberrations, and the dotted to allowance for diffusion aberration. When fewer than four distributions are shown on the figures, this means that the remaining aberrations are negligibly small, and the corresponding distributions coincide with the aberration-free distributions. The distribution in Fig. 3a was calculated for the following parameters: $P_0 = 1$ W, $\vartheta_z = 2.2\cdot 10^5$ cm/s, $a = 0.25 w_0$, $f = 5 z_R = 15.6\lambda$, and $\Delta\vartheta_z/\vartheta_z = 10^{-3}$. It can be seen from the figure that the beam dimensions at the focus, with allowance for aberration, do not differ greatly from the limiting diffraction value and amount to several angstroms. Figure 3b shows the distribution for the same beam caustic, but at a lower laser power, $P_0 = 10$ mW. In this case the atom velocity $\vartheta_z = 6.9\cdot 10^4$ cm/s is close to the average velocity of the thermal-beam atoms. The fluctuation distribution is the same as before. The conditions on the chromaticity of the beam are less stringent: the distribution with allowance for chromatic aberration was plotted for $\Delta\vartheta_z/\vartheta_z = 4\cdot 10^{-3}$. Decreases in the laser power and in the atom velocity led to a worse resolution: the beam diameter in the focus was about 10 Å. Figure 3c shows the influence of the

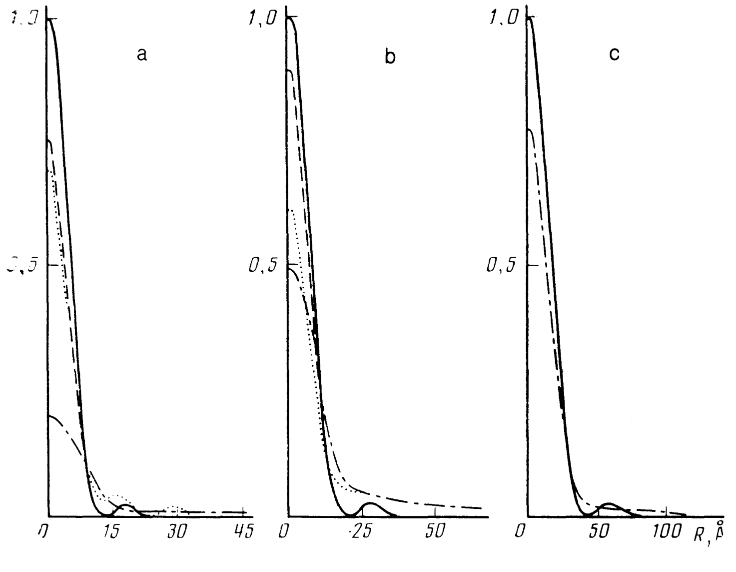

FIG. 4. The same as Fig. 3, for the case $w_0 = 10\lambda$, $f = 1570\lambda = 4.9\cdot 10^{-2}$ cm, $a = w_0/6$, $\Delta\vartheta_z/\vartheta_z = 10^{-3}$; a—$P_0 = 1$ W, $\vartheta_z = 6.9\cdot 10^5$ cm/s; b—$P_0 = 10^{-1}$ W, $\vartheta_z = 3.9\cdot 10^5$ cm/s; c—$P_0 = 10^{-2}$ W, $\vartheta_z = 2.2\cdot 10^5$ cm/s.

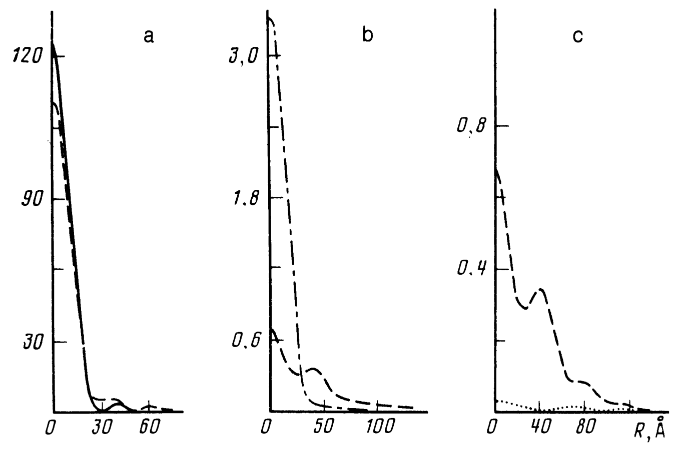

FIG. 5. The same as Fig. 3, for the case $w_0 = 100\lambda$, $f = 9.4$ cm, $a = w_0/10$: a—$P_0 = 1$ W, $\vartheta_z = 2.1\cdot 10^6$ cm/s, $\Delta\vartheta_z/\vartheta_z = 10^{-4}$; b—$P_0 = 10$ W, $\vartheta_z = 3.9\cdot 10^6$ cm/s, $\Delta\vartheta_z/\vartheta_z = 10^{-3}$; c—the same parameters as for case b.

chromaticity of the beam on the distribution width. Deterioration of the beam chromaticity from 10^{-3} to 10^{-2} leads to a substantial spreading of the beam.

Figure 4 shows the same distributions, but with a different value of the laser-beam diameter, $w_0 = 10\lambda$. It can be seen that at all three values of the power ($P_0 = 1$, 0.1, and 0.01 W) one can find parameters at which the distributions are close to diffractive. To prevent spherical aberrations from greatly broadening the distribution, the atom-beam diameter was in this case $a = w_0/6$. The role of this kind of aberration decreases substantially with decrease of the laser power. In Fig. 4c all other types of aberrations are negligibly small, and the calculated distributions practically coincide with the aberration-free curve. From a comparison of the three curves with three values of laser power it is seen that the resolution of an atomic objective made up of a lower-power beam is worse. The resolution decreases also on going from $w_0 = \lambda$ to $w_0 = 10\lambda$. The reason is that we have fixed the focus at the distance $f = 5z_R$.

The curves of Fig. 5 correspond to $w_0 = 100\lambda$. This case is quite difficult to implement in practice. Indeed, it follows from Fig. 5a that the chromaticity of the beam must be very high, $\Delta\vartheta/\vartheta = 10^{-4}$. Figure 5b shows the influence of spherical aberration at a small diameter of the atomic beam compared with the beam diameter ($a = 0.1w_0$), and a steep decrease of the atom density is seen. The same figure shows the distribution with allowance for chromatic aberration at $\Delta\vartheta_z/\vartheta_z = 10^{-3}$. Figure 5c duplicates Fig. 5b in an enlarged scale, and shows a curve with allowance for diffusive aberration. It can be seen that the diffusive aberration washes out completely the atomic beam at the focus.

5. CONCLUSION

We conclude by listing the requirements that must be met by the laser radiation and by the atomic beam to be able to focus the latter into a spot of several angstroms. The focusing potential field is produced by using a TEM_{01}^* laser mode strongly focused to a size on the order of the wavelength of light. The radiation power needed to focus beams having thermal velocity is several dozen milliwatts. Diffractive resolution of the atomic objective is realized at an atomic-beam monochromaticity $\Delta\vartheta/\vartheta = 10^{-3}$.

Using the considered method of focusing an atomic beam down to several angstroms, it is difficult to conceive of an atomic microscope similar to a raster electron microscope working in transmission or reflection. The atoms scattered or reflected from the investigated object can be recorded by all possible methods that are sensitive enough, especially by laser methods of detecting single atoms.[15] Of course, to realize such a scanning atomic microscope with angstrom resolution it is necessary to have just as small an atomic-beam source. Such small apertures can be readily obtained by modern methods.

In conclusion, the authors thank S. V. Shul'ga for help with the computer calculations, Yu. E. Lozovik and V. G. Minogin for a helpful discussion of the results, and to the referee for remarks concerning the role of laser-radiation fluctuations.

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Translated by J. G. Adashko