Inelastic resonant tunneling of electrons through a potential barrier

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An exact solution is obtained of the problem of resonant tunneling of electrons at an arbitrary value of the electron-phonon interaction at the resonance center. If the strain accompanying the formation of the polaron state on the center is large compared with the amplitude of the zero-point vibrations of the crystal, a substantial smearing takes place in the energy dependence of the resonance transparency, and intense inelastic tunneling channels are produced. Nonetheless, the integrated probability of tunneling for a "beam" of electrons with a broad energy distribution does not depend on the electron-phonon coupling. The results can be used to analyze experimental data on resonant tunneling of electrons through an amorphous dielectric layer.

1. INTRODUCTION

The transparency of a barrier to tunneling increases drastically if it contains localized states that are at resonance with the tunneling electron.[1] The probability of an electron of energy ε tunneling "through" a localized state is determined[1)] by the Breit-Wigner formula[2]

$$T_{res}(\varepsilon-\varepsilon_0)=\frac{\Gamma^2}{(\varepsilon-\varepsilon_0)^2+\Gamma^2}, \tag{1}$$

where ε_0 is the energy of the localized state, and Γ the width of the level produced by this state in a barrier of finite thickness d. The value of Γ depends mainly on the ratio of d and the impurity-state radius a_0:

$$\Gamma\propto\exp(-d/a_0). \tag{1.2}$$

Being a quantum coherent effect, resonant tunneling should be sensitive to the loss of phase coherence of the electron wave function. This circumstance, postulated in the phenomenological approach,[3] leads to the conclusion that the resonance-level width and the tunneling probability depend strongly on temperature. The conclusions of the phenomenological approach,[3] however, do not agree with the results of Ref. 4, where an attempt was made at a microscopic analysis of tunneling with the aid of a model potential that varies harmonically (i.e., in accordance with a prescribed law) with time.

In fact, electron-phonon interaction (EPI) on a center leads to the dynamic effect of binding the states of an electron and phonons into a state of the polaron type. This effect of formation of a coherent electron-phonon state does not reduce to loss of phase coherence of the electron wave function, as suggested in Ref. 3, and the result of the influence of the EPI on resonant tunneling is thus not obvious. The present paper is devoted to a consistent quantum-mechanical analysis of tunneling through resonant centers that interact with lattice vibrations. The analysis that follows shows that a strong EPI leads to a substantial smearing of the resonant $T(\varepsilon)$ line and to the onset of inelastic resonant-tunneling channels, but does not change significantly its integral intensity. The line shape is not Lorentzian in the presence of EPI.

The magnitude of the EPI at the center is determined by the dimensionless parameter $(u_\Lambda/u_0)^2$, where $u_0=(\hbar/M\omega_D)^{1/2}$ is the amplitude of the zero-point vibrations and $u_\Lambda\approx\Lambda/M\omega_D^2 a$ is the characteristic magnitude of the lattice polaron deformation due to the presence of electrons at the center (ω_D is the Debye frequency, Λ is the strain-potential constant, M is the ion mass, and a is the lattice parameter). The EPI at the center leads to substantial hybridization of the quasilocal electronic state and of the phonon degrees of freedom of the crystal, under the condition

$$(u_\Lambda/u_0)^2\gg 1. \tag{1.3}$$

As a result, in place of a single impurity level, the electron-phonon states at the center are distributed over an energy band whose width is estimated at

$$\varepsilon_\Lambda\sim\hbar\omega_D u_\Lambda/u_0\gg\hbar\omega_D. \tag{1.4}$$

The previously mentioned modification of the picture of the resonant tunneling is due to the onset of wide resonance band (1.4). In contrast to (1.1), the width of the resonance line is determined not by Γ but by $\varepsilon_\Lambda\gg\Gamma$. The maximum value of the total tunneling probability $T_t(\varepsilon)$ as a function of the initial energy ε is estimated by the expression

$$\max T_t(\varepsilon)\sim\Gamma/\varepsilon_\Lambda. \tag{1.5}$$

The magnitude of the EPI does not influence the barrier integral resonant transparency $\int d\varepsilon T_t(\varepsilon)$. It is of the order of Γ, as before, and exceeds substantially the nonresonant tunneling probability which is proportional to Γ^2.

Resonant tunneling of band electrons through localized states in a barrier is best analyzed by tunnel-Hamiltonian method (all the energies are reckoned here and elsewhere from the bottom of the conduction band for electrons outside the barrier, $\hbar=1$):

$$H=H_a+H_d+H_c+H_{ac}+H_{cd}. \tag{1.6}$$

$$H_a=\sum_k\varepsilon_k a_k^+ a_k,\quad H_d=\sum_p\varepsilon_p d_p^+ d_p,$$

$$H_c=\left[\varepsilon_0+\frac{1}{N^{1/2}}\sum_q\alpha_q(b_q^+-b_q)\right]c^+c+\sum_q\omega_q b_q^+ b_q. \tag{1.7}$$

$$H_{ac}=\sum_k g_{ca}(\mathbf{k})\,(a_k^+ c+c^+ a_k),$$

$$H_{cd}=\sum_p g_{cd}(\mathbf{p})\,(d_p^+ c+c^+ d_p). \tag{1.8}$$

Here ε_p, a_k^+, and d_p^+ are the energies and the creation operators of an electron in the left and right edges of the tunnel junction (we assume here for simplicity that H_a and H_d obey the same dispersion law), ε_0, and c^+ are the same for

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the electron in the localized state, ω_q and b_q^+ are the phonon dispersion law and creation operators, and N is a normalization factor (the number of atoms in the lattice). The matrix element α_q is connected in the usual manner with deformation-interaction constant Λ of the localized electrons (M is the atom mass):

$$\alpha_q = -i\Lambda q/(M\omega_q)^{1/2}. \tag{1.9}$$

The hybridization constants g_{ca} and g_{cd} are due to the below-barrier "tail" of the wave functions of the band electrons. If the resonant impurity is located at a distance x from the center of the barrier, we have

$$g_{ca},\ g_{cd} \propto \exp\{-(d \pm 2x)/2a_0\}.$$

(The values of g for an impurity with a small-radius potential were calculated in Ref. 5). No account is taken in the Hamiltonian (1.6) of the interaction of the band states with the phonons. This simplification is permissible because the phonon renormalizations of the itinerant states are small compared with analogous renormalization for localized states to the extent that the parameter $\omega_D/q_D v \sim s/v \ll 1$ (s is the speed of sound and v is the electron velocity). To the same extent that this parameter is small, we can neglect also the dependences of g_{ca} and g_{cd} on the phonon variables. Bearing in mind the case of a sufficiently sharp resonance, we assume that the hybridization constants are small compared with the characteristic energies of the electronic states:

$$g_{ca},\ g_{cd} \ll \varepsilon_0. \tag{1.10}$$

The Hamiltonian H_c is diagonalized in terms of the new operators β_q^+ and γ^+:

$$\beta_q^+ = U b_q^+ U^+ = b_q^+ + \frac{\alpha_q}{N^{1/2}\omega_q} c^+c,$$

$$\gamma^+ = U c^+ U^+ = \exp\left\{\sum_q \frac{\alpha_q}{N^{1/2}\omega_q}(b_q^+ + b_q)\right\} c^+. \tag{1.11}$$

$$U = \exp\left\{\sum_q \frac{\alpha_q}{N^{1/2}\omega_q}(b_q^+ + b_q) c^+c\right\}. \tag{1.12}$$

The operator H_c is then transformed into

$$H_c = \left(\varepsilon_0 - \sum_q \frac{|\alpha_q|^2}{N\omega_q}\right)\gamma^+\gamma + \sum_q \omega_q \beta_q^+ \beta_q \tag{1.13}$$

and has eigenfunctions

$$\varphi\{m,\ n_q\} = (\gamma^+)^m \prod_q (\beta_q^+)^{n_q}|0\rangle,\ m=0,\ 1,\ n_q=0,\ 1,\ 2, \ldots \tag{1.14}$$

It can be seen from (1.11)–(1.14) that in the case of one phonon frequency ω_D phonon replicas are produced in the density of the quasilocal states.[6-8] The hybridization operators H_{ac} and H_{cd} correspond to transitions with creation or annihilation of a large number of electron-phonon states. With allowance for (1.11) we obtain, say for H_{ac}

$$H_{ac} = \sum_k g_{ca}(\mathbf{k}) a_k^+ \gamma \left\{\alpha_0 + \sum_{n=1}^{\infty} [\alpha_n (\beta_D^+)^n - \alpha_n^* \beta_D^n]\right\} + \text{H.a.}, \tag{1.15}$$

$$\alpha_n = \frac{1}{n!}\left(-\frac{\alpha_D}{\omega_D}\right)^n \exp\left\{-\frac{|\alpha_D|^2}{2\omega_D^2}\right\}. \tag{1.16}$$

It follows from (1.15) and (1.16) that at $|\alpha_D/\omega_D| \gg 1$ the hybridization matrix element[2)] $V_n = g(n!)^{1/2}\alpha_n$ is a nonmonotonic function of the subscript n, and reaches a maximum at

$$n \approx n_0 = |\alpha_D/\omega_D|^2.$$

In the region $n \gg n_0$ the value of V_n decreases like

$$V_n \propto (n_0/n)^{n/2}.$$

Equation (1.13) shows that the resonant energy ε_0 of the electronic state undergoes a polaron shift. In view of the structure of V_n, however, the hybridization of the itinerant and impurity states takes place effectively in an energy band whose center coincides as before with the resonant value ε_0. The width of this band, according to (1.16), is of the order of α_D and it can be seen from (1.9) that it coincides with ε_Λ, see Eq. (1.4). The transitions of the itinerant electrons into states of this band determine the smearing of the resonant tunneling line and its inelastic character. The cause of the latter is easily understood if it is recognized that the matrix elements of the transitions under the action of H_{ac} and H_{cd} between states with different phonon occupation numbers $\{m_q\}$ and $\{n_q\}$ differ from zero. From conservation law for the total system energy

$$\varepsilon_k + \sum_q n_q\omega_q = \varepsilon_p + \sum_q m_q\omega_q$$

follows the possibility of energy nonconservation of the electron in the course of tunneling ($\varepsilon_p \neq \varepsilon_k$). The characteristic electron energy loss in tunneling is determined by the value of n_0 at which the hybridization matrix element is a maximum. Under condition (1.3), the energy loss turns out to be of the order of the polaron-shift energy ε_{pol}:

$$\varepsilon_{\text{pol}} = \frac{1}{N}\sum_q |\alpha_q|^2/\omega_q. \tag{1.17}$$

At finite temperature, the tunneling process is complicated by the contributions of processes in which real thermally excited phonons participate. Analysis of these processes calls for a more rigorous treatment, which will be carried out in the next section.

2. DYNAMIC EVOLUTION OF ELECTRONIC STATES IN THE COURSE OF TUNNELING

Calculation of the probability of resonant tunneling of an itinerant electron from the left to the right edge requires an analysis of the decay of the "pure" itinerant state $\mathbf{k}$ due to its hybridization with a set of local electron-phonon states. To describe the dynamics of this process it is convenient to introduce the amplitudes $\hat{u}_{p\sigma}$ ($\sigma = a,d$) and $\hat{v}$ of the probabilities of the transition of a state $a_k^+|0\rangle$ specified at the initial instant of time into the states $d_p^+|0\rangle$ and $c^+|0\rangle$. We represent the wave function of the system with the Hamiltonian H [see (1.6)] in the form

$$\Psi(t) = \left\{\sum_p (\hat{u}_{pa}(t) a_p^+ + \hat{u}_{pd}(t) d_p^+) + \hat{v}(t) c^+\right\}|0\rangle \tag{2.1}$$

($\hat{u}_{p\sigma}$ and $\hat{v}$ are operators in the phonon variables). The fact that $c^+|0\rangle$ is not an eigenstate for the Hamiltonian H_c leads to a nontrivial dependence of $u_{p\sigma}$ and v on the operators b_q^+ and b_q. This dependence is determined by the evolution

equations that follow from the Schrödinger equation expressed in the interaction representation (relative to the free-phonon Hamiltonian

$$H_{ph}^{(0)}=\sum_{\mathbf{q}}\omega_{\mathbf{q}}b_{\mathbf{q}}^{+}b_{\mathbf{q}}):$$

$$i\frac{\partial\hat{u}_{p\sigma}}{\partial t}=\varepsilon_{p}\hat{u}_{p\sigma}+g_{c\sigma}(\mathbf{p})\hat{v},$$

$$i\frac{\partial\hat{v}}{\partial t}=\left[\varepsilon_{0}+\frac{1}{N^{1/2}}\sum_{\mathbf{q}}\alpha_{\mathbf{q}}\left(b_{\mathbf{q}}^{+}e^{i\omega_{\mathbf{q}}t}-b_{\mathbf{q}}e^{-i\omega_{\mathbf{q}}t}\right)\right]\hat{v}+\sum_{p\sigma}g_{c\sigma}(\mathbf{p})\hat{u}_{p\sigma}. \tag{2.2}$$

In accordance with the considered tunneling problem, we choose the initial conditions for Eqs. (2.2) in the form

$$\hat{u}_{p\sigma}(t=0)=\delta_{pk}\delta_{c\alpha},\ \hat{v}(t=0)=0. \tag{2.3}$$

The evolution equations (2.2) can be greatly simplified if the energies transferred by the scattering are low compared with the characteristic energies of the electrons

$$|\varepsilon_{k}-\varepsilon_{p}|\ll\varepsilon_{k}. \tag{2.4}$$

This simplification is easiest to track by transforming in relations (2.2) to Fourier components $\hat{u}_{k\sigma}(\omega)$ and $\hat{v}(\omega)$ with respect to time. After solving the first equation of the system (2.2) for $\hat{u}_{k\sigma}(\omega)$ we obtain a closed equation for the Fourier component $\hat{v}(\omega)$:

$$(\omega-\hat{\varepsilon}+i\Gamma)\hat{v}(\omega)=i\sum_{\mathbf{k}}g_{c\alpha}(\mathbf{k})\frac{\hat{u}_{k\alpha}(t=0)}{\omega-\varepsilon_{k}}. \tag{2.5}$$

The operator of the electron "energy" renormalized by the hybridization is defined by the relation

$$\hat{\varepsilon}\hat{v}(\omega)=\left[\varepsilon_{0}+\text{V. P.}\sum_{p\sigma}|g_{c\sigma}(\mathbf{p})|^{2}\frac{1}{\omega-\varepsilon_{p}}\right]\hat{v}(\omega)+\int_{-\infty}^{\infty}d\omega_{1}\int_{0}^{\infty}dte^{i\omega_{1}t}\sum_{\mathbf{q}}\frac{\alpha_{\mathbf{q}}}{N^{1/2}}\left[e^{-i\omega_{\mathbf{q}}t}b_{\mathbf{q}}^{+}-e^{i\omega_{\mathbf{q}}t}b_{\mathbf{q}}\right]\hat{v}(\omega_{1}+\omega). \tag{2.6}$$

The damping Γ of the electronic states at the center is of the form

$$\Gamma=\pi\sum_{k\sigma}|g_{c\sigma}(\mathbf{k})|^{2}\delta(\omega-\varepsilon_{k})=\Gamma_{l}+\Gamma_{r},$$

$$\Gamma_{l,r}\propto\exp\{-(d\pm2x)/a_{0}\}. \tag{2.7}$$

Γ_l and Γ_r are the partial widths of the resonance level and are due to electron departure to the left or right edges. Under the condition (1.10) one can neglect the hybridization-renormalization of the electron energy [the term proportional to g^2 in (2.6)]. The condition (2.4) allows us to neglect the frequency dependence of the damping Γ [$\Gamma(\omega)=\text{const}$]. This neglect simplifies substantially the analysis of the dynamics of the electronic states. Putting $\Gamma(\omega)\equiv\Gamma$ and transforming to a temporal representation, we obtain simplified equations for $\hat{u}_{p\sigma}(t)$ and $\hat{v}(t)$ in the form

$$i\frac{\partial\hat{u}_{p\sigma}}{\partial t}=\varepsilon_{p}\hat{u}_{p\sigma}+g_{c\sigma}(\mathbf{p})\hat{v},$$

$$i\frac{\partial\hat{v}}{\partial t}=\left[\varepsilon_{0}-i\Gamma+\sum_{\mathbf{q}}\frac{\alpha_{\mathbf{q}}}{N^{1/2}}\left(b_{\mathbf{q}}^{+}e^{i\omega_{\mathbf{q}}t}-b_{\mathbf{q}}e^{-i\omega_{\mathbf{q}}t}\right)\right]\hat{v}+\sum_{\mathbf{p}}g_{c\alpha}(\mathbf{p})\hat{u}_{\mathbf{p}}(t=0)e^{-i\varepsilon_{p}t}. \tag{2.8}$$

The solution of the first equation of the system (2.8) under the initial condition (2.3) takes the form

$$\hat{u}_{p\sigma}(t)=\hat{u}_{p\sigma}(0)e^{-i\varepsilon_{p}t}-g_{c\sigma}(\mathbf{p})g_{c\alpha}(\mathbf{k})e^{-i\varepsilon_{p}t}\times\int_{0}^{t}d\tau_{1}\exp\{-i(\varepsilon_{0}-\varepsilon_{p}-i\Gamma)\tau_{1}\}\int_{0}^{\tau_{1}}d\tau_{2}\exp\{i(\varepsilon_{0}-\varepsilon_{k}-i\Gamma)\tau_{2}\}\hat{v}_{0}(\tau_{1})\hat{v}_{0}^{-1}(\tau_{2})\hat{u}_{k}(0). \tag{2.9}$$

The operator $\hat{v}_0(\tau)$ is then the solution of the Cauchy problem

$$i\frac{\partial\hat{v}_{0}}{\partial t}=\sum_{\mathbf{q}}\frac{\alpha_{\mathbf{q}}}{N^{1/2}}\left(b_{\mathbf{q}}^{+}e^{i\omega_{\mathbf{q}}t}-b_{\mathbf{q}}e^{-i\omega_{\mathbf{q}}t}\right)\hat{v}_{0},\quad\hat{v}_{0}(t=0)=1. \tag{2.10}$$

Equation (2.10) can be reduced to a linear differential equation of first order by changing to the coordinate representation for the operators $b_{\mathbf{q}}^{+}$ and $b_{\mathbf{q}}$ (the Bargmann representation[9]). Solving the last equation by the trajectory method we get

$$\hat{v}_{0}(t)=\exp\left\{\frac{1}{i}\sum_{\mathbf{q}}\frac{\alpha_{\mathbf{q}}^{2}}{N\omega_{\mathbf{q}}^{2}}(\omega_{\mathbf{q}}t-\sin\omega_{\mathbf{q}}t)+\sum_{\mathbf{q}}\frac{\alpha_{\mathbf{q}}}{N^{1/2}\omega_{\mathbf{q}}}\left[(1-e^{i\omega_{\mathbf{q}}t})b_{\mathbf{q}}^{+}+(1-e^{-i\omega_{\mathbf{q}}t})b_{\mathbf{q}}\right]\right\}. \tag{2.11}$$

Relations (2.9) and (2.10) make it possible to find the probability amplitude $\hat{U}_{k\to p}(t)$ of the resonant tunneling:

$$\hat{U}_{\mathbf{k}\to\mathbf{p}}(t)=-g_{c\alpha}(\mathbf{k})g_{c\delta}(\mathbf{p})\exp\{i(\varepsilon_{k}-\varepsilon_{p})t\}\times\int_{0}^{t}d\tau_{1}\exp\{-i(\varepsilon_{0}-\varepsilon_{p}-i\Gamma)\tau_{1}\}\times\int_{0}^{\tau_{1}}d\tau_{2}\exp\{i(\varepsilon_{0}-\varepsilon_{k}-i\Gamma)\tau_{2}\}\hat{v}_{0}(\tau_{1})v_{0}^{-1}(\tau_{2}). \tag{2.12}$$

Calculation of the $\mathbf{k}\to\mathbf{p}$ transition probability per unit time requires averaging over the operator $\hat{U}_{\mathbf{k}\to\mathbf{p}}^{+}\hat{U}_{\mathbf{k}\to\mathbf{p}}$ over the initial states of the phonon system at the instant of time $t=0$. Assuming these to be equilibrium states and recognizing that at the initial instant the local level is empty and there is no renormalization of the phonon modes [see (1.12)] we obtain for the transition probability per unit time $W_{\mathbf{k}\to\mathbf{p}}$ the expression

$$W_{\mathbf{k}\to\mathbf{p}}=\lim_{t\to\infty}\frac{1}{t}\frac{\text{Sp}\{\exp[-\beta H_{ph}^{(0)}]\hat{U}_{\mathbf{k}\to\mathbf{p}}^{+}(t)\hat{U}_{\mathbf{k}\to\mathbf{p}}(t)\}}{\text{Sp}\{\exp[-\beta H_{ph}^{(0)}]\}}. \tag{2.13}$$

The mean values in (2.13) are calculated in the same manner as in the theory of the Mössbauer effect.[10] Taking (2.11) and (2.12) into account, we obtain ultimately from (2.13)

$$W_{\mathbf{k}\to\mathbf{p}}=|g_{c\alpha}(\mathbf{k})|^{2}|g_{c\delta}(\mathbf{p})|^{2}F(\varepsilon_{k}\to\varepsilon_{p}),$$

$$F(\varepsilon_{k}\to\varepsilon_{p})=2\pi\int_{-\infty}^{\infty}dt_{1}\exp\{i(\varepsilon_{p}-\varepsilon_{k})t_{1}\}\times\int_{0}^{\infty}dt_{2}\exp\{-i(\bar{\varepsilon}_{0}-\varepsilon_{k}-i\Gamma)t_{2}\}\times\int_{0}^{\infty}dt_{3}\exp\{i(\bar{\varepsilon}_{0}-\varepsilon_{k}+i\Gamma)t_{3}\}V(t_{1},t_{2},t_{3}), \tag{2.14}$$

$$V(t_1, t_2, t_3) = \exp\Big\{-\sum_{\mathbf{q}} \frac{|\alpha_{\mathbf{q}}|^2}{2\omega_{\mathbf{q}}^2} |(1 - e^{-i\omega_{\mathbf{q}} t_2})$$

$$+ e^{i\omega_{\mathbf{q}} t_1}(e^{-i\omega_{\mathbf{q}} t_2} - 1)|^2 \operatorname{cth} \frac{\omega_{\mathbf{q}}}{2T}$$

$$- \sum_{\mathbf{q}} \frac{|\alpha_{\mathbf{q}}|^2}{2\omega_{\mathbf{q}}^2} [e^{i\omega_{\mathbf{q}} t_2} - e^{-i\omega_{\mathbf{q}} t_3}$$

$$+ e^{i\omega_{\mathbf{q}} t_1}(1 - e^{i\omega_{\mathbf{q}} t_3})(e^{-i\omega_{\mathbf{q}} t_2} - 1)) - \text{c.c.}]\Big\}, \tag{2.15}$$

where $\tilde{\varepsilon}_0$ is the energy of the resonance level with allowance for the polaron shift (1.17):

$$\tilde{\varepsilon}_0 = \varepsilon_0 - \varepsilon_{\text{pol}}. \tag{2.16}$$

[The factors $1/N$ preceding the sums over $\mathbf{q}$ in (2.14) and elsewhere will henceforth be omitted; we assume these factors to be included in the definition of the summation procedure.] It is easy to verify that in the absence of EPI ($\alpha_{\mathbf{q}} = 0$) the relations (2.14) and (2.15) determine the elastic-tunneling probability described by the Breit-Wigner formula. At $\alpha_{\mathbf{q}} \neq 0$ the inelastic-processes probabilities become different from zero. For $T = 0$, Eq. (2.15) takes the form

$$V(t_1, t_2, t_3) = \exp\Big\{-\sum_{\mathbf{q}} \frac{|\alpha_{\mathbf{q}}|^2}{\omega_{\mathbf{q}}^2} [a(t_2, t_3) - b(t_2, t_3) e^{i\omega_{\mathbf{q}} t_1}]\Big\},$$

$$a(t_2, t_3) = 2 - e^{i\omega_{\mathbf{q}} t_2} - e^{-i\omega_{\mathbf{q}} t_3},$$

$$b(t_2, t_3) = (e^{-i\omega_{\mathbf{q}} t_2} - 1)(e^{i\omega_{\mathbf{q}} t_3} - 1). \tag{2.17}$$

Expanding formally V in powers of $\exp(i\omega_{\mathbf{q}} t_1)$ we obtain terms of the form $\exp(in\omega_{\mathbf{q}} t_1)$, but with integer positive n. It is seen from (2.14) that this leads to $W_{k\to p} = 0$ at $\varepsilon_p > \varepsilon_k$. This property agrees with the general premise that there is no phonon absorption[3)] at $T = 0$.

Relation (2.14) permits calculation of the partial probabilities of the tunneling:

$$T(\varepsilon \to \varepsilon_1) = \sum_{\mathbf{k},\mathbf{p}} W_{\mathbf{k}\to\mathbf{p}} \delta(\varepsilon_{\mathbf{k}} - \varepsilon)\, \delta(\varepsilon_{\mathbf{p}} - \varepsilon_1) = \frac{1}{\pi^2} \Gamma_l \Gamma_r F(\varepsilon \to \varepsilon_1). \tag{2.18}$$

and the total tunneling probability. We present here equations for each of these quantities.

a) Total tunneling probability

Integrating (2.18) over the energy transfer, we obtain the total probability for tunneling of an electron of given initial energy ε:

$$T_t(\varepsilon) = \int d\varepsilon_1 T(\varepsilon \to \varepsilon_1)$$

$$= 2\frac{\Gamma_l \Gamma_r}{\Gamma_l + \Gamma_r} \int_{-\infty}^{\infty} dt \exp\{-\Gamma|t| + i(\varepsilon - \tilde{\varepsilon}_0) t\}$$

$$\times \exp\Big\{-\sum_{\mathbf{q}} \frac{|\alpha_{\mathbf{q}}|^2}{\omega_{\mathbf{q}}^2} [(2n_{\mathbf{q}} + 1) - (n_{\mathbf{q}} + 1) e^{-i\omega_{\mathbf{q}} t} - n_{\mathbf{q}} e^{i\omega_{\mathbf{q}} t}]\Big\}, \tag{2.19}$$

where

$$n_{\mathbf{q}} = [\exp(\omega_{\mathbf{q}}/T) - 1]^{-1}.$$

At $|a_{\mathbf{q}}| \ll \omega_{\mathbf{q}}$ the contributions to $T_t(\varepsilon)$ with participation of n phonons decrease like $|\alpha_{\mathbf{q}}/\omega_{\mathbf{q}}|^{2n}$. Taking only one-phonon processes into account, we get

$$T_t(\varepsilon) = (1 - C) T_{res}(\varepsilon - \tilde{\varepsilon}_0) + \sum_{\mathbf{q}} A_{\mathbf{q}} T_{res}(\varepsilon - \tilde{\varepsilon}_0 - \omega_{\mathbf{q}}) + \sum_{\mathbf{q}} B_{\mathbf{q}} T_{res}(\varepsilon - \tilde{\varepsilon}_0 + \omega_{\mathbf{q}}),$$

$$T_{res}(\varepsilon) = \frac{4\Gamma_l \Gamma_r}{\varepsilon^2 + (\Gamma_l + \Gamma_r)^2}, \quad A_{\mathbf{q}} = \left|\frac{\alpha_{\mathbf{q}}}{\omega_{\mathbf{q}}}\right|^2 (n_{\mathbf{q}} + 1), \tag{2.20}$$

$$B_{\mathbf{q}} = \left|\frac{\alpha_{\mathbf{q}}}{\omega_{\mathbf{q}}}\right|^2 n_{\mathbf{q}}, \quad C = \sum_{\mathbf{q}} (A_{\mathbf{q}} + B_{\mathbf{q}}).$$

In the case $|\alpha_{\mathbf{q}}| \gg \omega_{\mathbf{q}}$ the multiphonon contributions are important. The integral (2.19) can be determined by asymptotic expansion of the argument of the exponential in powers of $\omega_{\mathbf{q}} t \sim |\omega_{\mathbf{q}}/\alpha_{\mathbf{q}}|^2 \ll 1$ up to second-order terms. Under the condition

$$\Gamma^2 \ll \varepsilon_\Lambda^2(T) \equiv \sum_{\mathbf{q}} |\alpha_{\mathbf{q}}|^2 (2n_{\mathbf{q}} + 1) \tag{2.21}$$

we obtain

$$T_t(\varepsilon) = 2^{1/2}\pi^{1/2} (\Gamma_l \Gamma_r / \Gamma \varepsilon_\Lambda(T)) \exp\{-(\varepsilon - \varepsilon_0)^2 / 2\varepsilon_\Lambda^2(T)\}. \tag{2.22}$$

As seen from relation (2.22), the electron-phonon interaction leads to a substantial broadening of the resonance. It will be shown below that this is due to inclusion of inelastic scattering channels. It is important to note that such radical changes of the character of the scattering do not change the integrated transparency of the barrier. Using (2.19), we can directly verify the correctness of the following sum rule:

$$\int d\varepsilon\, T_t(\varepsilon) = \int d\varepsilon\, T_{res}(\varepsilon) = 4\pi \Gamma_l \Gamma_r / (\Gamma_l + \Gamma_r), \tag{2.23}$$

which is obtained for any value of the parameter $|\alpha_{\mathbf{q}}|/\omega_{\mathbf{q}}$.

b) Probability of elastic tunneling

We can separate in the total probability $T_t(\varepsilon)$ a term $T_{el}(\varepsilon)$ corresponding to the contributions of elastic processes that conserve the electron energy, and the partial contributions of the inelastic processes:

$$T(\varepsilon \to \varepsilon_1) = T_{el}(\varepsilon) \delta(\varepsilon - \varepsilon_1) + T_{in}(\varepsilon \to \varepsilon_1).$$

According to (2.14) and (2.18) we obtain for $T_{el}(\varepsilon)$

$$T_{el}(\varepsilon) = 4\Gamma_l \Gamma_r \int_0^\infty dt_1 \int_0^\infty dt_2 \exp\{i(\tilde{\varepsilon}_0 - \varepsilon)(t_2 - t_1) - \Gamma(t_1 + t_2)\} V(t_1, t_2), \tag{2.24}$$

where

$$V(t_1, t_2) = \lim_{t\to\infty} \frac{1}{2t} \int_{-t}^{t} d\tau\, V(\tau, t_1, t_2). \tag{2.25}$$

From expression (2.15) for the function $V(\tau, t_1, t_2)$ and from

Eqs. (2.24) and (2.25) it follows that

$$T_{el}(\varepsilon)=4\Gamma_l\Gamma_r|\mathcal{A}_{el}(\varepsilon)|^2,$$

$$\mathcal{A}_{el}(\varepsilon)=\int_0^\infty dt\exp\{-\Gamma t-i(\varepsilon-\tilde{\varepsilon}_0)t\}\times\exp\left\{-\sum_q\left|\frac{\alpha_q}{\omega_q}\right|^2[(2n_q+1)-(n_q+1)e^{-i\omega_q t}-n_q e^{i\omega_q t}]\right\}. \quad (2.26)$$

Comparison of (2.19) with (2.26) allows us to express the total probability $T_t(\varepsilon)$ in terms of the elastic-tunneling probability amplitude:

$$T_t(\varepsilon)=\frac{2\Gamma_l\Gamma_r}{\Gamma_l+\Gamma_r}[\mathcal{A}_{el}(\varepsilon)+\mathcal{A}_{el}^*(\varepsilon)],$$

this being a consequence of the optical theorem.[2]

The difference between (2.26) and the Breit-Wigner formula is determined in the weak-EPI limit by small corrections of order $|\alpha_q|^2/\omega_q^2$, which correspond to a decrease of the probability of elastic tunneling on account of inclusion of parallel inelastic channels.

In the multiphonon scattering regime ($|\alpha_q/\omega_q|^2\gg 1$), when inequality (2.21) is satisfied, the elastic cross section can be represented in the form

$$T_{el}(\varepsilon)=(4\Gamma_l\Gamma_r/\varepsilon_\Lambda^2(T))\left|\int_0^\infty d\tau\exp\{-\tau^2+i\tau(\varepsilon-\varepsilon_0)/\varepsilon_\Lambda(T)\}\right|^2. \quad (2.27)$$

It follows from (2.27) that under conditions of strong EPI the relative contribution of the elastic processes to the total tunneling probability at $|\varepsilon-\varepsilon_0|\lesssim\varepsilon_\Lambda(T)$ is small in terms of the parameter $\Gamma/\varepsilon_\Lambda(T)\ll 1$ and the principal role is played by the elastic channels (see Fig. 1).

The transformation of $T_{el}(\varepsilon)$ with increase of the EPI force is easily seen. At $|\alpha_q/\omega_q|\ll 1$ we have a resonant Breit-Wigner peak at $\varepsilon\approx\varepsilon_0$, with wings corresponding to resonant tunneling with participation of virtual phonons. With increase of $|\alpha_q|$ the intensity of the wings increases and the position of the resonance peak shifts in accord with the increase of the polaron shift ε_{pol}. For strong EPI, the main contribution (2.27) to $T_{el}(\varepsilon)$ is due in fact to the wings. The "memory" of the Breit-Wigner peak remains only in the form of the correction

$$\delta T_{el}(\varepsilon)=\frac{4\Gamma_l\Gamma_r}{(\varepsilon-\tilde{\varepsilon}_0)^2+\Gamma^2}\exp\left\{-\sum_q\frac{|\alpha_q|^2}{\omega_q^2}(2n_q+1)\right\}. \quad (2.28)$$

This correction is integrally small. Under the condition $|\alpha_q/\omega_q|^2\gg\ln(\varepsilon_\Lambda(T)/\Gamma)$, which differs from the strong-EPI condition (1.3) only by a logarithmic factor, the amplitude of the correction (2.28) is also small compared with the maximum value of (2.27).

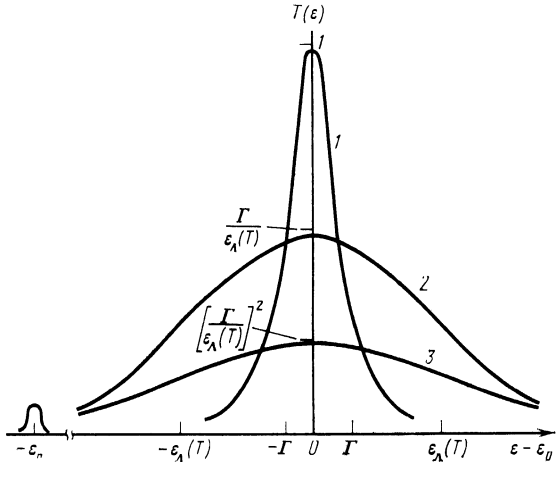

FIG. 1. Energy dependences of the resonant-tunneling probabilities. Curve (1) corresponds to the Breit-Wigner formula, which is valid in the absence of EPI; curves (2) and (3) correspond to the total and elastic cross sections $T_t(\varepsilon)$ and $T_{el}(\varepsilon)$ for strong EPI, $\Gamma_l=\Gamma_r$.

c) Partial cross sections in inelastic channels

The quantity $T_{in}(\varepsilon\to\varepsilon_1)$ depends on the detuning $\varepsilon-\varepsilon_0$ and on the energy $E=\varepsilon-\varepsilon_1$ transferred to the phonons. In the case of strong EPI the T_{in} are largest when the absolute value of the detuning does not exceed $\varepsilon_\Lambda(T)$. The characteristic energy transfers E are due to multiphonon processes and therefore exceed ω_D substantially. We therefore present first an expression for $T_{in}(\varepsilon\to\varepsilon_1)$ obtained for the values $\varepsilon=\varepsilon_0$ and $E\gg\omega_D$. Using the saddle-point method and the last inequality to calculate the integrals in (2.14), we get

$$T_{in}(E)|_{\varepsilon=\varepsilon_0}=\frac{\pi^{1/2}(\Gamma_l\Gamma_r/\Gamma)}{\varepsilon_\Lambda(T)(\Delta\varepsilon|E|)^{1/2}}\times\exp\left\{-\left(\frac{|E|}{\Delta\varepsilon}\right)^{1/2}-\frac{\varepsilon_\Pi E}{\varepsilon_\Lambda^2(T)}\theta(-E)\right\}. \quad (2.29)$$

Here

$$\Delta\varepsilon=\sum_q|\alpha_q|^2\omega_q/\Gamma^2. \quad (2.30)$$

The saddle-point asymptote in (2.14) and expression (2.19) are valid under the condition (2.21) and for energy transfers $|E|\ll\varepsilon_{pol}$. In the case of a sufficiently short electron lifetime at the center ($\Gamma^{-1}\ll\omega_D^{-1}$) there is not enough time for polaron renormalization of the quasilocal-state energy and the characteristic value of $|E|$ is determined by Eq. (2.30). In the opposite case $\Gamma\ll\omega_D$ the characteristic $|E|\approx\varepsilon_{pol}$. At these values, the saddle point method cannot be used in (2.14), but an order-of-magnitude estimate of the cross section $T_{in}(E\sim\varepsilon_{pol})$ can be obtained as before with the aid of Eq. (2.29). The "tail" of T_{in} is exponentially small for large energy transfers $E>2\varepsilon_{pol}$ and is determined at $\Gamma\ll\omega_D$ by the relation $\ln T_{in}(E)\sim-(E-2\varepsilon_{pol})/\varepsilon_\Lambda(T)$. Note that the cross section (2.29) is asymmetric in E all the way to temperature $T\sim\varepsilon_{pol}$ that exceed the Debye temperature substantially. This is due to the decisive contribution of multiphonon processes to the scattering. At small energy transfers $E\lesssim\omega_D$ and low temperatures $T\ll\omega_D$ the correct relation is not (2.29) but

$$T_{in}(E)\sim\frac{\Gamma_l\Gamma_r}{\varepsilon_\Lambda^2\omega_D}\ln\left(\frac{\omega_D}{E}\right)\theta(E). \quad (2.31)$$

In the case of weak EPI the formula for $T(\varepsilon\to\varepsilon_1)$ can be simplified by expanding V in (2.15) in powers of the small parameter $|\alpha_q/\omega_q|^2$. Greatest interest attaches to the limit

$\Gamma \ll \omega_D$, when the central peak is much narrower than the resonance-line wings. The cumbersome equation for $T(\varepsilon \to \varepsilon_1)$ takes then the simpler form

$$T(\varepsilon \to \varepsilon_1) = \frac{4\Gamma_l\Gamma_r}{(\varepsilon-\tilde{\varepsilon}_0)^2+(\Gamma_l+\Gamma_r)^2} + \left[\frac{4\Gamma_l\Gamma_r}{(\varepsilon-\tilde{\varepsilon}_0)^2+(\Gamma_l+\Gamma_r)^2}+\frac{4\Gamma_l\Gamma_r}{(\varepsilon_1-\tilde{\varepsilon}_0)^2+(\Gamma_l+\Gamma_r)^2}\right] \times\left[\sum_q A_q\delta(\varepsilon-\varepsilon_1-\omega_q)+\sum_q B_q\delta(\varepsilon-\varepsilon_1+\omega_q)-C\delta(\varepsilon-\varepsilon_1)\right]. \quad (2.32)$$

The quantities A_q, B_q, and C are defined in (2.20).

3. TUNNELING THROUGH AN AMORPHOUS DIELECTRIC LAYER

We have clarified above the character of the influence of the EPI on the elementary resonant-tunneling act. The manifestations of the EPI in experiments depend on the specific method of formation of the tunnel layer and the resonance states. On the basis of the experiments of Beasley *et al.*[13,14] we consider resonant tunneling through an amorphous dielectric interlayer containing a large number of localized states. Their energies are distributed in a wide interval. The states participating in the transport of the charge through the junction are those whose energies are close to the Fermi level within the limits of the temperature-spreading or of the energy unbalance due to the voltage $\mathcal{U}$ applied to the junction.

We consider first the manifestations of the resonant tunneling in the conductance G of the junction in the absence of EPI. The states actively participating in resonant processes are those localized near the midpoint of the barrier ($|x| \lesssim a_0$) and located (for $T=0$) within an energy band $\sim\Gamma$ near the Fermi level. The two-dimensional density of such states is $n_\square \sim N(\varepsilon_F)a_0\Gamma$, where $N(\varepsilon_F)$ is the density of states in the amorphous layer and ε_F is the Fermi energy determined by the edges of the junction. The conductance

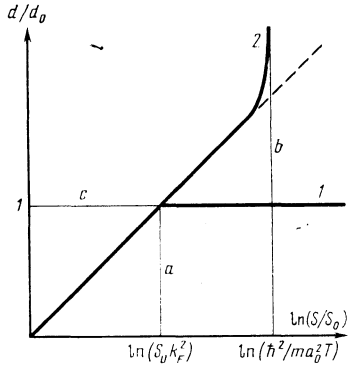

FIG. 2. Different regions of the characteristic areas S of the junction and the dielectric thickness d. The conductivity is determined by direct tunneling in the region a and by resonant tunneling in region b; the fluctuations of G are large in region c. The dashed line separates the regions b and c at $T=0$.

concept has meaning for junctions of area $S \gg n_\square^{-1}$. In the opposite case the characteristics of the junction depend on the specific arrangement of the impurities in it (the fluctuations of G are large). Taking into account the order-of-magnitude estimate $\Gamma \sim (\hbar^2/ma_0^2)\exp(-d/a_0)$, we obtain the conditions that the parameters S and d must meet for the fluctuations of G to be small (see Fig. 2):

$$\ln(S/S_0) > d/a_0, \quad S_0 = ma_0/\hbar^2 N(\varepsilon_F). \quad (3.1)$$

At a finite temperature $T > \Gamma$ the fluctuations are small for samples with area $S > (N(\varepsilon_F)a_0T)^{-1}$.

Consider now the case of large-area junction, when the fluctuations of G are small. For the resonant component of the current

$$I_r \propto S n_\square \propto \exp(-d/a_0)$$

to predominate in G over the usual tunnel component

$$I_{nr} \propto S k_F^2 \exp(-2d/a_0),$$

the insulator thickness should satisfy the condition[4)] (see Fig. 2)

$$d > d_0, \quad d_0 = a_0 \ln(S_0 k_F^2). \quad (3.2)$$

If the inequality is reversed, direct tunneling through the barrier prevails. Thus, for a set of junctions with increasing value of d and fixed area $S > S_0^2 k_F^2$ a crossover should be observed, viz., a transition from a relation $\ln G(d) \propto -2d/a_0$ at $d < d_0$ to $\ln G(d) \propto -d/a_0$ at $d > d_0$ (the crossover corresponds to passage through line 1 of Fig. 2).

We illustrate now the modification of the junction behavior by the EPI. The contribution from each resonant state to the current through the junction is given by

$$I_i(\mathcal{U}) = \frac{e}{\pi\hbar}\int d\varepsilon_1\, d\varepsilon_2\, T_i(\varepsilon_1 \to \varepsilon_2) \times\left\{f_0\left(\varepsilon_1-\frac{e\mathcal{U}}{2}\right)\left[1-f_0\left(\varepsilon_2+\frac{e\mathcal{U}}{2}\right)\right] -f_0\left(\varepsilon_1+\frac{e\mathcal{U}}{2}\right)\left[1-f_0\left(\varepsilon_2-\frac{e\mathcal{U}}{2}\right)\right]\right\} \quad (3.3)$$

(f_0 is the Fermi distribution function of the electrons at the edges). For a large junction area [see (3.1)] the fluctuations are small and a reasonable mean value of $I(\mathcal{U})$ is the value of $T_i(\varepsilon_1 \to \varepsilon_2)$ averaged over the locations and energies ε_i of the defects:

$$\langle T(\varepsilon_1 \to \varepsilon_2)\rangle = N(\varepsilon_F) S\Gamma^2 \int_{-d/2}^{d/2} dx \int_{-\infty}^{\infty} dt_1 \int_0^{\infty} dt_2 \exp\{i(\varepsilon_2-\varepsilon_1)t_1 - 2\Gamma \operatorname{ch}(x/a_0)t_2\} V(t_1,t_2,t_2). \quad (3.4)$$

It can be seen from (3.3) that at temperatures $T \ll \max(\varepsilon_\Lambda, \omega_D)$ the order of magnitude of G is determined by the elastic part $\langle T(\varepsilon_1 \to \varepsilon_1)\rangle_{el}$ of the transition probability:

$$G \approx (e^2/\pi\hbar)\langle T(\varepsilon_F \to \varepsilon_F)\rangle_{el}.$$

In the case of strong EPI (1.3) we easily obtain with the aid of (2.15), (2.21), and (3.4)

$$\langle T(\varepsilon_F \to \varepsilon_F)\rangle_{el} = N(\varepsilon_F) S a_0 \ln(\varepsilon_\Lambda/\Gamma)\Gamma^2/\varepsilon_\Lambda \quad (3.5)$$

and consequently $\ln G(d) \propto -2d/a_0$, just as in the case of nonresonant tunneling (only the pre-exponential factors in G differ), and there is no crossover.

Crossover as a function of $\ln G(d)$ was distinctly observed[13,14] in samples with $S \sim 10^{-4}$ cm^2. An estimate of d_0

from Eq. (4.2) using the values[14] $a_0 \approx 8$ Å, $N(\varepsilon_F) \approx 10^{19}$ $\text{eV}^{-1} \cdot \text{cm}^{-3}$ agrees very well with the experimentally observed[14] value[5)] $d_0 \approx 70$ Å. The crossover is evidence of sufficiently weak EPI in the experiment of Ref. 14. We restrict ourselves to this case. Using (2.32) and (3.4) we get

$$\langle T(\varepsilon_1 \to \varepsilon_2) \rangle = N(\varepsilon_F) S \Gamma a_0 \cdot 4\pi^2 \Big\{ (1-C)\delta(\varepsilon_1 - \varepsilon_2) + \sum_q A_q \delta(\varepsilon_1 - \varepsilon_2 - \omega_q) + \sum_q B_q \delta(\varepsilon_1 - \varepsilon_2 + \omega_q) \Big\}. \tag{3.6}$$

It is seen from (3.6) that the conductivity agrees, apart from small corrections $\sim (\alpha/\omega_D)^2$, with the value of G in the absence of EPI, $G_0 \approx (e^2/\pi\hbar) N(\varepsilon_F) \Gamma a_0 S$. It is just these corrections, however, which determine the temperature dependence of $G(t)$. Being interested in the region of sufficiently low temperatures $T \ll \omega_D$, we take into account in (3.6) only acoustic phonons. The sign of the increment $\delta G(T) \equiv G(T) - G(0)$ is not obvious beforehand, since the increase in the number of inelastic channels with rise of temperature can be offset by the decrease of the intensity of an elastic channel on account of the temperature dependence of $C(T)$ in (3.6). Simple calculations enable us to verify that there is no complete cancellation and that $\delta G(T) > 0$. The quantity $\delta G(T)$ can be represented in the form

$$\delta G(T)/G(0) \sim (\alpha_D/\hbar\omega_D)^2 (T/\hbar\omega_D)^2. \tag{3.7}$$

The characteristic value of α_D is determined here by the relation

$$(\alpha_D/\hbar\omega_D)^2 = \Lambda^2/M\hbar s^3 q_D.$$

The conclusion that $G(T)$ is a quadratic function at low temperatures agrees with the experimental results of Ref. 13. The temperature dependence of $G(T)$ in the case of strong EPI differs substantially from (3.7), for it follows from (2.31) and (3.4) that

$$\delta G(T)/G(0) \sim (T/\hbar\omega_D) \ln (\hbar\omega_D/T).$$

Another manifestation of EPI is its influence on the behavior of the current-voltage characteristics (IVC) of the junction. For large-area junctions, at small displacements $e\mathcal{U}$, the nonlinearity is due entirely to the EPI. At $e\mathcal{U} \ll \hbar\omega_D$ the nonlinearity of the conductivity can be estimated from (3.7) by replacing T in it by $e\mathcal{U}$. More interesting manifestations of EPI appear, however, in the case

$$S \lesssim S_0 (S_0 k_F^2)^{d/d_0},$$

when the IVC of the junction reflects a definite realization of a random distribution of the resonant states in energy and in space. Let us assess the influence exerted on the IVC at low temperatures ($T \lesssim \Gamma$) by one resonant level for which $\varepsilon_0 - \varepsilon_F \equiv \Delta > 0$. In the absence of EPI it would lead to the appearance on the IVC of a step of height

$$\delta I_0 \sim e\Gamma/\pi\hbar \tag{3.8}$$

at a voltage $\mathcal{U} = 2\Delta/e$ on the junction. The width of this step is $\sim \Gamma/e$, and the derivative $dI/d\mathcal{U}$ is equal to zero both ahead of and behind the step (see Fig. 3). Weak EPI, while hardly changing the size of the step (3.8), determines the change of $I(\mathcal{U})$ past the step. With the aid of expressions (2.32) and (3.3) we easily obtain

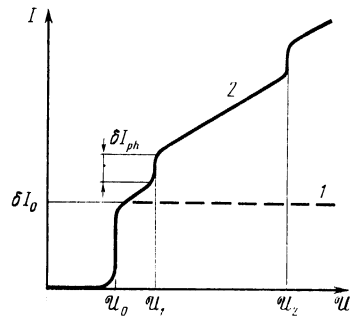

FIG. 3. $I(\mathcal{U})$ dependence for one resonance center: 1) $I(\mathcal{U})$ for $\alpha_q = 0$; 2) $I(\mathcal{U})$ for weak EPI with acoustic modes and a local one.

$$\delta I \sim \frac{e\Gamma}{\pi\hbar} \theta\left(\frac{e\mathcal{U}}{2} - \Delta\right) \Big\{ 1 - \sum_q \left|\frac{\alpha_q}{\hbar\omega_q}\right|^2 \Big[\theta\left(\hbar\omega_q - \frac{e\mathcal{U}}{2} + \Delta\right) + \theta\left(\hbar\omega_q - \frac{e\mathcal{U}}{2} - \Delta\right) \Big] \Big\}. \tag{3.9}$$

It follows from (3.9) that acoustic phonons lead to a smooth growth of $\delta I(\mathcal{U})$ at $e\mathcal{U} > 2\Delta$. If the resonance level is close enough to the Fermi level ($2\Delta < \hbar\omega_D$), the current jump δI is accompanied by a derivative jump

$$\delta \frac{dI}{d(e\mathcal{U})}\Big|_{\mathcal{U}=2\Delta} \sim \frac{e\Gamma}{\pi\hbar} \left|\frac{\alpha_D}{\hbar\omega_D}\right|^2 \frac{\Delta}{(\hbar\omega_D)^2} \tag{3.10}$$

(see Fig. 3). Under the condition $2\Delta > \hbar\omega_D$ the last factor in (3.10) must be replaced by $(e\mathcal{U} - 2\Delta)/\hbar\omega_D$, i.e., $d^2I/d\mathcal{U}^2$ also has a jump. An increase of the derivative $dI/d\mathcal{U}$ after each step on the $I(\mathcal{U})$ plot was experimentally observed in Ref. 14.

From relations (2.32) and (3.3) follows a small value $\sim |\alpha_D/\hbar\omega_D|^2 (T/\hbar\omega_D)^2$ of the temperature corrections to the value of (3.8), i.e., the heights of the resonance peaks on the plot of $dI/d\mathcal{U}$ vs $\mathcal{U}$ should vary insignificantly with temperature. This agrees likewise with the experimental data of Ref. 14.

If the phonon spectrum contains a local mode of frequency ω_l, which interacts with the resonance center, the main step on the $I(\mathcal{U})$ plot is accompanied by additional steps—phonon replicas (Fig. 3). Analyzing expression (3.9), we easily see that under the condition $\hbar\omega_l > 2$ there are two replicas at displacements $e\mathcal{U}_1 = 2(\hbar\omega_l - \Delta)$ and $e\mathcal{U}_2 = 2(\hbar\omega_l + \Delta)$. The height of the additional steps is $\delta I_{ph} \sim |\alpha_l/\hbar\omega_l|^2 \delta I_0$. We point out that the positions of the main step $e\mathcal{U} = 2\Delta$ and of the replicas are related by

$$\mathcal{U}_2 - \mathcal{U}_1 = 2\mathcal{U}_0. \tag{3.11}$$

If, however, $\hbar\omega_l < 2\Delta$, there remains only one phonon replica at $\mathcal{U} = \mathcal{U}_2$.

Well resolved individual $I(\mathcal{U})$ steps and, correspondingly, peaks on the plot of $dI/d\mathcal{U}$ vs $\mathcal{U}$ were observed in Ref. 14 in samples with $S \sim 1.8 \cdot 10^{-9}$ cm^2 and $d = 51$ Å. A sharp peak was observed for one of the samples at a rather low voltage $\mathcal{U}_0 \approx 3.8$ mV. One cannot exclude the possibility that the two succeeding peaks of lower intensity, at voltages $\mathcal{U}_1 \simeq 10$ mV and $\mathcal{U}_2 \approx 17.5$ mV and corresponding to the condition (3.11) are phonon replicas of the first. The cited voltages lead to a reasonable value $\hbar\omega_l \approx 6.9$ mV. The ratio of the peak amplitudes leads to an estimated ratio $|\alpha_l/\hbar\omega_l| \approx 0.6$.

4. CONCLUSION

The results of the present paper for electron tunneling through a barrier can also be used, following obvious modifications, to calculate the cross section for resonant scattering of carriers by impurities in a conducting medium. We note in this context that, as shown above, strong EPI with the impurity leads to a considerable smearing of the line and to the appearance of inelastic scattering channels. A similar configuration of the energy dependence of the scattering cross section and onset of a temperature dependence of the cross section should influence strongly the temperature and field dependences of transport processes in conductors with resonant scatterers. In addition, at sufficiently high density of the centers (e.g., in compounds with intermediate valence), such a broadening should be accompanied by a temperature-dependent broadening of the peak in the density of the electronic state, and be reflected by the same token in the kinetics of such systems. An experimental study of the discussed effect of polaron renormalization of resonant scattering is possible with the aid of tunnel and microjunction injection of hot carriers. Note that besides the contributions to transport effects, a specific "phonon emission" from resonant impurities is produced and records the passage of the electron through scattering centers.

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[1)]Equation (1.1) is written for the case of a symmetric location of the impurity center relative to the barrier boundaries.

[2)]Note that at $q \approx q_D \approx 1/a$ the parameter $|\omega_q/\omega_q|$ is of the same order as the previously introduced parameter u_A/u_0 [see Eqs. (1.3) and (1.9)].

[3)]Note that equations similar to (2.14) and (2.15), derived less rigorously (by a modification of second-order perturbation theory[11,12]), were used by V. V. Khizhnyakov to investigate Raman scattering of light by excitons. This circumstance was pointed out to the author by I. B. Levinson.

[4)]At much larger thicknesses $d \sim S_0^{1/2}$, which we shall not consider, two-impurity configurations become important.[5]

[5)]Under the conditions of Ref. 14, $S_0 \approx 10^{-11}$ cm^2 and the characteristic area (see Fig. 2) is $S_0(S_0 k_F^2) \approx 10^{-11}$ cm^2.

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