Energy relaxation of degenerate hot electrons: population waves and Burgers turbulence along the energy axis

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The transport equation for the motion of a degenerate gas along the energy axis during interactions with a thermalized distribution is discussed. The equation is analogous to the Burgers equation for the one-dimensional flow of a compressible gas. The structure of the stationary state is investigated in the presence of a pump and recombination. Population waves, similar to shock waves in the gas, and the propagation of "sound" along the energy axis are examined. One-dimensional Burgers turbulence develops along the energy axis when the pump intensity is a random function of time.

INTRODUCTION

The advent of femtosecond lasers has resulted in rapid advances in the kinetics of hot photoelectrons in semiconductors because it is now possible to investigate the time-dependence of the distribution function.[1–3] The significant feature of experiments with femtosecond lasers is that a large number of electrons is produced and, although the electrons do not reach the bottom of the band, degeneracy usually appears during the operation of the pump or during relaxation. It is then interesting to consider the effect of local degeneracy on relaxation and to examine the type of collective behavior induced by the nonlinearity of the transport equation due to degeneracy.

In contrast to transport in coordinate space, transport in momentum space is characterized by nonlocal behavior due to the collision integral. We shall consider the situation where it is possible to select directions in momentum space along which transport is due to small momentum transfer (modulus of momentum) and the transport equation can be written in the diffusion approximation.

At the same time, the entire behavior of a gas (liquid) in coordinate space, described by the Navier-Stokes equations, can be simulated by giving the molecules a repulsion at short distances and attraction at large distances in the microscopic description. The Pauli principle plays an analogous role in momentum space. Two electrons effectively "repel" because they cannot occupy the same point in momentum space, which means that neighboring points effectively "attract" them. This means that many of the properties of flows in coordinate space can be established for flows in momentum space. In particular, there is an analogy between the "flow" of electrons along the energy axis ε in quasielastic scattering by a thermostat at temperature T and the one-dimensional flow of a compressible gas. The transport equation is identical with the hydrodynamic equation which, in this case, is the Burgers equation.[4,5] The temperature T plays the part of viscosity (or, more precisely, dissipation) in the transport equation, and degeneracy plays the part of compressibility (nonlinearity). The Reynolds number is introduced as a measure of nonlinearity and viscosity (degeneracy).

We shall examine in detail the form of the stationary distribution in the presence of a pump and of recombination.

The Burgers equation describes the propagation of shock waves which, in the case of flow in momentum space, will be referred to as population waves. In this sense, the Fermi distribution is a standing population wave. We shall discuss some of the properties of these waves, such as the propagation of a single crest produced by the pump pulse, propagation of a kind of "sound" along the energy axis, and attenuation of the sound as a result of viscosity and nonlinearity. When the pump power is a random function of time, the statistical properties of the distribution function reduce to the so-called Burgers turbulence (Refs. 4, 6, 7). The turbulence develops in a self-similar manner as electrons relax downward along the energy axis.

1. TRANSPORT EQUATION

The transport equation in the diffusion approximation for quasielastic scattering by the thermostat will be derived in the usual way.[8] In the initial collision integral

$$S(\varepsilon)=-\frac{1}{g(\varepsilon)}\frac{d}{d\varepsilon}[g(\varepsilon)J(\varepsilon)],\tag{1}$$

written in terms of the flux $J(\varepsilon)$ along the ε axis and the density of states $g(\varepsilon)$, i.e.,

$$\begin{aligned}-g(\varepsilon)J(\varepsilon)&\\ =\int_\varepsilon^\infty\int_0^\varepsilon d\varepsilon'\,d\varepsilon''g(\varepsilon')g(\varepsilon'')&[f(\varepsilon')W(\varepsilon',\varepsilon'')(1-f(\varepsilon''))\\ &-f(\varepsilon'')W(\varepsilon'',\varepsilon')(1-f(\varepsilon'))],\end{aligned}\tag{2}$$

we assume that the scattering probability $W(\varepsilon,\varepsilon')$ satisfies the principle of detailed balancing and has the form

$$W(\varepsilon,\varepsilon')=\begin{cases}K(-\omega), & \varepsilon>\varepsilon'\\ K(\omega)e^{-\omega/T}, & \varepsilon<\varepsilon'\end{cases};\quad \omega=\varepsilon'-\varepsilon.\tag{3}$$

The symmetric part of $K(\omega)$ must be large for values of ω that are small in comparison with the characteristic scales of the distribution, e.g., in comparison with the temperature T. This enables us to expand $f(\varepsilon)$ in the integrand of (2), so that the flux assumes the differential form

$$-J(\varepsilon)=A(\varepsilon)\left[f(\varepsilon)-f^2(\varepsilon)+T\frac{df(\varepsilon)}{d\varepsilon}\right],\tag{4}$$

where the transport coefficient $A(\varepsilon)$ can be expressed in terms of the second moment of $K(\omega)$:

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$$A(\varepsilon)=\frac{g(\varepsilon)}{T}\int_0^\infty d\omega\,\omega^2K(\omega). \tag{5}$$

This integral must be evaluated for $\omega \ll T$.

The expression for the flux (4) differs from the flux in the nondegenerate case by the term $f^2(\varepsilon)$. Hence, degeneracy affects dynamic friction, which now occurs at the rate $A(\varepsilon)[1-f(\varepsilon)]$, but does not affect diffusion, the diffusion coefficient being given by $D(\varepsilon) = TA(\varepsilon)$. The flux associated with dynamic friction has the maximum value $J = A/4$ at $f=\frac{1}{2}$.

Let us now compare the transport equation containing the collision integral (1), (4)

$$\frac{\partial f}{\partial t}=\frac{1}{g}\frac{\partial}{\partial\varepsilon}\left[gA\left(f-f^2+T\frac{\partial f}{\partial\varepsilon}\right)\right], \tag{6}$$

with the hydrodynamic Burgers equation describing the one-dimensional flow of weakly-dissipating, weakly-nonlinear gas (Ref. 5, p. 495):

$$\frac{\partial v}{\partial t}=s\frac{\partial v}{\partial x}-\alpha_v v\frac{\partial v}{\partial x}+as^3\frac{\partial^2 v}{\partial x^2}; \tag{7}$$

where $v(x,t)$ is the flow velocity of the gas, s is the velocity of sound, α_v is determined by compressibility and is given by α_v $(\gamma+1)/2$ for a polytropic gas, $\gamma = c_p/c_v$ (the ratio of specific heats at constant pressure and volume, respectively), a is a measure of dissipation and is given by

$$a=\frac{1}{2\rho s^3}\left[\left(\frac{4}{3}\eta+\zeta\right)+\varkappa\left(\frac{1}{c_v}-\frac{1}{c_p}\right)\right], \tag{8}$$

ρ is the density of the gas, η, ζ are the viscosity coefficients, and $\varkappa$ is the thermal conductivity.

If we neglect the dependence of the coefficients g, A on ε, then (6) and (7) are found to be identical. This means that the variation in compressibility with pressure is analogous to degeneracy and, owing to viscosity and thermal conduction, dissipation is analogous to diffusion along the energy axis, determined by the temperature T. It follows from (6) that the relative contribution of nonlinear and dissipative effects can be characterized by the Reynolds number $\mathrm{Re} = j\Delta\varepsilon T^{-1}$, where f and $\Delta\varepsilon$ are the characteristic amplitude and the scale of the distribution, respectively.

Let us briefly consider the validity of (6), e.g., for a three-dimensional electron gas. Phonons and thermalized electrons and/or holes play the role of the thermostat described by the probability (3). It is readily verified that quasielastic scattering by phonons (acoustic or optical) usually predominates at low concentrations at which the degeneracy of the electrons is not significant. To allow for scattering of carriers by carriers, it is simplest to consider a p-type sample in which the concentration of nonequilibrium electrons is less than that of equilibrium holes ($n_e \ll n_p$). We shall confine our attention to this situation. The segment of the energy axis along which the relaxing electrons travel after photoproduction must lie in the passive region $\varepsilon < \hbar\Omega_0$, where Ω_0 is the frequency of an optical phonon) if the concentration of thermalized holes is relatively low, and can be extended to the active region ($\varepsilon > \hbar\Omega_0$) if there are enough holes.[3]

2. STRUCTURE OF STATIONARY STATE

In this section, we shall examine the form of the distribution function $f(\varepsilon)$ that is established for given rates of pumping and recombination. For a nondegenerate gas, the distribution function $f(\varepsilon)$ is usually determined for a given flux $J(\varepsilon)$ along the energy axis. It will be seen below that this problem is not properly defined for a degenerate gas.

We wish to find the stationary solution of the equation

$$\frac{\partial f}{\partial t}=\frac{1}{g}\frac{\partial}{\partial\varepsilon}\left[gA\left(f-f^2+T\frac{\partial f}{\partial\varepsilon}\right)\right]-\frac{f(\varepsilon)\delta(\varepsilon)}{g(0)\tau_R}+\frac{G_0L^3}{g(\varepsilon_0)}(1-f(\varepsilon))\delta(\varepsilon-\varepsilon_0), \tag{9}$$

which explicitly includes recombination at $\varepsilon = 0$ at the rate τ_R^{-1} and a pump at the point $\varepsilon = \varepsilon_0$ with a pumping rate G_0L^3 (L is a linear dimension).

We shall consider this problem in the simplest case, for which $(g(\varepsilon)A(\varepsilon) = \mathrm{const})$. This occurs, for example, for electron-electron (electron-hole) scattering in a nondegenerate $3D$-gas, or for scattering by acoustic phonons in a $2D$-gas.[3] Integrating (9) with respect to ε, we obtain, in the time-independent case, a Riccati equation

$$f_y'+f-f^2=\varkappa,\quad y=\varepsilon/T \tag{10}$$

and the condition

$$\alpha f(0)=\mu(1-f(\varepsilon_0)), \tag{11}$$

where, by definition, the dimensionless flux is given by

$$\varkappa=\begin{cases}0, & y<0 \quad\text{or}\quad y>y_0\\ \alpha f(0), & 0<y<y_0, \qquad y_0=\varepsilon_0/T\end{cases} \tag{11a}$$

and we have introduced the dimensionless pump and recombination rate constants

$$\mu=G_0L^3/g(\varepsilon)A(\varepsilon),\ \alpha=1/\tau_R g(\varepsilon)A(\varepsilon).$$

After the integration of (10), the result, i.e., the function $f(\varepsilon)$, depends parametrically on $\varkappa$ and on the undetermined constant $\bar{y} = \bar{\varepsilon}/T$. Conditions (11) and (11a) are required to determine these parameters.

Consider the quadratic three-term expression $\mathcal{T}(f) = f^2 - f + \varkappa$ with the discriminant $1/4 - \varkappa$. When the discriminant is positive, $\lambda^2 \equiv 1/4 - \varkappa$, $0 \leqslant \lambda \leqslant \frac{1}{2}$, we have $\mathcal{T}(f) = (f - 1/2 - \lambda)(f - 1/2 + \lambda)$, and the solution has the form

$$f(y)={}^1/_2-\lambda\ \mathrm{th}\ [\lambda(y-\bar{y})],\quad {}^1/_2-\lambda<f<{}^1/_2+\lambda, \tag{12}$$

$$f(y)={}^1/_2-\lambda\ \mathrm{cth}\ [\lambda(y-\bar{y})],\quad f<{}^1/_2-\lambda\ \text{ or }\ f>{}^1/_2+\lambda. \tag{13}$$

This shows that f_y' does not vanish for finite y, so that f does not cross the point $1/2 \pm \lambda$. In the case of a negative discriminant, $\gamma^2 \equiv \varkappa - 1/4$, $\gamma > 0$, we have the solution

$$f(y)={}^1/_2+\gamma\ \mathrm{tg}\ [\gamma(y-\bar{y})],\quad -\pi/2<\gamma(y-\bar{y})<\pi/2. \tag{14}$$

The regions in which the individual solutions of (12)–(14) are realized for $0 < y < y_0$ (this occurs in a unique manner) are conveniently represented on the α, μ plane (Fig. 1). It is clear, in view of (12)–(14), that conditions (11) and (11a) are transcendental equations in $\varkappa$ and $\bar{y}$. The analysis can be extended further in the case of the strong pump $\varepsilon_0 \gg T$,

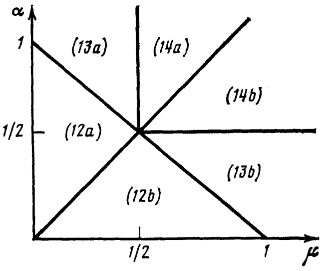

FIG. 1. Different types of stationary solution $f(\varepsilon)$ of (12)–(14) on the α, μ plane with $\varepsilon_0 \gg T$.

$y_0 \gg 1$ in which we are interested here. According to (10)–(14), the determination of $\varkappa$ and $\bar{y}$ is not difficult, but turns out to be very laborious. Here, we merely note that the original equation (6) transforms into itself when we introduce the replacement $f \to 1 - f$, $\varepsilon \to \varepsilon_0 - \varepsilon$, $\alpha \leftrightarrow \mu$, so that it is sufficient to consider the region $\alpha \geqslant \mu$, which is labeled with the index a in Fig. 1. The answers are as follows:

$$\lambda = {}^1/_2 - \mu, \quad \bar{y} = \lambda^{-1}\,\mathrm{Arth}\,\{\lambda^{-1}[\mu\alpha^{-1}(1-\mu) - {}^1/_2]\},$$
$$f(0) = \mu\alpha^{-1}(1-\mu), \; f(y_0) = \mu; \tag{12a}$$

$$\bar{y} = \lambda^{-1}\,\mathrm{Arcth}\,\{\lambda^{-1}[\mu\alpha^{-1}(1-\mu) - {}^1/_2]\}, \tag{13a}$$

$\lambda, f(0), f(y_0)$ are given by Eqs. (12a);

$$\gamma = \pi\left\{y_0 + 4\frac{4\alpha\mu - \alpha - \mu}{(2\mu-1)(2\alpha-1)}\right\}^{-1},$$

$$\bar{y} = \frac{y_0}{2} + 2\frac{\alpha - \mu}{(2\mu-1)(2\alpha-1)},$$

$$f(0) = 1/4\alpha, \; f(y_0) = 1 - 1/4\mu. \tag{14a}$$

In the region in which $\alpha < \mu$ (labeled as b in Fig. 1), the formulas are obtained by introducing the above replacement. The distribution at $y = y_0$ does not depend on α for $\alpha > \min(\mu, 1/2)$. The function $f(y)$ depends on the rate of recombination for all y, and this means that the search for a stationary solution with a given flux is an undefined problem for $\alpha < \min(\mu, 1/2)$. We note that, in the nondegenerate case, the analogous condition is much more stringent $\alpha \lesssim e^{-y_0}$, and is absent altogether in the limit that we are considering here ($y_0 \gg 1$).

Analysis of the solutions given by (12)–(14) shows that they are stable and that the stability is absolute or "convective." In the latter case, before the fluctuating perturbation reaches an appreciable magnitude, it is either carried outside the region $0 < y < y_0$ or to the point $f(y) = \frac{1}{2}$. The absolutely stable solutions (13), (14) are those for which $f'_y > 0$, and convective stability sets in when the sign in (12) is reversed. Solutions (12), (13) are characterized by Reynolds number $\mathrm{Re} \sim 1$, and solution (14) by $\mathrm{Re} \sim y_0 \gg 1$.

3. POPULATION WAVES

Among the nonstationary problems that can be solved analytically for (6), we begin with the problem of the propagation of the wave $f(\varepsilon, t) = f(\varepsilon - Ut)$, subject to the initial condition $f(-\infty) = f_1, f(+\infty) = f_2, f_1 > f_2$. Equation (6) is formally extended to the entire ε axis: $-\infty < \varepsilon < \infty$. In reality, the width of the region in which the wave propagates must be much greater than the width of the wave. The problem arises, for example, when the rate of generation changes from $\mu = \alpha - \delta\mu$ to ($\mu = \alpha + \delta\mu$ ($\delta\mu \ll \alpha$, $\alpha < 1/2$)). Although the pump intensity has increased only slightly, the point $\bar{\varepsilon} = \bar{y}T$ (12a), which has an undetermined position for $\mu = \alpha$, leaves the neighborhood of $\varepsilon = 0$ for the neighborhood of $\varepsilon = \varepsilon_0$. A population wave therefore propagates from $\bar{\varepsilon} \sim T$ to $\varepsilon_0 - \bar{\varepsilon} \sim T$.

For simplicity, we shall neglect the dependence of $g(\varepsilon)$ and $A(\varepsilon)$ on ε. Substituting $\tau = tAT^{-1}, y = \varepsilon T^{-1}$ in (6), we then obtain

$$f_\tau' = (f - f^2 + f_y')_y'. \tag{15}$$

The required solution has a form analogous to (12):

$$f(y, \tau) = \frac{f_1 + f_2}{2} + \frac{f_1 - f_2}{2}\,\mathrm{th}\left[\frac{(f_1 - f_2)(y - u\tau)}{2}\right],$$
$$u = f_1 + f_2 - 1, \quad U = Au. \tag{16}$$

When $u = 0$, we have a standing wave. The Fermi distribution is a special case of this for $f_1 = 1, f_2 = 0$.

Equation (15) enables us to investigate the time evolution of different initial perturbations, in the same way as in hydrodynamics.[5,7] As an example, we note that a single-hump distribution $f(\varepsilon)$ of width Td and height f_0, produced by the pump, transforms into a triangular wave in the course of time. Its high-energy wing turns out to be a population wave, and propagates in time in accordance with the law $y_h(\tau) - y_0 = (2d\tau)^{1/2} - \tau$. The low-energy wing becomes longer, and occupies the region $y_l < y < y_h$ where $y_l(\tau) - y_0 = -\tau$. When $d \gg 1$, the distribution is

$$f(y, \tau) = (f_0/2\tau)(y - y_l(\tau)), \quad y_l < y < y_h, \quad f(y, \tau) = 0,$$
$$y < y_l \quad \text{or} \quad y > y_h.$$

The wave with the opposite sign, $f_1 < f_2$, cannot propagate. When the initial conditions are chosen in this form, the solution depends on the ratio of f_2 and $\frac{1}{2}$. When $f_2 < \frac{1}{2}$, the distribution tends to f_2 for any ε, and the width of the front increases diffusively. When $f_2 > \frac{1}{2}$, the region $f(\varepsilon) = \frac{1}{2}$ appears at the center and extends in both directions.

In dimensionless units, the width of the front of the wave (16) is $\Delta\varepsilon = 2T/(f_1 - f_2)$, and the Reynolds number is $\mathrm{Re} = 2$.

4. "SOUND"

We shall now determine the dispersion relation for harmonic waves propagating along the ε-axis in time. Such waves can be excited by variations in the rates of pumping or recombination. For simplicity, we shall suppose that the wavelength $2\pi k^{-1}$ is much less than the scale of $f(\varepsilon)$, and that the frequency ω is much greater than the reciprocal of the relaxation time of $f(\varepsilon)$. The distribution then has the form

$$f(\varepsilon) = f + \delta f e^{ik\varepsilon - i\omega t}, \quad \delta f \ll f.$$

Substituting this in (6), we obtain the dispersion relation

$$\omega = A[(2f - 1)k - iTk^2]. \tag{17}$$

Thus, when $\omega \ll AT^{-1}(2f-1)^2$, "sound" waves can propagate along the ε-axis. The propagation takes place in the direction of high (low) energies for $f > 1/2 (< 1/2)$. The "sound" waves are attenuated by dissipation, in accordance with (17). They can also attenuate as a result of a nonlinearity,[5,6] which does not manifest itself in the linear harmonic analysis. Each wave train eventually transforms into a triangular wave whose high-energy wing is a population wave. When attenuation by dissipation occurs in a time $t_d = T/Ak^2$, attenuation due to nonlinearity (degeneracy) occurs in a time $t_n = T/2Ak\delta f$. The ratio of these two times gives the Reynolds number for the sound waves.

5. TURBULENCE

Let us now consider the propagation of random waves, produced by the pump with a time dependence of the form $\mu(\tau) = \mu + \delta\mu(\tau)$, where $\mu =$ const, $|\delta\mu| \ll \mu$. The random function $\delta\mu(\tau)$ is specified by the correlator $\langle \delta\mu(\tau)\delta\mu(\tau+\theta)\rangle = M(\theta)$, which has the characteristic scale $\bar{\theta}$. A sawtooth profile appears after the waves flip over as a result of the nonlinearity, and the profile moves downward along the energy axis with velocity $1-2\mu$. Each population front has its own height h and its own relative velocity u, so that wavefront collisions take place in which the two colliding fronts form a single front. In these collisions, the wave fronts behave like particles with mass and momentum. The collisions are perfectly inelastic. The quantity indicated[7] in Fig. 2 plays the part of mass. The particle masses and separations increase with time, and the number of particles decreases. The state of the system after a time equal to the relaxation time $\tau_r = y_0/(1-2\mu)$ can, in principle, be determined from the exact solution of (6), but this procedure is equivalent to the solution of the set of equations of motion for all the particles of the one-dimensional gas. We shall only be interested in averages and, in particular, the average separation between the particles (wavelength), the average mass, the correlator of the distribution function, and the average characteristics of recombination luminescence. For simplicity, we shall put $\alpha = 1-\mu$ (i.e., $f = \mu$), $\mu < 1/2$.

Near $y = y_0$, where θ is not small, the correction to the distribution function is $\delta f(y,\tau) = f - \mu$ and satisfies the linearized equation

$$\frac{\partial \delta f}{\partial \tau} = \frac{\partial \delta f}{\partial y}(1-2\mu) \tag{18}$$

subject to the boundary condition $\delta f(y_0,\tau) = \delta\mu(\tau)$. Hence, we have

$$\delta f(y,\tau) = \delta\mu[\tau + (y-y_0)/(1-2\mu)].$$

It is clear from this formula that the pump operates like a strip chart recorder: $\delta\mu(\tau)$ moves the "pen" across the "paper" transported at the rate $1-2\mu$. Let us transform to the moving coordinate frame $\tilde{y} = y + (1-2\mu)\tau$ and introduce the function $\varphi(\tilde{y},\tau) = \frac{1}{2}f(y,\tau)$, which satisfies the equation

$$\varphi'_\tau = \varphi''_{\tilde{y}\tilde{y}} - \varphi\varphi'_{\tilde{y}} \tag{19}$$

with the random initial condition

$$\varphi(\tilde{y}) = \varphi(\tilde{y},0) = {}^1/_2\delta\mu[(\tilde{y}-y_0)/(1-2\mu)]. \tag{20}$$

The line drawn by the "recorder" is the initial condition for (19). We now define the Loitsyanskiĭ-Burger invariant by

$$\Xi = \int_0^\infty \langle \varphi(\tilde{y})\varphi(\tilde{y}+z)\rangle\, dz = \frac{1-2\mu}{4}\int_0^\infty M(\theta)\, d\theta. \tag{21}$$

The problem defined by (19) and (20) is examined for $\Xi \neq 0$ in Ref. 4 and for $\Xi = 0$ in Ref. 6. We shall confine our attention to the case where $\Xi \neq 0$. According to Burgers, the distribution then develops so that the triangular waves exhibit a self-similar behavior in time. The mean length of the wave is

$$\langle l\rangle = \zeta^{-1}\Xi^{1/3}\tau^{2/3}, \quad \zeta = 1.053\ldots, \tag{22}$$

and the mean "mass" is $\langle m\rangle = \langle l\rangle$. The mean height of the wave is $\langle h\rangle = \langle l\rangle/\tau$, and the Reynolds number is $\mathrm{Re}(\tau) \propto \tau^{1/3}$. The correlator $F = \langle f(y,\tau)f(y+\Delta y,\tau)\rangle$ is not evaluated theoretically in Ref. 4. However, it is known that its dependence on time τ and Ξ is of the form $(\Xi/\tau)^{2/3}$, and the characteristic scale in Δy is $\langle l\rangle$.

As far as recombination is concerned, we note that, near $y = 0$, the diffusion term cannot be neglected, as in (18). For quasistationary propagation of the wave with $l \gg 1$, the equation for the flux

$$\partial\delta f/\partial y + \delta f(1-2\mu) = \alpha\delta f(y)\delta(y), \quad \delta f(\infty) = \delta f_\infty$$

near $y = 0$ yields

$$\delta f(y,\tau) = \delta f_\infty(\tau)\left[1 - \frac{\mu}{1-\mu}e^{-(1-2\mu)y}\right],$$

which is always valid except for the short times taken by the triangular wavefronts to cross $y = 0$. Hence, the recombinational luminescence correlator is

$$\langle I(t)I(t+\Delta t)\rangle \propto \langle \delta f(0,\tau)\delta f(0,\tau+\Delta\tau)\rangle \propto \langle \delta f_\infty(\tau)\delta f_\infty(\tau+\Delta\tau)\rangle \propto F \propto (\Xi/\tau_r)^{2/3}. \tag{23}$$

The necessary condition for the validity of (18)–(23) is $|\delta\mu| \gg \max[1/(1-2\mu)\bar{\theta},\ (1-3\mu)^2\bar{\theta}/y_0]$. The triangular wave train should also be observed in the case of hot luminescence.

In conclusion, I wish to express my gratitude to V. F. Gantmakher for suggesting this topic, A. F. Dite for drawing Ref. 1 to my attention, I. B. Levinson for discussions and for pointing out Ref. 8, and S. V. Iordanskiĭ and D. E. Khmel'nitskiĭ for their interest in the early stages of this research.

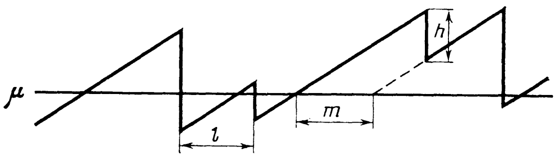

FIG. 2. Profile of developed Burgers turbulence for large Reynolds numbers.[4,7]

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