Edge magnetoplasmons in an electron system at a helium surface; long-wavelength asymptotic spectrum

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The spectrum of edge magnetoplasmons (EMPs) in a bounded, charged $2D$ system above helium is analyzed. The actual profile of the equilibrium electron density, $n_0(x)$, near the edge of the electron disk is taken into account. An approximate analytic expression is derived for the EMP spectrum for weak magnetic fields, corresponding to small values of the parameter $\gamma = d/d_H$ (d is the screening length, and $d_H = s\omega_c^{-1}$, where s is the sound velocity in the unbounded, screened $2D$ plasma, and ω_c is the cyclotron frequency). Since the $n_0(x)$ profile is not flat, additional acoustic branches appear in the EMP spectrum. These branches are not seen in the limiting case in which $n_0(x)$ is described by a θ-function. In addition, a numerical solution of the problem of the EMP spectrum for arbitrary values of γ and for a semi-infinite geometry is reported. At values $\gamma < 1$, this solution leads to results which agree qualitatively with the analytic predictions. In the region $\gamma \gtrsim 1$, the numerical data on the EMP spectrum agree well with the experimental data available.

A recent advance in research on the dynamics of charged $2D$ systems was the discovery of edge magnetoplasmons (EMPs), whose spectrum has several specific features which make it simple to identify oscillations of this sort. A theory of EMPs which has developed in parallel with the experimental research (e.g., Refs. 1–9) has succeeded in deriving many interesting results on the properties of EMPs in a variety of limiting cases. On the other hand, the various existing versions of the theory use several assumptions which hinder the interpretation of experimental data[1,2,8,9] on the behavior of EMPs in an electron system at the surface of liquid helium. For example, Volkov and Mikhaĭlov[4,5] and also Fetter[6,7] postulate a θ-function (stepped) behavior of the equilibrium electron density profile $n_0(x)$ at the boundary of a bounded, charged two-dimensional ($2D$) system. A hypothesis of this sort simplifies the calculations considerably, making possible some special mathematical approaches. The ultimate result is the derivation of some fairly general expressions for the EMP spectrum over broad ranges of the parameter values. The θ-function approximation for the $n_0(x)$ profile, however, is unacceptably crude when applied to a situation with electrons above helium. In a weak magnetic field H directed normal to the surface, the length scale for the damping of an edge plasmon with distance into the electron system is $d_H = s\omega_c^{-1}$, where ω_c is the cyclotron frequency, and s is the velocity of a $2D$ plasmon in the screened plasma in the absence of a magnetic field. Falling off rapidly with increasing magnetic field, d_H becomes comparable to the length scale d over which there is a substantial variation in the profile of the equilibrium electron density, $n_0(x)$. In actual experiments we would have $d \approx 1$ mm, and the relation $d_H = d$ would hold at H values as low as a few tens of oersteds. Under the condition $d_H < d$, the EMP theory must naturally incorporate the actual behavior of the function $n_0(x)$.

An alternative method for describing the dynamics of EMPs was proposed by Glattli[2,9] who took the actual $n_0(x)$ profile into account to some extent. That method, however, is applicable only if the magnetic field is not too strong, and even under this restriction it does not cover all the qualitative features of the EMP spectrum, as we will see below.

There is accordingly a need for a systematic account of the effect of the real $n_0(x)$ profile on the EMP spectrum in both weak and strong magnetic fields. The solution of this problem in the long-wavelength approximation is the content of the present paper. The paper is organized in the following way. In Sec. 1 we write the basic equations and briefly discuss the impedance approximation, in which the effect of the boundary and the profile can be dealt with by introducing some effective boundary condition on the logarithmic derivative of the electric potential of a plasmon. Far from the boundary, it is described by an ordinary wave equation. In Sec. 2 we take up the case of weak magnetic fields, in which the presence of the small parameter γ, proportional to H, makes it possible to find an approximate analytic solution of the system of equations describing EMPs [see (8a) for the definition of γ]. In Sec. 3 we report the results of a numerical solution of the problem of the EMP spectrum in the long-wavelength approximation for arbitrary values of the parameter γ. Finally, in Sec. 4 we discuss the results and compare them with experimental data.

1. BASIC EQUATIONS

We consider a $2D$ electron system which occupies the half-plane $x \geqslant 0$, $z = 0$ of a Cartesian coordinate system. Screening electrodes are positioned symmetrically in the $z = \pm d$ planes. A guard electrode with a potential W, which keeps the edge of the $2D$ electron system in an equilibrium state, is in the plane $x = -\zeta$, away from the edge of the electron system (Fig. 1). As was shown in Ref. 9, the equilibrium electron density $n_0(x)$ is described by the expression

$$n_0(x) = Nn(x), \qquad n(x) = \left\{ \operatorname{th}\frac{\pi x}{2d}\left[\operatorname{th}\frac{\pi(x+\zeta)}{2d} + \operatorname{th}\frac{\pi\zeta}{2d}\right] \cdot \left(\operatorname{th}\frac{\pi x}{2d}\operatorname{th}\frac{\pi\zeta}{2d} + 1\right)^{-1}\right\}^{1/2} \qquad (1)$$

Here $N = n_0(\infty)$, so we have $n(x) \to 1$ in the limit $x \to +\infty$. We assume that the conductivity of the electron layer is de-

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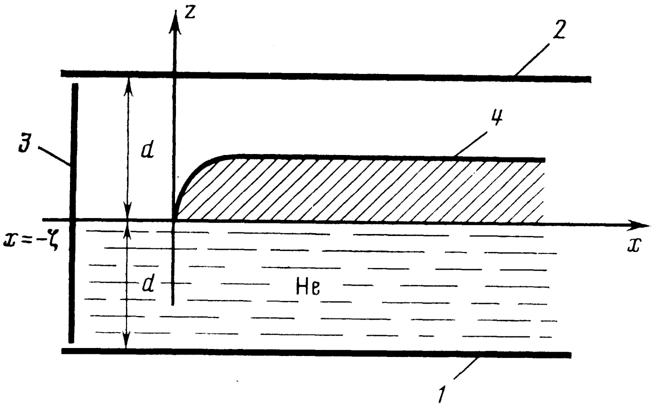

FIG. 1. Semi-infinite geometry. The electrons fill the half-plane $z=0$, $x>0$. 1, 2—Screening electrodes 3—guard electrode; 4—profile of the equilibrium electron density.

scribed in the dissipationless one-electron approximation; i.e.,

$$\sigma_{xx}=\frac{i\omega e^2 n_0(x)}{m(\omega^2-\omega_c^2)},\quad \sigma_{xy}=\frac{\omega_c}{i\omega}\sigma_{xx}, \tag{2}$$

where e and m are respectively the charge and mass of the electron, and ω is the EMP frequency.

The system of equations determining the EMP dynamics relates a perturbation of the electrostatic potential in the $z=0$ plane,

$$\varphi(x,\ y,\ t)=\varphi(x)\exp(-i\omega t+iqy),$$

to a deviation of the electron density from its equilibrium value $n_0(x)$:

$$\delta n(x,\ y,\ t)=\delta n(x)\exp(-i\omega t+iqy),$$

where q is the wave vector of the plasmon along the y axis. To first order in δn and φ, the continuity equation takes the form

$$\frac{m(\omega^2-\omega_c^2)}{eN}\delta n(x)=\left\{(n\varphi')'-nq^2\varphi+n'q\frac{\omega_c}{\omega}\varphi\right\}. \tag{3}$$

An electrostatic equation containing δn and φ is

$$\varphi(x)=-e\int_0^\infty L_q(x-x')\delta n(x')\,dx', \tag{4}$$

$$L_q(x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}K(k_x,q)\exp(ik_x x)\,dk_x, \tag{5}$$

$$K(k_x,k_y)=\frac{2\pi}{|k|}\,\mathrm{th}\,|k|d,\quad |k|=(k_x^2+k_y^2)^{1/2}. \tag{6}$$

In this form, the equation ignores the effect of the guard electrode, which can be kept quite small by choosing an appropriate potential W for the guard ring—corresponding to a distance ζ which is large in comparison with d. In (5) the quantity $K(k_x,k_y)$ is the Fourier transform to the Green's function of the Laplace equation describing the field of a point charge in the space between the screening electrodes under the assumption that the dielectric constant is unity everywhere.

Going over to the long-wavelength approximation, we can quite accurately replace L_q by

$$L_0(x)=2\pi L(x),\quad L(x)=\frac{1}{\pi}\ln\mathrm{cth}\frac{\pi|x|}{4d},\quad qd\ll 1. \tag{7}$$

The function $L(x)$ has a logarithmic singularity as $x\to 0$, and it falls off exponentially at infinity (at $x\gtrsim d$).

We now introduce the dimensionless parameter x/d (which we will denote by the same letter, x) and the new function $\nu(x)=-2\pi de\delta n(x)$. In the long-wavelength approximation, corresponding to the limit $q\to 0$, $\omega\to 0$, $q/\omega=\mathrm{const}$, the system (3), (4) then takes the form

$$(n\varphi')'+n'\lambda\gamma\varphi=\gamma^2\nu(x), \tag{8}$$

$$\gamma=\frac{\omega_c d}{s}=\frac{d}{d_H},\quad \lambda=\frac{qs}{\omega}, \tag{8a}$$

$$\varphi(x)=\int_0^\infty L(x-x')\nu(x')\,dx'. \tag{9}$$

Here $s=(2\pi Ne^2d/m)^{1/2}$ is the plasmon velocity in the screened $2D$ plasma in the absence of a magnetic field. The parameter γ plays a significant role in the EMP theory for screened systems, determining the regions of weak ($\gamma\ll 1$) and strong ($\gamma\gg 1$) magnetic fields for the given problem. The quantity λ serves as a spectral parameter in (8). The requirements $q\to 0$, $\omega\to 0$ acutally mean that the inequalities $qd\ll 1$, $qd_H\ll 1$, and $\omega\ll\omega_c$ hold.

To formulate our impedance approximation we need to reduce the system of equations (8)–(9) to a single differential equation for φ. To do this, we first examine the asymptotic behavior of the function $\varphi(x)$ at large $x\gg 1$. In this region we have $n=1$ and $n'=0$, and an EMP is described by the system of equations

$$\varphi''=\gamma^2\nu(x),$$

$$\varphi(x)=\int_0^\infty L(x-x')\nu(x')\,dx',$$

from which it follows that $\varphi(x)$ satisfies the equation

$$\gamma^2\varphi(x)=\int_0^\infty L(x-x')\varphi''(x')\,dx'. \tag{10}$$

The asymptotic behavior of φ at $x\gg 1$ does not depend on the detailed form of the kernel $L(x)$; it is determined by the qualitative characteristics of this kernel, e.g., its zeroth moment and its effective radius. For the problem of seeking the behavior of φ at infinity we can then replace L by the simple kernel

$$\tilde{L}(x)=\frac{\pi}{4}\exp\left(-\frac{\pi|x|}{2}\right). \tag{11}$$

After this replacement, Eq. (10) can be solved exactly (it reduces to a differential equation after the left and right sides are differentiated twice). We find

$$\varphi=\mathrm{const}\exp(-Ex),\quad E=E(\gamma)=\gamma\frac{\pi}{(\pi^2+4\gamma^2)^{1/2}}. \tag{12}$$

At small values of γ, the potential φ and also the perturbation of the electron density, $\nu(x)$, are therefore proportional to $\exp(-\gamma x)$; i.e., they vary slowly over distances ~ 1. This result means that at $x\gtrsim 1$ the equality $\varphi=\nu$ holds [since the function ν, which varies slowly over distances on the order of the effective radius of the kernel L, can be taken out of the integral], and throughout the region $0<x<\infty$ we can write[1)]

$$\nu(x)=\varphi(x)+V(x)\varphi(x), \tag{13}$$

where $V(x)$ is some function which is nonzero at $x\lesssim 1$ and which falls off exponentially at large values of x. Substituting (13) in to (8), we find the equation which we have been seeking for φ and which describes the EMPs:

$$(n\varphi')'=-n'\lambda\gamma\varphi+\gamma^2\varphi(x)+\gamma^2V(x)\varphi. \tag{14}$$

After dividing by φ, we can relate (14) as

$$(n\varphi'/\varphi)'=-n'\lambda\gamma+\gamma^2V(x)+\{\gamma^2-n(\varphi'/\varphi)^2\}. \tag{14a}$$

It is obvious from Eq. (14) that for $x\gtrsim 1$ (where $n'\approx V\approx 0$) we are left with the following equation for φ: $\varphi''-\gamma^2\varphi=0$. The solution of this equation is a damped exponential function $\varphi=\exp(-\gamma x)$. Now integrating both sides of (14a) from 0 to some value $\alpha\gtrsim 1$ [formally, we can extend the integration to the entire interval $0<x<\infty$, since the right side of (14a) falls off exponentially as $x\to\infty$, and the integral over it is insensitive to the choice of the upper limit, if the latter is greater than unity], we find the following value for the logarithmic derivative φ'/φ:

$$(\varphi'/\varphi)_a=-\lambda\gamma+A\gamma^2, \tag{15}$$

where we have, under the assumption $\varphi'/\varphi\approx\gamma$ at $x\lesssim 1$,

$$A\approx\int_0^\infty V(x)\,dx+\int_0^\infty(1-n(x))\,dx\sim 1. \tag{15a}$$

Both of the integrals on the right side of (15a) converge exponentially at $x\gtrsim 1$. The first leads to a value $A\neq 0$ even in the case $n(x)=\theta(x)$.

The conditions for joining (15) with the damped exponential function lead to the following equation, after a factor of γ is cancelled out:

$$-1=-\lambda+A\gamma,\qquad \lambda=qs/\omega.$$

Hence

$$\omega=sq/(1+A\gamma)\approx s(1-A\gamma)q\,; \tag{16}$$

i.e., if $A>0$ the EMP velocity falls off with increasing H, in qualitative agreement with the experimental results.

The equation for Ref. 2 for the EMP spectrum reduces in our half-plane case to the impedance approximation, presented here, with $A=0$. Consequently, as was pointed out in Ref. 2, that equation yields an H-independent EMP velocity.

2. WEAK FIELDS: APPROXIMATE ANALYTIC SOLUTION

A. We now consider the system (8), (9) under the condition $\gamma\ll 1$, without restricting the analysis to the impedance approximation. It turns out that to within terms of second order in γ we can replace $\nu(x)$ by $n(x)\varphi(x)$ on the right side of (8). Specifically, since φ and ν vary slowly at $x\gtrsim 1$, and we have $n(x)=1$, it follows from (9) that we have $\nu(x)=n(x)\varphi(x)$ in the region $x\gtrsim 1$. With regard to the region $x\lesssim 1$, we note that the quantity $\nu(x)=n(x)\varphi(x)$ appears on the right side of (8) with a small coefficient $\gamma^2\ll 1$, while the term on the left, proportional to n', does not contain this small factor. Accordingly, for 0, $x\lesssim 1$ the right side is totally unimportant, and we can replace $\nu(x)$ here by any expression which is correct in order of magnitude. Consequently, the approximation $\nu(x)=n(x)\varphi(x)$ can be regarded as satisfactory over the entire interval $0<x<\infty$. Substituting $\nu=n\varphi$ into (8), and dividing by $n(x)$, we find the differential equation

$$\varphi''+g(x)\varphi'(x)+[-\gamma^2+\lambda\gamma g(x)]\varphi(x)=0, \tag{17}$$

where $g(x)=n'(x)/n(x)$.

The question of the boundary condition for Eq. (17) deserves special attention. The natural requirement that the normal component of the current vanish at the free boundary of the bounded system is satisfied automatically in this case, since the equilibrium electron density is zero at $x=0$ [we recall that in the case of electrons above helium we have $n(x)\sim x^{1/2}$ in the region $x\lesssim d$; see (1)]. On the other hand, for any function $n(x)$ which vanishes as a power of x as $x\to+0$ the coefficients of $\varphi'(x)$ and $\varphi(x)$ in (17) have $1/x$ singularities. Accordingly, as conditions for assistance in extracting physically meaningful solutions of Eq. (17) we adopt the requirements that the function $\varphi(x)$ be regular near $x=0$ and that $\varphi(x)$ decay exponentially at infinity.

It is pertinent at this point to take a look at the procedure used to linearize the initial equations in the EMP theory. From the continuity equation (3) we have

$$\frac{\delta n}{n}\infty\,\varphi''-(qd)^2\varphi+\frac{n'}{n}(\varphi'+\lambda\gamma\varphi).$$

Imposing on $\varphi(x)$ the natural requirements of continuous differentiability at $x\in[0;\infty)$ (i.e., that the electric field be finite) and infinite differentiability at $x\in(0;\infty)$, we can formulate the following conditions on $\varphi(x)$, under which it is legitimate to linearize the continuity equation with respect to δn [the inequality $\delta n(x)\ll n(x)$ holds for all $x\in(0;\infty)$]. In the case $n(x)=\theta(x)$, in which the last term is absent from the expression for $\delta n/n$, we require that $\varphi''(x)$ remain finite as $x\to+0$. If, on the other hand, we have $n(x)\sim x^p$ as $x\to+0$ with $p>0$, then n'/n behaves as x^{-1} as $x\to+0$, and $\varphi''(x)$ is $O(x^{-1})$ in any case [since otherwise the integral over $\varphi''(x)$ would diverge near the origin, and the derivative $\varphi'(0)$ would be infinite]. Consequently, the requirement that $\delta n/n$ be bounded as $x\to+0$ means in this case that $\varphi''(x)$ is bounded near the origin, so the following condition holds:

$$\varphi'+\lambda\gamma\varphi=O(x),\qquad x\to+0. \tag{17a}$$

We will let this terminate our discussion of the boundary conditions on Eq. (17), and we will move onto a study of this equation

B. It is not possible to solve Eq. (17) for the function g with the real $n(x)$ profile. However, for

$$g(x)=\frac{\mu e^{-\mu x}}{1-e^{-\mu x}}=\mu(e^{\mu x}-1)^{-1},\qquad \mu=\text{const} \tag{18}$$

Eq. (17) can be solved completely. The function $g(x)$ from (18) gives a qualitatively correct description of the behavior of the function n'/n calculated for the actual equilibrium electron density $n_0(x)$. Specifically, it follows from (1) that we have $n(x)\sim x^{1/2}$ as $x\to+0$ and $n(x)\sim 1-b\exp(-\pi x)$, with $b=$ const, at $x\gtrsim 1$. In other words, we have $n'/n\sim X^{-1}$ as $x\to+0$ and $n'/n\sim e^{-\pi x}$ at $x\approx 1$. Clearly, the function $g(x)$ given in (18) meets these requirements with $\mu = \pi$, so the approximation (18) will presumably lead to results which are correct at least qualitatively.

The successive substitutions $\varphi(x) = e^{-\gamma x}\psi(x)$, $e^{-\mu x} = \xi$ reduce Eq. (17), with g from (18), to a hypergeometric equation. As a result we find that the solutions of interest here (which are regular at the origin and which decay rapidly at infinity) form a discrete set of functions φ_M, which we will number with the index M:

$$\varphi_M(x) = e^{-\gamma x} P_M \quad (2e^{-\mu x} - 1), \quad \beta = \gamma/\mu, \quad M = 0, 1, 2, \ldots, \tag{19}$$

where $P_M^{(\alpha,\beta)}(u)$ are Jacobi polynominals. These solutions correspond to the following values of the constant λ in (17):

$$\lambda_M = 1 + [M^2 + M(1+2\beta)]/\beta. \tag{20}$$

It is easy to verify that $\varphi_M(x)$ from (19) satisfies condition (17a) with $\lambda = \lambda_M$ from (20). It is also obvious that $\varphi''_M(0)$ is finite. Consequently, the corresponding value of $\delta n/n$ is bounded over the entire interval $0 < x < \infty$, so the linearization of the initial equations is correct in this case.

The EMP spectrum for electrons above helium corresponds to the value $\mu = \pi$. Returning to the definition of λ in (8a), we find the long-wavelength asymptotic behavior of $\omega(q)$ with $\mu = \pi$, based on (20):

$$\omega_M(q) = s_M q, \quad s_0 = s, \quad s_M = \frac{\gamma s}{(M^2+M)\pi} \ll s, \tag{21}$$

$$M > 0, \quad \gamma \ll 1.$$

In weak magnetic fields, we thus have not a single branch but many "acoustic" branches in the EMP spectrum in the region $qd \ll 1$. One of these branches has a velocity which is the same as the plasmon velocity in an unbounded $2D$ plasma [the approximation which we are using here is insufficient for determining the H dependence of s_0; as can be seen from (16), this dependence appears only in the next order in γ]. For all the other branches, the velocities are found to be proportional to the magnetic field. The fact that the relation $\lambda_M > 0$ holds for all M means that the EMPs propagate in only one direction along the plasma boundary.

The approximation used here allows us to answer the qualitative question of the role played by the profile $n_0(x)$ in shaping the EMP spectrum in a nonzero magnetic field. Specifically, the function $g(x)$ from (18) corresponds to $n = 1 - e^{-\mu x}$; i.e., the parameter μ determines the rate at which the model function $n(x)$ reaches the $n \doteq 1$ plateau as x varies from 0 (where we have $n = 0$) to infinity. In this case the limit $\mu \to \infty$ corresponds to a transition to $n(x) = \theta(x)$. The EMP spectrum for arbitrary μ is given by (21), in which π is replaced by μ in the expression for s_M. It follows that all the branches other than the zeroth degenerate to $\omega_M \equiv 0$ when we go to a stepped distribution of the equilibrium electron density. In other words, the θ approximation for $n_0(x)$ reveals only one of the branches of the EMP spectrum.

3. ARBITRARY MAGNETIC FIELDS: NUMERICAL SOLUTION

At this point we lift the restriction on the strength of the magnetic field. Substituting $v(x)$ from (8) into (9), we find the equation

$$\gamma^2\varphi(x) - \int_0^\infty L(x-x')\,(n\varphi')'\,dx' = \lambda\gamma \int_0^\infty L(x-x')\,n'\varphi\,dx'. \tag{22}$$

In other words, the problem of the long-wavelength asymptotic behavior of the EMP spectrum leads to a generalized eigenvalue problem for $\lambda = qs/\omega$. In order to reduce the integral equation (22) to a matrix equation, we need to choose some complete system of functions which is orthogonal on the interval $(0; \infty)$, and we need to expand $\varphi(x)$ in this system, retaining the first l terms in the expansion. Making use of the results of Sec. 2, we naturally choose the following set of functions as this complete system:

$$\Phi_k(x) = [\pi(1 + \tilde\beta + 2k)]^{1/2} e^{-Ex} P_k^{(0,\tilde\beta)}(2e^{-\pi x} - 1), \tag{23}$$

$$k = 0, 1, 2, \ldots \quad E = \gamma\pi(\pi^2 + 4\gamma^2)^{-1/2}, \quad \tilde\beta = 2E/\pi.$$

These functions are orthogonal on the interval $0 < x < \infty$ with a weight

$$w(x) = e^{-\pi x}. \tag{24}$$

Note of the functions $\Phi_k(x)$ satisfies the requirement (17a). However, there is the hope that a linear combination

$$\sum_{k=1}^{l} c_k \Phi_k(x),$$

which solves Eq. (22) approximately, will have this property for corresponding values of λ, as in the derivation of solutions (19) and (20). The matrix problem was solved for $l = 4$, 6, and 8 with the help of two different expressions for the equilibrium electron density: $n(x)$ from (1) and $n_1(x) = 1 - e^{-\pi x}$, which leads to the function g given by (18). The results found confirm the conclusion, reached in Sec. 2, that the EMP spectrum contains some additional acoustic branches, whose velocity is proportional to $\gamma \sim H$ at $\gamma \ll 1$. A calculation with the profile $n_1(x)$ makes it possible to compare the numerical data with the analytic results, (21) for the EMP spectrum in the region $\gamma \ll 1$. It turns out that the behavior of the velocity of the zeroth branch is described by $s_0(\gamma) \approx 1 - 0.8\gamma$, and for $M = 1$, 2, and 3 the error of the approximate values of s_M is less than 2%. As we go from $l = 4$ to $l = 8$, the velocity of the first three branches changes by $\sim 0.1\%$, showing that there is rapid convergence in terms of l.

In most of the numerical calculations which we will be using below for comparison with experimental data, we used $n(x)$ from (1). We wish to emphasize that the quantitative changes in the calculated results which occur when the profile $n_1(x)$ is replaced by $n(x)$ from (1) do not affect the qualitative structure of the EMP spectrum. For example, the velocity of the first branch ($M = 1$) is reduced by $\sim 30\%$ when this replacement is made, but the ratio S_{M+1}/S_M remains essentially constant at $M > 0$.

Figure 2 shows curves of $s_0(\gamma)$ and $s_1(\gamma)$ for $0.1 \leqslant \gamma \leqslant 10$ in full logarithmic scale. We see that at large values of γ the velocity of the zeroth branch is inversely proportional to γ. These results were obtained for $n(x)$ from (1) with $\zeta = 1.65d$; as ζ was varied from d to $2d$, the values of λ_M changed by no more than 2%. In the region $\gamma \gtrsim 3$ the dependence $s_0(\gamma)$ can be approximated well by

$$s_0(\gamma) = s\frac{\alpha}{\gamma} = \frac{2\pi nec}{H}\alpha, \quad \alpha = 0.84. \tag{25}$$

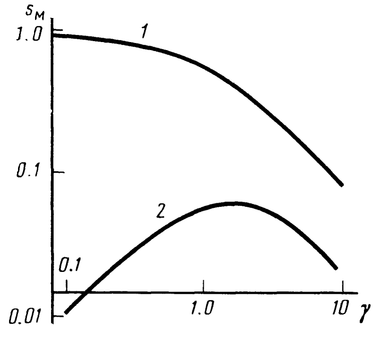

FIG. 2. Velocities of the zeroth and first branches of the EMP spectrum versus $\gamma = d/d_H$. 1—$s_0(\gamma)/s$; 2—$s_1(\gamma)/s$ [$s_1(\gamma)$ reaches a maximum ($\sim 0.06s$) at $\gamma \sim 1.6$].

4. DISCUSSION OF RESULTS; COMPARISON WITH EXPERIMENTAL DATA

Edge magnetoplasmons in a system of electrons above helium were studied experimentally in Refs. 1, 2, 8, and 9. In Ref. 1, the electron layer was a rectangle, while in Refs. 2, 8, and 9 it was a disk. In the case of a disk, an excitation in which the electrostatic potential φ is described by an expression of the form (r and θ are polar coordinates)

$$\varphi(r, \theta, t) = \varphi(r) \exp(-i\omega t + in\theta) \qquad (26)$$

can be thought of as an EMP which is traveling along the boundary of the disk with a wave number $q = nR^{-1}$, where R is the radius of the region filled by the electrons. We do not know of an analogous and comparatively simple structure of low-index EMPs for the case of rectangular geometry, so we will compare the results derived above with the measurements of Refs. 2, 8, and 9. We cannot hope to find a good agreement between the theory, which describes the EMPs in a half-plane, with experimental data obtained from an electron disk of radius of R except when the length scale for the damping of the EMPs along the normal to the boundary (which is given at small values of q by the expression $d_H = \omega_0^{-1} s$, $\gamma \ll 1$) is quite small in comparison with R. In other words, the magnetic field must be quite strong. This condition held for the experiments of Ref. 8. Figure 3 reproduces some of the measurements of Ref. 8, for the mode with azimuthal index $n = 1$ (a comparison with the modes with $n = 2$, 3, and 4 shows that at a fixed magnetic field we have $\omega \propto n$ to within 1–3%; i.e., the dispersion law for the EMPs is essentially linear). Also shown here are a curve plotted from the results of the numerical calculations of Sec. 3 and a curve plotted from the figures in Fetter's paper,[7] on a numerical analysis of the EMP spectrum in a disk under the approximation $n(r) = \theta(R - r)$. As the radius of the electron disk [which determines the value of the parameter ζ in (1)] we selected $R = 7$ mm. (An accurate determination of R would require knowing the potentials of the screening electrodes and of the guard ring, which were not given in Ref. 8. However, the applied potentials in a similar experiment, whose results are shown in Fig. 2 of Ref. 2, corresponded to the value $R = 7.4$ mm. This is the justification for the choice which we have made.) Fetter's paper[7] reports calculations only for the values $d/R = 0$, 0.5, and ∞. In Ref. 8, the value was $d/R \approx 0.14$. Curves $F1$ and $F2$ in Fig. 3 correspond to different methods of interpolating the EMP frequency to the value in $d/R = 0.14$ ($F1$ shows the results of a linear interpolation from 0 to 0.5; $F2$ corresponds to a plausible smooth curve). The calculations carried out in Ref. 7 span the magnetic field region $H \leqslant 40$ G (for the n and R values corresponding to Ref. 8). It can be seen from this figure that our numerical results agree quite satisfactorily with the experimental data in the region $d_H/R < 0.2$.

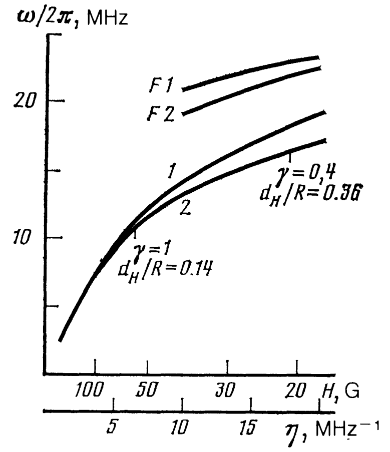

FIG. 3. Experimental results of Ref. 8 (curve 1) for the mode with the azimuthal index $n = 1$, along with the results of a numerical calculation (curve 2) for the semi-infinite geometry with $q = R^{-1}$. The lower scales show the values of the magnetic field H and of the parameter $\eta = 2\pi \cdot 1000/\omega_c$, which was used in Ref. 8. For $\eta < 5$ ($H \gtrsim 80$ G), curves 1 and 2 merge at the scale of this figure. Curves $F1$ and $F2$ are the results of a recalculation of Fetter's results[7] for the situation of Ref. 8

A branch of the EMP spectrum with $M > 1$ has yet to be discovered. The apparent explanation is that such branches, with comparatively low frequencies, are easily suppressed by dissipative effects. For them, the value of the parameter $\omega\tau$ (τ is the momentum relaxation time of an electron), which determines the efficiency of one of the EMP damping mechanisms is comparable to unity at $M = 1$–3 and exceeds it at $M \gg 1$. Difficulties of this sort can be avoided by making use of the fact that the electrons above the helium can easily be heating by an electric field. In the process, there motion along the z direction goes into states which interact comparatively weakly with oscillations of the surface of the liquid. The result is to increase τ and the value of the parameter $\omega\tau$. Furthermore, in the region $1 \leqslant \gamma \lesssim 3$ the velocity s_1 is proportional to $N^{1/2}$. In other words, if we wish to increase the value of the parameter $\omega\tau$ we should work with electrons densities as high as possible. Consequently, we would be interested in seeing an experiment designed to detect branches of the EMP spectrum with $M > 1$.

There is one final point we would like to discuss. As we mentioned earlier (Sec. 2), the combination $\varphi' + \lambda\gamma\varphi$ should vanish rapidly as $x \to +0$. Otherwise, the fluctuation of the electron density near the origin induced by the plasmon will exceed the equilibrium density, which has an $x^{1/2}$ behavior as $x \to +0$. This would mean that the linear approximation in δn would be incorrect near the origin. The eigenfunctions found in the course of the solution of the corresponding matrix eigenvalue (λ) problem exhibit the following property: at $\gamma \lesssim 1$, the condition $(\varphi'/\varphi)_0 = -\lambda\gamma$ holds to within an error of order 10% for the zeroth branch, while for the branches with $M = 2$, 3, and 4 it holds to within a few tenths of 1%. As γ is increased, this error increases. At $\gamma \sim 10$, it amounts to $\sim 80\%$ for the zeroth branch, and it improves monotonically with increasing M. For the fourth branch it is $\sim 10\%$.

In summary, the validity of the procedure used for linearization in the region $\gamma \gtrsim 1$ remains questionable. One possibility (and an extremely likely one) is that the method used here, involving an expansion in eigenfunctions, while yielding good results in a search for the integral characteristics of Eq. (22) (the matrix elements are calculated through the use of a double integration, and are not very sensitive to the local variations in the functions in which the expansion is carried out), is poorly suited for searching for such quantities as the logarithmic derivative at the origin.[2)] Another possibility is that condition (17a) is indeed violated as γ is increased. In the latter case, the situation is similar to that which prevails in the case of a θ-function approximation of $n(x)$, where $\varphi(x)$ contains an $x^{3/2}$ term in the limit $x \to +0$ (Ref. 5). In this case a contribution to δn which diverges as $x \to +0$ comes from the term $n\varphi''$ (we have $n \equiv 1$ in the θ approximation). It would apparently be possible to avoid violating the inequality $\delta n \ll n$ by including a term of diffusive origin in the electron current. We have not checked out this version quantitatively, since it significantly complicates the numerical calculations.

[1)]The solution of Eq. (8) for $\nu(x)$ (under the additional condition that there are no charges at $x<0$) should be of the form $v(x) = \varphi(x) + \int_0^\infty K(x,x')\varphi(x')dx'$, where $K(x,x')$ is some kernel which is concentrated primarily in the region $|x - x'| \lesssim 1, x \lesssim 1$. The assumption that φ varies slowly ($\varphi'/\varphi \sim \gamma \ll 1, 0 < x < \infty$) allows to extract the potential φ from the integral in this expression; as a result we find for $\nu(x)$ expression (13) with $V(x)\int_0^\infty K(x,x')dx'$.

[2)]In precisely the same way, the variational method in quantum mechanics yields the discrete spectrum quite reliably, but it is very inaccurate in describing the details of the behavior of the wave functions.

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